



Axions From SN 1987a

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Abstract. We consider the emission of axions from SN 1987a by nucleon-nucleon axion bremsstrahlung (the dominant emission process). Based upon the observation of neutrinos from SN 1987a we require the axion luminosity to be $\lesssim 10^{53}$ erg s⁻¹. This limit can be satisfied if: (1) axions couple very weakly, $m_a \lesssim 0.75 \times 10^{-3}$ eV; or (2) axions couple strongly enough to be 'trapped' and radiated thermally with a temperature $\lesssim 10$ MeV, which occurs for $m_a \gtrsim 3.7$ eV. In general, 'axion trapping' occurs for $m_a \gtrsim 1.6 \times 10^{-2}$ eV. Our mass constraints are probably reliable to within a factor of ~ 3 .



Introduction. The axion is the pseudo-Nambu-Goldstone boson associated with the spontaneous breakdown of the Peccei-Quinn quasi-symmetry. In 1977 Peccei-Quinn (PQ) symmetry was proposed to solve the ‘strong CP problem’, and ten years later is still probably the most attractive solution to this very nagging problem. [For further discussion of the strong CP problem, PQ symmetry, and the axion see Refs. 1.] The original axion with symmetry breaking scale equal to the electroweak scale was quickly ruled out by laboratory experiment and on astrophysical grounds (axion emission from the sun and red giants²). To wit, the ‘invisible axion’ was introduced^{3,4}, with symmetry breaking scale $f_a \gg 300$ GeV. Generically, invisible axions are of two types: DFS³ and hadronic⁴. The DFS axion has fundamental couplings to all fermions with strength $\sim (m_f/f_a)$, while the hadronic axion only has fundamental couplings to quarks, and possibly only to heavy, exotic quarks beyond the usual 6 ‘light’ quarks. Both types of axions couple (through anomalies) to photons and nucleons.

Cosmology and astrophysics have been used to set stringent bounds on the axion mass. In order that cosmologically-produced axions not contribute excessive mass density today, the axion mass must satisfy⁵:

$$m_a \gtrsim 3.6 \times 10^{-6} \text{eV} \gamma^{-0.85} (\Lambda_{QCD}/200\text{MeV})^{-0.6} \quad (1)$$

where Λ_{QCD} is the QCD scale parameter, and γ accounts for any entropy production after axion production: $\gamma \equiv (\text{entropy per comoving volume after}/\text{entropy per comoving volume before})$. Light axions are emitted in copious numbers from stars, thereby affecting stellar evolution, especially that of red giants. For the DFS axion the most stringent limit is⁶: $m_a \lesssim 10^{-2}$ eV; for the hadronic axion the corresponding limit is⁷: $m_a \lesssim 2 - 3$ eV. [Note the limit for the hadronic axion depends upon its anomalous coupling to two photons. In simple, unified models this coupling is fixed; however in some exotic models it can be significantly smaller,⁸ and the mass limit less stringent (perhaps by a factor of 15).] For further discussion of stellar axion emission we refer the reader to Refs. 6, 7, and 9.

Ellis and Olive¹⁰ have considered axion emission from SN 1987a through processes involving electrons. However, the dominant process is nucleon-nucleon, axion bremsstrahlung¹¹, and the limits we set here are based upon this process. Since the axion-nucleon coupling results primarily from axion-pion mixing, it is nearly model-independent^{8,12,13}, and so our bounds apply essentially equally to both hadronic and DFS axions. In order that axion emission not too rapidly cool the hot neutron star (in a time \lesssim few sec), and thereby quench the emission of thermal neutrinos, neutrinos which were observed in at least two underground detectors^{14,15}, we require that the axion luminosity Q_a be less than 10^{53} erg sec⁻¹. This can occur in one of two ways: first, if the axion

coupling is very small: $m_a \lesssim 0.75 \times 10^{-3}$ eV; second, if the axion coupling is sufficiently strong so that axions are ‘trapped’ and thermalized in the hot core, and the ‘axion-sphere’ has a temperature $\lesssim 8 - 10$ MeV: this occurs for $m_a \gtrsim 3.7$ eV.

The Invisible Axion. Throughout we will follow Srednicki¹², but using the normalization conventions of Kaplan⁸ and Sikivie.¹³ [Note $(f_a/N)_{Srednicki} = 2(f_a/N)_{Kaplan, Sikivie} \equiv (f_a/N)$, where N is the color anomaly of the PQ symmetry.] The axion mass and symmetry breaking scale are related by: $m_a \simeq 0.62$ eV/ $[(f_a/N)/10^7 \text{GeV}]$. The effective interaction Lagrangian of the axion with electrons, nucleons, and photons is

$$\mathcal{L}_{int} = ig_{aee}(\bar{e}\gamma_5 e)a + ig_{aNN}(\bar{n}\gamma_5 n)a + ig_{aPP}(\bar{p}\gamma_5 p)a + g_{a\gamma\gamma}a\vec{E} \cdot \vec{B} \quad (2)$$

where a is the axion field. The couplings are: $g_{aee} = [X'_e/N + (3\alpha^2/4\pi)(E \ln(f_a/m_e)/N - 1.93 \ln(\Delta_{QCD}/m_e))]m_e/(f_a/N)$; $g_{a\gamma\gamma} = (\alpha/2\pi)(N/f_a)(E/N - 1.93)$; $g_{aNN} = [(-F_{A0} - F_{A3})(X'_d/N - 0.18) + (-F_{A0} + F_{A3})(X'_u/N - 0.32)][m/(f_a/N)]$; $g_{aPP} = [(-F_{A0} - F_{A3})(X'_u/N - 0.32) + (-F_{A0} + F_{A3})(X'_d/N - 0.18)][m/(f_a/N)]$. Here X'_i are the PQ charges of the electron, and u and d quarks, $\alpha \simeq 1/137$, E is the electromagnetic anomaly of the PQ symmetry ($= 8N/3$ when the axion is incorporated into simple unified models), m_e is the electron mass, m is the nucleon mass, and F_{A0} and F_{A3} are the axial isoscalar and isovector pion-nucleon couplings. Experiment suggests $F_{A3} \simeq -1.25$ and theory that $F_{A0} = 0.6F_{A3} \simeq -0.75$ (see Ref. 12). [The couplings derived by Sikivie¹³ and Kaplan⁸ are consistent with these of Srednicki¹².] For the DFS axion: $X'_e = \cos^2 \beta/3$, $X'_u = 1 - \cos 2\beta$, and $X'_d = 1 + \cos 2\beta$, where β parameterizes the relative sizes of the ‘up’ and ‘down’ PQ vacuum expectation values¹². For the hadronic axion: $X'_e = X'_u = X'_d = 0$. The coupling of the hadronic axion to the electron arises only through radiative corrections, and the couplings to nucleons only through axion-pion mixing.

For axion emission from the supernova we will only be interested in the axion-nucleon couplings, $g_{aNN} \simeq [(2X'_d/N - X'_u/2N) - 0.20]m/(f_a/N)$ and $g_{aPP} \simeq [(2X'_u/N - X'_d/2N) - 0.55]m/(f_a/N)$. Lacking precise knowledge of X'_u/N , X'_d/N , F_{A0} , and F_{A3} , where necessary we take $g_{aPP} \simeq g_{aNN} \simeq 0.5m/(f_a/N)$, for both types of axions. [We note that for $X'_u/N = 0.32 \simeq 1/3$ and $X'_d/N = 0.18 \simeq 1/6$ both g_{aNN} and g_{aPP} actually vanish.]

Axions from SN 1987a. SN 1987a confirmed astrophysicists’ most cherished belief about Type II supernovae¹⁶—namely, that the bulk of the $\sim (2 - 4) \times 10^{53}$ ergs of gravitational binding energy released during the core collapse is carried away by neutrinos. Assuming that all neutrino species were emitted in roughly equal numbers, the detection of $\bar{\nu}_e$ ’s by the Kamiokande II¹⁴ and IMB¹⁵ detectors indicates that neutrinos with a characteristic temperature of $\sim (3 - 5)$ MeV carried off $\sim \text{few} \times 10^{53}$ ergs from the supernova¹⁷.

Since the bulk of the neutrinos were detected in the first few sec, the inferred neutrino luminosity is $\sim 10^{53}$ erg s $^{-1}$.

According to the generally-accepted, and now basically-confirmed, theory of core collapse¹⁸, a Type II supernova is initiated when the $\sim 1.4M_{\odot}$ Fe iron core of a massive star collapses (on a time scale of msec). The collapse is halted when the core reaches a few times nuclear density ($\simeq 8 \times 10^{14}$ g cm $^{-3}$), the precise value depending upon the nuclear matter equation of state at supernuclear densities. The hydrodynamic shock resulting from the core bounce propagates outward, eventually leading to the optical fireworks. Because of the very high densities, neutrinos are trapped in the hot core ($T \sim 30 - 70$ MeV), and are radiated from the ‘neutrino-sphere’ ($R \simeq 2 - 3 \times 10^6$ cm) where the density is $\sim 10^{12}$ g cm $^{-3}$, and the temperature is $\sim (3 - 5)$ MeV. In the standard scenario neutrino emission cools the core and releases the binding energy in a few sec.

The inner core, which contains most of the mass, has approximately constant density. In the outer core the density and temperature decrease: $T \propto \rho^{1/3}$, as the core is nearly isentropic. The ‘observed’ neutrino-sphere temperature of $\sim (3 - 5)$ MeV then indicates a central temperature of 30-70 MeV. Rather than use a detailed model of the hot core (with accompanying theoretical uncertainties), we use the following simple and transparent model for the newly-born neutron star: mass $\simeq 1.4M_{\odot}$; density $\rho_{14} = (\rho/10^{14}$ g cm $^{-3}) \simeq 8$; temperature $T \simeq 30$ MeV $c_1 \rho_{14}^{1/3} \simeq 60$ MeV (c_1 expected to be 0(1) allows for uncertainty in the adiabat of the hot core); radius of the inner constant density region, $R \simeq 1.9 \times 10^6$ cm $\rho_{14}^{-1/3} \simeq 10^6$ cm. During its initial cooling phase, before the star’s lepton number is carried away by ν_e ’s, the core should have approximately equal numbers of neutrons and protons, with number density, $n \simeq 2.3 \times 10^{-4}$ GeV $^3 \rho_{14}^{1/3}$ ($\hbar = k_B = c = 1$ throughout).

If axions exist, the hot core can also cool itself by axion emission. At such high densities and temperatures the dominant emission process is nucleon-nucleon, axion bremsstrahlung. Since neutrinos were observed to have come from the supernova over a time interval of a few sec, axions had better not cool the core in a time less than this. If the axion luminosity were say $\gtrsim 10^{54}$ erg s $^{-1}$, axions would cool the core in less than a sec—clearly inconsistent with the observation of neutrinos from SN 1987a. On the other hand, if the axion luminosity were lower than $\sim 10^{52}$ erg s $^{-1}$, axion emission would have only a slight effect on the cooling of the core. At the intermediate luminosity of $\sim 10^{53}$ erg s $^{-1}$, axions should affect the cooling significantly, perhaps even enough to be inconsistent with the observation of neutrinos. We shall use 10^{53} erg s $^{-1}$ as the maximum, permissible axion luminosity, and note that our constraints scale only as the square root of this luminosity.

Axion emission from hot neutron stars through nucleon-nucleon axion bremsstrahlung has been calculated in the degenerate limit by Iwamoto¹⁹. In our case the Fermi

momentum $p_F \simeq 190\text{MeV}\rho_{14}^{1/3}$ and the temperature $T \simeq 30\text{ MeV}c_1\rho_{14}^{1/3}$, so that $\varepsilon_F/(3T/2) \simeq 0.4\rho_{14}^{1/3}/c_1$: clearly, a newly-born, hot neutron star is not strongly degenerate. [Because of the approximations Iwamoto makes, his rate cannot be correctly extrapolated to the nondegenerate regime.] Using the matrix element computed by Iwamoto¹⁹, we have calculated the axion bremsstrahlung cross section in the NR, non-degenerate limit:

$$\langle \sigma \rangle = (3/80\pi^3)(T/m)^2 f^4 g_i^2 m^2/m_\pi^4 \simeq 1.2 \times 10^{-27} \text{cm}^2 g^2 (T/\text{GeV})^2 \quad (3)$$

where m_π is the pion mass and $f \sim 1$ the pion-nucleon coupling. The cross section has been averaged both thermally and over initial spins, and a factor of $\frac{1}{4}$ has been included to account for identical particles in the initial and final states. We have also made the approximation, $3mT \gg m_\pi^2$. Here g_i^2 is the appropriate axion-nucleon coupling squared: g_{aNN}^2 for $n + n \rightarrow n + n + a$; g_{aPP}^2 for $p + p \rightarrow p + p + a$; and $\simeq 2(g_{aNN}^2 + g_{aPP}^2)$ for $p + n \rightarrow p + n + a$ (here, the extra factor of 4 to ‘undo’ the previous factor of $\frac{1}{4}$). From the cross section it is simple to compute the axion luminosity from the core²⁰:

$$Q_a = n^2 \langle \sigma |v_1 - v_2| \rangle E_a V \simeq 5.1 \times 10^{72} \text{erg sec}^{-1} (\rho_{14}/8)^{13/6} c_1^{7/2} g^2 \quad (4)$$

where $E_a \simeq 3T$ is the average energy per axion emitted, $V \simeq 3.9 \times 10^{60} \text{ GeV}^{-3} \rho_{14}^{-1}$ is the volume of the core, $g^2 = 3(g_{aNN}^2 + g_{aPP}^2)$ accounts for all the axion bremsstrahlung processes mentioned above, and the core temperature $T \simeq 30 \text{ MeV } c_1 \rho_{14}^{1/3}$.

Adopting $\rho_{14} \simeq 8$ and using our limit to the axion luminosity $Q_a \lesssim 10^{53} \text{ erg s}^{-1}$ we obtain the limit: $g \lesssim 1.4 \times 10^{-10} (\rho_{14}/8)^{-13/12} c_1^{-7/4}$. Furthermore, if we assume that $g_{aNN} \simeq g_{aPP} \simeq \frac{1}{2}m/(f_a/N)$ we find the bounds:

$$(f_a/N) \gtrsim 8 \times 10^9 (\rho_{14}/8)^{13/12} c_1^{7/4} \text{GeV}, \quad (6a)$$

$$m_a \lesssim 0.75 \times 10^{-3} (\rho_{14}/8)^{-13/12} c_1^{-7/4} \text{eV} \quad (6b)$$

Note the strong dependence on the core adiabat: taking a lower adiabat, $c_1 = 0.5$, changes the mass limit to: $m_a \lesssim 2.5 \times 10^{-3} \text{ eV}$. Recall that changing the maximum allowed axion luminosity by a factor c_2 only increases the axion mass bound by a factor $c_2^{1/2}$. [Note, for $g_{aNN} \simeq g_{aPP} \simeq 0.5m/(f_a/N)$: $m_a \simeq 5.4g\text{MeV}$ and $(f_a/N) \simeq 1.15g^{-1} \text{ GeV}$.]

Next, we must consider axion reabsorption to check our implicit assumption that once emitted, axions just ‘stream out’. [Had one computed the neutrino luminosity of the hot core ignoring neutrino trapping, one would overestimate the neutrino luminosity by about a factor of 10^6 .] The axions produced should have an approximately thermal spectrum. For a thermal distribution of axions it follows from the Boltzmann equation that their thermally-averaged, mean free path l is given by:

$$l^{-1} \simeq n^2 \langle \sigma |v_1 - v_2| \rangle / (T^3/\pi^2). \quad (7)$$

To assess the possible importance of reabsorption we compare l to the size of the core, $d \simeq V^{1/3} \simeq 2 \times 10^6$ cm (for $\rho_{14} = 8$): $d/l \simeq 4.2 \times 10^{14} g^2 \rho_{14}^2 (T/\text{GeV})^{-1/2}$. Taking $\rho_{14} \simeq 8$ and $T \simeq 60$ MeV we find that $d/l \gtrsim 1$ for $g \gtrsim 3.0 \times 10^{-9}$. That is, for $m_a \gtrsim 1.6 \times 10^{-2}$ eV axions are ‘trapped’ and should thermalize in the hot core.

In the ‘trapped regime’ ($m_a \gtrsim 1.6 \times 10^{-2}$ eV) there will be an ‘axion-sphere’ with temperature, T_a , and radius, r_a , determined by the condition: $\tau_a \simeq 2/3$. The axion ‘optical depth’, τ_a , is given by: $\tau_a = \int_{\infty}^r dr/l$. To compute τ_a one needs to know $\rho(r)$ and $T(r)$ outside the constant density inner core. In the spirit of our simple model we assume: $\rho_{14} = (r/r_{14})^{-n}$ ($r_{14} \simeq 1.5 \times 10^6$ cm, $n \simeq 3 - 7$), and as before, $T \simeq 30\text{MeV} c_1 \rho_{14}^{1/3}$. Then it follows that: $\tau_a \simeq 1.8 \times 10^{15} c_1^{-6} g^2 (T_a/30\text{MeV})^{5.5-3/n} / (11n/6 - 1)$, or²¹:

$$T_a \simeq 4.7 \times 10^{-2} \text{MeV} c_1^{12/11} g^{-4/11} \zeta^{-2/11} \quad (8)$$

where $\zeta = (T_a/30\text{MeV})^{-3/n} / (11n/6 - 1)$ depends upon n and is $\sim 0.1 - 0.7$. Note again the strong dependence on the core adiabat c_1 . In this regime ($g \gtrsim 3 \times 10^{-9}$) thermal axions (of temperature T_a) are radiated from the ‘axion-sphere’. Our criterion $Q_a \lesssim 10^{53}$ erg s⁻¹ translates to $T_a \lesssim 8 - 10$ MeV (depending upon the radius of the ‘axion-sphere’). This constrains g to be: $g \gtrsim 6.8 \times 10^{-7}$ (we have taken $T_a \lesssim 10$ MeV, $c_1 = 1$, and $\zeta = 1/3$). That is to say, for $3.0 \times 10^{-9} \gtrsim g \gtrsim 6.8 \times 10^{-7}$ axions thermalize in the core and are radiated from the ‘axion-sphere’ with luminosity greater than 10^{53} erg s⁻¹. Owing to their strong trapping in the core and correspondingly lower ‘axion-sphere’ temperature, axions with $g \gtrsim 6.8 \times 10^{-7}$ ($f_a/N \lesssim 1.7 \times 10^6$ GeV, $m_a \gtrsim 3.7$ eV) are permissible. For the DFS axion $m_a \gtrsim 3.7$ eV is certainly ruled out⁶. However, for the hadronic axion, $m_a \gtrsim 3.7$ eV, may just be allowed (see *Introduction*), especially when the uncertainties of this and the red giant limit⁷ are taken into account. In addition, relic hadronic axions from the Big Bang of about this mass may actually be detectable from their decays into 2 photons²².

Summary and Discussion We find that axion emission from SN 1987a restricts the axion mass to be either less than $\sim 0.75 \times 10^{-3}$ eV or greater than ~ 3.7 eV (the later range, of interest only for the hadronic axion). Once again we call to the reader’s attention uncertainties which could affect our calculations and hence limits: (i) the relationship assumed between g_{aNN} , g_{aPP} and $m/(f_a/N)$; (ii) the neglect of collective nuclear effects in computing $\langle \sigma \rangle$ and Q_a ; (iii) the criterion $Q_a \lesssim 10^{53}$ erg s⁻¹ and the assumed ρ - T relationship. In short, our limits are probably only reliable to within a factor of 3.

Raffelt and Seckel¹¹ have also considered axion emission from SN 1987a by nucleon-nucleon, axion bremsstrahlung. Using Iwamoto’s emission rates¹⁹ (valid in the NR, non-degenerate regime) and a more detailed cooling model which takes into account the effect of axion emission on the core itself, they obtain a similar bound: $(f_a/N) \gtrsim 10^{10}$ GeV, in

the 'freestreaming regime'. Since the hot core is neither strongly-degenerate, nor strongly-nondegenerate, it is reassuring that the use of the two different rates leads to comparable bounds. While they have noted the possibility of an allowed mass range where axions are 'trapped' and thermalized, they have not addressed this issue quantitatively.

Ellis et al²³ have also studied axion production by nucleon-nucleon axion bremsstrahlung. Using very detailed numerical models of the core collapse and subsequent cooling of the core they obtain a preliminary bound in the 'freestreaming regime' which is slightly more stringent than ours.

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 20. Correcting Iwamoto's axion emission rates for a factor of 1/2 for identical particles in the final state, we find that his rates are about a factor of 10 higher than ours for $\rho_{14} \simeq 8$. This factor probably owes to the fact that his degenerate limit approximation overestimates the nucleon density (axion emission \propto (nucleon density)²).
 21. In addressing the issue of trapping and in computing the temperature of the 'axion-sphere' we have used our NR, nondegenerate cross section, and assumed that the nuclear composition is equal numbers of free neutrons and protons. At these densities and temperatures ($\rho_{14} \simeq 10^{-2}$, $T \simeq 10$ MeV), $\epsilon_F/(3T/2) \simeq 0.09$, so that the nondegenerate regime applies. Shortly after core bounce ($\gtrsim 0.5$ sec), the nuclear composition at the densities of interest, $\rho_{14} \gtrsim 10^{-2}$, should be free nucleons (see J. Lattimer et al, *Nucl.Phys.* **A432**, 646 (1985)), with free neutrons outnumbering protons as time goes on. In the spirit of our simple model we will neglect this last fact.
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