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Baryon Number Generation From Cosmic String Loops

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Abstract

Baryon number generation due to the decay of particle-antiparticle pairs created from cosmic string loops is studied. In the situation when cusp evaporation occurs a significant baryon asymmetry (baryon/entropy $\sim 10^{-4}\epsilon$: ϵ is a net baryon asymmetry generated from one pair of particle-antiparticle) can be generated.

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1 Introduction

Cosmic strings [1] are topological defects which are generated at the grand unification phase transition in the early universe. In particular cosmic string loops, which are produced by an intersection of two infinite strings or a self-intersection of an infinite string, can work as seeds in the process of galaxy formation [2]. Cosmic string loops oscillate with period $\sim L/2$ (L: size of a loop) and their sizes are reduced by emitting gravitational radiation. The radiation rate is estimated as $\sim G\mu^2$ where μ is the line density of the string and is expressed by using grand unification scale σ ($\sim 10^{-3}M_{pl}$) as $\mu \sim \sigma^2$.

Recently Srednicki and Theisen [3] and Brandenberger [4] have discussed energy loss by particle-antiparticle emission from string loops. They have considered the coupling between the Higgs field responsible for strings and another "Higgs" field and have found that "Higgs" particle-antiparticle pairs are emitted mostly from so-called cusps. But they concluded that energy loss due to particle-antiparticle emission is negligible in comparison with that by gravitational radiation.

In this paper we study baryon number generation from cosmic strings assuming that emitted "Higgs" particles decay into quarks and leptons with violation of baryon number conservation. Although particle-antiparticle emission is energetically negligible, it may generate a significant baryon number asymmetry in our universe. The baryon number generation from cosmic strings was studied by Bhattacharjee, Kibble and Turok [5] but from a different context in which the baryon number is generated in the collapsing phase of string loops. Their scenario is based on the assumption that the string loops collapse by the self-intersections rapidly but this assumption contradicts with the galaxy formation scenario by cosmic strings.

The existence of cusps where strings move with the velocity of light is very important in considering particle-antiparticle emission. In reality cusps cannot exist because strings have a finite width ($\sim \sigma^{-1}$) but a part moving close to the speed of light may exist. Furthermore in such a region microscopic interactions may dominate over the macroscopic dynamics of the strings and the cusp region may be evaporated by particle-antiparticle emission.

It seems that strings without cusps do not play any important role in the emission, but it is not the case for a new type of cosmic string loops which were found by Garfinkle and Vachaspati [6]. Those loops have kinks instead of cusps. We will show that kinky loops emit particle-antiparticle pairs at the rate similar to the loops with cusps.

The particle-antiparticle emission rate is estimated in two ways. First we calculate an emission rate by a perturbative method when the string state is changed little by the emission. However non-perturbative processes may be more important. We cannot estimate the emission rate precisely in this case, but we can obtain a rough estimate by considering the evaporation of the cusp region [4]. Baryon number generation in both processes is considered in this paper and it is concluded that a significant baryon number can be generated when cusp evaporations occur.

2 Perturbative process

We first consider the perturbative emission from cosmic loops following Srednicki and Theisen [3] and Brandenberger [4]. A complex scalar field ϕ which is responsible for strings may couple to various other fields. Here we assume that ϕ couples to another scalar field ψ via the interaction lagrangian:

$$L_{int} = -\lambda|\phi|^2\psi^2 \tag{1}$$

Let us introduce a real field φ defined by

$$\varphi = |\phi| - \sigma \tag{2}$$

where $\varphi = 0$ far from the string and $\varphi = -\sigma$ at the center of the string. The amplitude for a string state $|S>$ to emit two "Higgs" particles and to become a state $|S'>$ is given by

$$< S'; \psi_1\psi_2|S >\sim \lambda \int d^4x \exp[i(k_1 + k_2)x] < S'|\varphi^2|S > \tag{3}$$

If the string state is not changed by the emission process, we can replace $< S'|$ with $< S|$. Since strings are very thin, φ deviates from zero only on the strings. Hence $< S|\varphi^2|S >$ is expressed by

$$< S|\varphi^2|S >\sim \int_0^L d\varsigma|\vec{s}\,'(\varsigma,t)|^2\delta^3(\vec{x} - \vec{s}(\varsigma,t)) \tag{4}$$

where $\vec{s}$ is the vector describing the string location and is a function of two parameters ς and t. $\vec{s}$ satisfies the equation of motion and gauge conditions :

$$\ddot{\vec{s}} = \vec{s}\,'', \qquad (\dot{\vec{s}} \pm \vec{s}\,')^2 = 1 \tag{5}$$

Eq.(3) is expressed by using the periodicity of $\vec{s}$ as

$$< S' : \psi_1\psi_2|S > \simeq 2\pi\lambda \sum_n a_n \delta(E - 4\pi n/L) \tag{6}$$

$$a_n = \frac{2}{L}\int_0^L d\varsigma |\vec{s}'(\varsigma,t)|^2 \exp[-i\vec{k}\cdot\vec{s}(\varsigma,t)] \tag{7}$$

where E and $\vec{k}$ are the total energy and momentum of a particle-antiparticle pair. The total number of particle-antiparticle pairs from one loop with size L is given by

$$\begin{aligned} N &= \sum_n N_n \\ N_n &= 2\pi \int \frac{d^3k_1}{(2\pi)^3(2k_1^0)} \frac{d^3k_2}{(2\pi)^3(2k_2^0)} |a_n|^2 \delta(E - 4\pi n/L) \end{aligned} \tag{8}$$

where k_1 and k_2 are the momenta of the emitted particle and antiparticle. The dominant contribution to N comes from a cusp for an ordinary loop and N is estimated approximately by

$$\begin{aligned} &N \sim \frac{1}{L}\sum \frac{1}{n} \sim \frac{1}{L}\ln n_{max}/n_{min} \sim \frac{1}{L} \\ &n_{max} \sim \sigma L, \qquad\qquad n_{min} \sim (m_\psi L)^{3/2} \end{aligned} \tag{9}$$

provided that $L \leq \sigma^2/m_\psi^3$, *i.e.* the minimum energy of emitted particles $(m_\psi^{3/2}L^{1/2})$ [3] is smaller than σ.

3 Kinky loops

There are families of string loops which satisfy eq.(5) and have no cusps. Among them, a family of loops with kinks is very interesting because those may be generated naturally by an intersection of two strings. The kink runs around the loop with the velocity of light in one direction and is present at all times. The simplest kinky solution of eq.(5) is [6]:

$$\begin{aligned}
\vec{s}(\varsigma,t) &= \frac{1}{2}[\vec{a}(\varsigma-t)+\vec{b}(\varsigma+t)] \\
\vec{a} &= \left(\frac{L}{2\pi}\xi-\frac{L}{4}\right)\vec{A} && 0\le\xi\le\pi \\
&= \left(\frac{-L}{2\pi}\xi+\frac{3L}{4}\right)\vec{A} && \pi\le\xi\le 2\pi \\
\vec{b} &= \left(\frac{L}{2\pi}\eta-\frac{L}{4}\right)\vec{B} && 0\le\eta\le\pi \\
&= \left(\frac{-L}{2\pi}\eta+\frac{3L}{4}\right)\vec{B} && \pi\le\eta\le 2\pi
\end{aligned} \tag{10}$$

where $\vec{A}$ and $\vec{B}$ are constant unit vectors and ξ and η are defined by $\xi=(2\pi/L)(\varsigma-t)$ and $\eta=(2\pi/L)(\varsigma+t)$. Substituting this into eq.(7) we get

$$\begin{aligned}
a_n &= \frac{L}{2\pi^2}\left[\frac{n^2+\kappa_1\kappa_2}{(n^2-\kappa_1^2)(n^2-\kappa_2^2)}[1-\vec{A}\cdot\vec{B}]+\frac{n^2-\kappa_1\kappa_2}{(n^2-\kappa_1^2)(n^2-\kappa_2^2)}[1+\vec{A}\cdot\vec{B}]\right] \\
&\quad\times\sin\left(\frac{\pi}{2}(n+\kappa_1)\right)\sin\left(\frac{\pi}{2}(n-\kappa_2)\right)
\end{aligned} \tag{11}$$

where κ_1 and κ_2 are defined by $\kappa_1=L(\vec{k}\cdot\vec{A})/4\pi$ and $\kappa_2=L(\vec{k}\cdot\vec{B})/4\pi$. Since $n\gg\kappa_1,\kappa_2$, $|a_n|^2$ is approximately given by

$$\begin{aligned}
|a_n|^2 &= \left(\frac{L}{2\pi^2}\right)^2\frac{1}{n^4}\sin^2\left(\frac{\pi}{2}(n+\kappa_1)\right)\sin^2\left(\frac{\pi}{2}(n+\kappa_1)\right) \\
&\simeq \frac{1}{4}\left(\frac{L}{2\pi^2}\right)^2\frac{1}{n^4}
\end{aligned} \tag{12}$$

Hence from eq.(8) we get expression of N as

$$N=\sum_n(1/12\pi^4 Ln)\sim\frac{1}{L} \tag{13}$$

This is the same order of magnitude as that obtained in the string loops with cusps.

4 Cusp evaporation

So far we have neglected the width of the string. This is because the width of strings is very thin ($\sim \sigma^{-1}$) and may not affect their global motion. However in the region where cusps arise the width of the string cannot be neglected and the cusp region may be smoothed. Near the cusp the string solution is given by [7]

$$
\begin{aligned}
s_x &= t - \frac{1}{2}(c_1^2 + c_2^2 + c_3^2)\left(\frac{t^3}{3} + \varsigma^2 t\right) - c_1 c_2 \left(t^2\varsigma + \frac{\varsigma^3}{3}\right) \\
s_y &= c_1 \frac{t^2 + \varsigma^2}{2} + c_2 \varsigma t + c_4 \left(\frac{t^3}{3} + \varsigma^2 t\right) + c_5 \left(t^2 \varsigma + \frac{\varsigma^3}{3}\right) \\
s_z &= c_3 \varsigma t + c_6 \left(\frac{t^3}{3} + \varsigma^2 t\right) + c_7 \left(t^2 \varsigma + \frac{\varsigma^3}{3}\right)
\end{aligned}
\tag{14}
$$

where c_1, c_2 and c_3 are constant of the order $1/L$ and c_4, c_5, c_6 and c_7 are constants of the order $1/L^2$. As is seen from eq.(14) at the cusp $(t, \varsigma) = (0, 0)$ two branches of the string are very close. Therefore if the string width is taken into account, two branches are overlapped in the cusp region. In such a region the particle-antiparticle production may proceed very rapidly through the interaction of eq.(1). This process cannot be treated perturbatively. We, however, do not yet know how to treat this non-perturbative process properly.

In the extreme limit the cusp region may evaporate. Then the emission power of particle-antiparticle pairs is estimated as [4]

$$P \sim \mu \varsigma_c / L \sim \mu(\sigma L)^{-1/3} \tag{15}$$

where $\varsigma_c = \sigma^{-1/3} L^{2/3}$ is the length of the cusp region and is defined by

$$|\vec{s}(0, \varsigma_c) - \vec{s}(0, -\varsigma_c)| \simeq \sigma^{-1} \tag{16}$$

Then the emission rate N is given by using the average energy of particle-antiparticle pair $< E >$ as

$$N \sim \frac{\mu}{(\sigma L)^{1/3} < E >} \tag{17}$$

$< E >$ is expected to be between m_ψ and σ.

5 Baryon number generation

We can now estimate baryon number generation from string loops. Baryon number asymmetry is produced if emitted particle-antiparticle pairs decay in a way which violates baryon number. Here we do not specify any model or any interaction which violates baryon number conservation. Instead we introduce a parameter ϵ which is the average baryon asymmetry produced by the decay of one particle-antiparticle pair. The mechanism of baryon number generation we consider here is almost the same as that due to Higgs decay in the inflationary universe. The difference is whether Higgs particles are produced by inflaton decay or by cosmic string decay.

The created particle-antiparticle pairs do not contribute to the baryon asymmetry when the temperature is higher than the mass of particles. This is because the inverse decay processes erase the baryon asymmetry at such a high temperature. When the temperature cools below the mass of the particles, the inverse decay processes are suppressed by the Boltzmann factor $\sim \exp(-m_\psi/T)$. Therefore we assume that the inverse decay is neglected after the temperature drops below $m_\psi/10$. Then the baryon number produced by cosmic string loops is given by

$$n_B \simeq \int_{t_s}^{\infty} dt \int dL \epsilon N(L) n_{st}(t,L) \left(\frac{R(t)}{R(t_s)}\right)^3 \tag{18}$$

where $n_{st}(t,L)$ is the distribution function of loops and R is a cosmic scale factor and t_s is the time at which the temperature becomes $m_\psi/10$ and is given by

$$t_s = 100\sqrt{\frac{45}{16\pi^3 N_g}} \left(\frac{m_\psi}{M_{pl}}\right)^{-2} t_{pl} \tag{19}$$

where N_g is the effective number of helicity states ($\sim$ 100) .

(a) perturbative process

In the case of perturbative emission considered in sections 2 and 3 the particle-antiparticle emission can be neglected energetically in comparison with the gravitational radiation. Then the distribution of string loops is determined by gravitational energy loss [2]:

(1) $t_* < \ t < (\gamma G\mu)^{-1} t_*$

$$n_{st} = \begin{cases} \nu t^{-3/2} L^{-5/2} & t_* < L < t \\ 0 & L < t_* \end{cases} \tag{20}$$

(2) $t > \ (\gamma G\mu)^{-1} t_*$

$$n_{st} = \begin{cases} \nu t^{-3/2} L^{-5/2} & (\gamma G\mu) t < L < t \\ \nu (\gamma G\mu)^{-5/2} t^{-4} & L < (\gamma G\mu) \end{cases} \tag{21}$$

where t_* is the time when effects of friction between strings and surrounding matter become negligible. Before t_* the motion of string loops inside the horizon is highly damped by the friction and string loops may shrink rapidly. Using eqs.(18) -(21)

we get the final ratio η_B of baryon to entropy :

$$\eta_B = \frac{n_B}{s} \simeq 8\pi\epsilon\nu \left(\frac{45}{16\pi^3 N_g}\right)^{1/4} (G\mu)^3 (\gamma G\mu)^{-1} \tag{22}$$
$$\text{for} \quad m_\psi > m_1 \equiv 10 \left(\frac{45}{16\pi^3 N_g}\right)^{1/4} (G\mu)(\gamma G\mu)^{-1/2} M_{pl}$$

and

$$\eta_B \simeq 8 \times 10^{-3}\pi\epsilon\nu \left(\frac{45}{16\pi^3 N_g}\right)^{-1/2} \left(\frac{m_\psi}{M_{pl}}\right)^3 (G\mu)^3 (\gamma G\mu)^{-1} \tag{23}$$
$$\text{for} \quad m_\psi < m_1$$

Inserting $G\mu = 10^{-6}, \gamma G\mu = 10^{-4}$ and $N_g = 100$ into eqs.(22) and (23), η_B is estimated as

$$\eta_B \sim 10^{-14}\epsilon\nu \ \min\left(1, \left(\frac{m_\psi}{10^{11}\text{GeV}}\right)^3\right) \ll 10^{-10} \tag{24}$$

Therefore this contribution to the present baryon asymmetry ($\sim 10^{-10}$) is negligible.

(b)Cusp evaporation

Energy loss by cusp evaporation is not negligible in the early stage of string evolution ($\sim t_*$). Let us consider the lifetime of loops for this energy loss. Since the energy loss per unit time is $\mu(\sigma L)^{-1/3}$, the lifetime is then $\mu L/(\mu(\sigma L)^{-1/3}) = L^{4/3}\sigma^{1/3}$. Hence the minimum size of loops which survive more than one cosmic expansion at time t is $\sim t^{3/4}\sigma^{-1/4}$ while that for energy loss by gravitational radiation is $\sim \gamma G\mu t$. As a result the minimum size of loops is determined by cusp evaporation as $\sim t^{3/4}\sigma^{-1/4}$ until the cosmic time equals to $t_3 = (\gamma G\mu)^{-4}(G\mu)^{-1/2}t_{pl}$. The distribution function of loops is then modified by

(1) $t_* < \ t < (G\mu)^{-5/2} t_{pl} \equiv t_2$

$$n_{st} = \begin{cases} \nu t^{-3/2} L^{-5/2} & t_* < L < t \\ 0 & L < t_* \end{cases}$$

(2) $t_2 < \ t < t_3$

$$n_{st} = \begin{cases} \nu t^{-3/2} L^{-5/2} & t^{3/4}\sigma^{-1/4} < L < t \\ 0 & L < t^{3/4}\sigma^{-1/4} \end{cases} \tag{25}$$

(3) $t > \ t_3$

$$n_{st} = \begin{cases} \nu t^{-3/2} L^{-5/2} & (\gamma G\mu) t < L < t \\ \nu (\gamma G\mu)^{-5/2} t^{-4} & t^{3/4}\sigma^{-1/4} < L < (\gamma G\mu) \\ 0 & L < t^{3/4}\sigma^{-1/4} \end{cases}$$

where t_2 is the time at which loops with size t_* decay out. Then the final ratio of baryon to entropy is given by

$$\eta_B \simeq 9\epsilon\nu \left(\frac{< E >}{M_{pl}}\right)^{-1} (G\mu)^2 \tag{26}$$

$$\text{for} \quad m_\psi \ll m_2 \equiv 10 \left(\frac{45}{16\pi^3 N_g}\right)^{1/4} (G\mu)^{5/4} M_{pl}$$

$$\begin{aligned} \eta_B &\simeq 4\epsilon\nu \left(\frac{< E >}{M_{pl}}\right)^{-1} \left(\frac{m_\psi}{M_{pl}}\right)^{3/4} (G\mu)^{17/16} \\ &+ 10\epsilon\nu \left(\frac{< E >}{M_{pl}}\right)^{-1} (\gamma G\mu)^{3/2} (G\mu)^{5/4} \end{aligned} \tag{27}$$

$$\text{for} \quad 10 \left(\frac{45}{16\pi^3 N_g}\right)^{1/4} (G\mu)^{1/4} (\gamma G\mu)^2 M_{pl} \equiv m_3 \ll m_\psi \ll m_2$$

$$\eta_B \simeq 10^3 \pi\epsilon\nu \left(\frac{m_\psi}{M_{pl}}\right)^{5/3} \left(\frac{< E >}{M_{pl}}\right)^{-1} \tag{28}$$

$$\text{for} \quad m_\psi \ll m_3$$

If we take $G\mu = 10^{-6}$ and $\gamma G\mu = 10^{-4}$ then we get the final ratio of baryon to entropy as is shown in fig.1. η_B increases as the average energy of emitted particle-antiparticle pairs decreases. It turns out that $\eta_B \sim O(10^{-4})\epsilon\nu$ if the mass m_ψ is about 10^{10} GeV and $< E > \sim m_\psi$. It is possible to account for present baryon asymmetry in this case. While η_B reduces to $(10^{-8} \sim 10^{-10})\epsilon\nu$ for $< E > = \sigma$ and $m_\psi > 10^{10}$GeV. Furthermore the estimation is based on uncertain assumption that cusp evaporation really occurs. We should regard this estimation as an upper limit for non-perturbative emission from string loops.

6 Conclusion

As was seen in the previous section the perturbative emission has a negligible effect on the baryon asymmetry in our universe. Only non-perturbative effect may contribute to the present baryon asymmetry. It is, however, difficult to perform a precise estimation for non-perturbative effect.

In this paper, assuming that cusp evaporation occurs, we have estimated baryon number generation, and have found that a significant baryon number can be generated. Unfortunately, so far, we have no positive evidence of cusp evaporation. If it really occurs, those cosmic string loops may account for not only the galaxy formation but also the baryon asymmetry in our universe.

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Figure Caption

Fig.1 Final baryon-entropy ratio η_B as the function of "Higgs" mass m_ψ for $< E >= m_\psi$, σ and M_{pl}.

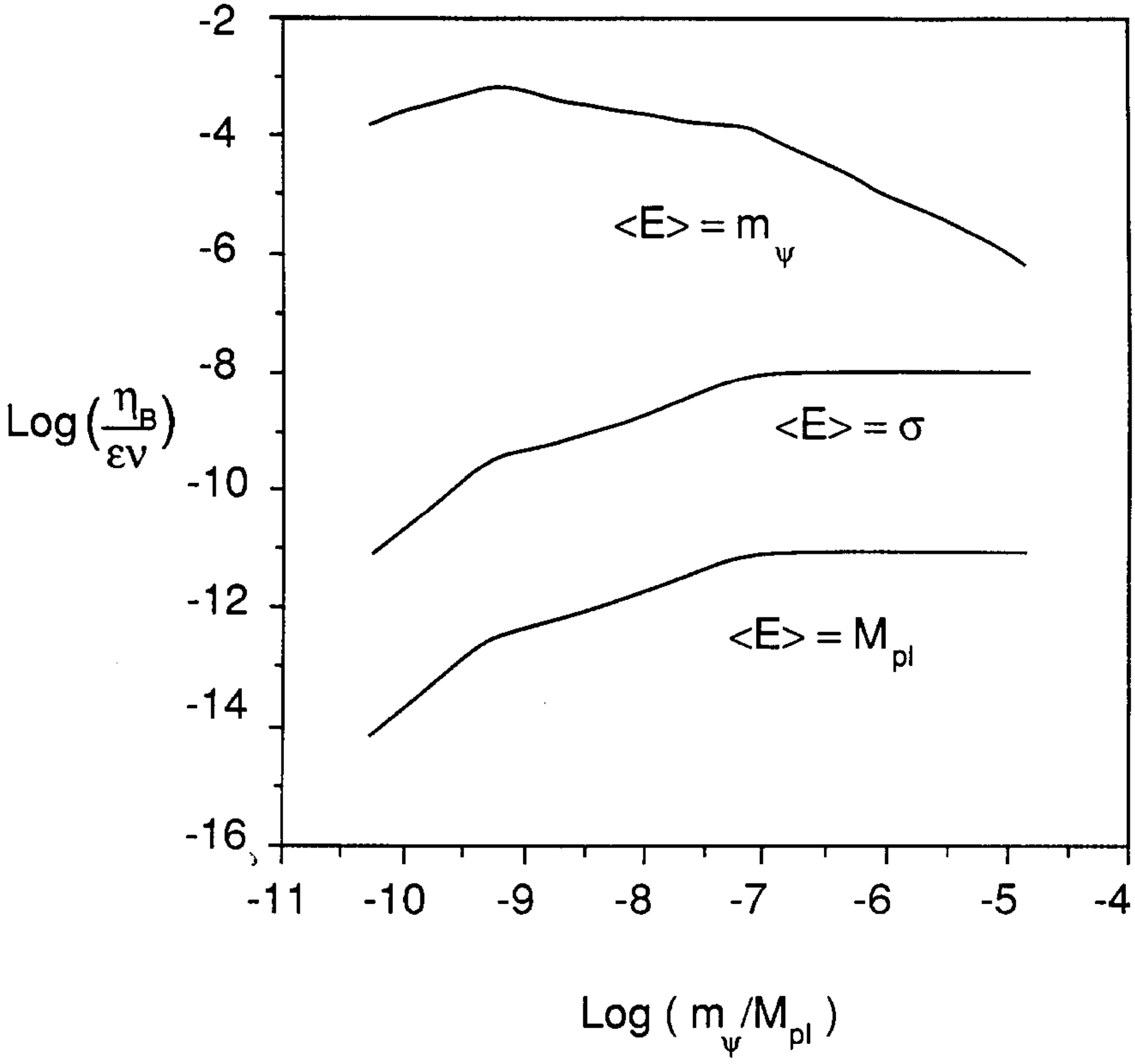
-2
-4
-6
-8
-10
-12
-14
-16
Log $(\frac{\eta_B}{\varepsilon\nu})$
$<E> = m_\psi$
$<E> = \sigma$
$<E> = M_{pl}$
-11
-10
-9
-8
-7
-6
-5
-4
Log (m_ψ/M_{pl})

Figure 1