QUARKONIUM ANNIHILATION RATES

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ABSTRACT

Recent measurements of ratios of quarkonium annihilation rates are used to evaluate the strong fine structure constant α_s. Expressions are presented for QCD radiative corrections with α_s referred to the quark mass scale. We find $\alpha_s(m_b) = 0.180^{+0.009}_{-0.008}$ from the ratio $\Gamma(\Upsilon \to \gamma gg)/\Gamma(\Upsilon \to ggg)$. The corresponding range of $\Lambda^{(4)}_{\overline{MS}}$ (the QCD scale factor for four light quark flavors) is 150–215 MeV. The experimentally more precise but theoretically more questionable ratio of the gluonic and muonic widths of J/ψ and Υ yields $\alpha_s(m_c) = 0.29 \pm 0.02$, $\alpha_s(m_b) = 0.189 \pm 0.008$ when v^2/c^2 corrections to these ratios for J/ψ and Υ are parametrized linearly. Averaging the two determinations, we find $\Lambda^{(4)}_{\overline{MS}} = 199 \pm 22\ MeV$, $\alpha_s(m_c) = 0.278 \pm 0.014$, $\alpha_s(m_b) = 0.185 \pm 0.006$. Further predictions are made for ratios of rates.

The annihilation of a heavy quark-antiquark pair ("quarkonium") into final states consisting of leptons, photons, and light quarks can provide useful information on the strong fine structure constant $\alpha_s(\mu)$.[1,2] Here μ is a renormalization scale, for which various prescriptions have appeared in the literature.[3,4] Annihilation rates typically depend on $\alpha_s(\mu)$ to some power p, times a correction factor:

$$\Gamma(Q\overline{Q} \to [final\ \ state]) = A\,[\alpha_s(\mu)]^p\,[1 + B(\mu)\alpha_s(\mu) + 0(\alpha_s^2)]\ . \qquad (1)$$

Unless $p = 0$, the coefficient B depends both on the scale μ and on the exact definition of the coupling constant (the "renormalization scheme"). This double ambiguity means that a scale choice which is reasonable in one scheme is unreasonable in another, and has led to some confusion about whether the power series expansion is well behaved in most processes. In Ref. 4, a technique was introduced for probing physical momentum scales in QCD processes so as to allow an intelligent and informed guess for the renormalization scale in a scheme-independent way. This analysis concluded that with the important exception of the ratio of the gluonic and muonic widths of the Υ, the perturbation series for most QCD processes is quite well behaved and can be used for phenomenology.

In this article we provide a concise summary of QCD corrections to rates based on the simple and reasonable choice $\mu = m_Q$ in the modified-minimal-subtraction ($\overline{MS}$) scheme.[5] At the same time we make use of the most recent measurements of Υ annihilation rates to evaluate $\alpha_s(m_b)$ precisely. We find

$$\alpha_s(m_b) = 0.180^{+0.009}_{-0.008} \quad , \tag{2}$$

corresponding to a QCD scale factor of

$$\Lambda^{(4)}_{\overline{MS}} = 182^{+33}_{-32}\ MeV \ , \tag{3}$$

from the ratio of partial widths into two gluons and a photon and into three gluons. The superscript on $\Lambda_{\overline{MS}}$ denotes the number of light quark flavors. The result (3) is completely supported by the experimentally more precise but theoretically more questionable ratio of the gluonic and muonic widths, which yields $\alpha_s(m_b) = 0.173 \pm 0.005$. An attempt to describe the ratios of gluonic and muonic widths of J/ψ and Υ simultaneously is made by parametrizing v^2/c^2 corrections to these ratios in a linear fashion. The results are

$$\alpha_s(m_c) = 0.29 \pm 0.02 \quad , \quad \alpha_s(m_b) = 0.189 \pm 0.008 \quad , \tag{4}$$

$$\Lambda^{(4)}_{\overline{MS}} = 216 \pm 31\ MeV \quad , \tag{5}$$

in accord with the ranges in Eqs. (2) and (3). Averaging the determinations (3) and (5), we find

$$\Lambda^{(4)}_{\overline{MS}} = 199 \pm 22\ MeV \quad , \tag{6}$$

$$\alpha_s(m_c) = 0.278 \pm 0.014 \quad , \tag{7}$$

$$\alpha_s(m_b) = 0.185 \pm 0.006 \quad . \tag{8}$$

We shall compare the result (7) with crude determinations of $\alpha_s(m_c)$ from two-gluon to two-photon partial width ratios of η_c and χ_2, and predict more precise values for these ratios.

We relate $\alpha_s(m_Q)$ to α_s at some other mass scale μ using the expression

$$\alpha_s^{-1}(m_Q) = \alpha_s^{-1}(\mu) + \frac{\beta_0}{4\pi} \ln(m_Q^2/\mu^2) \ , \tag{9}$$

where

$$\beta_0 = 11 - \frac{2}{3}\, n_f \quad , \tag{10}$$

and n_f is the number of light flavors with mass less than m_Q: $n_f = 3$ for $m_Q = m_c$, $n_f = 4$ for $m_Q = m_b$. Equating

$$[\alpha_s(\mu)]^p\,[1 + B(\mu)\alpha_s(\mu)] = [\alpha_s(m_Q)]^p\,[1 + B(m_Q)\alpha_s(m_Q)] \quad . \tag{11}$$

to leading order in α_s, we find

$$B(m_Q) = B(\mu) + \frac{p\beta_0}{2\pi} \ln\left(\frac{m_Q}{\mu}\right) \quad . \tag{12}$$

Expressions for QCD radiative corrections are taken from Refs. 4 and 5 for three-gluon and two-gluon plus photon decays of $^3S_1$ states, and from Ref. 6 (see also the last two of Refs. 2) for two-gluon decays of $^1S_0$, $^3P_0$, and $^3P_2$ levels. These expressions are summarized in Table 1.

In Table 1, B is of course independent of μ for purely electromagnetic decays. The number of light flavors is to be taken as $n_f = 3$ for charmonium states and 4 for $b\bar{b}$ ones. The logarithmic correction factors $\ln[m_Q R_c]$, where R_c are confinement radii, are taken as approximate expressions for $\ln[4m_Q^2/|M^2 - 4m_Q^2|]$. The prescription of renormalization at the scale $\mu = m_Q$ leads to some differences in terms (2/3)ln 2 for two-gluon processes or ln 2 for three-gluon processes from coefficients of α_s/π cited in the literature.

We now calculate the expression quoted in Table 1 using the quarkonium parameters shown in Table 2. We use $m_c = 1.5\ GeV/c^2$ and $m_b = 4.9\ GeV/c^2$ here and in what follows. The results (including familiar expressions for total rates) are summarized in Table 3. Ambiguities in the definition of the logarithmic terms in corrections to $\Gamma(^3P_{0,2} \to glue)$ are such that the coefficient of α_s/π should not be regarded as known to better than about $\pm$ 1.

We next summarize relevant partial decay widths and branching ratios of quarkonium, and give the values of α_s extracted from various ratios. The results are shown in Table 4. We quote a few details of the determinations. Unless otherwise noted, experimental values are taken from Ref. 7.

1. η_c *decays*. The total width of η_c can be assumed to be dominated by two-gluon decay:

$$\Gamma(\eta_c \to gg) \approx \Gamma_{tot}(\eta_c) = 11.5 \pm 4.3\ MeV \quad . \tag{13}$$

An average of several experiments involving e^+e^- [8] and $\bar{p}p$ collisions[9] gives[8]

$$\Gamma(\eta_c \to \gamma\gamma) = 9 \pm 4\ keV \quad . \tag{14}$$

The predicted ratio of these two quantities is

$$\frac{\Gamma(\eta_c \to gg)}{\Gamma(\eta_c \to \gamma\gamma)} = \frac{9[\alpha_s(m_c)]^2}{8\alpha^2}\left(1 + 8.2\,\frac{\alpha_s}{\pi}\right) \quad . \tag{15}$$

Here and subsequently we omit the argument (μ) of α_s in the correction term, since a change in μ only affects higher-order corrections.

The $\eta_c \to \gamma\gamma$ width can be expressed in terms of that for $J/\psi \to \mu^+\mu^-$ if $|\Psi(0)|^2$ is the same for the two states. The magnetic transition $J/\psi \to \gamma\eta_c$ is substantially weaker than one estimates nonrelativistically, however. This suggests that the J/ψ and η_c wave functions may not be identical, with their overlap reduced by hyperfine and coupled-channel effects. Ignoring such effects, we would predict

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(J/\psi \to \mu^+\mu^-)} = \frac{4}{3}(1 + 1.96\,\frac{\alpha_s}{\pi}) \ , \tag{16}$$

or $\Gamma(\eta_c \to \gamma\gamma) \approx 7\ keV$ for $\Gamma(J/\psi \to \mu^+\mu^-) = 4.7 \pm 0.3\ keV$.[8] This value is compatible with the present experimental range (14).

2. *J/ψ decays*. The total decay width $\Gamma_{tot}(J/\psi)$ is composed of e^+e^-, $\mu^+\mu^-$, $\gamma^* \to q\bar{q}$, $\gamma\eta_c$, ggg, and γgg contributions. From Ref. 7 we find

$$\Gamma(ee) = \Gamma(\mu\mu) = (6.9 \pm 0.9)\%\ \Gamma_{tot}\ ; \tag{17a}$$

$$\Gamma(\gamma^* \to q\bar{q}) = (2.4 \pm 0.2)\ \Gamma(\mu\mu) \tag{17b}$$

(The latter value is estimated from e^+e^- cross section measurements around the J/ψ mass); and

$$\Gamma(\gamma\eta_c) = (1.27 \pm 0.36)\%\ \Gamma_{tot}\ . \tag{18}$$

Then

$$\Gamma(\gamma gg) + \Gamma(ggg) = (68.4 \pm 4.2)\%\ \Gamma_{tot}\ . \tag{19}$$

Now we use the measured value[10]

$$\Gamma(\gamma gg) = (10 \pm 4)\%\ \Gamma(ggg) \tag{20}$$

to conclude

$$\Gamma(ggg) = (62.2 \pm 4.4)\%\ \Gamma_{tot}\ . \tag{21}$$

Combining this with Eq. (17a), we find the ratio shown in Table 4. The theoretical expectation is

$$\frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to \mu\mu)} = \frac{5}{18}\left(\frac{M}{2m_c}\right)^2 \frac{(\pi^2 - 9)[\alpha_s(m_c)]^3}{\pi\alpha^2}\ (1 + 1.6\,\frac{\alpha_s}{\pi})\ . \tag{22}$$

The rather tightly constrained value of $\alpha_s(m_c)$ noted in Table 4 is probably in fact a crude estimate, since v^2/c^2 corrections have been neglected in Eq. (22).

The $\gamma gg/ggg$ ratio in Eq. (20) is expected to be

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}\ (1 - 2.9\,\frac{\alpha_s}{\pi})\ . \tag{23}$$

3. *χ decays*. The most precise measurements of the total and $\gamma\gamma$ widths of χ_2 (the $J^{PC} = 2^{++}$ $c\bar{c}$ state at 3556 MeV) come from an ISR experiment. They yield[9]

$$\Gamma(\chi_2 \to gg) = 2.6^{+1.4}_{-1.0}\ MeV\ , \tag{24}$$

$$\Gamma(\chi_2 \to \gamma\gamma) = 2.9^{+1.3}_{-1.0} \pm 1.7\ keV\ . \tag{25}$$

The Crystal Ball collaboration[11] obtains $2.8 \pm 2.0\ keV$ for this last value. Averaging the two, we obtain $\Gamma(\chi_2 \to \gamma\gamma) = 2.85 \pm 1.43\ keV$. The predicted ratio of the gg and $\gamma\gamma$ rates is

$$\frac{\Gamma(\chi_2 \to gg)}{\Gamma(\chi_2 \to \gamma\gamma)} = \frac{9[\alpha_s(m_c)]^2}{8\alpha^2}\,(1 + 1.0\,\frac{\alpha_s}{\pi}) \ . \tag{26}$$

We also present in Table 4 predictions for experimental ratios based on the value of $\alpha_s(m_c)$ quoted in Eq. (7). Future measurements[12] of η_c and χ widths to gg and $\gamma\gamma$ will be able to check these predictions much more closely than in the past.

4. Υ *decays*. We use the branching ratios

$$\Gamma(ee) = \Gamma(\mu\mu) = (2.8 \pm 0.2)\%\ \Gamma_{tot} \tag{27}$$

and calculate[13]

$$\Gamma(\tau\tau) = (2.76 \pm 0.2)\%\ \Gamma_{tot} \ , \tag{28}$$

$$\Gamma(\gamma^* \to q\bar{q}) = (10.1 \pm 0.9)\%\ \Gamma_{tot} \ . \tag{29}$$

This implies

$$\Gamma(\gamma gg) + \Gamma(ggg) = (81.6 \pm 1.4)\%\ \Gamma_{tot} \ . \tag{30}$$

There are three determinations of $\Gamma(\gamma gg)/\Gamma(ggg)$:

$$\frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)} = \begin{cases} (2.93 \pm 0.12 \pm 0.18)\%\ (ARGUS)^{14} \\ (2.54 \pm 0.18 \pm 0.14)\%\ (CLEO)^{15} \\ (2.99 \pm 0.59)\%\ (CUSB)^{16} \end{cases} \tag{31}$$

Here it appears important[14] to use the photon energy spectrum calculated by Field;[17] others[18] are not in accord with the data. Averaging these values, we find the $\gamma gg/ggg$ ratio quoted in Table 4, and so

$$\Gamma(ggg) = (79.4 \pm 1.4)\%\ \Gamma_{tot} \ , \tag{32}$$

leading to the $ggg/\mu\mu$ ratio cited in Table 4. We expect

$$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = \frac{10}{9}(\frac{M}{2m_b})^2\,\frac{(\pi^2 - 9)[\alpha_s(m_b)]^3}{\pi\alpha^2}\,(1 + 0.43\,\frac{\alpha_s}{\pi}) \ . \tag{33}$$

The implied experimental value of $\alpha_s(m_b)$ in Table 4 is in accord with the estimates of Ref. 19:

$$\Upsilon: \qquad \alpha_s([0.48\ M_\Upsilon]^2) = 0.172^{+0.008}_{-0.007} \tag{34}$$

$$\Upsilon': \qquad \alpha_s([0.48\ M_{\Upsilon'}]^2) = 0.177^{+0.015}_{-0.012} \tag{35}$$

$$\Upsilon'': \qquad \alpha_s([0.48\ M_{\Upsilon''}]^2) = 0.170^{+0.015}_{-0.012} \ . \tag{36}$$

The theoretical expression for the $\gamma gg/ggg$ ratio is

$$\frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)} = \frac{4}{5}\,\frac{\alpha}{\alpha_s(m_b)}\,(1 - 2.6\,\frac{\alpha_s}{\pi}) \ . \tag{37}$$

The experimental average for this ratio leads to a value of $\alpha_s(m_b)$ almost precisely equal to that obtained from Eq. (33).

The agreement of the two determinations of α_s is especially important since concerns have been raised about the convergence of the perturbation series for the ratio of Eq. (33), but not for the ratio of Eq. (37). The prediction for the ratio of Eq. (37) based on Eq. (8), shown in square brackets, is also in accord with the present experimental value.

We now present details of the calculation that led to Eqs. (4) and (5). The relation between $\alpha_s(\mu)$ and $\Lambda_{\overline{MS}}^{(n_f)}$, to two-loop accuracy, is

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}} \left(1 - \frac{\beta_1 \ln \ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}}{\beta_0^2 \ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}}\right), \tag{38}$$

where

$$\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta_1 = 102 - \frac{38}{3} n_f \tag{39}$$

and we use $n_f = 4$ for $m_c \le \mu \le m_b$.

We parametrize v^2/c^2 corrections to Eqs. (22) and (33) by a factor $(1 + C v^2/c^2)$, with $v^2/c^2 = 0.24$ for charmonium, 0.073 for Υ. (See the first of Refs. 3). The experimental $ggg/\mu\mu$ ratios are both reproduced to within one standard deviation over the range of parameters in Eqs. (4) and (5), with C ranging between ≈ -2.9 and ≈ -3.5. The large magnitude of the v^2/c^2 correction for charmonium means this exercise is best a qualitative one, but it does lead to a value of $\Lambda_{\overline{MS}}^{(4)}$ [Eq. (5)] consistent with that implied by the $\gamma gg/ggg$ ratio [Eq. (3)]. Other determinations[20] of $\Lambda_{\overline{MS}}^{(4)}$ are consistent with values around 200 MeV, but with wide error limits.

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Table 1.

Numerical or analytic expressions for first-order corrections to decay rates, of the form $\Gamma/\Gamma^{(0)} = 1 + B(\mu)\alpha_s/\pi$.

Process	$B(\mu)$
${}^1S_0 \to \gamma\gamma$	$\pi^2/3 - 20/3 = -3.38$
$\to glue$	$\beta_0 \ln(\mu/m_Q) + 159/6 - 31\,\pi^2/24 - 11 \ln 2$ $+ n_f(-8/9 + [2/3] \ln 2)$
${}^3S_1 \to e^+e^-$	$-16/3 = -5.33$
$\to \gamma\gamma\gamma$	-12.61 ± 0.03
$\to glue$	$(3\beta_0/2) \ln(\mu/m_Q) - 0.26 - 1.16\, n_f$
$\to \gamma + glue$	$\beta_0 \ln(\mu/m_Q) - 4.37 - 0.77\, n_f$
${}^3P_0 \to \gamma\gamma$	$\pi^2/3 - 28/9 = 0.18$
$\to glue$	$\beta_0 \ln(\mu/m_Q) + 370/27 + 5\pi^2/16 - 11 \ln 2$ $+ n_f(-16/27 + [2/3] \ln 2 + [4/27] \ln[m_Q R_c])$
${}^3P_2 \to \gamma\gamma$	$-16/3 = -5.33$
$\to glue$	$\beta_0 \ln(\mu/m_Q) + 1855/72 - 337\pi^2/128 - 6 \ln 2$ $+ n_f(-11/18 + [2/3] \ln 2 + [5/9] \ln[m_Q R_c])$

Table 2. Quarkonium parameters affecting first-order corrections to P-wave annihilation rates.

State	R_c (GeV^{-1})
$c\bar{c}(1P)$	3.17
$b\bar{b}(1P)$	1.86
$b\bar{b}(2P)$	3.05

Table 3. Lowest order expressions and first order QCD corrections with α_s computed at the mass scale of the constituent quark ($m_c = 1.5\ GeV,\ m_b = 4.9\ GeV$) for decay processes of $c\bar{c}$ and $b\bar{b}$ quarkonium states. Here we assume three colors of quarks. Note that corrections to ratios of ^3P_J decay widths are known more precisely than individual values.

Process	Rate	Correction Factor
$^1S_0 \to \gamma\gamma$	$12\pi e_Q^4 \alpha^2 \lvert\Psi(0)\rvert^2/m_Q^2$	$1 - 3.4\ \alpha_s/\pi$
$\to glue$	$8\pi\alpha_s^2 \lvert\Psi(0)\rvert^2/3m_Q^2$	$1 + 4.8\ \alpha_s/\pi\ \ (\eta_c)$
		$1 + 4.4\ \alpha_s/\pi\ \ (\eta_b)$
$^3S_1 \to e^+e^-$	$16\pi\alpha^2 e_Q^2 \lvert\Psi(0)\rvert^2/M^2$	$1 - 16\ \alpha_s/3\pi$
$\to \gamma\gamma\gamma$	$16(\pi^2-9)\alpha^3 e_Q^6 \lvert\Psi(0)\rvert^2/3m_Q^2$	$1 - 12.6\ \alpha_s/\pi$
$\to glue$	$40(\pi^2-9)\alpha_s^3 \lvert\Psi(0)\rvert^2/81\ m_Q^2$	$1 - 3.7\ \alpha_s/\pi\ \ (J/\psi)$
		$1 - 4.9\ \alpha_s/\pi\ \ (\Upsilon)$
$\to \gamma + glue$	$32(\pi^2-9)e_Q^2 \alpha\alpha_s^2 \lvert\Psi(0)\rvert^2/9m_Q^2$	$1 - 6.7\ \alpha_s/\pi\ \ (J/\psi)$
		$1 - 7.4\ \alpha_s/\pi\ \ (\Upsilon)$
$^1P_1 \to glue^{a)}$	$(20/9\pi)\alpha_s^3 \lvert R_{nP}'(0)\rvert^2 \ln(m_Q\langle R_c\rangle)/m_Q^4$	Not known
$^3P_0 \to \gamma\gamma$	$27 e_Q^4 \alpha^2 \lvert R_{nP}'(0)\rvert^2/m_Q^4$	$1 + 0.2\ \alpha_s/\pi$
$\to glue$	$6\alpha_s^2 \lvert R_{nP}'(0)\rvert^2/m_Q^4$	$1 + 9.5\ \alpha_s/\pi\ (\chi)$
		$1 + 10.0\ \alpha_s/\pi\ \ (\chi_b)$
		$1 + 10.2\ \alpha_s/\pi\ \ (\chi_b')$
$^3P_1 \to q\bar{q} + glue^{a)}$	$(8/9\pi)n_f\,\alpha_s^3\ \lvert R_{nP}'(0)\rvert^2/m_Q^4 \ln(m_Q\langle R_c\rangle)$	Not known
$^3P_2 \to \gamma\gamma$	$36 e_Q^4 \alpha^2 \lvert R_{nP}'(0)\rvert^2/5m_Q^4$	$1 - 16\ \alpha_s/3\pi$
$\to glue$	$8\alpha_s^2 \lvert R_{nP}'(0)\rvert^2/5m_Q^4$	$1 - 2.2\ \alpha_s/\pi\ \ (\chi_c)$
		$1 - 0.1\ \alpha_s/\pi\ \ (\chi_b)$
		$1 + 1.0\ \alpha_s/\pi\ \ (\chi_b')$

[a] $\langle R_c\rangle$ is the average radius of the $^1P_1$ or $^3P_1$ state.

Table 4. Information on α_s obtained from various ratios of quarkonium annihilation rates.

Ratio	Expression (Eq. no.)	Value[a)]	Parameter	Value
$\frac{\Gamma(\eta_c \to gg)}{\Gamma(\eta_c \to \gamma\gamma)}$	(15)	$(1.28 \pm 0.74) \times 10^3$ $[(2.8 \pm 0.3) \times 10^3]$	$\alpha_s(m_c)$	$0.20^{+0.04}_{-0.06}$
$\frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to \mu\mu)}$	(22)	9.0 ± 1.3	$\alpha_s(m_c)$	0.175 ± 0.008
$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)}$	(23)	0.10 ± 0.04 $[0.062 \pm 0.005]$	$\alpha_s(m_c)$	$0.19^{+0.10}_{-0.05}$
$\frac{\Gamma(\chi_{c2} \to gg)}{\Gamma(\chi_{c2} \to \gamma\gamma)}$	(26)	$(0.91 \pm 0.62) \times 10^3$ $[(2.1 \pm 0.2) \times 10^3]$	$\alpha_s(m_c)$	$0.19^{+0.05}_{-0.08}$
$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to \mu\mu)}$	(33)	$28.4^{+2.7}_{-2.4}$	$\alpha_s(m_b)$	0.173 ± 0.005
$\frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)}$	(37)	$(2.76 \pm 0.15)\%$ $[(2.67 \pm 0.09)\%]$	$\alpha_s(m_b)$	$0.180^{+0.009}_{-0.008}$

[a)]Quantities in brackets denote predictions based on Eqs. (7) and (8).