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Testing the Fritzsch Quark Mass Matrices with the Invariant Function Approach

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Abstract

The invariant function approach using projection operator techniques is applied to the Fritzsch mass matrices to calculate exactly the absolute squares of the Kobayashi - Maskawa matrix elements and the invariant J-value associated with CP violation. The overlap of the experimentally-allowed annular KM region and the $B_d^0 - \bar{B}_d^0$ band in the $\phi_{B'}$ vs m_t plane is small but consistent with a light strange quark and heavy top quark with mass, $m_t^{phys} \sim 100$ GeV. Mixing in the $B_s^0 - \bar{B}_s^0$ channel is predicted to be nearly complete, and the amplitude for the $(b \to u)/(b \to c)$ transition is calculated to be about 0.06.

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As the top quark continues to escape detection and its mass remains undetermined,[1] it is clear that the pattern of quark masses and mixings observed in nature is still much of a mystery. Various attempts have been made to gain deeper insight into the flavor dynamics of the standard model by the introduction of specific forms[2] of the quark mass matrices and judicious parametrization[3] of the Kobayashi - Maskawa (KM) matrix. But until recently, these mass matrices have not been severely tested due to a lack of information about the V_{ub}, V_{tu} and V_{ts} KM matrix elements. The recent data on $B_d^0 - \bar{B}_d^0$ mixing,[4] the $b \to u$ transition[5] in the $B \to p\bar{p}\pi(\pi)$ decay modes and CP violation[6] in the ϵ'/ϵ ratio now begin to make such tests convincing.

One particularly attractive set of quark mass matrices is that suggested by Fritzsch[2] who assumes the $U(3)_L \otimes U(3)_R$ chiral symmetry, in the case of 3 families, is broken in stages such that only nearest neighbors mix to yield the observed spectrum. The Fritzsch model has been studied[7] in some detail by many people in the past. Recently Harari and Nir[8] have analyzed it in light of the new data on $B - \bar{B}$ mixing[4] and have concluded that the model is nearly ruled out unless the top quark mass is close to 88 GeV. In this letter, we analyze the KM matrix elements based on the Fritzsch mass matrices by using the invariant function approach developed by one of us (CJ) in a series of papers.[9] This technique, algebraic in character, is particularly well suited to exhibiting correlations among the mass matrix entries, the mass eigenvalues and the KM matrix elements which tend to escape attention in more direct numerical approaches. We present our results in such a fashion that one can readily discern their dependence on the various quark masses. Given the present uncertainties for these masses, our results indicate that the 3-family Fritzsch model is viable if $m_t \sim 100$ GeV.

Our starting point is the Fritzsch mass matrices[2] in the 3-family up and down quark sectors

$$\mathbf{M} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}, \qquad \mathbf{M}' = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B' \\ 0 & B'^* & C' \end{pmatrix} \tag{1}$$

in the weak bases $\{u', c', t'\}$ and $\{d', s', b'\}$, respectively. The matrices are Hermitian, and the phases of A and B can be rotated away by the diagonal unitary phase transformation, $X\mathbf{M}X^\dagger$, where $X = diag(exp\{-i\phi_A\}, 1, exp\{i\phi_B\})$. We thus regard A and B in $\mathbf{M}$ above as real, and let A' and B' have phases $\phi_{A'}$ and $\phi_{B'}$, respectively, since these phases can not be simultaneously rotated away with the same phase transformation. As noted earlier, the Fritzsch form of the mass matrices is suggested by complete chiral symmetry breakdown in successive steps,[10] with the top and bottom quarks getting the first large masses and the others picking up masses by nearest neighbor interactions.

The mass matrices can be brought to diagonal form by means of the U and U' unitary transformations

$$U\mathbf{M}U^\dagger = \mathbf{D} = diag(m_u, -m_c, m_t) \tag{2a}$$

$$U'\mathbf{M}'U'^\dagger = \mathbf{D}' = diag(m_d, -m_s, m_b) \tag{2b}$$

where m_c and m_s have been taken to be positive. The cubic eigenvalue equations can be solved exactly or by successive approximations in terms of the absolute squares of the mass matrix elements. Alternatively, we can express the absolute values of the mass matrix elements in terms of the mass eigenvalues by means of invariant traces and determinants, obtaining

$$C = m_t - m_c + m_u \tag{3a}$$

$$|A|^2 = m_u m_c m_t/(m_t - m_c + m_u) \tag{3b}$$

$$|B|^2 = (m_c - m_u)(m_t + m_u)(m_t - m_c)/(m_t - m_c + m_u) \tag{3c}$$

and likewise for their primed counterparts.

It is important to realize that since QCD predicts running quark masses, all masses and matrix elements should be evaluated at the same energy scale. Following the analysis of Gasser and Leutwyler,[11] we adopt the following quark masses at $\mu = 1$ GeV:

$$\begin{aligned} m_u &= 5.1 \pm 1.5\ MeV, & m_d &= 8.9 \pm 2.6\ MeV \\ m_c &= 1.35 \pm 0.05\ GeV, & m_s &= 175 \pm 55\ MeV \\ m_t &= ? & m_b &= 5.3 \pm 0.1\ GeV \end{aligned} \tag{4}$$

The unknowns are the top quark mass, $m_t(1GeV)$, and the two phases, $\phi_{A'}$ and $\phi_{B'}$, two of which are independent. In (4) above, one can determine $m_b(1GeV)$ from $m_b(m_b) = 4.25$ GeV by selecting $\Lambda_3 = 100$ MeV for the 3-flavor QCD scale and running m_b down to 1 GeV by using the corresponding 4-flavor parameter $\Lambda_4 = 76$ MeV. If instead one were to select $\Lambda_3 = 200$ MeV for the 3-flavor scale parameter, one would use the corresponding 4-flavor parameter $\Lambda_4 = 165$ MeV and obtain $m_b(1GeV) = 5.9 \pm 0.1$ GeV. See ref. 11. Although the light quark masses have sizable errors, their ratios are also constrained by quark mass expansions and the charged meson and baryon masses to be[11]

$$\frac{m_d}{m_u} = 1.76 \pm 0.13, \qquad \frac{m_s}{m_d} = 19.6 \pm 1.6, \qquad \frac{m_s}{m_u} = 34.5 \pm 5.1 \tag{5}$$

We now use the invariant function approach described in a series of papers by one of us (CJ) to determine the absolute squares[9] of the KM matrix elements and the J-value[12,13] associated with CP violation. This method allows us to derive exact expressions by direct calculation rather than first determining approximately the unitary transformations U and U' which diagonalize $\mathbf{M}$ and $\mathbf{M}'$ and then computing the KM matrix V from the definition $V = UU'^{\dagger}$. For this purpose we introduce the flavor projection operators given by

$$P_\alpha(\mathbf{D}) = \begin{cases} diag(1,0,0), & \alpha = 1 = u \\ diag(0,1,0), & \alpha = 2 = c \\ diag(0,0,1), & \alpha = 3 = t \end{cases} \tag{6a}$$

$$P'_i(\mathbf{D}') = \begin{cases} diag(1,0,0), & i = 1 = d \\ diag(0,1,0), & i = 2 = s \\ diag(0,0,1), & i = 3 = b \end{cases} \tag{6b}$$

Hence we can write the absolute square of an individual KM matrix element in terms of the flavor projection operators according to

$$\begin{aligned} |V_{\alpha i}|^2 &= V^{\dagger}_{i\alpha} V_{\alpha i} \\ &= Tr\ (V^{\dagger} P_\alpha(\mathbf{D}) V P'_i(\mathbf{D}')) \\ &= Tr\ (U^{\dagger} P_\alpha(\mathbf{D}) U U'^{\dagger} P'_i(\mathbf{D}') U') \\ &= Tr\ (P_\alpha(\mathbf{M}) P'_i(\mathbf{M}')) \end{aligned} \tag{7}$$

where $\alpha = 1,2,3 = u,c,t$ and $i = 1,2,3 = d,s,b$. In the last form, the trace is over the product of the flavor projection operators in the weak eigenbases associated with (1). These operators are given explicitly in terms of the mass matrices $\mathbf{M}$ and

$\mathbf{M}'$ by

$$P_\alpha(\mathbf{M}) = \begin{cases} (m_t - \mathbf{M})(m_c + \mathbf{M})/[(m_t - m_u)(m_c + m_u)], & \alpha = 1 = u \\ -(m_t - \mathbf{M})(\mathbf{M} - m_u)/[(m_t + m_c)(m_c + m_u)], & \alpha = 2 = c \\ (\mathbf{M} + m_c)(\mathbf{M} - m_u)/[(m_t + m_c)(m_t - m_u)], & \alpha = 3 = t \end{cases} \tag{8}$$

and similarly for $P_i'(\mathbf{M}')$ as shown in ref. 9. Hence from (7) and (8) above, the absolute squares of the KM matrix elements can be calculated exactly in terms of the quark masses, the absolute values of the mass matrix entries which are in turn related to the quark masses as in (3) above, and the two phases $\phi_{A'}$ and $\phi_{B'}$.

The invariant function, the so-called J-value, for CP violation is obtained from the determinant of the commutator

$$J = \frac{i}{2}\frac{det\ [\mathbf{M},\mathbf{M}']}{(m_t + m_c)(m_t - m_u)(m_c + m_u)(m_b + m_s)(m_b - m_d)(m_s + m_d)} \tag{9}$$

as given in ref. 12. With the Chau - Keung, Harari - Leurer, and Fritzsch parametrization[3] of the KM matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \tag{10}$$

the J-value in (10) can be expressed in terms of

$$J = |V_{us}||V_{ub}||V_{cs}||V_{cb}|\sin\delta \tag{11}$$

from which $\sin\delta$ can be determined.

Schubert[14] has recently summarized the latest information available on the KM matrix. This is given by

$$V = \begin{pmatrix} 0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0 \pm 0.0076 \\ -0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0474 \pm 0.0066 \\ 0.0104 \pm 0.0075 & -0.0462 \pm 0.0067 & 0.9989 \pm 0.0003 \end{pmatrix}$$

$$+i \begin{pmatrix} 0 & 0 & 0 \pm 0.0076 \\ 0 \pm 0.0004 & 0 \pm 0.0001 & 0 \\ 0 \pm 0.0075 & 0 \pm 0.0017 & 0 \end{pmatrix} \qquad (12)$$

after requiring unitarity of the matrix. The invariant function technique then enables us to compare the absolute squares of the KM elements, $|V_{\alpha i}|^2$, with the absolute squares of the entries in (12) above. For this purpose, we run through the range of top quark masses, $25 \leq m_t(1GeV) \leq 180$ GeV for independent values of $\phi_{A'}$ and $\phi_{B'}$.

We illustrate the results in Fig. 1 for four sets of quark masses selected from (5) above. We plot the allowed region of $\phi_{B'}$ vs. $m_t(1GeV)$ for which all nine calculated $|V_{\alpha i}|^2$ lie within one standard deviation of those values determined from the best experimental fit in (12). The central values of the tightly constrained $\phi_{A'}$ intervals (typically $\pm 2^o$) were used for the plots. The allowed region has an annular shape, the size of which depends critically upon the value taken for the strange quark mass input, but much less so for the allowed masses of the other quarks.

To demonstrate this point, in Fig. 1a we have plotted the allowed region for all central quark mass values as given by Gasser and Leutwyler[11] at the 1 GeV scale with $\Lambda_3 = 100$ MeV. If m_s is lowered from 175 MeV to 120 MeV as in Fig. 1b, the allowed region expands from $m_t(1GeV) \lesssim 90$ GeV to $m_t(1GeV) \lesssim 160$ GeV. But changing the strange quark mass alone reduces the m_s/m_d and m_s/m_u ratios below their permitted ranges. In Fig. 1c we have also lowered the u and d masses, so the ratios again agree with those in (5). The main effect has been to raise the phase angle $\phi_{A'}$ back to a value around 85°.

In order to calculate the "physical" top mass, m_t^{phys}, we first define the running top mass, $m_t(\mu)$, to be equal to m_t at $\mu = m_t$, then run it down to $m_b(m_b)$ with $N_f = 5$ flavors and the 5-flavor QCD scale $\Lambda_5 = 47$ MeV, and then down to 1 GeV with $N_f = 4$ flavors and the 4-flavor scale $\Lambda_4 = 76$ MeV, both corresponding to $\Lambda_3 = 100$ MeV. For a given m_t, $m_t(1GeV)$ can then be determined from the ratio product

$$\frac{m_t}{m_t(m_b)} \cdot \frac{m_t(m_b)}{m_t(1GeV)} \tag{13a}$$

where the running mass is given by

$$m_t(\mu) = \bar{m}_t \left(\frac{L}{2}\right)^{-2\gamma_0/\beta_0} \left\{1 - \frac{2\beta_1\gamma_0}{\beta_0^3}\frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L} + O\left[\frac{(\ln L)^2}{L^2}\right]\right\} \tag{13b}$$

in terms of the renormalization group invariant mass $\bar{m}_t$ and scale Λ,

$$\begin{aligned} &L = \ln \tfrac{\mu^2}{\Lambda^2} \\ &\beta_0 = 11 - \tfrac{2}{3}N_f, \qquad \gamma_0 = 2 \\ &\beta_1 = 102 - \tfrac{38}{3}N_f, \qquad \gamma_1 = \tfrac{101}{12} - \tfrac{5}{18}N_f \end{aligned} \tag{13c}$$

for the appropriate number of flavors N_f. Finally, m_t^{phys} is defined in terms of $m_t(m_t)$ with a first order QCD correction[11]

$$m_t^{phys} = m_t(m_t)\left\{1 + \frac{4}{3\pi}\alpha_s + O(\alpha_s^2)\right\} \tag{13d}$$

where

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 L}\left\{1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L}{L} + O\left(\frac{(\ln L)^2}{L^2}\right)\right\} \tag{13e}$$

is evaluated at $\mu = m_t$. In this way the m_t^{phys} scale is plotted in Fig. 1 by correspondence with the $m_t(1GeV)$ scale.

The most recent results on $B_d^0 - \bar{B}_d^0$ mixing have been reported[4] by the ARGUS group as

$$r_d = \frac{P(B_d^0 \rightarrow \bar{B}_d^0)}{P(B_d^0 \rightarrow B_d^0)} = 0.17 \pm 0.05 \tag{14a}$$

corresponding to

$$x_d = \sqrt{\frac{2r}{1-r}} = 0.64 \pm 0.12 \tag{14b}$$

This in turn can be related to the physical top mass and V_{td} by

$$\begin{aligned} x_d &\simeq 0.15 \left(\frac{\tau_b}{3.3 \times 10^{-16}}\right) \left(\frac{B_B f_B^2}{(0.14GeV)^2}\right) \left(\frac{m_t^{phys}}{40GeV}\right)^2 \\ &\simeq 0.34 |\, V_{td}^2 \,| m_t^2 \end{aligned} \tag{15a}$$

from work of Altarelli and Franzini,[15] from which we find

$$(m_t^{phys} \mid V_{td} \mid)^2 = 1.9 \pm 0.4 \tag{15b}$$

Curves of constant $(m_t^{phys} \mid V_{td} \mid)^2$ are plotted in Fig. 1 along with the experimentally allowed band given in (15b). We observe that this band fails to overlap the KM allowed annulus in Fig. 1a by 3 standard deviations. By lowering the strange quark mass to $m_s(1GeV) = 120$ MeV, so that the annulus is greatly enlarged, an overlap or near overlap is obtained in Fig. 1b,c. In Fig. 1d we have chosen quark masses consistent with (4) and (5) which maximize the overlap of the two experimentally allowed regions. The best fit is obtained with a physical top quark mass around 100 GeV.

In Table 1 we present additional information obtained from the cases illustrated in Fig. 1a-d for the regions of best fit. In particular, the invariant J-values obtained are close to 0.3×10^{-4}, the preferred value found in the analysis of Donaghue, Nakada, Paschos and Wyler.[16] In the Chau - Keung, Harari - Leurer, and Fritzsch representation[3] of the KM matrix, $\sin\delta$ is closely related to the phase angle $\phi_{A'}$ and is found to be greater than 0.9 in the physical cases of interest. The predicted values for $\frac{|V_{ts}|^2}{|V_{td}|^2}$ suggest a $B_s^0 - \bar{B}_s^0$ mixing parameter $x_s \gtrsim 10x_d$ corresponding to $r_s \gtrsim 0.95$, nearly maximal mixing in this channel. The amplitude ratio

$$\frac{b \to u}{b \to c} = \frac{|V_{ub}|}{|V_{cb}|} \simeq 0.06 \tag{16}$$

is in reasonable agreement with the allowed range $0.07 - 0.23$ recently determined[5] by the ARGUS group on the basis of the observed decay modes, $B^+ \to p\bar{p}\pi^+$ and $B^0 \to p\bar{p}\pi^+\pi^-$. Finally we note that the bag parameter, B_K, determined[17] from the ϵ parameter in K decay acquires values near 0.70 in good agreement with the predictions of Bardeen, Buras and Gérard[18] only for values of $m_t^{phys} \sim 100$ GeV.

In summary, we have used the invariant function approach (projection operator technique) of one of the authors (CJ) to determine the KM matrix elements squared $|V_{\alpha i}|^2$ in terms of the parameters of the Fritzsch mass matrices. From the experimentally allowed region we can relate the free parameters $\phi_{A'}$ and $\phi_{B'}$ to m_t, the top quark mass. The $B_d^0 - \bar{B}_d^0$ mixing results limit this allowed region to a small range of $m_t^{phys} \sim 100$ GeV for special choices of the light quark masses. Our results are in rough agreement with those recently presented by Harari and Nir,[8] though our technique is quite different, as is the presentation of the critical regions. Details of our calculations along with additional numerical results will be presented in a manuscript under preparation. It is our conclusion that the Fritzsch quark mass matrices for three families are consistent with the present experimental information, given the uncertainties in the quark masses themselves as defined at the common 1 GeV scale. The crucial test for the Fritzsch scheme remains the discovery of the top quark with $m_t^{phys} \sim 100$ GeV.

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Figure Caption

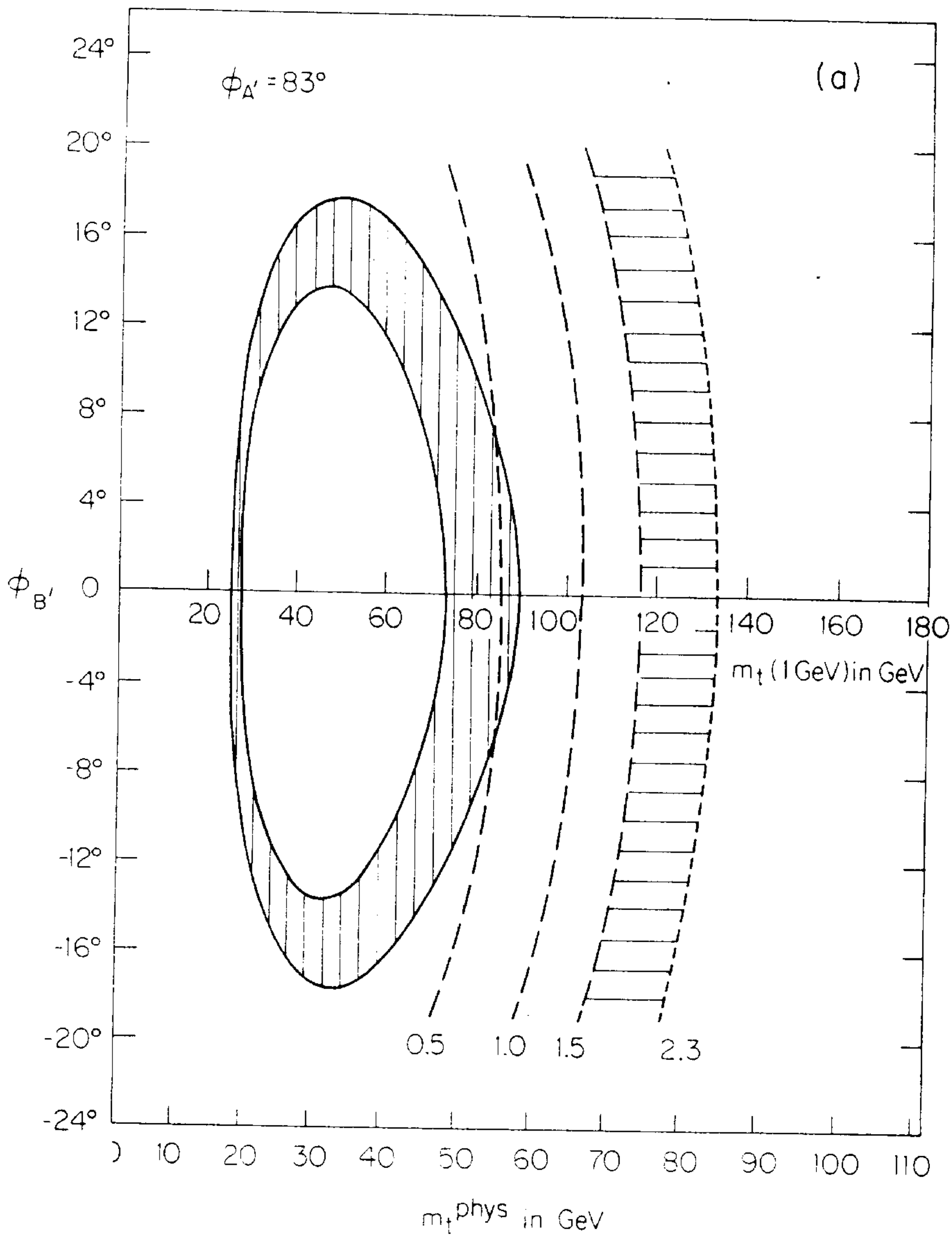
Figure 1: Phase angle $\phi_{B'}$ vs. $m_t(1GeV)$ and m_t^{phys} plots showing the physically allowed annular regions for the KM matrix elements squared and bands for the $B_d^0 - \bar{B}_d^0$ mixing result with one standard deviation tolerance. The sets of quark masses for the four plots are

(a) $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 175$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV;

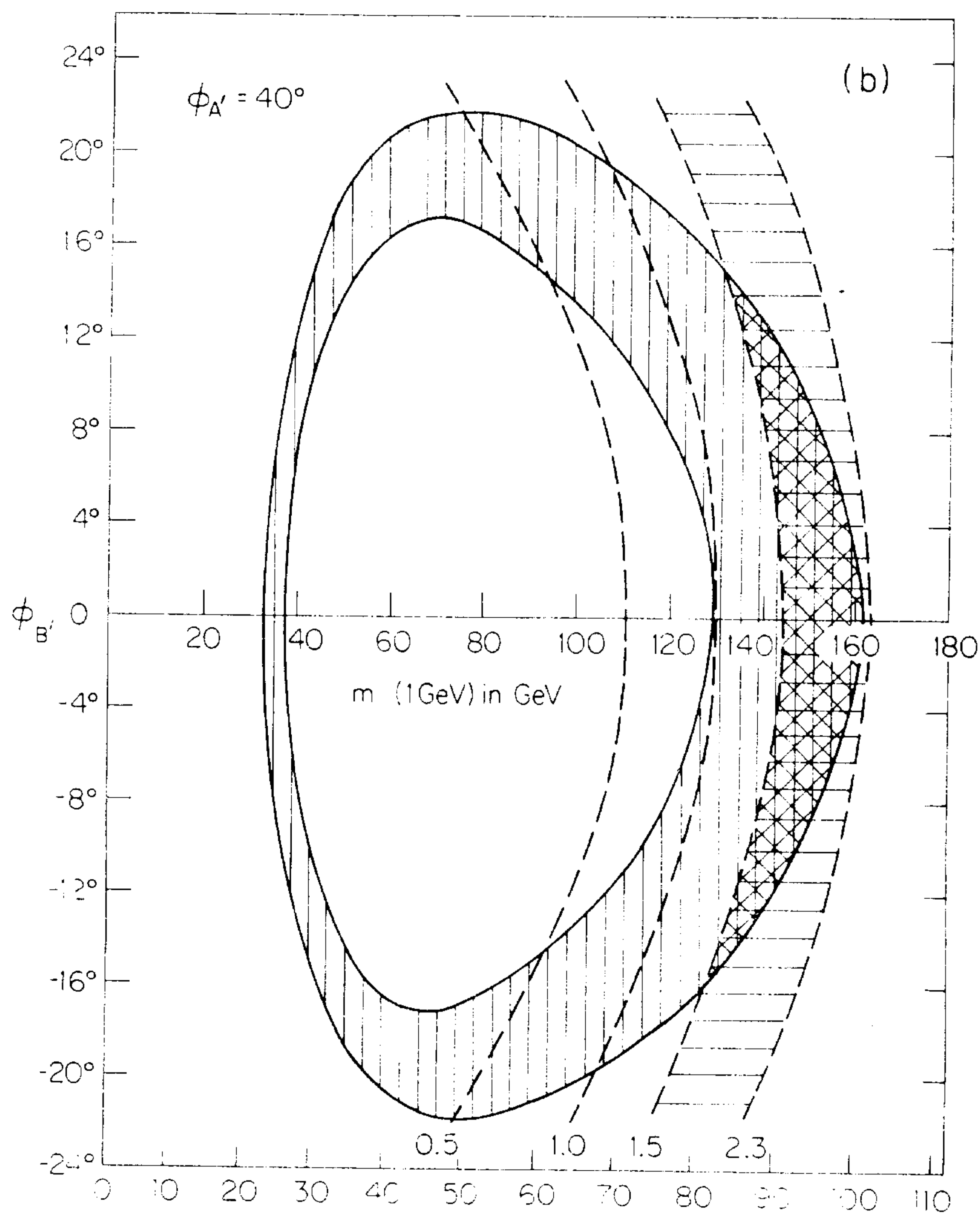
(b) $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 120$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV;

(c) $m_u = 3.5$ MeV, $m_d = 6.1$ MeV, $m_s = 120$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV;

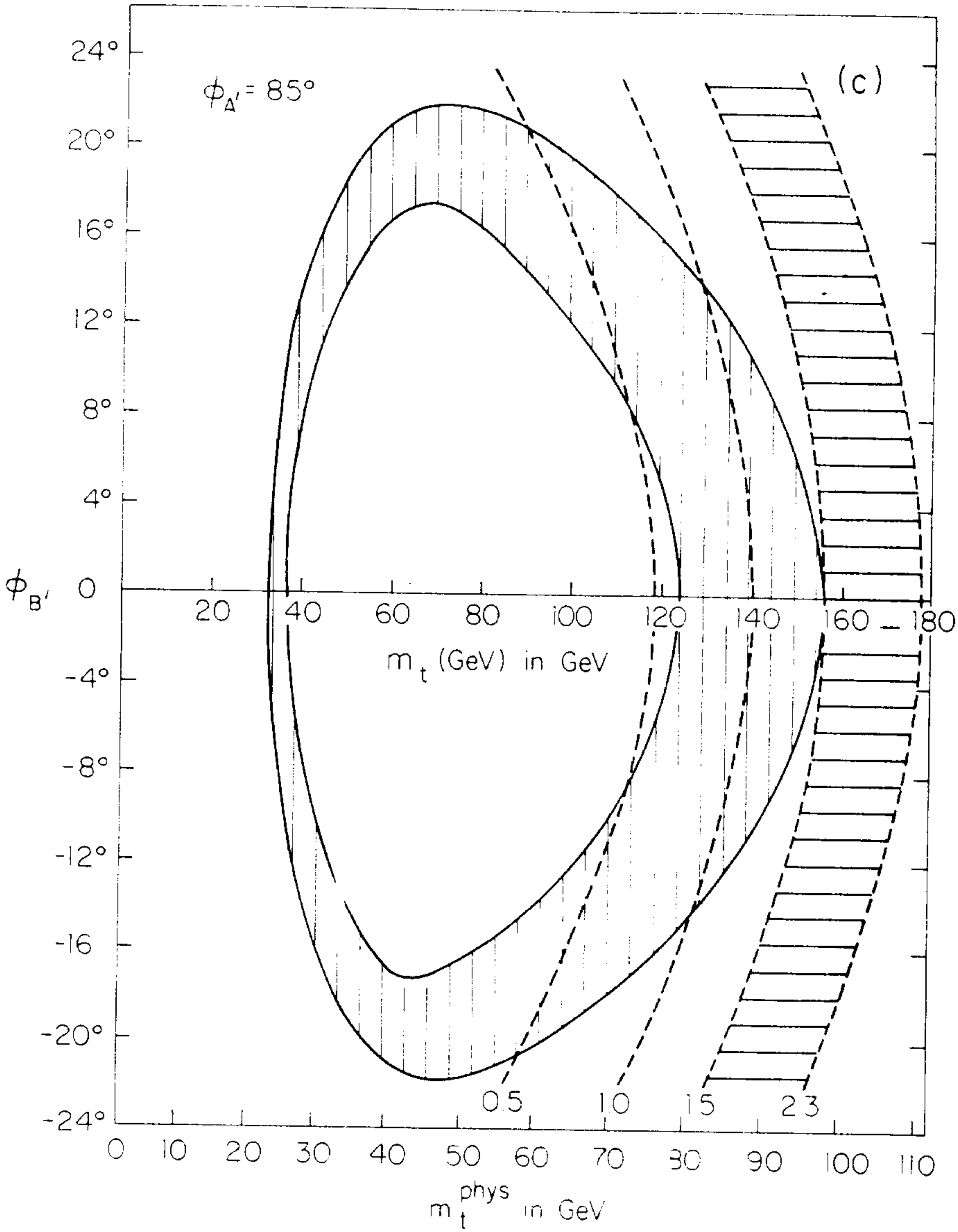
(d) $m_u = 4.1$ MeV, $m_d = 6.7$ MeV, $m_s = 120$ MeV, $m_c = 1.40$ GeV and $m_b = 5.4$ GeV.



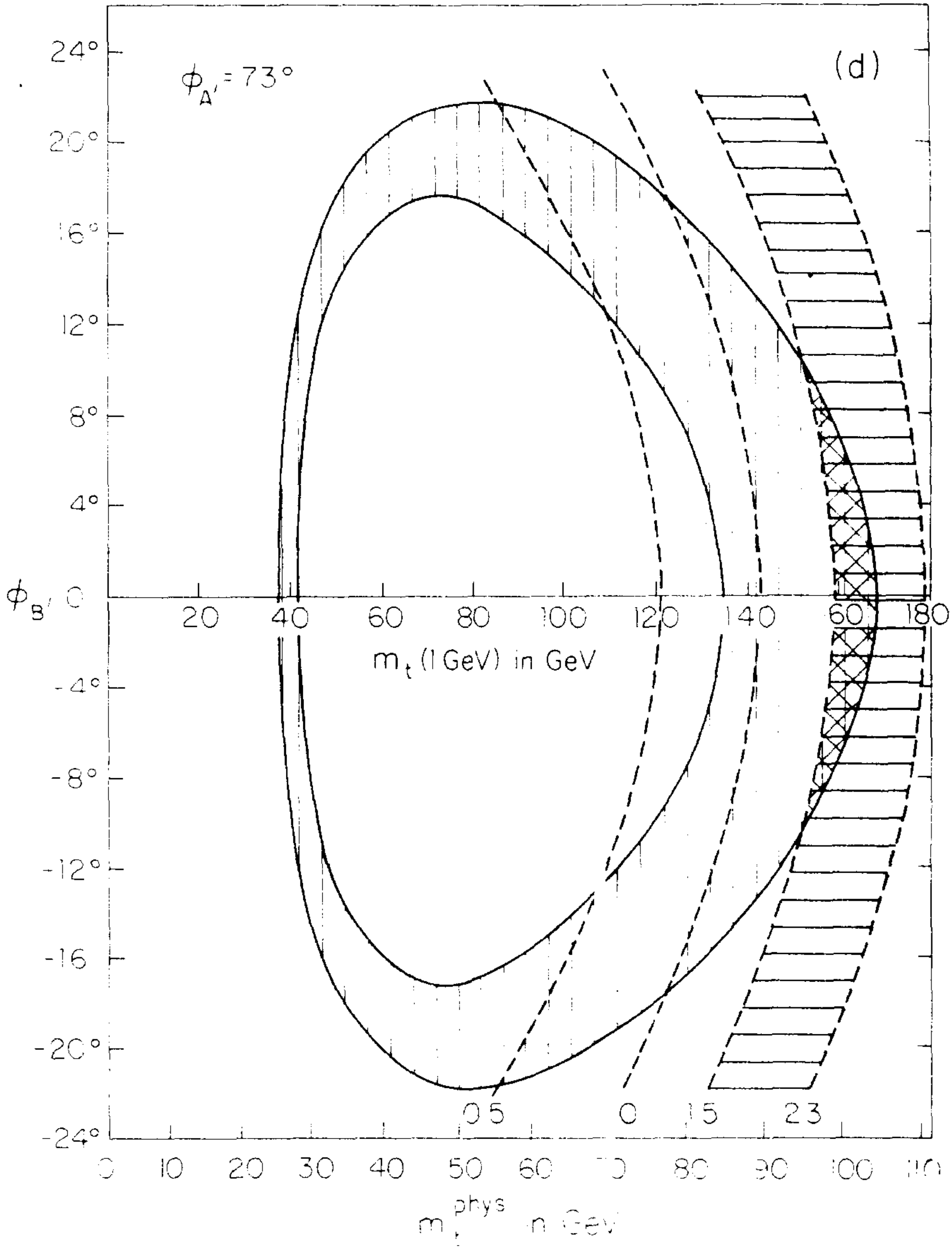
(a)
$\phi_{A'} = 83°$
$\phi_{B'}$
24°
20°
16°
12°
8°
4°
0
-4°
-8°
-12°
-16°
-20°
-24°
20 40 60 80 100 120 140 160 180
m_t (1 GeV) in GeV
0.5
1.0
1.5
2.3
0 10 20 30 40 50 60 70 80 90 100 110
m_t^{phys} in GeV



(b)
$\phi_{A'} = 40°$
$\phi_{B'}$
24°
20°
16°
12°
8°
4°
0
-4°
-8°
-12°
-16°
-20°
20
40
60
80
100
120
140
160
180
m (1GeV) in GeV
0.5
1.0
1.5
2.3
0
10
20
30
50
60
70
80
100
110
m_t^{phys} in GeV



24°
20°
16°
12°
8°
4°
$\phi_{B'}$ 0
-4°
-8°
-12°
-16°
-20°
-24°
$\phi_{A'} = 85°$
(c)
20
40
60
80
100
120
140
160
180
m_t (GeV) in GeV
0.5
1.0
1.5
2.3
0
10
20
30
40
50
60
70
80
90
100
110
m_t^{phys} in GeV



$\phi_{A'} = 73°$
(d)
24°
20°
16°
12°
8°
4°
$\phi_{B'}$ 0
-4°
-8°
-12°
-16°
-20°
-24°
20
40
60
80
100
120
140
160
180
m_t (1 GeV) in GeV
0.5
0
15
23
0
10
20
30
40
50
60
70
80
90
100
110
m_t^{phys} in GeV

Table 1: Values of additional parameters associated with the optimal points in the plots of Fig. 1.

Fig. 1	$m_t(1GeV)$	m_t^{phys}	$\phi_{A'}$	$\phi_{B'}$	J	δ	$m_t^2\|V_{td}\|^2$	$\frac{\|V_{ts}\|^2}{\|V_{td}\|^2}$	r_s	$\frac{\|V_{ub}\|}{\|V_{cb}\|}$	B_K
(a)	85 GeV	56 GeV	83°	0°	0.35×10^{-4}	105°	0.47	15.6	0.75	0.068	1.40
(b)	152 GeV	95 GeV	40°	0°	0.26×10^{-4}	140°	1.75	11.8	0.96	0.075	0.90
(c)	156 GeV	98 GeV	84°	0°	0.31×10^{-4}	96°	1.46	17.3	0.97	0.053	0.77
(d)	163 GeV	102 GeV	73°	0°	0.32×10^{-4}	109°	1.63	15.8	0.97	0.060	0.72