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Cosmic String Wakes[1]

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ABSTRACT. The accretion of matter onto the wakes left behind by horison-sise pieces of cosmic string is studied. We find that in a universe containing cold dissipationless matter (CDM), accretion onto wakes produces a network of sheet-like regions with a nonlinear density enhancement. For an $\Omega_0 \approx 1$ CDM dominated universe containing strings with string mass parameter $G\mu = 10^{-6}\mu_6$ and momentum $\beta_s\gamma_s$ we find: (1) the average separation between the sheets is $\approx 40h_{50}{}^{-2}$ Mpc, (2) the width of the nonlinear region is $\approx 3.0\gamma_s\beta_s\mu_6$ Mpc, (3) the infall velocity of $70\gamma_s\beta_s\mu_6 h_{50}$ km/sec is coherent over approximately the interwake separation, and (4) the surface density is $2\times10^{11}\gamma_s\beta_s\mu_6 h_{50}{}^2\,\mathrm{M}_\odot\mathrm{Mpc}^{-2}$. We estimate the fraction of matter fallen into wakes to be $\sim 6\,f\,\mu_6\beta_s{}^2\gamma_s{}^2\,\%$, where f is the ratio of the length in infinite strings to the horison length at the epoch of formation of the wake. Most of the CDM, however, collapses onto string loops which may be the seeds for the formation of galaxies and clusters of galaxies, and some fraction of these loop condensations accrete onto wakes. The coherence length of wakes with the highest surface density is about that of the typical distance between the sheet-like formations of galaxies that are being observed in the ongoing CfA redshift survey. While it is tempting to associate these wakes with the large-scale distribuion of galaxies, there is no straightforward or obvious connection between the two. For instance, we would not predict that the voids were empty of galaxies unless either galaxy formation in the voids was supressed by some mechanism, or galaxy formation required additional special conditions that were found only in the wake structures. Though we do briefly consider plausible mechanisms to achieve this separation between the matter and the light, we stress that these are speculative and invoke highly uncertain astrophysical biassing schemes. Wakes created by infinite cosmic strings do not straightforwardly produce the observed large-scale galaxy and cluster distribution.

I. INTRODUCTION

Recent results from an ongoing deep galaxy redshift survey suggest that galaxies are located on the surfaces of large ($\sim 50\,h_{50}{}^{-1}$ Mpc) bubble-like structures (de Lapparent, Geller, and Huchra 1986). These observations have proven difficult to reconcile with presently popular theories of the nature of cosmological inhomogeneities (Silk 1986). One possible source of cosmological inhomogeneities would be the existence of cosmic strings (see Vilenkin 1985 for a review). Silk and Vilenkin (1984) pointed out that the wakes of long cosmic strings would produce sheet-like inhomogeneities and Vachaspati (1986) suggested that these structures could be associated with the surfaces on which galaxies lie. In this paper we explore this possibility in detail.

Cosmic strings are one-dimensional stable concentrations of energy density which occur in many particle physics theories. These strings have a constant mass per unit length, μ. In theories which allow cosmic strings, a network of such strings will form in the very early universe (Kibble 1976). The evolution of the network of cosmic strings in the radiation era has been studied numerically by Albrecht and Turok (1985). They conclude that at any given time there will be a network of infinite strings roughly tracing out a random walk with step length $\sim 2t$ and a distribution of loops with radius smaller than $2t$.

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Matter may accrete around loops and it is possible to form bound objects with the density and mass of both galaxies and clusters of galaxies in this way (Turok and Brandenberger 1986, Stebbins 1986, and Sato 1986). Using the results of numerical simulations, Turok (1985) has shown that the number density and correlation function of loops large enough to accrete clusters of galaxies match the number density and correlation function of Abell clusters as measured by Bahcall and Soneira (1983), if $\Omega_0 \approx 1$ and the intercommutation probability is about unity. It has also been argued that the present correlation function of galaxy-size objects accreted around loops is in rough agreement with the observed correlation function of galaxies (Brandenberger and Turok 1986). However, nonlinear gravitational clustering is undoubtedly important in these objects so that this result is somewhat puzzling. Furthermore, if galaxies are arranged in very large two-dimensional structures, then explaining these large structures is as important and probably more fundamental than explaining the galaxy-galaxy correlation function. Estimates of the microwave anisotropies from strings are below present observational limits (Brandenberger and Turok 1986; Traschen, Turok, and Brandenberger 1986; and Brandenberger, Albrecht, and Turok 1986) in cold dark matter models and marginally consistent with hot dark matter models.

In this paper we investigate accretion of matter onto wakes and determine the effects of wakes on the large scale structure of the universe. We conclude that the most important wakes are those which were formed at the time t_{eq} of equal matter and radiation density. This will lead to sheet-like overdense regions of galaxies with a mean separation of 40 $h_{50}{}^{-1}$ Mpc, in agreement with the scale of the bubbles of de Lapparent, Geller, and Huchra (1986). However, for the value of $G\mu \approx 10^{-6}$ favored from galaxy formation considerations in a universe with cold dark matter, a wake will have accreted matter from a distance of only about 1.5 Mpc, which is much less than the the distance between the wakes. Hence the regions between the wakes will retain most of their matter losing only a small fraction to the wakes.

The outline of the paper is as follows. In §2 we give a brief review of wakes, the density perturbations induced by long string segments moving with respect to the Hubble flow. We then discuss accretion of cold matter onto wakes in §3. We discuss the effect of a long string on fluids with finite velocity dispersion (hot matter like neutrinos) or sound speeds (baryons after the epoch of recombination) in §4. In §5, the interactions between loops and wakes are discussed, and the conditions for wakes to survive disruption by loops. In §6 we present some general considerations for galaxy formation in wakes. The geometry of wake surfaces is explored in §7. Finally §8 includes a summary of our results and conclusions.

II. THE ROLE OF WAKES

The peculiar velocity of any given segment of string is typically close to speed of light (Vilenkin 1985), whether the segment is part of a loop or part of an infinite string. Moving strings attract matter and as a string passes it will give nearby matter a boost in direction of the surface swept out by the string. As the matter moves toward this surface from both sides it will pile up at this surface thus forming some sort of two-dimensional structure, at least temporarily. The properties of the structure, such as its thickness and the period of time for which it persists, will depend on the exact nature of matter that the string moves through. If they persist, they will expand with the universe and thus retain constant comoving size. A moving gravitating point-like object such as star leaves behind similar structures, which have been dubbed accretion wakes (Bondi 1944). Here we will call the surfaces traced out by the strings the "wake surfaces" and any structure that forms in the ambient medium we will call "wakes". In addition to the wakes there is the large scale velocity field which, at least initially, extends far beyond the region where structure has formed. It is this large scale perturbed velocity field which can in some cases cause the wakes to grow.

At any time t, the network of strings and hence the network of wakes, may conceptually be divided into two parts. The first part, which we shall call "long" strings, consists of infinite strings and loops whose sizes are much bigger than the horizon. The typical radius of curvature of this component is $\sim 2t$ and the typical distance one must go transverse to a piece of long string to find another piece of long string is also $\sim 2t$ (Albrecht and Turok 1985). The string velocity is coherent over about an expansion time and is close to the speed of light. Thus the curve traced out by a given point on a string has a radius of curvature approximately the horizon size. It follows that a wake surface itself has radii of curvature $\sim 2t$ and is separated by a distance $\sim 2t$, where t is the time when the wake is produced. The long string network is not at all periodic. In particular, it is unlikely that a long string would trace out a path close to the path traced out previously by a piece of long string (this is not true for loops as we shall see), although it might pass through a wake surface. Thus once the wake is produced it will evolve without further amplification by the string network. In §3 we discuss how such an isolated wake evolves.

The other component of the string network consists of smaller-than-horizon loops. These loops are borne of the long string component. This may happen in one of two ways. A sub-horizon loop is produced when a super-horizon loop persists long enough for the horizon to grow larger than the loop. This process is probably subdominant since the long string component is dominated by infinite strings and not loops (Vachaspati and Vilenkin 1984, Albrecht and Turok 1985). Secondly, a long string can intersect itself and chop off a loop (intercommute). A loop, once produced, may cut itself into two smaller loops and either or both of those may also get cut up, and so on. Eventually a stable set of non-intersecting loops remains. These loops will retain their physical size and mass and will oscillate almost exactly periodically. Each loop grows smaller in comoving coordinates and the surface it traces out in comoving coordinates will be a sequence of succesively smaller "shells". The string, in general, is moving and the shells are only roughly concentric. On these shell surfaces wakes will form. It easy to see that the string segments moving on opposite sides of the shell will produce impulses which add constructively for positions outside the shell and destructively for positions on the inside. The time averaged acceleration field for matter outside the radius of the loop is a net attraction toward the loop, whereas matter within the loops experiences a small net acceleration which depends on the exact loop configuration. As a result of the net inward pull of the loops, at some time the region around the loop will collapse, and subsequently, more matter will continue to fall on the central condensations. This infall has been studied by Brandenberger and Turok (1986), Stebbins (1986), and Sato (1986) using approximations to the time averaged fields. In fact, descriptions of loop accretion in terms of superpositions of wakes are probably a more accurate description of the accretion than the static models used so far. Jeans instability in the wakes of baryons, with or without CDM, may produce interesting structures, perhaps even stars, long before other nonlinear structures associated with the loops are formed (see also Rees 1986).

The wakes produced by a loop would also be swept into the condensation around it and would be disrupted by violent relaxation. In a CDM universe, wakes survive until today only around large loops (we calculate their size in §5), and are therefore rare. For this reason we confine our attention exclusively to wakes produced by long strings.

As we shall show, wakes persist only in a cold matter component. The criterion for coldness is calculated in §4. Assuming there is cold matter around, the geometry of the surviving wakes approximates the trajectories (in comoving coordinates) of strings at some earlier epoch. As mentioned above, the scale for the system of wakes is the horizon size at the time of formation of the dominant wakes. As the comoving horizon size becomes arbitrarily small at arbitrarily early times one might expect arbitrarily small scale wakes. However, as we show in §3, the most prominent wakes in a CDM dominated universe are produced during the era of matter-radiation equality, and in §5 we show that the less prominent wakes are disrupted by loops. Thus in a CDM universe the scale for the prominent wakes is the horizon at matter-radiation equality. We take the wake scale as a function of the redshift z in a CDM universe to be

$$\lambda_i(z) \equiv \frac{1}{H(z)}(1+z) = \frac{1}{H_0}\left(1+\Omega_0 z+\Omega_0\frac{(1+z)^2}{1+z_{eq}}\right)^{-\frac{1}{2}} \qquad \frac{1}{H_0} = 6.0\times 10^3 {h_{50}}^{-1}\,\mathrm{Mpc} \qquad (2.1).$$

Here we have used the convention $c = 1$. λ_i differs from the actual horizon size by $\mathcal{O}(1)$ in an $\Omega_0 = 1$

universe, and is a good enough estimate of the scale of the wake system. In a universe with three light species of neutrinos

$$1 + z_{eq} = 6.26 \times 10^3 \Omega_0 {h_{50}}^2. \tag{2.2}$$

Thus the scale of the prominent wakes is approximately $\lambda(z_{eq}) = 54{\Omega_0}^{-1}{h_{50}}^{-2}$ Mpc. This is also the approximate scale of the structure found by de Lapparent, Geller, and Huchra (1986). Of course, this is a natural scale in any theory so the rough equality of the two scales cannot be considered to be strong evidence for cosmic strings, but rather a motivation for the idea that wakes of long strings at z_{eq} could dominate the large scale geometry. The other wakes even if they do not survive as sheet-like structures to the present will be an important contributor to the overall amplitude of the fluctuations on small scales.

III. ACCRETION OF COLD MATTER ONTO ISOLATED WAKES

The string-induced wakes which are the subject of this paper are the perturbations produced by infinite (greater than the size of the horizon) strings as they sweep across the universe at relativistic speeds. Note that the wake, unlike a loop-induced accretion, must grow from the initial perturbation without the help of a continuous gravitational pull. In a realistic universe with strings there will be loops accreting matter as well as horizon-size strings producing wakes. The matter in the universe will feel the pull of both. We will assume and later justify that a universe containing strings will produce objects similar to the isolated wakes which we will now describe.

In order to determine the structures produced by string wakes we must first determine the specific nature of the perturbation initially induced, or equivalently the gravitational field of a long string. The gravitational field of a string at a distance comparable to the horizon (to our knowledge) has not been previously calculated and it will be discussed in a later paper (Veeraraghavan, Stebbins and Silk, 1987). As we shall show, given the finite value of z_{eq}, the matter which has accreted nonlinearly onto a wake would have started at a position much smaller than a horizon distance from the string that formed the wake. The gravitational field of a string at this distance is well approximated by the Minkowski space gravitational field of the moving string. Thus for the purpose of calculating the present nonlinear structure of wakes we do not need to know how the cosmology modifies the gravitational field of a vacuum string near the horizon.

The long strings in the string network are typically moving at velocities close to the speed of light and so for particles closer to the string than the horizon the time for a string to pass by is less than an expansion time. As noted by Vachaspati (1986) there will be nonlinear structure produced immediately behind the wake, but most of the matter that has collapsed by today would have taken very many expansion times to do so. For this matter, which collapses long after the wake was formed, it is not a bad approximation to shift the starting time of growth by a factor close to unity. We may thus simplify the problem by calculating the total impulse given to a particle as the string moves by and give this impulse to each particle instantaneously at some initial time, t_i. Finally, to simplify the problem even more we will assume that the string which produces the wake is straight and will thus move at a constant velocity. This is justified since the typical radius of curvature of strings which are not part of loops is the horizon size, and so the distance from the string of the typical particle that we are interested in is much less than the curvature radius. Through these two approximations we have reduced the problem at hand to one that has plane-parallel symmetry.

To calculate the impulse we follow the method of Turok (1984). We take the perturbed Minkowski metric to be

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

with $h_{\mu\nu} \ll 1$ and $\eta_{\mu\nu} = diag(+,-,-,-)$. For a peculiar gravitational force $h_{\mu\nu} \ll 1$ of finite duration the impulse given to a non-relativistic particle at a fixed position is

$$\Delta\mathbf{v}(\mathbf{x}) = -\frac{1}{2}\nabla \int_{-\infty}^{+\infty} h_{00}\, dt. \tag{3.1}$$

It was shown by Turok (1984), that in the harmonic gauge the gravitational field of a string in Minkowski space is

$$h_{00}(\mathbf{x},t) = -4G\mu \int \frac{|\dot{\mathbf{r}}(\sigma,t')|^2}{|\mathbf{x}-\mathbf{r}(\sigma,t')| - (\mathbf{x}-\mathbf{r}(\sigma,t'))\cdot\dot{\mathbf{r}}(\sigma,t')} d\sigma, \tag{3.2}$$

where $t'(t,\sigma)$ is the retarded time : $t - t'(t,\sigma) = |\mathbf{x} - \mathbf{r}(\sigma, t'(t,\sigma)|$, and σ is a parameter along the string. Using

$$dt' = \frac{|\mathbf{x}-\mathbf{r}(\sigma,t')|}{|\mathbf{x}-\mathbf{r}(\sigma,t')| - (\mathbf{x}-\mathbf{r}(\sigma,t'))\cdot\dot{\mathbf{r}}(\sigma,t')} dt \tag{3.3}$$

we find

$$\Delta\mathbf{v}(\mathbf{x}) = -2G\mu\nabla \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{|\dot{\mathbf{r}}(\sigma,t')|^2}{|\mathbf{x}-\mathbf{r}(\sigma,t')|}\, dt'\, d\sigma. \tag{3.4}$$

For an infinitely long straight string moving in the x-direction in the x-y plane we may take $\dot{\mathbf{r}} = \beta_s \hat{\mathbf{x}}$ and $\sigma = \gamma_s \hat{\mathbf{y}}$ where $\gamma_s \equiv (1-{\beta_s}^2)^{-1/2}$ so $\mathbf{r}(\sigma,t') = \beta_s t' \,\hat{\mathbf{x}} + {\gamma_s}^{-1}\sigma\, \hat{\mathbf{y}}$. It is clear that the time integral of h_{00} can only depend on z so the impulse given can also only depend on z and must be in the z direction. For simplicity we take $\mathbf{x} = (0,0,z)$. We thus obtain

$$\Delta v_z = 2G\mu \frac{d}{dz}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{{\beta_s}^2}{\sqrt{z^2 + {\gamma_s}^{-2}\sigma^2 + {\beta_s}^2 t'^2}}\, dt'\, d\sigma. \tag{3.5}$$

Although the integral is divergent, the gradient which gives the impulse is well defined, and there is nothing unphysical about using the above expression. The integral is infinite because we are considering an infinitely long straight string, but the impulse given to a particle near a moving string does not depend on the actual size of the the string as long as it is big enough and flat enough. This is analagous to the gravitational field of a large flat sheet of matter. The potential difference between the sheet of matter and infinity diverges for very large sheets but the gravitational acceleration above the sheet just depends on the surface density and not the size of the sheet. Taking the gradient and then performing the integral we obtain:

$$\Delta v_z = -4\pi G\mu\beta_s\gamma_s \,\mathrm{sgn}(z) = -3.8\mu_6\beta_s\gamma_s\,\mathrm{sgn}(z)\,\mathrm{km/s}, \tag{3.6}$$

where μ_6 is $G\mu$ in units of 10^{-6}. Thus the impulse given to a particle is independent of distance from the string as long as it is much closer to the string than both the horizon and radius of curvature of the string. In the following we shall use $v_{\mathrm{i}} = |\Delta v_z|$.

Our cosmological setting is a Friedmann-Robertson-Walker cosmology containing as its major mass components radiation and a form of cold, dissipationless matter (CDM). The behavior of the baryons and the interaction with other perturbations are discussed in later sections. The evolution of the expansion rate, from which we may determine the scale factor, $a(t)$, is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_{\mathrm{eq}}}{3}\left(\left(\frac{a_{\mathrm{eq}}}{a}\right)^4 + \left(\frac{a_{\mathrm{eq}}}{a}\right)^3 + \frac{{\Omega_0}^{-1}-1}{1+z_{\mathrm{eq}}}\left(\frac{a_{\mathrm{eq}}}{a}\right)^2\right), \tag{3.7}$$

where $a = a_{\mathrm{eq}}$ at $z = z_{\mathrm{eq}}$. The background CDM and radiation density are given by $\overline{\rho_{\mathrm{cdm}}}(t) = \rho_{\mathrm{eq}}(a_{\mathrm{eq}}/a)^3$ and $\overline{\rho_{\mathrm{rad}}}(t) = \rho_{\mathrm{eq}}(a_{\mathrm{eq}}/a)^4$, respectively. The peculiar gravitational potentials will be sufficiently weak and slowly changing that the radiation fluid density and pressure will not vary much from their background value (we will discuss this further in the discussion on wakes and hot matter). In considering the CDM flow we will assume that the density and pressure of the relativistic fluids remain at their background values at all times, i.e. $\rho_{\mathrm{rad}} \approx \overline{\rho_{\mathrm{rad}}}$ and $p_{\mathrm{rad}} \approx \overline{p_{\mathrm{rad}}}$.

As we are interested in scales much less than the horizon size we may use a Newtonian formalism for the gravitational field of the CDM and radiation. In this space-time the equation of motion for a free, non-relativistic particle such as a particle of CDM is (Peebles §8 1980)

$$\ddot{\mathbf{X}} + 2\frac{\dot{a}}{a}\dot{\mathbf{X}} = a\mathbf{g}(\mathbf{X}). \tag{3.8}$$

Here $\mathbf{X}$ is the particle's position in comoving coordinates scaled to today, and $\mathbf{g}$ is the peculiar gravitational acceleration. We normalize the scale factor a to 1 at $z = 0$. The peculiar acceleration satisfies an analog of Poisson's equation:

$$\frac{1}{a}\nabla \cdot \mathbf{g}(\mathbf{x}) = -4\pi G(\rho_{\rm tot} + 3p_{\rm tot} - \overline{\rho_{\rm tot}} - 3\overline{p_{\rm tot}}) \cong -4\pi G(\rho_{\rm cdm} - \overline{\rho_{\rm cdm}}). \tag{3.9}$$

In our simplified model of the string there will be no peculiar gravitational fields parallel to the plane of symmetry. The fluid velocity in those directions will just be the Hubble flow. Thus we only need follow the motion of CDM particles in the direction perpendicular to the plane of the wake. We shall denote the component of $\mathbf{X}$ in that direction by X, where $X = 0$ is the central plane of the wake. As the CDM has negligible velocity dispersion entire planes of CDM will have the same trajectory $X(t)$. Combining (3.8) and (3.9) we find that the trajectories satisfy

$$\ddot{X} + 2\frac{\dot{a}}{a}\dot{X} + 4\pi G\int_0^X (\rho_{\rm cdm}(X',t) - \overline{\rho_{\rm cdm}}(t))\, dX' = 0. \tag{3.10}$$

We may describe the entire flow by a function $X(Q,t)$ where Q is a Lagrangean coordinate which labels the CDM planes. In particular we take

$$Q = \tilde{Q}(X(Q,t_i),t_i) \qquad \text{where} \qquad \tilde{Q}(X,t) \equiv \int_0^X \frac{\rho_{\rm cdm}(X',t)}{\overline{\rho_{\rm cdm}}(t)}\, dX'. \tag{3.11}$$

This is a particularly nice choice because $Q = \tilde{Q}(X(Q,t),t)$ remains true as long as the plane in question does not cross the trajectory of another plane. This will not happen until after the perturbation goes nonlinear. Before plane-crossing (3.10) may be written in the simple form

$$\ddot{X}(Q,t) + 2\frac{\dot{a}}{a}\dot{X}(Q,t) + 4\pi G\overline{\rho_{\rm cdm}}(Q - X(Q,t)) = 0. \tag{3.12}$$

If there were no deviations from Hubble flow then $X = Q$ and the last term on the left-hand-side of (3.12) is zero. Equation (3.12) is essentially the Zel'dovich approximation (Zel'dovich 1970), which, as is well known, is exact for one-dimensional perturbations before shell crossing. We can rephrase (3.12) more elegantly with the following change of variables:

$$\Delta_Q(Q,t) \equiv (X(Q,t) - Q) \qquad \eta \equiv \ln\left(\frac{a}{a_{\rm eq}}\right) \qquad {}' \equiv \frac{\partial}{\partial\eta}.$$

$$\Delta_Q'' + [1-q]\Delta_Q' - \frac{3}{2}\Omega_{\rm cdm}\Delta_Q = 0, \qquad q \equiv -\frac{\ddot{a}\,a}{\dot{a}^2} \qquad \text{and} \qquad \Omega_{\rm cdm} \equiv \frac{8\pi G\overline{\rho_{\rm cdm}}}{3\left(\frac{\dot{a}}{a}\right)^2} \tag{3.13}$$

Here Δ_Q is the deviation of the plane with lagrangean coordinate Q from its initial position in comoving coordinates, q is the deceleration parameter and $\Omega_{\rm cdm}(\eta)$ is the ratio of the CDM density to the critical density. We have not used (3.7) in deriving (3.13) so this formula applies to perturbations of CDM in any cosmology where one may ignore the response of the other components to the CDM perturbation. Use of this Zel'dovich approach rather than the usual linearized density perturbations approach will prove much more convenient for calculating string perturbations. The reason for this is that strings, on scales less than the horizon, do not produce any *linear* density perturbations even though they do produce growing mode velocity perturbations. To see this, note that in the Newtonian approximation (Peebles 1980), the linearized CDM overdensity, δ, obeys

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\overline{\rho_{\rm cdm}}\delta = 4\pi G(\rho_{\rm string} + 3p_{\rm string}). \tag{3.14}$$

The right-hand-side of (3.14) is only non-zero at points through which a string has passed. These are, of course, two-dimensional surfaces and are of measure zero. Thus at almost all points there will be no first order density perturbation. Regions where caustics have occured or where $|\partial\Delta_i/\partial X_j| \not\ll 1$ have become nonlinear and may have significant overdensity. These nonlinear regions will not, in general, invalidate the linear analysis outside the nonlinear regions and for geometries with planar symmetry they definitely will not. On the other hand, equation (3.14) becomes a poor approximation on scales comparable to or greater than the horizon size. Wakes will produce large slightly underdense regions far from the wake, since these regions are the source of matter which falls on the wake. These very large scale perturbations will not be discussed further in this paper.

We may evolve (3.13) with the initial condition derived above. In terms of Δ_Q the initial conditions are

$$\Delta_Q(Q,t_i) = 0 \qquad \text{and} \qquad \frac{\dot{\Delta}_Q(Q,t_i)}{1+z_{\rm i}} = v_{\rm i} \tag{3.15}$$

where $z_{\rm i}$ is the redshift at the initial time t_i when the wakes form. The initial velocity $v_{\rm i}$ depends only on the string parameters and has no systematic dependence on the initial redshift. As soon as the wake forms, the innermost planes will cross and form a caustic structure at the center. This caustic structure will grow with time as more matter falls in. Since planes have crossed inside the caustics, equation (3.13) will not be valid in this inner region. For the moment we will concentrate on the regions outside caustics where (3.13) is valid. The peculiar velocity grows with time, but it will remain independent of position for planes that are outside the caustic structure. For such planes, Δ_Q will not depend on Q, but only on the initial redshift : $\Delta_Q(Q,t(z)) = \Delta(z)$. $\Delta(z)$ also represents the half-thickness of the nonlinear wake structure.

In certain relevant limits equation (3.13) may be solved exactly. One such case is the the matter era of an Einstein-de Sitter universe when $q = \frac{1}{2}$ and $\Omega_{\rm cdm} = 1$. With initial conditions (3.15) we obtain

$$\begin{aligned}\Delta(z) &= \frac{2}{5}\frac{v_{\rm i}}{H_0}\left(\frac{\sqrt{1+z_{\rm i}}}{1+z} - \frac{(1+z)^{\frac{3}{2}}}{(1+z_{\rm i})^2}\right) \qquad z_{\rm i} \ll z_{\rm eq}\,,\ \Omega_0 \approx 1 \\ &\to \frac{\Delta(0)}{1+z} \qquad z \ll z_{\rm i}\end{aligned} \tag{3.16}$$

In the matter era, all solutions with of (3.13) with constant peculiar velocity toward a central plane will approach the asymptotic form given above. Thus the second equation is appropriate for $z \ll z_{\rm eq}$ and $z \ll z_{\rm i}$ no matter what the value of $z_{\rm i}$ is. Of course, we have not yet determined how $\Delta(0)$ depends on $v_{\rm i}$ and $z_{\rm i}$ if $z_{\rm i} \gtrsim z_{\rm eq}$. We may also solve equation (3.13) during the radiation era of an Einstein-de Sitter universe when $q = 1$ and $\Omega_{\rm cdm} = 0$. With initial conditions (3.15) we obtain

$$\Delta(z) = \frac{v_{\rm i}}{H_0}\frac{\sqrt{1+z_{\rm eq}}}{1+z_{\rm i}}\ln\left(\frac{1+z_{\rm i}}{1+z}\right) \qquad z_{\rm eq} \ll z,\ z_{\rm i} \tag{3.17}$$

The z-dependence of (3.16) and (3.17) are just the well known growth rates for CDM perturbations in the matter era and radiation era, respectively.

We shall not discuss the details of accretion of CDM onto wakes during the radiation era because this may be irrelevant for galaxy formation. Baryons cannot be involved in nonlinear collapse at such early times and in any case almost all the matter that falls onto a given wake will do so

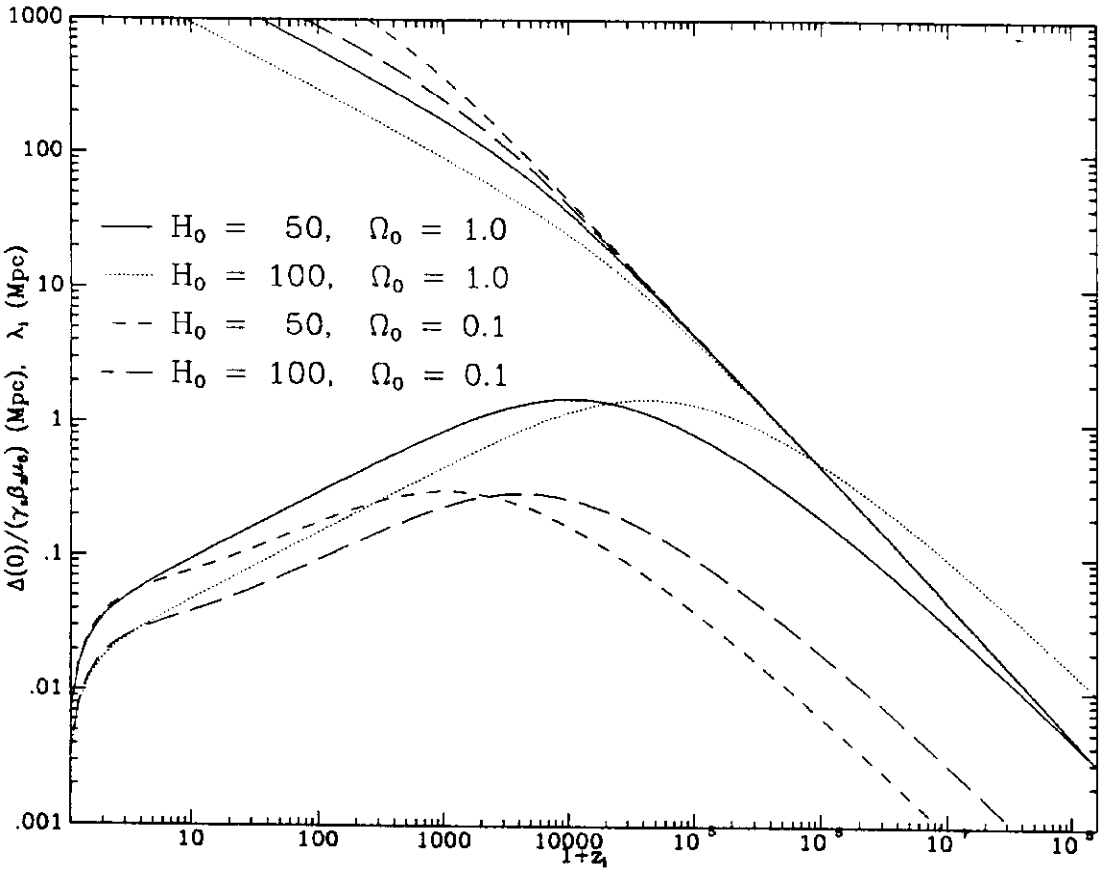

FIGURE 1. The lower curves give the distance moved by cold dissipationless matter falling into an isolated wake as a function of the redshift at which the wake was created, for various cosmological models as labelled. The upper curves are an approximation to the comoving horizon size as a function of the initial redshift, and are a rough estimate of the typical distance between wakes. Wakes created at redshifts when the lower curves are much smaller than the upper curves would only have accreted a small fraction of the matter in the universe, leaving the spaces in between them largely untouched. The earliest wakes which could have accreted all the matter would probably not have survived to do so.

during the matter era. We are however interested in accretion of CDM onto wakes during the matter era, whether or not the wake was formed before or after z_{eq}. To describe such infall we shall use the asymptotic form of equation (3.16). Comparing the asymptotic forms of (3.16) and (3.17) we may guess an approximate expression for $\Delta(0)$ of wakes which started in the radiation era:

$$\Delta(0) \approx \text{constant} \times \frac{v_i}{H_0}\frac{(1+z_{eq})^{\frac{3}{2}}}{1+z_i}\ln\left(\frac{1+z_i}{1+z_{eq}}\right) \qquad z_i \gg z_{eq}, \qquad \Omega_0 \approx 1. \tag{3.18}$$

Numerical integration shows that this expresion is accurate to a few tenths if we take the constant to be 1.0. Since $\Delta(0)$ is an increasing function of z_i for $z_i \ll z_{eq}$ and and decreasing function of z_i for $z_i \gg z_{eq}$ we conclude that $\Delta(0)$ is maximal for wakes that started at $z_i \approx z_{eq}$. This is just as predicted by Vachaspati (1986). To see this result in detail we have numerically integrated (3.13) with initial conditions (3.15) to obtain $\Delta(0)$ as a function of z_i. In Figure 1 we have plotted $\Delta(0)$ as a function of z_i with v_i given by equation (3.6). The maximum occurs for $z_i \approx 2z_{eq}$ and is given by $\max[\Delta(0)] \approx 0.25\, v_i\sqrt{1+z_{eq}}/H_0 = 1.5\,\gamma_s\beta_s\mu_6\,\text{Mpc}$ if $\Omega_0 \approx 1$. Also plotted in Figure 1 is the quantity λ_i ($\lambda_i \equiv (1+z_i)c\,a\dot{a}^{-1}$, equation (2.1)) which is an approximate measure of the comoving horizon size at z_i, and hence of the distance between wakes produced at that epoch. The largest wakes have a inter-wake distance of about $40\,h_{50}{}^{-1}\,\text{Mpc}$. The fact that $\Delta(0) < \lambda_i$ for most wakes tells us that these wakes have only accreted a small fraction of the matter in the universe. However, for wakes that formed very early, $\Delta(0) \gtrsim \lambda_i$ which means that these wakes could have accreted most of the matter in the universe, but we shall show that these early wakes will be rapidly disrupted by loops.

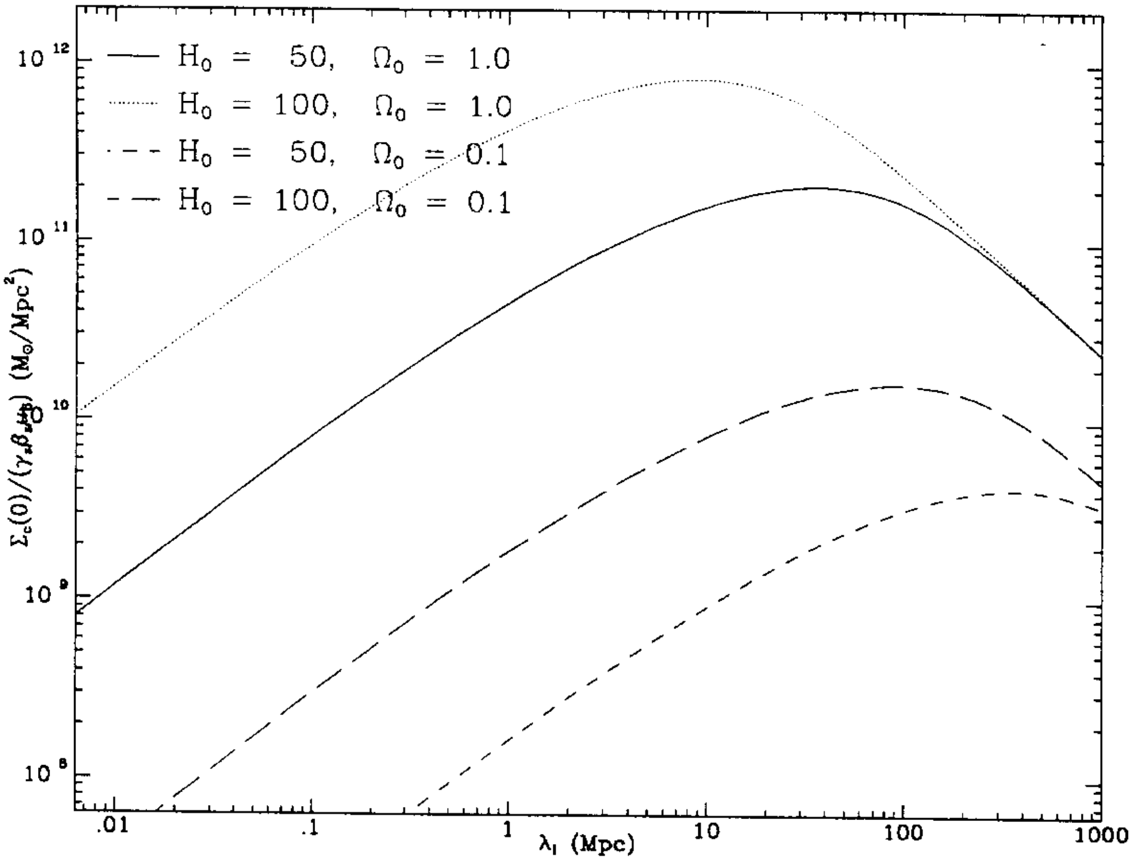

FIGURE 2. Plotted is the surface density of matter accreted by an isolated wake as a function of an estimate of the distance between them for various cosmological models. The surface density is mass per *physical* area, *not* comoving area. The distance between the wakes, λ_i, is determined by the redshift at which the wakes were created, as graphed in Figure 1. For $\Omega_0 = 1$ universes, which we consider to be most promising, the most prominent wakes are approximately $10 - 40$ Mpc apart. We expect that most of the mass accreted onto wakes will have already accreted around loops to form bound clumps which may be galaxies.

It is wakes with the largest $\Delta(0)$ which will have accreted the most matter and will therefore, presumably, be the most prominent. We will now estimate the surface density of wakes. Using the growing mode term in (3.16) one may show that the distance from the wake center at which CDM is just starting to fall back ("turn-around") is given by $\Delta(z)$. The caustic structure will always be confined within this distance from the center. The surface density (mass per physical area) interior to the turn-around distance is

$$\Sigma_t(z) = 4\frac{\Delta(z)}{1+z}\overline{\rho_{\rm cdm}} = 2.8 \times 10^{11}\frac{\Delta(0)}{1\,{\rm Mpc}}(1+z)\ M_\odot\,{\rm Mpc}^{-2}. \tag{3.19}$$

The CDM that recollapses will pass through caustics (collisionless shocks, where the density field is singular). From the viewpoint of galaxy formation, it is important to know just how much matter could have passed through the caustic structure (the "post-caustic matter"). Since this structure is confined to within $\Delta(z)$, (3.19) is an upper bound to the surface density of post-caustic matter. To obtain a lower bound, we extrapolate (3.16) to when the plane crosses the center, as would be valid if the CDM stopped when it reached the center and did not pass through to the other side. The matter which has just reached the center in the simple sticky dust model turned around at a redshift $2(1+z)$, and so our lower bound to the post-caustic surface density is

$$\Sigma_c(z) = 2\frac{\Delta(z)}{1+z}\overline{\rho_{\rm cdm}} = \frac{1}{2}\Sigma_t(z). \tag{3.20}$$

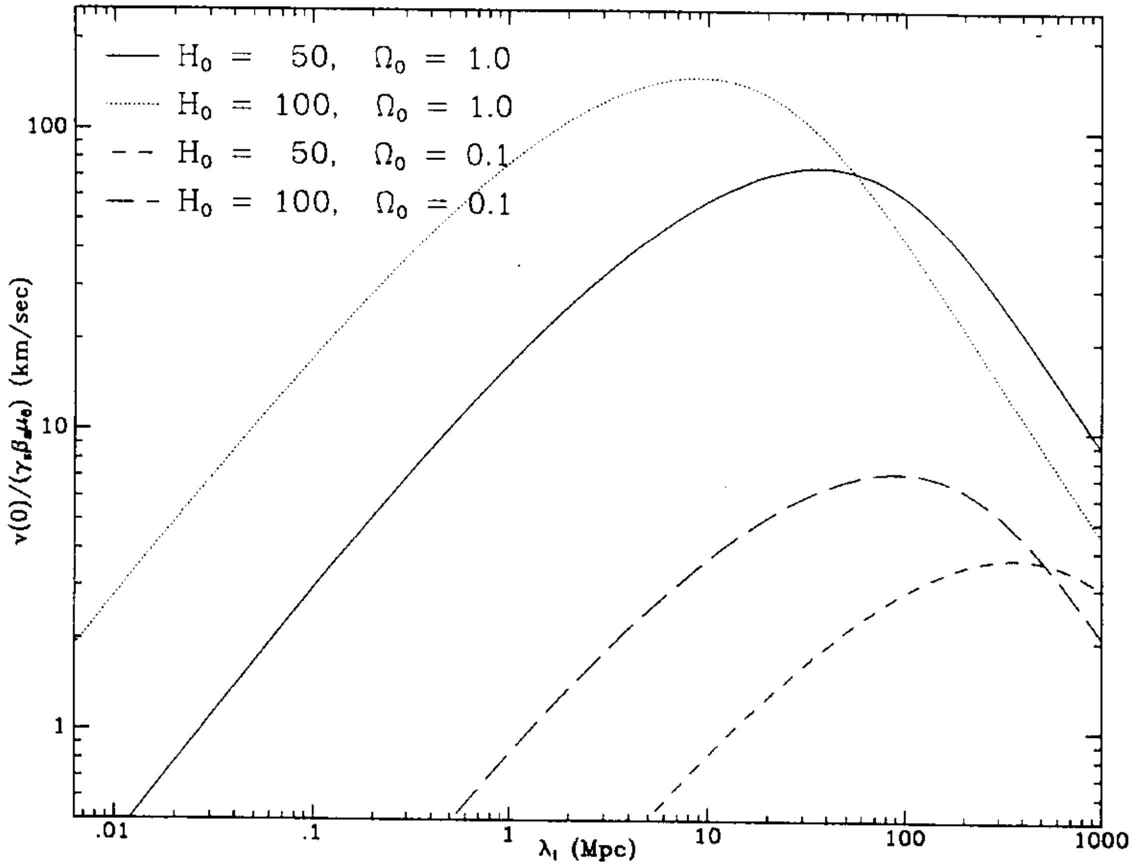

FIGURE 3. Wakes accrete matter by producing large scale peculiar velocity fields directed toward a central plane. These velocities are coherent over distances comparable to the horizon size at the time the wake was formed. The curves plotted are the present speed of these flows as produced by wakes formed at different epochs as a function of the distance over which the velocity field is coherent. The different curves are for different cosmological models as labelled. Beyond the accretion distance of a wake (see Figure 1) these large scale velocity fields would be the only effect of a nearby wake.

Thus the surface density of matter having gone through a caustic is confined to the narrow range between $\Sigma_c(z)$ and $\Sigma_t(z)$. The maximum surface density will be obtained for the same z_i which maximizes $\Delta(0)$. In Figure 2 we plot $\Sigma_c(0)$ as a function of z_i. The maximum surface density is $2 \times 10^{11}\, \gamma_s \beta_s \mu_6\, h_{50}{}^2\, M_\odot \mathrm{Mpc}^{-2}$ if $\Omega_0 \approx 1$. The peak surface densities depend on Ω_0 as $\Sigma \sim \Omega_0{}^2$ and, given the estimates of the observed surface density of walls, it seems unlikely that this model could explain these walls if $\Omega_0 \ll 1$. For this reason we concentrate on $\Omega_0 \approx 1$ universes for the rest of this paper.

Another quantity of interest is the large scale velocity fields produced by the wakes. The $z = 0$ CDM velocity toward the wakes for scales that are still expanding with the universe is plotted in Figure 3 against the estimated distance between wakes, λ_i. One hundred kilometer per second peculiar velocities coherent over a few tens of megaparsecs would be expected if $\Omega_0 \approx 1$. In an $\Omega_0 = 1$ universe the peculiar velocities grow as $v(z) = v(0)(1+z)^{-\frac{1}{2}}$, just as for any growing mode perturbation. The velocities due to a single wake are not sufficient to explain the recent observations of Burstein *et al.* (1986) or Collins, Joseph, and Robertson (1986) unless $G\mu h_{50} \approx 10^{-5}$ which is uncomfortably high and may already be excluded (see §II). Of course, a rather unlikely superposition of wakes could produce a very large velocity.

At redshifts $z \ll z_i$, it can be shown that the accretion has no preferred length or time scales, and therefore self-similar solutions to the equation of motion exist. Such one-dimensional self-similar collapse of CDM has been studied by Fillmore and Goldreich (1984, herafter FG). The initial conditions that FG used to motivate their work were pure density perturbations. In contrast, wakes have pure velocity perturbation initial conditions. However, at late times and on scales where the approximation of self-similarity is a good one, both these choices of initial conditions lead to the same scaling with time of the length scale turning around as a function of time. Thus the CDM flow for a wake should approach the CDM flow studied by FG with the appropriate choice of initial density profile. For a wake the fractional velocity perturbation from Hubble flow at any height above the symmetry plane is inversely proportional to the average surface density below it,

$$\delta_{v_i} = \frac{v_i}{4\pi G t_i} \Sigma_i^{-1}. \tag{3.21}$$

This is an $\epsilon = 1$, $n = 1$ perturbation in the notation of FG.

Of particular interest is the density structure of the wake. In our model the region far outside the collapsing matter remains at the background density (the infalling matter comes from "infinity"). This is an artifact of the plane parallel geometry and the assumption of an isolated wake. In a more realistic situation there would be underdensities produced between the wakes. We can justify our plane parallel geometry because the distance between wakes is ~ 40 Mpc (Vachaspati 1986), which is an order of magnitude bigger than the width of an individual collapsed structure. Although numerical integration is needed to obtain the detailed structure of the wake density profile on scales comparable to the turnaround scale, in the inner regions, FG's asymptotic power law approximations to the mass and density profiles can be used. In the inner regions, the surface density profile at redshift z is

$$\Sigma(x, z) = S\,\Sigma_t(0)(1+z)^{\frac{5}{2}} \left(\frac{x}{\Delta_0}\right)^{\frac{3}{4}}, \qquad x \ll \Delta_0((1+z))^{-\frac{4}{3}} \tag{3.22}$$

where the normalization S (≥ 1) must be determined numerically. We estimate it to be ~ 5. The density profile is,

$$\rho(x) = \frac{3}{2} S \rho_0 \left(\frac{x}{\Delta_0}\right)^{-\frac{1}{4}} (1+z)^{\frac{5}{2}} \qquad \rho_0 \equiv \frac{3{H_0}^2}{8\pi G} = 6.9 \times 10^{10} {h_{50}}^2 \ \mathrm{M_\odot \, Mpc^{-3}} \tag{3.23}$$

If $\Omega_0 \neq 1$ then the scales that collapse after $z = {\Omega_0}^{-1}$ will not follow this self-similar behavior but we would still expect the inner parts of the caustic region which collapsed earlier to retain their self-similar character. Note that this density profile is very shallow. However, the dissipation and concentration of the baryonic matter towards the central plane will lead to a somewhat steeper density profile, but sharp edges to the surfaces seen in the CfA survey are not obvious from our density profile.

In a realistic string scenario the wake would accrete an inhomogeneous medium due to the presence of other accreting strings and loops. Also, large nearby loops may disrupt wakes. These issues are considered in §5.

IV. EFFECTS OF FINITE VELOCITY DISPERSION OR SOUND SPEED

In this section we determine how a gas of of non-interacting particles with non-zero velocity dispersion or a fluid with non-zero sound speed reacts to a passing string. The former case is relevant if the universe is dominated by massive neutrinos and the latter will be relevant to the radiation-baryon fluid which exists before recombination and to the baryonic fluid after recombination. We will first discuss whether the perturbations will produce nonlinear structure by today. Later we will discuss what happens immediately around a passing string. This section is a preliminary investigation of cosmic string wakes in non-cold matter. As this is not the main subject of this paper we leave many questions unanswered and do not give as detailed a description of the wakes as we did in §3.

Wakes in the Radiation-Baryon Fluid before Recombination.

How will the fluid behave long after the string passes? Consider a spherical sub-horizon sized volume of a flat universe as viewed from its center-of-momentum frame. Because $\Omega_0 = 1$ the total Newtonian binding energy of of every fluid element is zero before it is perturbed. As a string passes by each fluid element will receive an impulse toward the string. If the string passes through the center of the sphere then for every fluid element the impulse given will have a component antiparallel to the Hubble flow and therefore every fluid element, except those very close to the string, will become, at least momentarily, bound. This will be true for fluid elements out to distances comparable to the horizon size. Beyond this distance the Newtonian approximation and the impulse approximation will both break down, and also there will be other competing wakes at this distance. Thus passing strings will produce horizon-sized region of gravitationally bound gas. However, the fluid may transfer its energy via sound waves to other collapsing regions thus damping the perturbations. This can only happen if the sound speed is large enough so that sound waves could move out of the initial horizon-sized region. The sound speed of the radiation-baryon gas is approximately constant at $c/\sqrt{3}$ in the radiation era and decays as $c\sqrt{z/z_{\rm eq}}$, in the matter era before recombination. Thus the comoving distance travelled by sound waves grows as z^{-1} in the rad era and as $\ln(1/z)$ in the matter era. Therefore the radiation-baryon fluid may damp perturbations on all sub-horizon scales in the radiation era. Due to the very slow logarithmic dependence on z and complications due to recombination (Silk 1969), damping in the matter era is a somewhat tricky subject. We shall not speculate about the degree of damping of wakes in the radiation-baryon fluid produced in between $z_{\rm eq}$ and $z_{\rm rec}$. After recombination the sound speed for the baryons is too low to prevent eventual collapse.

Neutrino Wakes in a Neutrino-Dominated Universe.

Everything that was stated in the previous paragraph applies equally well for a gas of non-interacting particles if one replaces the sound speed with the particle velocity dispersion. The damping mechanism in this case is "free-streaming" which means the actual motion of the particles rather than the motion of sound waves. For the case of a universe dominated by massive $10-30$ eV neutrinos the velocity dispersion behaves like that of the radiation-baryon fluid in radiation era and thus will damp the wake perturbations. However during the matter era the neutrinos become non-relativistic and their r.m.s. velocity is given by $v_{\rm rms} = 0.15\,c\,(1+z)/(1+z_{\rm eq})$ (assuming three species of neutrinos with two light and one massive enough to dominate the universe). The comoving scale damped after a given redshift, $z_{\rm i}$, is approximately the comoving distance moved by the average particle after that redshift. Taking the average particle to be moving at $v_{\rm rms}$, the damping scale is given by ($\Omega_0 \approx 1$)

$$\begin{aligned}\lambda_{\rm damp}(z_{\rm i}) &\approx 2\frac{v_{\rm rms}}{H_0\sqrt{\Omega_0}}(1+z_{\rm i})^{-\frac{1}{2}} \\ &= \frac{0.29}{h_{50}{}^3\Omega_0{}^{\frac{3}{2}}}\,{\rm Mpc}\sqrt{1+z_{\rm i}}, \quad (\Omega_0{}^{-1}-1) \ll 1+z_{\rm i} \ll 1+z_{\rm eq}.\end{aligned} \tag{4.1}$$

The wakes produced at $z \lesssim z_{\rm eq}$ will eventually collapse in a neutrino dominated universe if $\Omega_0 = 1$ but the interesting question is whether or not they will form nonlinear structure by today. In order for nonlinear collapse to occur the scale turning around must be greater than the damping scale at the time the wake is produced. i.e. $\lambda_{\rm nl} \gtrsim \lambda_{\rm i}$. It is probably roughly correct that once nonlinear collapse has begun it is similar to that for cold matter as describe in §4. If this is true then $\lambda_{\rm nl}(z)$ should be approximately given by equation (3.15), i.e.

$$\lambda_{\rm nl}(z) \approx \frac{8\pi}{5}\frac{\beta_s\gamma_s G\mu}{H_0\sqrt{\Omega_0}}\frac{\sqrt{1+z_{\rm i}}}{1+z} = \frac{0.030}{h_{50}\sqrt{\Omega_0}}\,{\rm Mpc}\,\beta_s\gamma_s\mu_6\frac{\sqrt{1+z_{\rm i}}}{1+z}, \tag{4.2}$$

for $\Omega_0{}^{-1}-1 \ll 1+z_{\rm i} \ll 1+z \ll 1+z_{\rm eq}$. This growth will continue until $z \approx \Omega_0{}^{-1}$. i.e. either until today or until the universe becomes open. Comparing (4.1) and (4.2) for present wakes we obtain

$$\frac{\lambda_{\rm nl}}{\lambda_{\rm damp}} \approx 0.1\,h_{50}{}^2\Omega_0{}^2\beta_s\gamma_s\mu_6. \tag{4.3}$$

which is independent of z_i. The fact that there is no z_i dependence suggest that all wakes produced after z_{eq} become nonlinear at the same time. We require that $\lambda_{nl} \gtrsim \lambda_{damp}$ for the wake to form nonlinear structure today. Keeping in mind that crudity of these calculations we see that the criterion for nonlinearity is ${h_{50}}^2{\Omega_0}^2\beta_s\gamma_s\mu_6 \gtrsim 10$. For some reasonable values of the parameters the inequality is satisfied and for others it is not. Thus whether neutrino wakes are important in a neutrino-dominated universe will depend on the details of the cosmology.

Baryon Wakes in a Neutrino-Dominated Universe.

Even if neutrino wakes do not collapse, the baryon wakes will. This is because the sound speed of the baryonic fluid is small and decreases rapidly after recombination. Below we shall show that the baryons immediately after recombination are slightly too hot to produce immediate nonlinear structure (if $G\mu \approx 10^{-6}$) but soon the baryons will start to produce wakes as in cold matter. The difference between these baryon wakes in a neutrino-dominated universe and the CDM wakes discussed in §3 is that the universe is matter-dominated but the cold fluid which is collapsing (the baryons) has a density much less than the critical density. These wakes are approximately described by (3.13) with $q = \frac{1}{2}$ and $\Omega_{cdm} \to \Omega_b/\Omega_0$. Here Ω_b is the fraction of the closure density in baryons. The equation for the displacement of baryons is thus

$$\Delta_Q'' + \frac{1}{2}\Delta_Q' - \frac{3}{2}\Omega_b\Delta_Q = 0, \tag{4.4}$$

which is only valid after recombination. Using the the initial conditions of (3.14) with $z_i < z_{eq}$ we obtain

$$\Delta_Q(z) = \frac{1}{(p_+ + p_-)}\frac{v_i}{H_0\sqrt{\Omega_0}}\left(\frac{(1+z_i)^{p_+-\frac{1}{2}}}{(1+z)^{p_+}} - \frac{(1+z)^{p_-}}{(1+z_i)^{p_-+\frac{1}{2}}}\right)$$

$$\text{where}\quad p_+ = \frac{1}{4}\left(\sqrt{1+24\Omega_b/\Omega_0}-1\right) \quad\text{and}\quad p_- = \frac{1}{4}\left(\sqrt{1+24\Omega_b/\Omega_0}+1\right). \tag{4.5}$$

The constraints from standard nucleosynthesis require $0.04 < \Omega_b{h_{50}}^2 < 0.14$ (Yang *et al.* 1984). The $\Omega_0 \approx 1$ neutrino cosmology with the largest baryon wakes have $h_{50} = 1$ and $\Omega_b = 0.14$, and the largest wakes are those with $z_i = z_{eq}$. The maximal wakes have present parameters

$$\frac{\Delta_Q(0)}{\beta_s\gamma_s\mu_6} = 9.8\,\mathrm{kpc} \qquad \frac{\Sigma_c(0)}{\beta_s\gamma_s\mu_6} = 9.5\times10^7\,\mathrm{M_\odot\,Mpc^{-2}} \qquad \frac{v(0)}{\beta_s\gamma_s\mu_6} = 0.14\,\mathrm{km/sec}. \tag{4.6}$$

If $\Omega_0 < 1$ then larger baryon wakes will occur because the exponent of the growing mode is increased. The limiting case is when $\Omega_b \approx \Omega_0$ in which case the neutrinos make a negligible contribution to the density and the universe is really baryon dominated. We will not discuss the baryon-dominated universe here because baryon wakes after z_{rec} are described by the CDM equation of §3. The neutrino cosmology with the minimal baryon wakes have $h_{50} = 2$ and $\Omega_b = 0.01$. In this minimal cosmology the maximal wakes have present parameters

$$\frac{\Delta_Q(0)}{\beta_s\gamma_s\mu_6} = 0.56\,\mathrm{kpc} \qquad \frac{\Sigma_c(0)}{\beta_s\gamma_s\mu_6} = 1.6\times10^6\,\mathrm{M_\odot\,Mpc^{-2}} \qquad \frac{v(0)}{\beta_s\gamma_s\mu_6} = 0.07\,\mathrm{km/sec}. \tag{4.7}$$

These parameters assume that the neutrino wakes themselves have not collapsed, which is a possiblity. If the neutrino wakes were to collapse than these structures would be totally disrupted. If this infall of baryonic matter is all that occurs then these structures are fairly cosmologically insignificant. These wakes grow only very slowly and they are much smaller and contain much less matter than the CDM wakes discussed in §3. In these models flow velocities actually decay with time, so the maximum post-shock temperature occurs when the wake is formed and is $\sim 10^3{\mu_6}^2$ K. These wakes could be significant in a neutrino-dominated universe if primordial star formation allows for explosive amplification. In this case one could get a large amount of star formation around the wakes, possibly leading to galaxies on the sheets traced out by these wakes. This scenario is not an entirely satisfactory explanation of the observed sheets of galaxies as the typical spacing of wakes produced at z_{rec} is $\sim 200\,h_{50}{}^{-1}\Omega_0{}^{-\frac{1}{2}}$ Mpc, which is somewhat too large in comparison to the observed structure. Furthermore, more baryons will collapse around small loops than wakes and structures formed via explosive amplification from these loop condensations would probably dominate the structure.

Fluid Flow near a Moving String.

We now turn our attention to what happens immediately around a piece of moving string. In §3 we have discussed the gravitational field of a string in terms of its form in the harmonic gauge of general relativity. This formalism has the most straightforward interpretation as a special relativistic extension of Newtonian gravity. It is, of course, equivalent to any other way of posing the theory of general relativity linearized about Minkowski space. In this section we will use a different choice of coordinates (gauge) in which the geometric interpretation of the gravitational fields close to the string is very simple. This gauge was first discovered by Vilenkin (1981). To summarize, Vilenkin showed that sufficiently near a string in Minkowski space the gravitational field may be represented by a flat space-time. The geometry is that of Minkowski space with a solid wedge cut out of each time-orthogonal hypersurface. The exposed faces are identified. The opening angle of the wedge is $8\pi G\mu$ and the string lies along the vertex of the wedge. The geometry described is singular at the location of the string. In an actual string there would be no singularity but there would be curvature inside the string (see Gott 1985). The approximations made in obtaining this geometry is that one is much closer to the string than its curvature radius and for a string in an expanding universe, one must also be much closer to the string than the horizon size. Using this geometry we will determine the fluid flow very near a piece of string. The question we are interested in is whether large overdensities are produced by focusing of streamlines near the string.

Since the space-time is everywhere flat we need not introduce gravitational fields into the fluid equations which greatly simplifies the problem. For streamlines close enough to the string we may approximate the upstream boundary conditions as a constant velocity, constant density flow toward the string. We will also ignore the Hubble flow which should also be a good approximation close enough to the string. The fluid flow in the "wedge geometry" is the same as fluid flow past a wedge which has been discussed by Landau and Lifshitz (1959, hereafter LL) §104. For supersonic strings the flow will have the form depicted in Figure 4. Here we will solve for the parameters of this relativistic flow. Let β_1 and β_2 be the pre- and post-shock fluid speed, respectively, as measured in the rest-frame of Figure 4. Using the notation of §3, β_s is the string speed in the fluid rest-frame. Therefore $\beta_1 = \beta_s$. It is required that there be no jump in the transvere velocity accross the shock so

$$\beta_2 = \beta_s \frac{\cos(\theta + 4\pi G\mu)}{\cos\theta} \cong \beta_s\,(1 - 4\pi G\mu \tan\theta)\,. \tag{4.8}$$

We denote the component of the pre- and post-shock fluid velocities perpendicular to the shock in the wedge geometry by $\beta_{\perp 1}$ and $\beta_{\perp 2}$, respectively. These are given by

$$\begin{aligned}\beta_{\perp 1} &= \beta_s \sin(\theta + 4\pi G\mu) \cong \beta_s \sin\theta(1 + 4\pi G\mu \cot\theta)\\ \beta_{\perp 2} &= \beta_2 \sin\theta \cong \beta_s \sin\theta(1 - 4\pi G\mu \tan\theta)\end{aligned} \tag{4.9}$$

The Rankine-Hugoniot jump conditions (LL §§82,126) read

$$\frac{(e_1+p_1)\beta_{\perp 1}}{1-\beta_{\perp 1}{}^2} = \frac{(e_2+p_2)\beta_{\perp 2}}{1-\beta_{\perp 1}{}^2}\,, \qquad \text{and} \qquad \frac{p_1 + e_1\beta_{\perp 1}{}^2}{1-\beta_{\perp 1}{}^2} = \frac{p_2 + e_2\beta_{\perp 2}{}^2}{1-\beta_{\perp 2}{}^2} \tag{4.10}$$

where e_1 and e_2 represent the pre- and post-shock energy density and p_1 and p_2 represent the pre- and post-shock pressure. Each quantity being as measured in the rest frame of the gas. If there is a

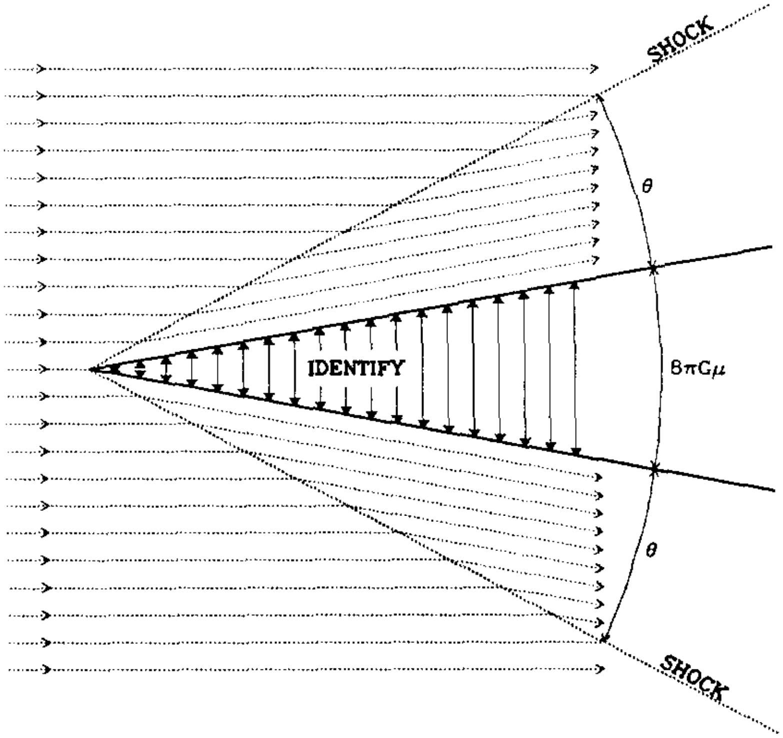

FIGURE 4. Here we plot a cross-section of the flow of a supersonic fluid near a cosmic string. The geometry is equivalent to flat space with a wedge removed. The solid lines are the two surfaces of the wedge. In the case of flow past a string the two planar surfaces of the wedge are actually the same plane. Using the reflection symmetry and single-valuedness of the flow we see that the boundary conditions at these surfaces is that the flow is parallel to the surfaces. Thus the flow is equivalent to flow past a solid wedge with a slip boundary condition. The opening angle of the wedge, $8\pi G\mu \sim 10-5$, is greatly exagerated in this figure. The light dotted lines are streamlines and the bold dotted lines are the positions of the two planar shock waves which will occur if the flow is sufficiently supersonic. The parameters of the flow are calculated in the text.

conserved density such as a particle number density, n, then an additional jump condition must be included:

$$\frac{n_1\beta_{\perp 1}}{\sqrt{1-\beta_{\perp 1}{}^2}} = \frac{n_2\beta_{\perp 2}}{\sqrt{1-\beta_{\perp 1}{}^2}}. \tag{4.11}$$

Equations (4.10) and (4.11) may be combined to give

$$\left(\frac{e_2+p_2}{n_2}\right)^2 - \left(\frac{e_1+p_1}{n_1}\right)^2 = (p_2-p_1)\left(\frac{e_1+p_1}{n_1^2}+\frac{e_2+p_2}{n_2^2}\right). \tag{4.12}$$

Given an equation of state $p(e,n)$ we can, in principle, solve these equations to obtain e_2, p_2, n_2, and θ in terms of e_1, p_1, n_1, β_s, and $G\mu$. We might expect that in many cases the shock would be weak because of the very small angle of the wedge. Let us therefore look for solutions that have small fractional changes in e, p, and n. Expanding to first order in $\Delta e \equiv e_2 - e_1$, $\Delta p \equiv p_2 - p_1$, and $\Delta n \equiv n_2 - n_1$ equations (4.10) and (4.12) yield

$$\frac{\Delta p}{\Delta e} \cong \beta_{\perp 1}{}^2 \qquad \text{and} \qquad \frac{\Delta n}{\Delta e} \cong \frac{n_1}{e_1+p_1}, \tag{4.13}$$

respectively. Combining with (4.9) and expanding to first order in $G\mu$ we obtain

$$\sin\theta \cong \frac{1}{\mathcal{M}_s} \qquad \frac{\Delta n}{n_1} \cong \frac{\Delta e}{e_1+p_1} \cong 4\pi G\mu \frac{\mathcal{M}_s{}^4}{(\mathcal{M}_s{}^2-\beta_s{}^2)\sqrt{\mathcal{M}_s{}^2-1}} \qquad \mathcal{M}_s \equiv \beta_s\sqrt{\frac{\Delta e}{\Delta p}} \tag{4.14}$$

It is well known that weak shocks are very nearly isentropic (LL §83) and to verify this for our shock note the equality

$$\frac{\Delta e}{\Delta n} \cong \left(\frac{\partial e}{\partial n}\right)_{s/n} = \frac{n}{e+p}, \tag{4.15}$$

where s is the entropy density and s/n is conserved for isentropic flow. Thus $\Delta p/\Delta e$ is the isentropic sound speed, i.e.

$$\frac{\Delta p}{\Delta e} \cong c_s{}^2 \equiv \left(\frac{\partial p}{\partial e}\right)_{s/n}. \tag{4.16}$$

This means that $\mathcal{M}_s$, defined in (4.14), is the Mach number of the string. It is apparent from (4.14) that our approximation of a weak shock is not self-consistent unless

$$(4\pi G\mu)^{-2} \gg \mathcal{M}_s{}^2 - 1 \gg (4\pi G\mu)^2. \tag{4.17}$$

Equation (4.14) indicates that the shock becomes strong as $\mathcal{M}_s{}^2 - 1 \rightarrow (4\pi G\mu)^2$, i.e. just barely supersonic flow. However, θ will never get larger than $\pi/2 - 4\pi G\mu$ when the shock front becomes a plane and not a wedge. At this point $\mathcal{M}_s \cong 1 + 4\pi G\mu$ and the shock is still weak. For smaller Mach numbers there are no realizable solutions with the geometry described in the figure. Rather a weak shock will proceed the string in what is called a bow wave. This point is discussed in LL§§86,104,114. For subsonic strings the flow around them will not include any shocks and will not contain large overdensities.

For very supersonic flow ($\mathcal{M}_s \gtrsim (4\pi G\mu)^{-1}$), or equivalently for a very cold fluid, the weak shock solution of equation (4.14) also breaks down. For fluids colder than this we will expect a strong shock and therefore large overdensities produced immediately behind the string. For example, a gas of monatomic hydrogen ($\beta_s \sim 1$) would undergo a strong shock only if $T \lesssim 10^3\,\mu_6{}^2$ K. This condition for a strong shock is not surprising, as it is equivalent to requiring that the sound speed of the gas be less than the velocity impulse given by the passing string. In considering strong shocks we will restrict our attention to a non-relativistic ($p \ll e$) ideal gas with constant ratio of specific heats: $\hat{\gamma}$. We will also neglect the Lorentz corrections to the jump conditions as the fluid velocities relative to the shock will be non-relativistic ($\beta_{\perp 1}, \beta_{\perp 2} \ll 1$) due to the smallness of θ. This problem has been done by LL §104 in the case of very strong shocks ($\mathcal{M}_s \gg (4\pi G\mu)^{-1}$) with the result

$$\theta \cong \frac{1}{2}(\hat{\gamma}+1)4\pi G\mu \qquad \frac{p_2}{p_1} \cong \frac{1}{2}\hat{\gamma}(\hat{\gamma}+1)\mathcal{M}_s{}^2(4\pi G\mu)^2 \qquad \frac{n_2}{n_1} \cong \frac{e_1}{e_2} \cong \frac{\hat{\gamma}+1}{\hat{\gamma}-1}$$

$$\mathcal{M}_s \equiv \beta_s \frac{c}{c_s} \qquad c_s \equiv \sqrt{\hat{\gamma}\frac{p_1}{e_1}}. \tag{4.18}$$

Thus a very thin region of shocked gas remains behind the string. This is just the beginning of the sheet-like wakes discussed in the previous section.

Flow of a Gas of Non-Interacting Particles near a Moving String.

Now we turn our attention to the behavior of a gas of non-interacting particles with a finite velocty dispersion as a string passes by. Let $v_{\rm d}$ be the velocity dispersion of the gas in its rest frame. In general, we expect such a gas to act similarly to a fluid with a sound speed equal to $v_{\rm d}$. From Liouville's theorem in general relativity we know that the phase-space density of gas is constant along trajectories in phase-space. The initial state of the gas before the string passes by is a homogeneous, isotropic distribution where the phase-space density only depends on the momentum. The velocity dispersion of the gas tells us something about this initial momentum distribution in the case of a non-relativistic gas, in particular if the initial distribution is

$$f(\mathbf{x},\mathbf{p}) = F(|\mathbf{p}|) \qquad \text{then} \qquad F(|\mathbf{p}_1|) \cong F(|\mathbf{p}_2|) \qquad \text{if} \qquad |\mathbf{p}_1 - \mathbf{p}_2| \ll m v_{\rm d}, \tag{4.19a}$$

at least in the region of momentum space where most of the particles are. Here m is the mass of the constituent particles. In the case of most relativistic gases, such as a Planck distribution or a relativistic Fermi-Dirac distribution,

$$F(x) \cong F(x+\delta) \qquad \text{if} \qquad \delta \ll x \tag{4.19b}$$

in the regions where most of the particles are. After the string passes, the distribution will not be isotropic or homogeneous but Liouville's theorem tells us that the the distribution may be written

$$f(\mathbf{x},\mathbf{p}) = F(|\mathbf{p} - \Delta\mathbf{p}(\mathbf{x},\mathbf{p})|) \tag{4.20}$$

where $\Delta\mathbf{p}(\mathbf{x},\mathbf{p})$ is momentum change of individual elements of phase-space fluid. While determining $\Delta\mathbf{p}(\mathbf{x},\mathbf{p})$ way be difficult, we may deduce certain things just by knowing the limits on the magnitude of $\Delta\mathbf{p}$. A passing string will only impart a momentum shift $|\Delta\mathbf{p}| \approx 4\pi G\mu\beta_s\gamma_s mc$ to a non-relativistic massive particle and $|\Delta\mathbf{p}|/|\mathbf{p}| \approx 4\pi G\mu\beta_s\gamma_s$ for a relativistic particle (see Kaiser and Stebbins 1984). Thus for all relativistic gases and for non-relativistic gases with $v_d \gg 4\pi G\mu\beta_s\gamma_s c$ the phase-space distribution is only slightly perturbed by a passing string, at least in the regions where most of the particles are. The real-space density, given by

$$\rho(\mathbf{x}) = \int f(\mathbf{x},\mathbf{p}) d^3\mathbf{p}, \tag{4.21}$$

can only be slightly perturbed. However, if $v_d \lesssim 4\pi G\mu\beta_s\gamma_s c$ then $\Delta\mathbf{p}$ will be large and the distribution function will be greatly perturbed. We would expect large overdensities immediately behind the string in this case, just as for a fluid with $c_s \lesssim 4\pi G\mu\beta_s\gamma_s c$.

V. INTERACTION OF WAKES WITH OTHER PERTURBATIONS

Comparison between the Mass Accreted by Loops and Wakes.

In this section it will be shown that most of the matter is in fact bound to loops of cosmic string, and this means that most of the matter collapsing onto string wakes has already accreted onto loops. In our estimates it is sufficient to use the simple asymptotic theory of the matter-dominated epoch that was discussed in §III, because accretion is efficient only in this epoch.

Our first task is to determine the fraction of the total matter that is swept up by wakes. Since the strings are initially moving at near-relativistic velocities, with the path of each string element being determined by both its local curvature and the expansion of the universe, the geometry of the wake surfaces will not possess any simple symmetry (see the discussion in §VII). We define the fraction ϕ of matter that collapses onto wakes created at a redshift $z_{i,w}$ to be the volume filling factor of those regions of the wake structures that have nonlinear overdensities, i.e., those regions bounded by the turnaround surfaces. This filling factor is given by,

$$\phi(z) \sim \left(\frac{\text{length in strings}}{\text{horizon length}}\right)\cdot\left(\frac{\text{distance moved by string in Hubble time}}{\text{horizon length}}\right)\cdot\left(\frac{\text{comoving thickness of nonlinear wake structure}}{\text{comoving horizon length at } z_{i,w}}\right) \tag{5.1}$$

The first two length ratios in equation (5.1) are scale-invariant in an asymptotically matter- or radiation-dominated universe. Let f be the ratio of the length in infinite strings to the horizon length (f is of course, always greater than unity. $f \sim 1$ is found in numerical simulations, Albrecht & Turok, 1985). In our Minowski approximation to the real spacetime, the ratio of the distance covered by a moving string to the distance out to the horizon is simply $\beta_s\gamma_s$. The comoving horizon length at $z_{i,w}$ is given in the matter era by

$$\lambda(z_{i,w}) = \frac{2}{3H_0(1+z_{i,w})^{\frac{1}{2}}} \tag{5.2}$$

(this agrees with figure 1 when $z_{i,w} = z_{eq}$). Further, the growing mode solution (3.16) for the thickness of the wake in comoving coordinates at redshift z is,

$$D(z) = \xi\frac{4v_i}{5H_0}\frac{(1+z_{i,w})^{\frac{1}{2}}}{(1+z)} = \xi\frac{16\pi}{5H_0}G\mu\beta_s\gamma_s\frac{(1+z_{i,w})^{\frac{1}{2}}}{(1+z)}. \tag{5.3}$$

Here $\xi \sim 0.63$ is a correction introduced so that (5.3) agrees with the maximum displacement shown in figure 1 when $z_{i,w} = z_{eq}$. The filling factor of the nonlinear wake structures today is therefore

$$\phi(0) = \frac{24\pi}{5}f\xi G\mu{\beta_s}^2{\gamma_s}^2(1+z_{i,w}) \tag{5.4}$$

which is about $0.06f\mu_6{\beta_s}^2{\gamma_s}^2$. Since f is greater than unity, a conservative estimate of the matter accreted onto the wakes is $\gtrsim 6\%$ for our typical choice of parameters. It is important to realize that we must be careful in our method of determining the ratio f because it strongly depends on the large-scale geometry. When numerical simulations of the evolution of string configurations with a sufficiently large dynamic range become available, it should be possible to determine f to better than our present order-of-magnitude estimate. Preliminary results from the CfA redshift survey indicate that the filling factor of the sheet-like distribution of galaxies is $\sim$ 20% (de Lapparent, talk given at 13th Texas Symposium, 1986). That the numbers compare favourably given all the uncertainties is probably fortuitous, as the rest of this section will show.

We now compare equation (5.4) with the fraction of matter which accretes onto loops. There is typically about one horizon-size loop per horizon volume, and the number density of loops that are smaller than the horizon is diluted both due to the expansion of the universe and due to gravitational radiation. For loops that have not yet radiated away, the scaling of the loop number density with loop radius depends on whether the loop entered the horizon before or after the epoch of matter-radiation equality. If $z_{i,l}$ is the redshift at which a loop of radius R_l enters the horizon, Brandenberger & Turok (1986) have shown that the number density of loops with radius between R_l and $R_l + dR_l$ at redshift z is

$$n(R_l,z)dR_l = \begin{cases} \nu_1 R_l^{-2} H_0^2 (1+z)^3 dR_l, & z_{i,l} < z_{eq} \\ \nu_2 R_l^{-\frac{5}{2}} H_0^{\frac{3}{2}} \frac{(1+z)^3}{(1+z_{eq})^{\frac{3}{4}}} dR_l, & z_{i,l} > z_{eq} \end{cases} \tag{5.5}$$

where

$$\begin{array}{llll} \nu_1 = \frac{9}{4}\nu, & z_{i,l} < z_{eq} & \text{(large loops)}, & \text{and} \\ \nu_2 = (\frac{3}{2})^{\frac{3}{2}}\nu, & z_{i,l} > z_{eq} & \text{(small loops)}. & \end{array} \tag{5.6}$$

The parameter ν has the value $\sim$ 0.01. The relations (5.5) hold whether the universe is radiation or mater dominated. The mass of a loop of radius R_l is $\beta\mu R_l$, with $\beta \sim 9$, according to numerical simulations by Albrecht and Turok (1985). The loops of interest to us, those that are seeds for the formation of galaxies and galaxy clusters, entered the horizon before z_{eq}. We will assume that the centre of motion of the loop is static because the its peculiar velocity is damped in a Hubble time (see Bertschinger 1986, for a discussion of accretion onto moving point masses). We will also assume that the loop began accreting at z_{eq}, since this assumption leads to an accreted mass that differs only by a factor of 2.5 from that obtained by taking into account accretion during the radiation era (Stebbins, 1986). The loop accretes a mass (Fillmore & Goldreich, 1984):

$$M(R_l,z) = \left(\frac{4}{3\pi}\right)^{\frac{2}{3}}\left(\frac{1+z_{eq}}{1+z}\right)\beta\mu R_l. \tag{5.7}$$

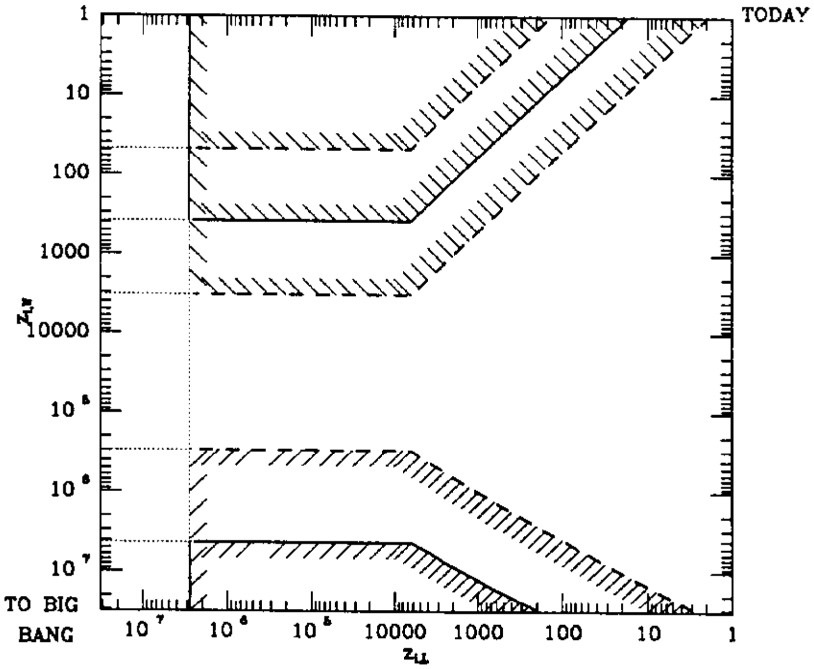

FIGURE 5. In this figure we show the funnel-like region where wakes are not disrupted by loops, in the plane of the initial redshift $z_{i,w}$ at which a wake is created, and the initial redshift $z_{i,l}$ at which a loop of radius R_l is created. We have used $\beta = 9$, $h_{50} = 1$ and $\epsilon = 1$, and the following three choices for ν and β_s : (a) 0.01, 0.2 (solid lines) (b) 0.01, 0.5 (outer dashed lines: the lower line is outside the plot boundaries) and (c) 0.05, 0.2 (inner dashed lines). The value $\nu \sim 0.01$ is found in numerical simulations of the radiation era, a higher value is possible during the matter era. $\beta_s = 0.2$ is also typical. A higher ν means a higher number density of loops, which causes the funnel to shrink inwards, whereas a higher value of β_s increases the surface area covered in wakes and so expands the funnel. Increasing any of h_{50}, β, or ϵ has the same effect as increasing ν. For the standard values $\beta_s = 0.2$ and $\nu = 0.01$, only the wakes formed at redshifts between about 7×10^6 and 4×10^2 are safe against disruption.

The mass density accreted onto loops is dominated by the smallest loops that survive until $z_{\rm eq}$. The smallest loop that survives past $z_{\rm eq}$ has a radius $R_l \sim \gamma G\mu t_{\rm eq}$, where $\gamma \sim 5$ (Brandenberger & Turok, 1986), so that

$$\rho_l(z) = \frac{9}{2}\left(\frac{4}{3\pi}\right)^{\frac{2}{3}} \frac{\beta\mu\nu}{\sqrt{\gamma G\mu}} H_0{}^2(1+z_{\rm eq})(1+z)^2. \tag{5.8}$$

The fraction of all matter in the loops is,

$$\frac{\rho_l(z)}{\rho_{\rm cdm}(z)} = 12\pi\left(\frac{4}{3\pi}\right)^{\frac{2}{3}} \beta\nu\sqrt{\frac{G\mu}{\gamma}}\left(\frac{1+z_{\rm eq}}{1+z}\right). \tag{5.9}$$

The numerical value of this ratio at the present time is $\sim 5.4\beta_9\,\nu_{0.01}\,\mu_6^{\frac{1}{2}}\,\gamma_5^{-\frac{1}{2}}\mathrm{h}_{50}^2$, which can be greater than unity for certain values of the parameters. The loops can, of course, accrete only as much CDM as available. What equation (5.9) tells us is that the loops are very efficient at accretion. A comparision with equation (5.5) tells us further that they are much more efficient than are the wakes. The matter which is now accreting onto wakes is not gas but the whole condensations around loops.

Competition between Wakes and Loops.

Since most of the matter falls into loops, it is quite conceivable that the coherent wake structure is disrupted by the existence of these loops. In this section, we show that the dominant wakes, i.e. those formed at about z_{eq}, are not destroyed by losing mass to loops. On the other hand, wakes produced in the radiation era, and those produced very recently, may be disrupted. These conclusions are based on a comparision of the magnitudes of the displacement suffered by a representative CDM particle due to the gravitational pull of loops and of wakes. Linear theory is adequate when comparing the displacements induced by loops and wakes of comparable initial redshift of formation. However, when considering very large loops, we include nonlinear evolution as well in deciding which objects survive.

We first recapitulate our results for the comoving displacement induced by a wake created at the redshift $z_{i,w}$ (equations 3.16 and 3.18)

$$\Delta_w(z) = \begin{cases} \frac{2}{5}\frac{v_i}{H_0}\frac{(1+z_{i,w})^{\frac{1}{2}}}{1+z}, & z_{i,w} < z_{eq} \\ \frac{v_i}{H_0}\frac{(1+z_{eq})^{\frac{3}{2}}}{1+z_{i,w}}\frac{1}{(1+z)}\ln\left(\frac{1+z_{i,w}}{1+z_{eq}}\right), & z_{i,w} > z_{eq}. \end{cases} \tag{5.10}$$

Since this is a result from linear theory, the displacement does not depend on the distance of the particle from the wake.

The mean separation in comoving coordinates between loops of the same radius R_l is

$$d(R_l) \sim \begin{cases} \nu_1^{-\frac{1}{3}} R_l^{\frac{1}{3}} H_0^{-\frac{2}{3}}, & z_{i,l} < z_{eq}, \\ \nu_2^{-\frac{1}{3}} R_l^{\frac{1}{2}} H_0^{-\frac{1}{2}} (1+z_{eq})^{\frac{1}{4}}, & z_{i,l} > z_{eq}. \end{cases} \tag{5.11}$$

Now we consider the effect of a single loop of radius R_l. The time-averaged gravitational field of the loop is simply the Newtonian field of a point object with mass $\beta\mu R_l$. The comoving displacement of a particle of CDM, which at the redshift $z_{i,l}$ of formation of the loop is at a comoving distance r_i from the centre of mass of the loop is (Peebles 1980, §19),

$$\Delta_l(z) = \frac{2}{5{H_0}^2}\frac{1+\min(z_{eq}, z_{i,l})}{(1+z)}\frac{\beta G\mu R_l}{r_i^2}. \tag{5.12}$$

This expression can be used during the matter era for loops created in the matter era, as well as for loops that started accreting in the radiation era, provided that the perturbation of the particle's position remains linear in the radiation era (with r_i always the comoving distance of the CDM particle from the loop centre of mass at $z_{i,l}$). If we take r_i to be the average interloop separation $d(R_l)$ given by equations (5.11), then

$$\Delta_l(z) = \begin{cases} \frac{2\nu_1^{\frac{2}{3}}}{5{H_0}^{\frac{2}{3}}}\beta G\mu R_l^{\frac{1}{3}}\left(\frac{1+z_{i,l}}{1+z}\right) & z_{i,l} < z_{eq} \\ \frac{2\nu_2^{\frac{2}{3}}}{5H_0}\beta G\mu\frac{(1+z_{eq})^{\frac{1}{2}}}{1+z} & z_{i,l} > z_{eq} \end{cases} \tag{5.13}$$

Notice that in our derivation the displacement caused by small loops does not depend on the properties of the loop such as its radius or equivalently, the redshift at which it entered the horizon. This is because most of the accretion occurs in the matter era. To determine whether loops created at $z_{i,l}$ can destroy wakes created at $z_{i,w}$, we will compare the magnitudes of their comoving displacements, $\Delta_l(z)$ and $\Delta_w(z)$. The wake structure would certainly survive if $\Delta_w(z)/\Delta_w(z) > 1$. Now, the displacement (5.13) is always less than the interloop distance for loops that enter much after a redshift $\sim 10^6$. This means that perturbations on the interloop distance scale are still very linear for these loops which are of greatest interest. The effect of such loops on wakes that were formed much earlier would be to simply pull over the entire wake structure without much disruption, and indeed we will see later that larger loops do not destroy wakes. On the other hand, loops that were formed early in the radiation era do induce a larger comoving displacement than a later wake. It is obvious on physical grounds, however, that in the limit of very small loops (assuming for the moment that they survive), a wake would not be destroyed, but would rather swallow up whole loop condensates. A loop created at a redshift of a few $\times 10^7$ will in isolation accrete all the matter within an interloop distance by the present epoch, and it is likely that these whole loop condensates, rather than homogeneous dust or gas, are accreted onto the later wakes. Now in fact, there is also a lower bound on the smallest size of loop that could be effective in destroying wakes. Since accretion of matter is efficient only in the matter era (there is actually some accretion onto loops even in the radiation era, but this can be ignored), we need consider only the effect of loops that have not gravitationally radiated away by z_{eq}. The smallest such loop has a redshift of formation,

$$z_{i,l}^{max} \sim \frac{z_{eq}}{\sqrt{\gamma G\mu}} \sim 3 \times 10^6 \, h_{50}^2 \, (\gamma_5 \mu_6)^{-\frac{1}{2}} . \tag{5.14}$$

Loops that formed earlier than this redshift would not alter significantly the wake structure. Therefore, we will assume that the wake structure created at $z_{i,w}$ is destroyed or badly distorted when the conditions $\Delta_l(0)/\Delta_w(0) > 1$ and $z_{i,l} < z_{i,l}^{max}$ are met.

We will find the following relation between the loop radius R_l and the redshift $z_{i,l}$ at its formation useful :

$$R_l = \epsilon \frac{2}{3H_0} \frac{\sqrt{1+\min(z_{eq},\ z_{i,l})}}{(1+z_{i,l})^2}. \tag{5.15}$$

The parameter ϵ is the ratio of the loop radius to the horizon ct at formation, and has been estimated in numerical simulations to be 0.2. Notice that matching equations (5.13) across z_{eq} gives $\epsilon \sim 0.45$.

Now we will consider the above conditions in more detail. We first consider the large loops created at redshift $z_{i,l}$ after z_{eq}. For large wakes, i.e. those with redshift $z_{i,w}$ smaller than z_{eq}, the first condition, *viz.*, $\Delta_l(0)/\Delta_w(0) < 1$ requires that

$$1 + z_{i,w} > \nu_1^{\frac{4}{3}} H_0^{\frac{2}{3}} \left(\frac{\beta}{4\pi\beta_s\gamma_s}\right)^2 (1+z_{i,l})^2 R_l^{\frac{2}{3}}, \qquad z_{i,w} \ \&\ z_{i,l} < z_{eq}$$

in order to avoid disruption by these large loops. Eliminating R_l yields :

$$1 + z_{i,w} > \nu^{\frac{4}{3}} \left(\frac{3\beta}{8\pi\beta_s\gamma_s}\right)^2 \epsilon^{\frac{2}{3}} (1+z_{i,l}) \sim 2.5 \times 10^{-3} (1+z_{i,l}) \nu_{0.01}^{\frac{4}{3}} \epsilon^{\frac{2}{3}} \beta_9^2 (\beta_s\gamma_s)^{-2}, \quad z_{i,w} \ \&\ z_{i,l} < z_{eq}. \tag{5.16}$$

The inequality (5.16) does not depend on the string parameter μ, because it enters in the gravitational field of both wakes and loops in the same way. The restriction on $z_{i,w}$ is very weak, so that effectively no matter era wakes are excluded.

Next, we will consider the wakes that were created in the radiation era. Let us define the quantity $\eta = (1+z_{i,w})/(1+z_{eq})$. The condition analogous to equation (5.16) that these wakes must satisfy in order that the displacement that they induce in the ambient matter is greater than that induced by the large matter era loops is,

$$\frac{\ln \eta}{\eta} \begin{array}{l} > \ \epsilon^{\frac{1}{3}} \nu^{\frac{2}{3}} \left(\frac{3\beta}{20\pi\beta_s\gamma_s}\right) \left(\frac{1+z_{i,l}}{1+z_{eq}}\right)^{\frac{1}{2}} \\ \sim \ 2.6 \times 10^{-4} (1+z_{i,l})^{\frac{1}{2}} \nu_{0.01}^{\frac{2}{3}} \epsilon^{\frac{1}{3}} \beta_9 (h_{50}\beta_s\gamma_s)^{-1}, \qquad z_{i,l} < z_{eq} < z_{i,w} \end{array} \tag{5.17}$$

It is immediately seen that the larger loops are not disruptive, both because the displacements they induce have not had much time to grow and because they are so rare.

Now, we can calculate the effect of smaller loops. If $\Delta_l(0)/\Delta_w(0) < 1$ for the survival of matter era wakes then :

$$1 + z_{i,w} > \nu^{\frac{4}{3}} \left(\frac{3\beta}{8\pi\beta_s\gamma_s} \right)^2 (1 + z_{eq}) \quad \sim 15.6 h_{50}^2 \nu_{0.01}^{\frac{4}{3}} \beta_9^2 (\beta_s\gamma_s)^{-2}, \qquad z_{i,w} < z_{eq} < z_{i,l} < z_{i,l}^{max}. \quad (5.18)$$

This is independent of the loop properties. Of course, physically it is unlikely that very small loops could actually disrupt very large wakes. It is more reasonable that the wakes accrete whole loop condensates, instead of homogeneous CDM, so that the condition (5.18) is meaningful provided the loop has not accreted all the matter within an interloop distance, which crudely requires simultaneously that $d(R_l > \Delta(0)$, i.e. that $z_{i,l} \leq 1.7 \times 10^7 \, (\beta_9 \mu_6 \nu_{0.01})^{-1}$, which is greater than $z_{i,l}^{max}$. Therefore, we can assume that the small loops in the shaded upper left region of figure 5 are indeed disruptive.

Lastly, small loops will not disrupt wakes created in the radiation era if

$$\frac{\ln \eta}{\eta} > \nu^{\frac{2}{3}} \left(\frac{3\beta}{20\pi\beta_s\gamma_s} \right) \quad \sim 0.02 \nu_{0.01}^{\frac{2}{3}} \beta_9 (\beta_s\gamma_s)^{-1}, \quad z_{eq} < z_{i,w} \;\&\; z_{i,l}, \quad \text{and} \quad z_{i,l} < z_{i,l}^{max}. \qquad (5.19)$$

In summing up, we see that wakes formed at redshifts between about $100\, z_{eq}$ and 10 for typical choices of all the parameters retain a coherent structure even though loops are far more numerous and more efficient at accretion. This is due to two reasons : (a) the loops retain their physical size once created and move away from the wakes of comparable spacing as the universe expands; and (b) small loops, though there is significant nonlinear accretion around them, can't destroy much bigger wakes. Big loops can't do much either, because their perturbations are very linear on typical i.e. interloop scales.

VI. BARYONS AND GALAXY FORMATION

From our discussion in the previous sections, it is clear that baryonic gravitational condensation around wakes must take place in either of two ways. If, as assumed in §III and §IV, the non-relativistic matter is a homogeneous mixture of CDM and baryonic gas, the coupling between the radiation and electrically charged baryon fluids will not allow the baryons to participate in the collapse onto a wake that is created before z_{rec}. However, after recombination the baryon sound speed drops tremendously. If the universe is dominated by CDM then the baryons will feel the gravitational field of the excess CDM mass accreted onto wakes and start to fall in. As long as $\Omega_b \ll 1$ the flow of the baryonic fluid will have essentially the same flow as the CDM after a few expansion times beyond recombination. Thus for $z < z_{eq}$ we may consider the flow of baryons into the wake, prior to their passage through caustics, to be same as the flow of CDM which was calculated in §III and §IV. This would be modified for $z > 100$ if the baryons were ionized because of Compton drag (Hogan 1979). Ionization of the universe is not implausible as there will be nonlinear structure at early times in string scenarios, however, the energy requirements for ionizing the universe at early times are rather formidable (Stebbins and Silk 1986) and in any case, we are most interested in baryonic infall at times after $z = 100$. In the nonlinear regime, there is an essential difference between the CDM flow and that of baryons : the latter would not pass through the caustics of the CDM but would be subject to shocks. Whether galaxy-size objects can form depends on whether the shocked gas can cool sufficiently and fragment. Provided that the shock velocity is in the range $10 - 30\,\mathrm{km/sec}$, the shocks are radiative if the baryonic component of the wake surface density exceeds $\sim 3 \times 10^{10}\, M_\odot\, \mathrm{Mpc}^{-2}$ (Silk 1985). This provides at least a necessary if not sufficient condition for galaxy formation. As can be seen from Figure 2 this condition is satisfied today for the largest wakes in an $\Omega \approx 1$, $\Omega_b > 0.01$ universe. As the surface density decreases with time this condition will certainly be satisfied at redshifts $\gtrsim 5$ when we expect galaxy formation occur. We can also draw upon the results of numerical simulations of the development of a one-dimensional sine wave perturbation in a dissipationless matter plus baryon universe (Shapiro and Struck-Marcell, 1985). The simulations confirm that the post-shock cooling of the baryons is quite efficient for smaller size 'pancakes', with a significant fraction cooling to form a dense condensation in the midplane. The pancake most relevant to our wakes (their case H) has a comoving wavelength of 1 Mpc in an $\Omega = 1$, $\Omega_b = 0.1$, $H_0 = 75\,\mathrm{km/sec}$ cosmology, in which the first accretion shock forms at $z \sim 6$, and $> 50\%$ of the baryons cool to temperatures below $10^4 K$ by $z \sim 3$. Presumably, the compression in the cool gas leads to Jeans instability and fragmentation into galaxy-size objects. If the universe is dominated by massive neutrinos, there is no nonlinear neutrino structure for the baryons to fall into at z_{rec}. However, if the baryon temperature falls below $10^3 \mu_6\, K$, as it will for at least some period at $z \sim 100$, then the baryons themselves will form nonlinear wakes just behind the strings. These wakes will grow but not as rapidly as CDM wakes because the baryons do not dominate the mass density of the universe.

However, as we have seen in §V, most of the mass actually accretes onto loops, and therefore it seems likely that the formation of galaxies is determined primarily by accretion onto loops and such accreting loops in the vicinity of a wake would fall in along with any unclumped matter. In this case, the physical significance of the surface density of the wake is not obvious (as it is for the scenario described previously) but it helps us in providing limits to the amount of matter that might be in galaxies associated with the wake. It is also certainly true that most loops will not have been accreted onto wakes. The recent observations of de Lapparent, Geller, and Huchra (1986) suggest that there may be very few luminous galaxies inside the individual bubbles. If this is indeed the case, then in order for string theories to survive, one must either explain how galaxy-sized clumps of baryons in galaxy-sized potential wells could exist for a Hubble time without making their presence known to human observers, or how these galaxy-sized potential wells managed to exclude baryons from remaining inside. The former conjecture seems the more plausible of the two. Even if galaxies only form from the condensations around loops, it is possible that the formation of luminous galaxies is enhanced in the overdense regions. For example, tidal interactions between gas-rich galaxies are known to greatly stimulate the rate of star formation. This would suggest that galaxies forming around loops that have fallen into wakes are prime candidates for developing into exceptionally luminous galaxies. The implication is that the luminosity distribution of galaxies is sensitive to the environment : low luminosity or nonluminous objects would be present everywhere, but the formation of luminous galaxies requires an additional stimulus like passage through a dense wake to turn on star formation. A potential problem with this scenario is that loops that are seeds for cluster formation would also accrete the galaxy-sized condensations (we thank the referee, Dr. Peebles, for pointing this out), and these cluster loops presumably have little to do with the wake distribution. This question could be resolved if we knew the loop-infinite string correlations, but this must await numerical simulations of the evolution of string configurations with a larger dynamic range than currently available. A more extreme possibility is that fragmentation of shocked wake gas may lead to the formation of massive objects of subgalactic scales that heat the surroundings and raise the Jeans mass, thereby seeding galaxy formation (Rees, 1986).

Thus it does not appear impossible to construct a viable theory of the galaxy formation and large-scale distribution based on cosmic string wakes. We *stress*, however, that such a theory does *not* follow immediately from our predictions of the large-scale matter distribution generated by cosmic strings. We find that it is essential to invoke astrophysical processes that can bias the light distribution to follow the wake structure rather than to follow the underlying matter distribution which is defined by the spectrum of loops. Whether this is indeed correct remains to be seen.

VII. THE GEOMETRY OF WAKES

The possibility that we may be able to observe wakes leads us to look in a little more detail at the geometry of the wake surfaces. As mentioned above the long string network consists of segments with typical radius of curvature close the the horizon size. Typically we expect the velocity of a string, $|\beta_s|$, to be $\sim .2 - .8$. If the period over which the velocity of a given string segment is coherent is also an expansion time then we might expect a tendency for the surfaces traced out to be more curved in the direction of motion of the string than in the transverse direction. This is because the radius of curvature in the transverse direction is $\sim t$ while the radius of curvature in the direction of motion is $\sim \beta_s t$ where $|\beta_s| < 1$. This can be tested quantitatively in numerical simulations.

The wake surface curvature in the direction transverse to the motion of the string is a measure of the curvature of the string. The curvature of the surface in the direction of motion of the string is a measure of the acceleration of the string in the directions perpendicular to the string's motion. The acceleration of a string segment perpendicular to its direction of motion is always in the direction of the curvature of the string (Vilenkin 1985). The surfaces traced out by strings will therefore never be negatively curved. The wake surface may have points of zero curvature and there will also be points where the curve is not smooth and the curvature is not defined. Both of these are expected to be sets of measure zero. To prevent confusion we mention that there is another acceleration term due to cosmological damping which we have not mentioned, but as it is anti-parallel to the direction of motion it will not affect the sign of the curvature. The positive curvature result applies to the wake surfaces and should be true for the wakes themselves if they are thin enough, at least initially. However, wakes may be distorted by the gravitational pull of nearby objects, and these distortions may cause the wake structure to become negatively curved at some places.

The wake surface may, of course, intersect other wake surfaces. This does not necessarily mean that two strings have actually collided. The place where the strings actually do collide will be points and not the one-dimensional lines of intersecting wake surfaces. When strings collide they may exchange partners (intercommute). Indeed intercommutation is thought to happen during most collisions (see Shellard 1986). If intercommutation does indeed take place, then two surfaces will join in a non-smooth way. The geometry of the wake surface starts to be singular at the point of collision and two lines of geometrical singularity will propagate out from this point on each of the two outgoing surfaces. The lines are points on which the two outgoing string segments are bent (i.e. the strings direction changes discontinuously along the string at a fixed time) and these bends propagate outward at the speed of light. The points on the string outside the bends have not yet received information about the intercommutation while the points within the two bends have had their trajectory changed by the intercommutation. While we expect that a wake formed from this rather convoluted geometry would disrupt itself to some extent there should still be some remnant of this geometry in the shapes of wakes at the present epoch.

VIII. SUMMARY AND CONCLUSIONS

In §III we have shown how perturbations can grow about an isolated wake in a universe dominated by cold dissipationless matter. We showed also that the wakes which have accreted the most matter are those that formed at $z \approx 2z_{eq}$. If $\Omega_0 \approx 1$, these large wakes are separated by about $40\, h_{50}{}^{-2}$Mpc and have a full width a tenth smaller in size : $3.0\mu_6 h_{50}{}^2$. This suggests that they have accreted between 8% and 20% of the matter in the universe. The present surface density of these large wakes is $\approx 2 \times 10^{11} h_{50}{}^2 \mu_6\, \mathrm{M}_\odot\, \mathrm{Mpc}^{-2}$. The velocity of matter falling towards these large wakes is about about $70 h_{50}{}^{-1}$ km/sec and the motion is coherent over distances comparable to $40\, h_{50}{}^{-2}$Mpc. The present density profile of the relaxed central region of these large wakes is roughly given by $\rho \approx 3 \times 10^3 h_{50}{}^2 (x/\mathrm{kpc})^{-1/4}\, \mathrm{M}_\odot \mathrm{kpc}^{-3}$. This expression ignores dissipation and the clumpiness of the medium, both of which would steepen the density profile.

In §IV we discussed how non-cold matter reacts to passing strings. Wakes in the radiation-baryon fluid formed before z_{eq} will be damped. Wakes in a neutrino-dominated universe were also discussed. Wakes in such a cosmology may or may not be important depending on H_0, Ω_0, and $G\mu$. Very weak wakes of baryons will form in a neutrino-dominated universe but will not be of much significance except possibly as seeds for self-amplified star formation. If the universe is dominated by cold dissipationless matter the baryons will be entrained in the CDM wakes after z_{rec}. We have studied the flow of fluids near a moving string. If the sound speed c_s is $\gg 4\mu_6$km/sec, only weak shocks or laminar flow will occur and no nonlinear structures are formed immediately around the string. If $c_s \ll 4\mu_6$km/sec, then nonlinear structure will form immediately. The conditions for strong or weak inhomogeneities around a string in a gas of non-interacting particles is the same as that for a fluid if one substitutes the particle velocity dispersion for the sound speed. The weak shocks which occur in the relativistic plasma during the early stages of our universe will be highly isentropic and will not produce significant entropy.

In §V we have estimated, in a highly idealized geometry, that the fraction of the closure density that accretes onto wakes is $\sim 22\%$. Most of the matter density accretes onto loops, of which only some fraction will be swept up by wakes. We have also confirmed by comparing the magnitudes of the displacements generated by wakes and loops in linear theory that the dominant wakes (those formed at $\sim, z_{eq}$), are not destroyed by loops of any size. The loops destroy wakes that are formed much earlier or later than z_{eq}.

In §VI we have discussed galaxy formation in a universe dominated by CDM. We have shown that baryons falling onto wakes in a CDM universe could cool to form galaxies. It is worth stressing that most loop condensations have *not* fallen into wakes and that star formation probably occurs in these condensations as well. However, as we have speculated, star formation might depend on the environment : it could be greatly enhanced in condensations around loops that have fallen into wakes, as compared to the condensations around loops in the ambient fluid. Galaxy formation aided by the presence of strings in the universe predicts that there should be low-luminosity galaxies in the 'voids'. If it is found that there are indeed neither galaxies or massive condensates inside the bubbles, then string theories with CDM may be ruled out.

Since it is possible that it is the galaxies on the wake surfaces of z_{eq} strings that are observed in the CfA redshift survey, in §VII we have used our knowledge of the equations of motion of infinite strings to comment on the geometry of these surfaces. The wake surfaces (or more exactly, the world-sheets traced out by the strings) always have positive curvature. The geometry of the wakes around two string segments that have intersected and exchanged partners was also discussed. Perhaps it is not too farfetched to hope that some remnant of this geometry persists even today, to be seen in the distribution of galaxies on the bubble surfaces.

The most prominent cosmic string wakes have a coherence length approximately equal to the coherence length of the surface of galaxies recently observed by de Lapparent, Geller, and Huchra (1986). While the wakes which are the subject of this paper may trace out sheets, most of the condensations of matter will occur around loops. There will be loops in between the wakes and we would therefore not expect there to be large regions empty of galaxy-size self-gravitating condensations. The observed voids could be reconciled with the string models only if it can be demonstrated that there are plausible mechanisms that bias the luminous galaxy formation to follow the wake structures, rather than the underlying mass distribution as defined by the spectrum of loop perturbations, *e.g.* through tidal interactions. A consequence of such a demonstration is that the probablity of quasar formation will also be enhanced: again tidal interaction with a neighboring object is the most likely mechanism to be responsible for fuelling the engine (commonly assumed to be a supermassive black hole) that powers quasars. Perhaps a pair of quasars forms in a wake: this would lead to physically associated pairs with mean separation up to ~ 40 Mpc and a very small velocity difference ($10 - 100$ km/sec) such objects may have already been detected (Turner *et al.* 1986, Shaver and Christiani 1986).

IX. ACKNOWLEDGEMENTS

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