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Heavy Quark Production in QCD*

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Abstract

I review the status of the theory of heavy quark production in QCD. The salient features of the recently published results on higher order corrections to the heavy quark production cross-section are described. New analytic results on high energy limit of the parton cross-sections are presented.

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1. Introduction

In this paper I review the status of the theory of heavy quark production. The production of heavy quarks in hadron-hadron and photon-hadron collisions continues to be a topic of great theoretical interest. The reasons for this enthusiasm are the existence of experimental data on charm production and the recent publication of data on bottom quark production in hadronic reactions. For a review of the data on the hadroproduction and photoproduction of heavy quarks I refer the reader to refs. [1] and [2] respectively. An additional motivation is provided by the need to relate the results of the search for the top quark[3] to a value of the heavy quark mass.

In the hadroproduction of heavy quarks there are two incoming strongly interacting particles. The produced quarks are coloured objects which subsequently fragment into the heavy mesons and baryons observed in the laboratory. Therefore heavy quark production is an important check of the QCD improved parton model in a complex hadronic environment. Because of their semi-leptonic decays, heavy quarks give rise to electrons, muons and neutrinos. Prompt leptons and missing energy are often used as signals for new phenomena. Therefore an accurate understanding of heavy quark production cross-sections is necessary to calculate background rates in the search for new physics.

The standard perturbative QCD formula for the inclusive hadroproduction of a heavy quark Q of momentum p and energy E,

$$H_A(P_1) + H_B(P_2) \to Q(p) + X \tag{1.1}$$

determines the invariant cross-section as follows,

$$\frac{E\, d^3\sigma}{d^3p} = \sum_{i,j} \int dx_1\, dx_2 \left[\frac{E\, d^3\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, p, m, \mu)}{d^3p}\right] F_i^A(x_1,\mu)\, F_j^B(x_2,\mu). \tag{1.2}$$

The functions F_i are the number densities of light partons (gluons, light quarks and antiquarks) evaluated at a scale μ. The symbol $\hat{\sigma}$ denotes the short distance cross-section from which the mass singularities have been factored. Since the sensitivity to momentum scales below the heavy quark mass has been removed, $\hat{\sigma}$ is calculable as a perturbation series in $\alpha_S(\mu^2)$. The scale μ is *a priori* only determined to be of the order of the mass m of the produced heavy quark. Variations in the scale μ lead to a considerable uncertainty in the predicted value of the cross-section. This uncertainty is diminished when higher order corrections are included. The photoproduction of heavy quarks is described by a similar formula.

The theoretical justification for the use of Eq. (1.2) in heavy quark production has been discussed in refs. [4, 5]. Although there is no proof of factorisation in heavy quark production, arguments based on the examination of low order graphs suggest that factorisation will hold. No flavour excitation contributions are included in Eq. (1.2), since the sum over partons runs only over light partons. Graphs having the same structure as flavour excitation are included as higher order corrections. Interactions with spectator partons give rise to terms not shown in Eq. (1.2) which are suppressed by powers of the heavy quark mass. If factorisation holds the power corrections should be suppressed relative to the leading order by $(\Lambda/m)^2$. Note however that in ref. [6] a non-relativistic calculation has been performed which finds terms which are only suppressed by (Λ/m). It would be useful (particularly in the interpretation of charm data) to establish that the power corrections in the full relativistic theory are in fact $(\Lambda/m)^2$.

The lowest order parton processes leading to the hadroproduction of a heavy quark Q are,

$$\begin{aligned} (a) \quad & q(p_1) + \overline{q}(p_2) \quad \rightarrow \quad Q(p_3) + \overline{Q}(p_4) \\ (b) \quad & g(p_1) + g(p_2) \quad \rightarrow \quad Q(p_3) + \overline{Q}(p_4). \end{aligned} \tag{1.3}$$

The four momenta of the partons are given in brackets. The invariant matrix elements squared for processes (a) and (b) have been available in the literature for some time[7,8,9] and are given by,

$$\overline{\sum} \left| M^{(a)} \right|^2 = \frac{g^4 V}{2N^2} \left(\tau_1^2 + \tau_2^2 + \frac{\rho}{2} \right) \tag{1.4}$$

$$\overline{\sum} \left| M^{(b)} \right|^2 = \frac{g^4}{2VN} \left(\frac{V}{\tau_1 \tau_2} - 2N^2 \right) \left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2} \right) \tag{1.5}$$

where the dependence on the $SU(N)$ colour group is shown explicitly, ($V = N^2 - 1$, $N = 3$) and m is the mass of the produced heavy quark Q. The matrix elements squared in Eqs. 1.4 and 1.5 have been summed and averaged over initial and final colours and spins. For brevity we have introduced the following notation,

$$\tau_1 = \frac{p_1 \cdot p_3}{p_1 \cdot p_2}, \quad \tau_2 = \frac{p_2 \cdot p_3}{p_1 \cdot p_2}, \quad \rho = \frac{2m^2}{p_1 \cdot p_2}. \tag{1.6}$$

For photoproduction of heavy quark the lowest order reaction is

$$(c) \quad \gamma(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4). \tag{1.7}$$

The matrix element squared for this reaction[10] is given in terms of the charge e_Q of the heavy quark as,

$$\overline{\sum} \left|M^{(c)}\right|^2 = g^2 e_Q^2 \left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right). \tag{1.8}$$

In terms of these lowest order matrix elements the invariant cross-section can be written as,

$$\frac{E_3\, d^3\hat{\sigma}}{d^3p_3} = \frac{1}{16\pi^2 s^2}\overline{\sum} |M|^2 \tag{1.9}$$

where s is the square of the total parton centre of mass energy.

The phenomenological consequences of the lowest order formulae can be summarised as follows. The average transverse momentum of the heavy quark or antiquark is of the order of its mass and the p_T distribution falls rapidly to zero as p_T becomes larger than the heavy quark mass. The rapidity difference between the produced quark and antiquark is predicted to be of order one. The theoretical arguments summarised above do not address the issue of whether the charmed quark is sufficiently heavy that the hadroproduction of charmed hadrons in all regions of phase space is described by the QCD parton model, neglecting terms suppressed by powers of the charmed quark mass. For the application of these formulae to heavy quark production at fixed target energy see ref. [11] and references therein.

There are arguments[12] which suggest that higher order corrections to heavy quark production could be large. These are mostly due to the observation that the fragmentation process,

$$\begin{array}{lll} g + g & \to & g + g \\ & & \hookrightarrow Q + \overline{Q} \end{array} \tag{1.10}$$

although formally of order α_S^3, can be numerically as important as the lowest order $O(\alpha_S^2)$ cross section. This happens because the lowest order cross section for the process $gg \to q\bar{q}$ is about a hundred times smaller than the cross section for $gg \to gg$. A gluon jet will fragment into a pair of heavy quarks only a fraction $\alpha_S(m^2)/2\pi$ of the time. Because of the large cross-section for the production of gluons, the gluon fragmentation production process is still competitive with the production mechanisms of Eq. (1.3). The description of heavy quark production by the gluon fragmentation mechanism alone is appropriate only when the produced heavy quark is embedded in a high energy jet[13].

The matrix elements squared for the hadroproduction of a heavy quark pair plus a light parton have all been calculated[12,9,14,15]. By themselves, they have physical significance only when the jet associated with the light parton has a large transverse momentum. When the produced light parton has small transverse momentum the matrix elements contain collinear and soft divergences, which cancel only when the virtual corrections are included, and the factorisation procedure is carried out. Corresponding results for the photoproduction of heavy quarks are given in ref. [16].

A partial $O(\alpha_S^3)$ calculation involving the quark gluon fusion process which is free from soft gluon singularities, but contains collinear singularities has been presented in ref. [9]. This calculation provides a concrete example of the factorisation scheme. However this calculation is valid when the quark gluon process dominates over the competing processes. In other regions one cannot use any partial calculation of higher order effects; both real and virtual diagrams contribute. They separately contain divergences which cancel in a complete calculation.

A full calculation of the inclusive cross section for heavy quark production to order α_S^3 is described in ref. [17]. Some aspects of this calculation are discussed in section 2. Corresponding results for photoproduction to order $\alpha_S^2\alpha_{em}$ are given in ref [18]. They are discussed briefly in section 3.

2. The hadroproduction of heavy quarks

The basic quantities calculated in ref. [17] are the short distance cross sections $\hat{\sigma}$ for the inclusive production of a heavy quark of transverse momentum p_T and rapidity y. This requires the calculation of the cross-sections for the following parton inclusive processes,

$$g+g \to Q+X,\ \ q+\bar{q} \to Q+X,\ \ g+q \to Q+X,\ \ g+\bar{q} \to Q+X$$
$$g+g \to \overline{Q}+X,\ \ q+\bar{q} \to \overline{Q}+X,\ \ g+\bar{q} \to \overline{Q}+X,\ \ g+q \to \overline{Q}+X. \tag{2.1}$$

The inclusive cross-sections for the production of an anti-quark $\overline{Q}$ differ from those for the production of a quark Q at a given y and p_T. This interference effect, which first arises in $O(\alpha_S^3)$, is small in most kinematic regions. From these results and Eq. (1.2) we can calculate the distributions in rapidity and transverse momentum of produced heavy quarks correct through $O(\alpha_S^3)$. At this point we list the parton sub-processes which contribute to the inclusive cross-sections.

$$\begin{aligned}
q+\bar{q} &\to Q+\overline{Q}, & \alpha_S^2, \alpha_S^3 \\
g+g &\to Q+\overline{Q}, & \alpha_S^2, \alpha_S^3 \\
q+\bar{q} &\to Q+\overline{Q}+g, & \alpha_S^3 \\
g+g &\to Q+\overline{Q}+g, & \alpha_S^3 \\
g+q &\to Q+\overline{Q}+q, & \alpha_S^3 \\
g+\bar{q} &\to Q+\overline{Q}+\bar{q}, & \alpha_S^3.
\end{aligned} \tag{2.2}$$

Note the necessity of including both real and virtual gluon emission diagrams in order to calculate the full $O(\alpha_S^3)$ cross-section.

In order to describe the results in a relatively concise way, I concentrate on the calculation of the total cross section for the inclusive production of a heavy quark pair. Integrating Eq. (1.2) over the momentum p we obtain the total cross section for the production of a heavy quark pair,

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2\ \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2)\ F_i^A(x_1,\mu) F_j^B(x_2,\mu) \tag{2.3}$$

where S is the square of the centre of mass energy of the colliding hadrons A and B. The total short distance cross section $\hat{\sigma}$ for the inclusive production of a heavy quark from partons i, j can be written as,

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} f_{ij}\Big(\rho, \frac{\mu^2}{m^2}\Big) \tag{2.4}$$

with $\rho = 4m^2/s$, and s the square of the partonic centre of mass energy. μ is the renormalisation and factorisation scale. In ref. [17] a complete description of the functions f_{ij} including the first non-leading correction was presented. These may be used to calculate heavy quark production at any energy and heavy quark mass.

Eq. (2.4) completely describes the short distance cross-section for the production of a heavy quark of mass m in terms of the functions f_{ij}, where the indices i and j specify the types of the annihilating partons. The dimensionless functions f_{ij} have the following perturbative expansion,

$$f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = f_{ij}^{(0)}(\rho) + g^2(\mu^2)\left[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m^2}\right)\right] + O(g^4). \tag{2.5}$$

In order to calculate the f_{ij} in perturbation theory we must perform both renormalisation and factorisation of mass singularities. The subtractions required for renormalisation and factorisation are done at mass scale μ. The dependence on μ is shown explicitly in Eq. (2.5). The energy dependence of the cross-section is given in terms of the ratio ρ,

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}. \tag{2.6}$$

The running of the coupling constant α_S is determined by the renormalisation group,

$$\frac{d\alpha_S(\mu^2)}{d\ln\mu^2} = -b_0\alpha_S^2 - b_1\alpha_S^3 + O(\alpha_S^4), \quad \alpha_S = \frac{g^2}{4\pi}, \quad b_0 = \frac{(33 - 2n_{\rm lf})}{12\pi}, \quad b_1 = \frac{(153 - 19n_{\rm lf})}{24\pi^2} \tag{2.7}$$

where $n_{\rm lf}$ is the number of light flavours.

The quantities $f^{(1)}$ depend on the scheme used for renormalisation and factorisation. Our results are obtained in an extension of the $\overline{MS}$ renormalisation and factorisation scheme. Full details are given in ref. [17]. In this scheme heavy quarks are decoupled at low energy. The light partons continue to obey the same renormalisation group equation as they would have done in the absence of the heavy quarks. Thus our results should be used in conjunction with the running coupling as defined in Eq.(2.7) and together with light parton densities evolved using the two loop $\overline{MS}$ evolution equations.

The functions $f_{ij}^{(0)}$ defined in Eqs. (2.4,2.5) are,

$$f_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}\left[2+\rho\right]$$

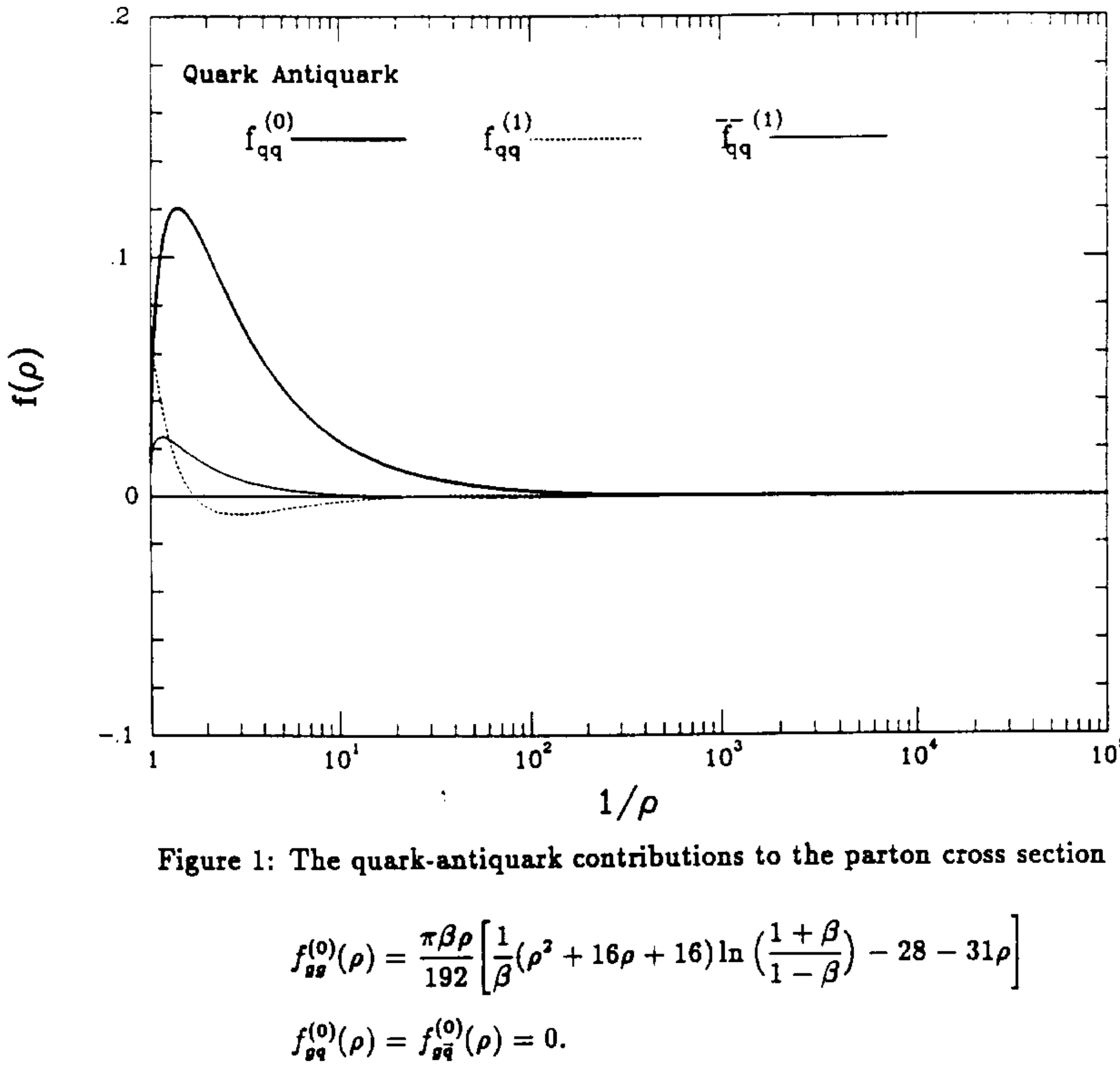

Figure 1: The quark-antiquark contributions to the parton cross section

$$f^{(0)}_{gg}(\rho) = \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}(\rho^2 + 16\rho + 16)\ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho\right]$$

$$f^{(0)}_{gq}(\rho) = f^{(0)}_{g\bar q}(\rho) = 0. \tag{2.8}$$

We now turn to the higher order corrections in Eq.(2.5) which are separated into two terms. The $\overline{f}^{(1)}(\rho)$ terms are the coefficients of $\ln(\mu^2/m^2)$ and are determined by renormalisation group arguments from the lowest order cross-sections,

$$\overline{f}^{(1)}_{ij}(\rho) = \frac{1}{8\pi^2}\left[4\pi\, b_0\, f^{(0)}_{ij}(\rho) - \int_\rho^1 dz_1 f^{(0)}_{kj}(\frac{\rho}{z_1})P_{ki}(z_1) - \int_\rho^1 dz_2 f^{(0)}_{ik}(\frac{\rho}{z_2})P_{kj}(z_2)\right]. \tag{2.9}$$

P_{ij} are the lowest order Altarelli-Parisi kernels. The quantities $f^{(1)}$ in Eq.(2.5) can only be obtained by performing a complete $O(\alpha_S^3)$ calculation. We do not have exact analytical results for the quantities $f^{(1)}$. In ref. [17] a physically motivated fit to the numerically integrated result is given. The fit agrees with the numerically integrated result to better than 1%. The functions $f^{(0)}, f^{(1)}$ and $\overline{f}^{(1)}$ are shown plotted in Figs. 1, 2 and 3 for the cases of quark-antiquark, gluon-gluon and gluon-quark fusion

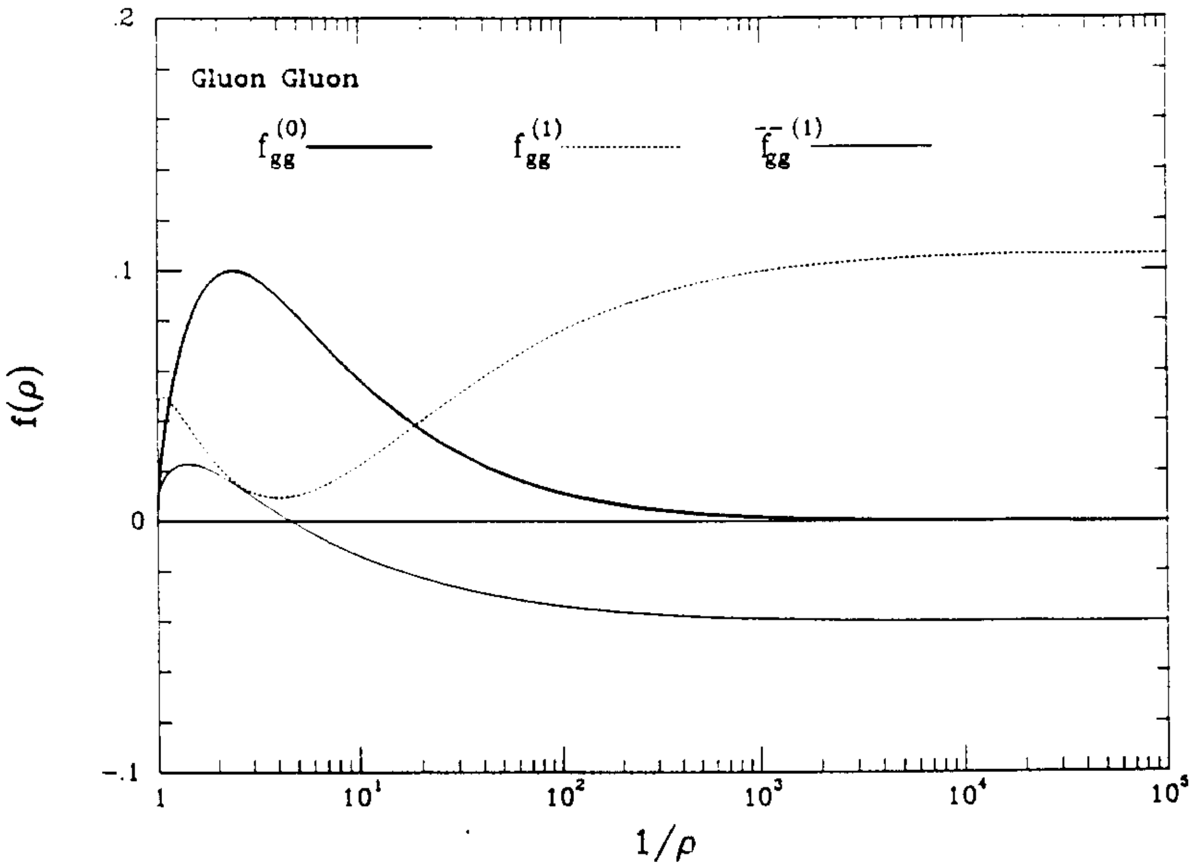

Figure 2: The gluon-gluon contributions to the parton cross section

respectively. Notice the strikingly different behaviour of the gluon-gluon and gluon-quark higher order terms in the high energy limit, $\rho \to 0$. These latter processes allow the exchange of a spin one gluon in the t-channel and are therefore dominant in the high energy limit. These cross sections tend to a constant at high energy. The lowest order terms involve fermion t-channel exchange and therefore fall off at large s as can be seen from Figs. 1 and 2. At high energy we find that,

$$f_{gg}^{(1)} \to 2Nk_{gg} + O(\rho \ln^2 \rho), \qquad \bar{f}_{gg}^{(1)} \to 2N\bar{k}_{gg} + O(\rho \ln^2 \rho)$$
$$f_{gq}^{(1)} \to \frac{V}{2N} k_{gg} + O(\rho \ln^2 \rho), \qquad \bar{f}_{gq}^{(1)} \to \frac{V}{2N} \bar{k}_{gg} + O(\rho \ln^2 \rho) \qquad (2.10)$$

where the constants k_{gg} and $\bar{k}_{gg}$ are,

$$k_{gg} = \frac{1}{\pi V}\left(\frac{V}{2N}\frac{41}{108} - N\frac{793}{43200}\right) \qquad \approx 0.018$$
$$\bar{k}_{gg} = -\frac{1}{\pi V}\left(\frac{V}{2N}\frac{7}{36} - N\frac{11}{360}\right) \qquad \approx -0.0067 \qquad (2.11)$$

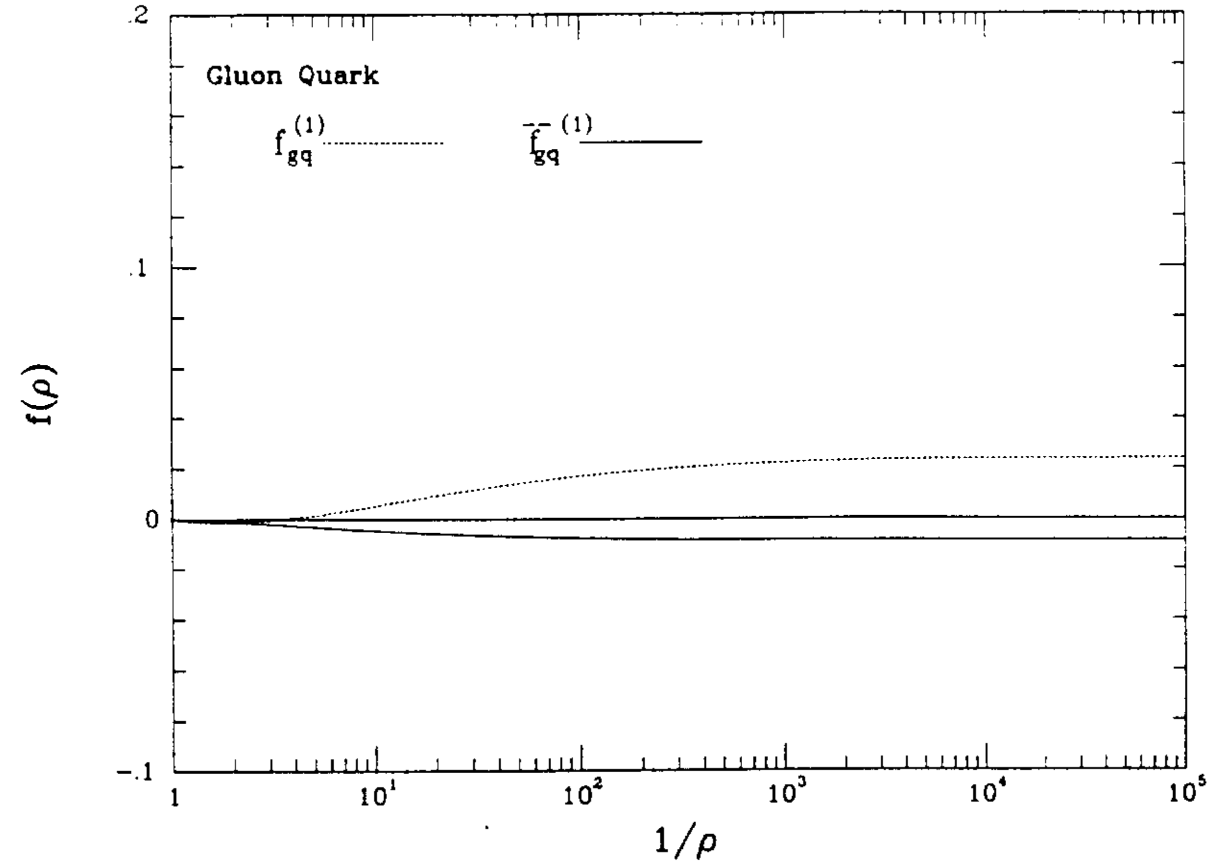

Figure 3: The gluon-quark contributions to the parton cross section

The colour factors V and N are defined after Eq. 1.5. The dominant diagrams are shown in Fig. 4. The asymptotic values of $f^{(1)}_{gg}$ and $f^{(1)}_{gq}$ are proportional to the colour charge of the line which provides the exchanged gluon, since in this limit the upper blob in Fig. 4 is the same for both diagrams and the lower vertex can be approximated by the eikonal form. In the gluon-gluon sub-process the exchanged spin one gluon can come from either incoming gluon, whereas in the gluon quark subprocess it can only come from the incoming quark line. This, together with the ratio of the gluon and quark charges, explains the relative factor of 9/2, shown in Eq.(2.10) and evident in Figs. 2 and 3.

A preliminary idea of the size of the corrections can be obtained from Figs. 1, 2 and 3 even before folding with the parton distribution functions. Taking a typical value for $g^2 \approx 2$, we see that the radiative corrections are large, particularly in the vicinity of the threshold. The significance of the constant cross-section region (gg, gq) at high energy will depend on the rate of fall-off of the structure functions with which the partonic cross-section must be convoluted.

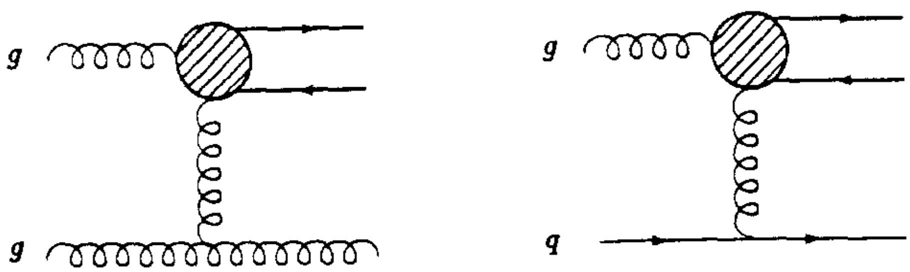

Figure 4: The diagrams responsible for the constant behaviour of the cross-section.

Near threshold,($\beta \to 0$), we have,

$$
\begin{aligned}
f^{(1)}_{q\bar{q}} &\to \mathcal{N}_{q\bar{q}}\Big[-\frac{\pi^2}{6}+\beta\Big(\frac{16}{3}\ln^2(8\beta^2)-\frac{82}{3}\ln(8\beta^2)\Big)+O(\beta)\Big]\\
f^{(1)}_{gg} &\to \mathcal{N}_{gg}\Big[\frac{11\pi^2}{42}+\beta\Big(12\ln^2(8\beta^2)-\frac{366}{7}\ln(8\beta^2)\Big)+O(\beta)\Big]\\
f^{(1)}_{gq} &\to O(\beta).
\end{aligned}
\tag{2.12}
$$

The normalisation, $\mathcal{N}_{ij}$ of the expressions in Eq.(2.12) is determined as follows,

$$
\mathcal{N}_{ij}=\frac{1}{8\pi^2}\frac{f^{(0)}_{ij}(\rho)}{\beta}\Big|_{\beta=0},\quad \mathcal{N}_{q\bar{q}}=\frac{1}{72\pi},\quad \mathcal{N}_{gg}=\frac{7}{1536\pi}. \tag{2.13}
$$

Notice that in this order in perturbation theory the cross-section is finite at threshold. This is due to the $1/\beta$ singularity which is responsible for the binding in a coulomb system. The coulomb attraction tends to increase the cross-section when the incoming partons are in a singlet state (gg), and decrease the cross-section when the incoming partons are in an octet state ($gg, q\bar{q}$). This results in a net positive term for the gg case. Note that the numerical importance of the term due to the coulomb singularity is quite small.

We now examine the region near threshold in more detail. The terms in Eq.(2.12) which are finite at threshold have already been explained. The $\ln^2(\beta^2)$ terms in

Eq. (2.12) have a general origin. They are due to terms of the form

$$f^{(1)}(\rho) \sim \frac{1}{8\pi^2}\int_\rho^1 dz \left[\frac{\ln(1-z)}{1-z}\right]_+ f^{(0)}(\rho/z) \tag{2.14}$$

For attempts to resum these terms of this form in Drell-Yan processes we refer the reader to ref. [19].

The phenomenological consequences of the co $O(\alpha_S^3)$ formulae have been investigated in ref. [1?], and to greater extent in ref. [20]. An attempt has been made to estimate the uncertainties due to the form of the gluon distribution and the value of Λ, the value of the heavy quark mass and the choice made for the scale μ. Predictions are given for ch bottom and top production at fixed target and collider energies.

3. The p otoproduction of heavy quarks

In this section I review the results which descri he -section for the photo-production of a heavy quark pair[18 he treatment closely parallels the hadroproduction results given in the previou tion. The interaction of the point-like component of the photon with a parton gives the cross-section,

$$\hat{\sigma}_{\gamma j}(s, m^2, \mu^2) = \frac{\alpha_S(\mu^2)\alpha_{em}}{m^2} f_{\gamma j}\Big(\rho, \frac{\mu^2}{m^2}\Big) \tag{3.1}$$

α_{em} is the fine structure constant. Knowledge of the functions $f_{\gamma j}$ (and the hadronic functions f_{ij}) gives a complete description of the short distance cross-section for the production of a heavy quark of mass m by a photon γ. The index j specifies the type of the incoming parton. The dimensionless functions $f_{\gamma j}$ have the following perturbative expansion,

$$f_{\gamma j}\Big(\rho, \frac{\mu^2}{m^2}\Big) = f^{(0)}_{\gamma j}(\rho) + g^2(\mu^2)\Big[f^{(1)}_{\gamma j}(\rho) + \overline{f}^{(1)}_{\gamma j}(\rho)\ln\Big(\frac{\mu^2}{m^2}\Big)\Big] + O(g^4) \tag{3.2}$$

The functions $f_{\gamma j}$ defined in eqs. (3.2) depend on the charge of the quark which interacts with the photon. In order to make these dependences explicit we define the quantities,

$$f_{\gamma g}(\rho, \frac{\mu^2}{m^2}) = e_Q^2 c_{\gamma g}(\rho, \frac{\mu^2}{m^2})$$

$$f_{\gamma q}(\rho, \frac{\mu^2}{m^2}) = e_Q^2 c_{\gamma q}(\rho, \frac{\mu^2}{m^2}) + e_q^2 d_{\gamma q}(\rho, \frac{\mu^2}{m^2}) \tag{3.3}$$

The charges of the heavy and light quarks are denoted by e_Q and e_q respectively. The interference term proportional to $e_Q e_q$ makes no contribution to the total cross-section. The perturbative expansions of c and d are defined by a formula with exactly the same structure as eq. (3.2). For the lowest order terms we find,

$$c_{\gamma g}^{(0)}(\rho) = \frac{\pi\beta\rho}{4}\left[\frac{1}{\beta}(3-\beta^4)\ln\left(\frac{1+\beta}{1-\beta}\right) - 4 + 2\beta^2\right] \tag{3.4}$$

$$c_{\gamma q}^{(0)}(\rho) = d_{\gamma q}^{(0)}(\rho) = c_{\gamma \bar{q}}^{(0)}(\rho) = d_{\gamma \bar{q}}^{(0)}(\rho) = 0. \tag{3.5}$$

We now turn to the higher order corrections in eq. (3.2) which are separated into two terms. The $\overline{f}^{(1)}(\rho)$ terms are the coefficients of $\ln(\mu^2/m^2)$ and are determined by renormalisation group arguments from the lowest order cross-sections,

$$\overline{f}_{\gamma j}^{(1)}(\rho) = \frac{1}{8\pi^2}\left[2\pi\, b_0\, f_{\gamma j}^{(0)}(\rho) - \sum_k \int_\rho^1 dz_1 f_{kj}^{(0)}(\frac{\rho}{z_1}) P_{k\gamma}(z_1) - \sum_k \int_\rho^1 dz_2 f_{\gamma k}^{(0)}(\frac{\rho}{z_2}) P_{kj}(z_2)\right]. \tag{3.6}$$

The quantities $f_{\gamma j}^{(1)}$ in eq. (3.2) can only be obtained by performing a complete $O(\alpha_{em}\alpha_S^2)$ calculation. The results of such a calculation are given in ref. [18].

The functions $c^{(0)}, c^{(1)}$ and $\overline{c}^{(1)}$ are shown plotted in fig. 5 for the case of photon-gluon scattering. In figs. 6 and 7 we plot $c^{(1)}, \overline{c}^{(1)}, d^{(1)}$ and $\overline{d}^{(1)}$ for the case photon-quark scattering. Notice that the term proportional to the square of the charge of the light quark is quite small.

At high energy we find that,

$$\begin{aligned} c_{\gamma g}^{(1)} &\to N k_{\gamma g} + O(\rho \ln^2 \rho), \qquad & \overline{c}_{\gamma g}^{(1)} &\to N \overline{k}_{\gamma g} + O(\rho \ln^2 \rho) \\ c_{\gamma q}^{(1)} &\to \frac{V}{2N} k_{\gamma g} + O(\rho \ln^2 \rho), \qquad & \overline{c}_{\gamma q}^{(1)} &\to \frac{V}{2N} \overline{k}_{\gamma g} + O(\rho \ln^2 \rho) \end{aligned} \tag{3.7}$$

where $k_{\gamma g}$ and $\overline{k}_{\gamma g}$ are,

$$\begin{aligned} k_{\gamma g} &= \frac{41}{108\pi} \qquad &\approx 0.121 \\ \overline{k}_{\gamma g} &= -\frac{7}{36\pi} \qquad &\approx -0.0619 \end{aligned} \tag{3.8}$$

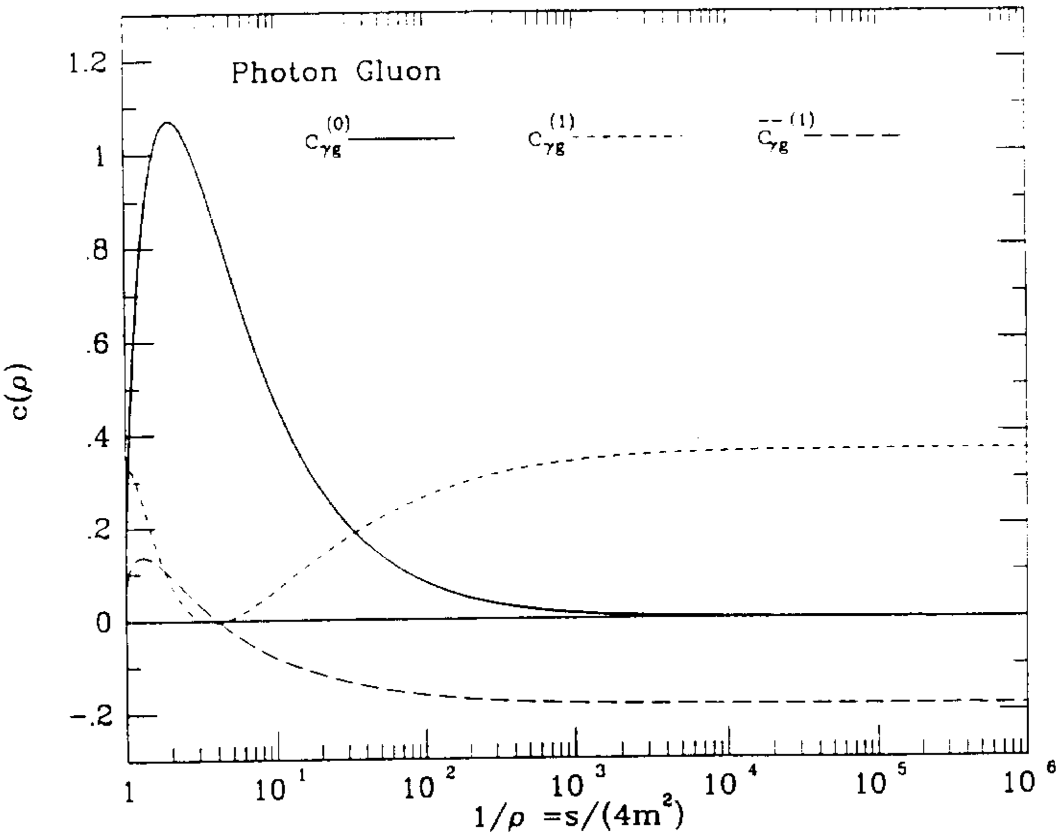

Figure 5: The photon gluon contributions to the parton cross-section

Thus the photon-parton cross-section goes to a constant as $s \to \infty$, in complete analogy with Eqs. 2.10, 2.11. Note that the constants $k_{\gamma g}$ and $\bar{k}_{\gamma g}$ are related to the 'Non-Abelian' parts of k_{gg} and $\bar{k}_{gg}$.

Near threshold we can extract the exact analytic behaviour,

$$\begin{aligned} c^{(1)}_{\gamma g} &\to \mathcal{N}_{\gamma g}\left[-\frac{\pi^2}{6}+\beta\left(6\ln^2(8\beta^2)-30\ln(8\beta^2)\right)+\mathcal{O}(\beta)\right] \\ c^{(1)}_{\gamma q} &\to \mathcal{O}(\beta) \\ d^{(1)}_{\gamma q} &\to \mathcal{O}(\beta). \end{aligned} \tag{3.9}$$

The normalisation, $\mathcal{N}_{\gamma g}$ of the expression in eq. (3.9) is determined as follows,

$$\mathcal{N}_{\gamma g} = \frac{1}{8\pi^2}\frac{c^{(0)}_{\gamma g}(\rho)}{\beta}\bigg|_{\beta=0} = \frac{1}{16\pi}. \tag{3.10}$$

The origin of these large terms near threshold has been explained in the previous section.

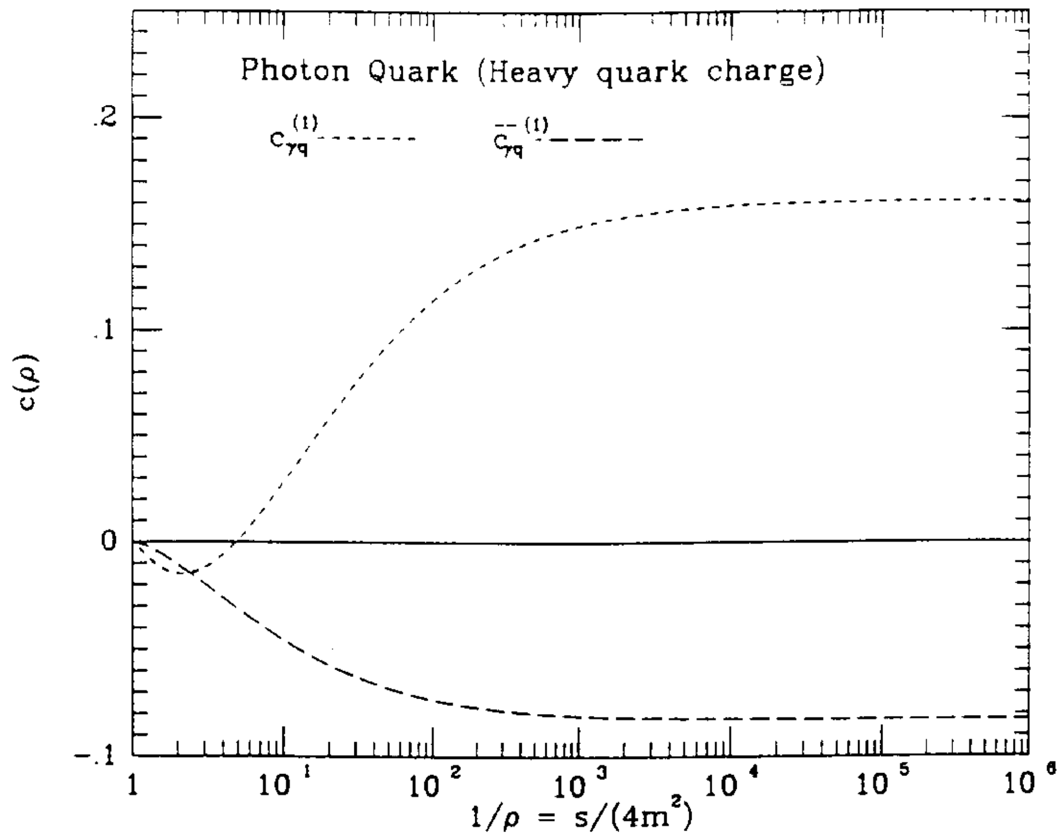

Figure 6: The photon quark contributions proportional to the square of the heavy quark charge.

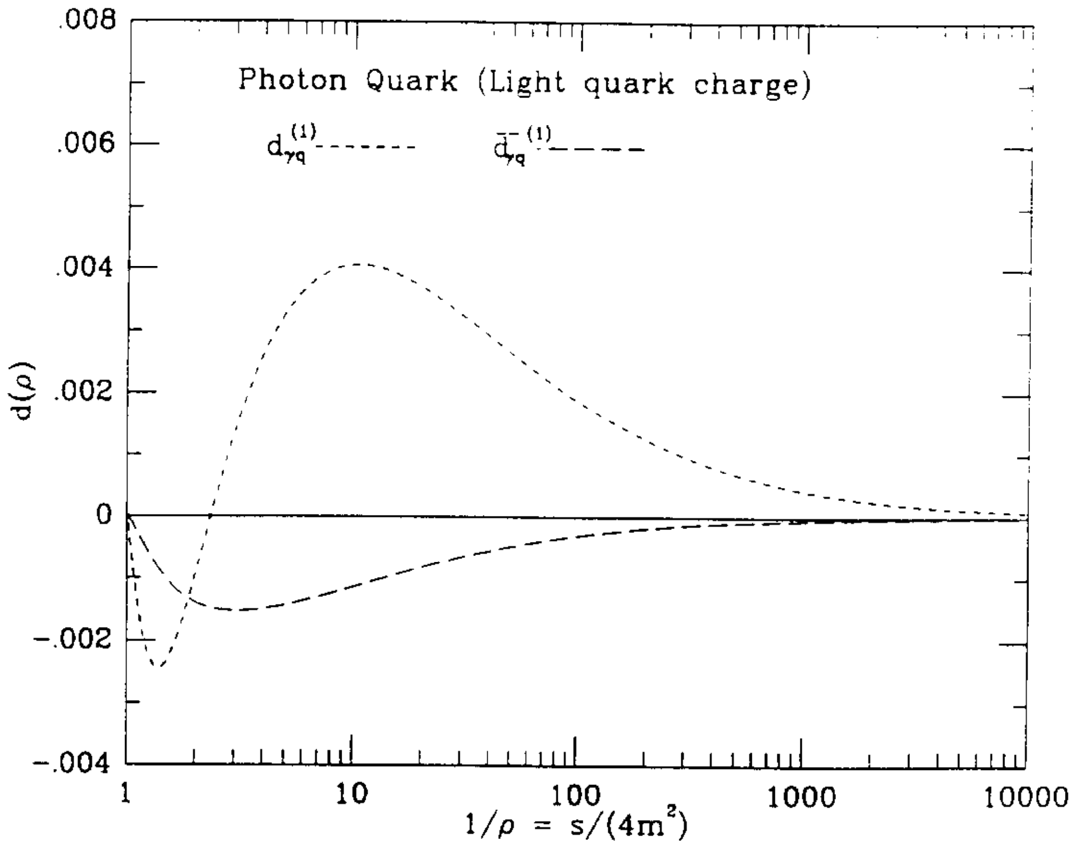

Figure 7: The photon quark contributions proportional to the square of the light quark charge.

The phenomenological consequences of these formulae have been described in ref. [18]. It is interesting to compare the higher order corrections to hadroproduction with the corrections to photoproduction. Once allowance has been made for the fact that two incoming gluons can participate in the hadroproduction of heavy quarks, whereas at most one gluon can participate in the photoproduction reaction, the corrections to the two processes are remarkably similar. This suggests that a much tighter theoretical prediction can be obtained by comparing the two processes.

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