Fermi National Accelerator Laboratory

MPI–PAE/PTh 21/87
FERMILAB-PUB-87/43-T
March 1987

Excited Weak Vector Bosons facing the High Energy and High Precision Frontier

U. BAUR*

Fermi National Accelerator Lab.
P.O.Box 500, Batavia, IL 60510 (USA)

M. LINDNER

Max-Planck-Institut für Physik und Astrophysik
- Werner-Heisenberg-Institut für Physik -
P.O.Box 40 12 12, Munich (Fed. Rep. Germany)

K.H. SCHWARZER

Sektion Physik, University of Munich
Theresienstrasse 37, Munich (Fed. Rep. Germany)

ABSTRACT

If the recently discovered weak bosons $W^{\pm}$ and Z were manifestations of a composite structure of the weak interactions rather than the commonly assumed gauge bosons of a fundamental gauge symmetry one is naturally led to expect a corresponding spectrum of excited weak vector bosons. Experiments at LEPI/SLC and LEPII may be uniquely suited to bring light into this question with their power to perform high precision tests of the standard model.

* Max-Kade-Fellow

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

In the course of recent years the belief has grown that with the Glashow-Weinberg-Salam model [1] one, finally, has uncovered a coherent picture of electroweak interactions. Low energy neutral current experiments [2] and the discovery of the $W^{\pm}$ and Z bosons at the CERN $p\bar{p}$ collider [3] have made this model in every sense of the word standard. However, triumph may not nearly be so near as one may be led to believe since important features of the standard picture are completely unwarranted by experiment so far – as the question of the self-couplings of the gauge bosons or of the crucial Higgs sector shows. For future testing of the standard picture it is therefore of tantamount importance to consider realistic alternatives and clearly isolate the standard predictions against the plethora of other models. Some alternatives are presented in superstring inspired models [4] and in models with a composite structure of the weak interactions [5]. In the latter it is most natural that one predicts excited partners of the $W^{\pm}$ and Z bosons; in a way one could even say that such particles were the essence of the idea of compositeness in these models.

In the following we will describe how this particular sort of compositeness would influence future highest energy experiments at the e^+e^- colliders now in the pipeline. It is clear that high energy and high precision together will give a most convincing testing ground. If experiments at new colliders like LEPI/II or SLC show no deviation from the standard model in sensitive measurables such as the mass of the Z, the leptonic decay width of the Z or various asymmetries, one will be able to set new stringent limits on the parameters of extensions of the standard model and thereby gain the important insight how accurately the standard model is tested. In this letter we would like to study this for the case of excited weak vector bosons.

For later use in our discussion it seems appropriate to review some of the basic properties of the excited weak vector bosons [6]. At the heart of such a discussion must be the hypothesis of generalized vector boson dominance (GVBD) which holds that the isovector part of the electromagnetic form factors of quarks and leptons is fully saturated by, in general n different isovector bosons. Here, as we want to isolate the most important physical effect, we will restrict ourselves to the lowest lying first excited isotriplet $\vec{W}'$ and can thus write [6] with e as the electric charge unit

$$\lambda_W g_W + \lambda_{W'} g_{W'} = e \quad . \tag{1}$$

The GVBD hypothesis is quite powerful and implies that the photon mixes with strengths λ_W and $\lambda_{W'}$ with the neutral bosons W_3 and W_3' which themselves couple to the fermions with strengths g_W and $g_{W'}$ and thus build up the source of the photon field in the Maxwell equations. The $\vec{W}'$ triplet takes part in the weak interactions and hence we get a handle on its parameters from the low energy neutral current data [2]. With the respective masses m_W and $M_{W'}$, the Fermi constant G_F, $\sin\theta_W$, θ_W being the Weinberg angle, and the parameter C_W describing non-standard additions to the neutral current we have [6]

$$\frac{g_W^2}{m_W^2} + \frac{g_{W'}^2}{M_{W'}^2} = \frac{8}{\sqrt{2}}\, G_F \quad , \tag{2}$$

$$\frac{\lambda_W g_W}{m_W^2} + \frac{\lambda_{W'} g_{W'}}{M_{W'}^2} = \frac{8}{\sqrt{2}}\, G_F\, \frac{\sin^2\theta_W}{e} \quad , \tag{3}$$

$$\frac{\lambda_W^2}{m_W^2} + \frac{\lambda_{W'}^2}{M_{W'}^2} = \frac{8}{\sqrt{2}}\, G_F\, \frac{1}{e^2}\left(C_W + \sin^4\theta_W\right) \quad . \tag{4}$$

For our subsequent discussion it is useful to reparametrize $\lambda_{W'}$ by

$$\lambda_{W'} = \lambda_W \left(\frac{m_W}{M_{W'}}\right)^n \quad . \tag{5}$$

with in principle arbitrary power n. Such a "duality relation" where n either equals 1 or 3/2 is well borne out in the context of the hadronic world which with the basic QCD behind serves as an illustrative guide to some hypercolor force binding the subconstituents of our $\vec{W}$ triplet and of its excited partner $\vec{W}'$. The case $n = 1$ corresponds to local duality while $n = 3/2$ is motivated by a bound state model with linearly rising confining potential (for a more detailed discussion see ref. [6]).

As we concern ourselves here more with the high energy implications of excited weak bosons we will leave the low energy discussion and just quickly note for our applications the following two facts. Firstly, through the mixing with the photon

[7] the physical neutral mass eigenstates, the Z and Z', are shifted upwards from the respective $W^{\pm}$ and $W'^{\pm}$ masses. One finds:

$$\left.\begin{array}{l} M_Z^2 \\ M_{Z'}^2 \end{array}\right\} = \frac{1}{2(1-\lambda_W^2-\lambda_{W'}^2)}\Big((1-\lambda_{W'}^2)m_W^2+(1-\lambda_W^2)M_{W'}^2$$

$$\mp\sqrt{((1-\lambda_{W'}^2)m_W^2-(1-\lambda_W^2)M_{W'}^2)^2+4\lambda_W^2\lambda_{W'}^2 m_W^2 M_{W'}^2}\ \Big) \ . \qquad (6)$$

For consistency, we have to require $\lambda_W^2+\lambda_{W'}^2<1$ in eq. (6). Secondly, the effective Lagrangian for the interactions of the Z and Z' with the fermions is obtained [6] after a diagonalization procedure:

$$\mathcal{L}_{Z,Z'} = g_Z\ (j_3^\mu - s_W^2\ j_{em}^\mu)\ Z_\mu + g_{Z'}\ (j_3^\mu - s_{W'}^2\ j_{em}^\mu)\ Z'_\mu \qquad (7)$$

where $(i = Z, Z')$

$$g_i = \frac{a_i}{b_i} \quad , \quad s_W^2 = \frac{e}{a_Z} \quad , \quad s_{W'}^2 = \frac{e}{a_{Z'}} \quad , \qquad (8)$$

$$a_i = \lambda_W g_W\ \frac{M_i^2}{M_i^2 - m_W^2} + \lambda_{W'} g_{W'}\ \frac{M_i^2}{M_i^2 - M_{W'}^2} \quad , \qquad (9)$$

$$b_i^2 = 1 - \lambda_W^2 - \lambda_{W'}^2 + \lambda_W^2\ \frac{m_W^4}{(M_i^2 - m_W^2)^2} + \lambda_{W'}^2\ \frac{M_{W'}^4}{(M_i^2 - M_{W'}^2)^2} \ . \qquad (10)$$

Here j_3^μ and j_{em}^μ denote the third component of the weak isospin current and the electromagnetic current, respectively. Due to the W_3'-photon mixing the coupling of the Z-boson deviates from the standard model prediction.

The observant reader will have gathered from the foregoing discussion how difficult it may become to pin down conclusively the excited weak vector boson hypothesis on the basis of low energy measurements alone. Granted that G_F, e and maybe an upper bound on C_W are quite well determined a big uncertainty remains in $\sin^2\theta_W$ where on the one hand the theoretical difficulties to deal with hadronic states and on the other low statistics in neutrino-lepton scattering leave us with relatively large error bars which reflect themselves in a poorly constrained W', Z' model. Adding the still broad uncertainties in m_W it becomes apparent how big a gain could be achieved in going to higher energy and high precision in our attempt to verify or rule out excited weak bosons. While a direct exploration of excited weak vector bosons probably needs to await the construction of a multi-*TeV* hadron collider [8], virtual effects of W' and Z' may already show up at much lower energies. As LEPI and SLC are getting readied now it is worthwhile to discuss the main advances one could achieve with these colliders. There are four quantities which are sensitive to the presence of additional heavy vector bosons :

i) The Z-mass M_Z. In the standard model including one-loop corrections it is given by

$$M_Z^2 = \frac{\pi\alpha(0)}{\sqrt{2}G_F}\,\frac{1}{\hat{s}^2\hat{c}^2}\,\frac{1}{1-\Delta r} \tag{11}$$

where $\hat{s}$ denotes the sine of the Weinberg-angle, $\hat{c}^2 = 1 - \hat{s}^2$, and Δr represents the radiative corrections. Once M_Z is determined to high precision at LEPI/SLC an excellent value of $\hat{s}^2$ follows.

ii) The ratio $\Delta = m_W/M_Z$. It can be determined either from a measurement of m_W at LEPII using the reaction $e^+e^- \to W^+W^-$ [9] and the value of M_Z coming from LEPI/SLC, or directly at the improved CERN $p\bar{p}$ collider (ACOL). Since Δ is supposed to be free from a systematic error on the energy scale, it will eventually provide a more precise test of the electroweak theory at ACOL than the separate measurements of m_W and M_Z. Both methods are expected to result in a similar precision for Δ. In the standard model $\Delta = \hat{c}$ whereas in our scheme (eq. (6)) it is a function of the excited vector boson parameters.

iii) The decay width of the Z into pairs of fermions with N_c color degrees of freedom,

$$\Gamma(Z \to f\bar{f}) = N_c\,\frac{M_Z}{48\pi}\,({v^i_{Zf}}^2 + {a^i_{Zf}}^2)\ . \tag{12}$$

Eq. (12) applies to both the standard and the alternative model but with different couplings v^i, a^i ($i = SM, Z'$). From eqs. (7) to (10) we find:

$$\begin{aligned} v^{SM}_{Zf} &= \frac{e}{\hat{s}\hat{c}}\,(T_{3f} - 2\hat{s}^2 Q_f)\ , \quad a^{SM}_{Zf} = -\,\frac{e}{\hat{s}\hat{c}}\,T_{3f} \\ v^{Z'}_{Zf} &= \frac{a_Z}{b_Z}\,T_{3f} - 2\frac{e}{b_Z}\,Q_f\ \ , \quad a^{Z'}_{Zf} = -\,\frac{a_Z}{b_Z}\,T_{3f}\ , \end{aligned} \tag{13}$$

where Q_f and T_{3f} denote the electric charge and the third component of the weak isospin of f, respectively. The cleanest Z decay is the one into charged leptons and is hence the one useful for high precision tests.

iv) The forward-backward and the left-right asymmetry, A_{FB} and A_{LR}, which are defined as

$$A_{FB} = \frac{F-B}{F+B} \quad , \qquad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{14}$$

with

$$F \pm B = \left[\int_0^x \pm \int_{-x}^0 \right] dcos\theta \quad \frac{d\sigma(e^+e^- \to f\bar{f})}{dcos\theta} \; .$$

Here $x \leq 1$ is the detector acceptance, θ the angle between the fermion f and the beam direction and $\sigma_{L,R}$ are the cross-sections for $e^-_{L,R} + e^+$ reactions. A_{LR} can only be measured if a longitudinally polarized e^--beam is available. In models with two massive vector bosons one obtains [10] in all generality

$$A_{FB}^{e^+e^- \to f\bar{f}} =$$

$$\frac{x}{1+\frac{x^2}{3}} \, \frac{(1-P_e)\sum_{h_f,h_e} h_f h_e \mid F(h_f,h_e) \mid^2 + 2P_e \sum_{h_f} h_f \mid F(h_f,1) \mid^2}{(1-P_e)\sum_{h_f,h_e} \mid F(h_f,h_e) \mid^2 + 2P_e \sum_{h_f} \mid F(h_f,1) \mid^2} \tag{15}$$

and

$$A_{LR}^{e^+e^- \to f\bar{f}} = P_e \, \frac{\sum_{h_f,h_e} h_e \mid F(h_f,h_e) \mid^2}{\sum_{h_f,h_e} \mid F(h_f,h_e) \mid^2} \tag{16}$$

with

$$F(h_f,h_e) =$$

$$-\frac{e^2}{s}Q_f + \frac{1}{4}\,\frac{(v_{if} - h_f a_{if})(v_{ie} - h_e a_{ie})}{s - M_i^2 + iM_i\Gamma_i} + \frac{1}{4}\,\frac{(v_{jf} - h_f a_{jf})(v_{je} - h_e a_{je})}{s - M_j^2 + iM_j\Gamma_j} \; ; \tag{17}$$

$h_{f,e} = \pm 1$ being the helicities of f and e, $f \neq e, \nu_e$ any fermion, M_i and Γ_i the mass and width of the respective vector boson $i = Z, j = Z'$, $\sqrt{s}$ is the *cms* energy and P_e the degree of longitudinal polarization of the e^--beam. To obtain the standard model asymmetries the last term in eq. (17) has to be dropped. The couplings v_{if} and a_{if} are given by eq. (13); v_{jf} and a_{jf} can be obtained by replacing the subscript Z by Z' in the second line of eq. (13). The asymmetries are very sensitive to new physics, especially A_{LR} is.

Quantities i) to iv) define a most appropriate arena in which the standard model can be tested against alternatives like ours. As we are interested in high precision tests at LEPI/SLC and LEPII energies we should take into account radiative corrections. Since the model discussed here is non-renormalizable due to its effective character a full renormalization program cannot be carried out. Nevertheless, the electromagnetic leading log corrections which in the standard model account for the main loop effects can be incorporated [11]. This treatment modifies e and $\sin^2\theta_W$, as determined from low energy experiments, in the same way as in the standard model provided that the particle spectrum below m_W is the same. Wherever e appears it is therefore understood as $e = \sqrt{4\pi\alpha(M_Z)}$ with $\alpha(M_Z) \approx 1/128$. $M_{W'}$, $g_{W'}$ and $\lambda_{W'}$ can in first approximation be assumed to evolve with q^2 in the same way as m_W, g_W and λ_W. Since we can only include electromagnetic corrections in the effective theory, composite W-boson model predictions should be compared with standard model calculations done to the same approximation. In the standard model this comes very close to the full one loop treatment for the quantities studied in this letter. Thus in eq. (11) $\alpha(0)/(1-\Delta r)$ is, to quite good accuracy, replaced by $\alpha(M_Z)$.

For our high precision comparison we suggest now the following strategy. The first aim in SLC or LEPI certainly will be an exact determination of the position of the Z-pole leading ultimately to a value of M_Z accurate within an error [12] of $\delta M_Z = 28\ MeV$. According to eq. (11) this leads to the excellent extraction of a value of $\hat{s}^2 = \sin^2\theta_W$ to within $\delta\hat{s}^2 = 0.0002$ accuracy. This value of $\hat{s}^2$ does not yet reveal anything about the validity of the standard model but represents nevertheless the most important landmark for high precision tests. As the Weinberg angle enters into many other physical processes it now becomes available for cross checks. A first such cross check would be done in looking at the value of $\hat{s}^2$ coming from low energy neutrino reactions $(\nu e, \nu N)$. Unfortunately, as was pointed out already in the above, the accuracy achievable within present and foreseeable low energy experiments [12] will not be better than $\delta\hat{s}^2 = 0.005$, i.e. 25 times worse than in the Z-pole determination. It would naturally be luring to speculate about a possible discrepancy between high and low energy $\hat{s}^2$ which clearly would point to new physics but let us assume here that the standard model has passed its first rough test. We will hence take the $\hat{s}^2$ and use it as the parameter "$\sin^2\theta_W$" in eq. (3) whereby we allow for a possible deviation $\Delta\hat{s}^2 = "\sin^2\theta_W" - \hat{s}^2$ of at most ± 0.005 corresponding to the low energy experimental accuracy.

For a given value of M_Z and of the parameter n we then determine numerically from eq. (6) the corresponding mass of the W, m_W, for specified excited vector mass $M_{W'}$ using the constraint eqs. (1), (2), (3) and (5). We have thus obtained the five unknowns λ_W, $\lambda_{W'}$, g_W, $g_{W'}$ and m_W from these five equations and have thus specified the couplings g_Z, $g_{Z'}$, s_W^2 and $s_{W'}^2$ appearing in eqs. (7) to (10) which are essential in the determination of the leptonic decay width of the Z and of the various asymmetries according to eqs. (12) to (17). Naturally the ratio m_W/M_Z is thus fixed as well and available for comparison with the standard $m_W/M_Z = \sqrt{1-\hat{s}^2}$. Eq. (4) serves to determine C_W which experimentally [13] has an upper limit ≤ 0.01 on it and is always non-negative in our model. If C_W exceeds this limit an excited vector boson of above given mass $M_{W'}$ is ruled out for specified M_Z and $\Delta\hat{s}^2$ – practically this means only, however, that a W' of mass less than 120 GeV is ruled out for given M_Z around 92 GeV. From the foregoing it is clear how we derive the non-standard and standard predictions for the quantities we now want to compare. In Fig. 1 we have collected the predictions for the ratio $\Delta = m_W/M_Z$ and the width $\Gamma(Z \to \ell^+\ell^-)$ and in Fig. 2 likewise for the asymmetries versus $\sqrt{s}$ for $n = 1$ in $e^+e^- \to \mu^+\mu^-$. Figs. 1a and c show the absolute values versus M_Z for the standard model and a Z' model with $M_{W'} = 500$ GeV, $n = 1$ and $\Delta\hat{s}^2 = 0$, $\pm$ 0.002, and $\pm$ 0.005 respectively. Throughout our further analysis we have taken as exemplary a value of $M_Z = 92$ GeV. Figs. 1b and d show how the deviations from the standard model develop with $\Delta\hat{s}^2$ and indicate the very weak dependence on the duality parameter n and the mass $M_{W'}$. For very small masses of the W', $M_{W'} \leq 200$ GeV, somewhat stronger disturbances would be obtained. In Fig. 2 the effects of lower and higher mass W' are characterized. Similar curves can be obtained for $n \neq 1$.

As one can see from Figs. 1 and 2, although $|\Delta\hat{s}^2| \leq 0.005$, quite clear signals for excited weak vector bosons could be observed in LEPI/II and SLC. Therefore, if the standard model was proven right within experimental accuracies, tight bounds on the W'-parameters would follow. In order to obtain those we take the respective standard model prediction as the central value and allow variations around it coming from the experimental accuracies in which a W' could still hide itself. We refer to a separate publication [10] for a more detailed discussion of those accuracies, here it may suffice to list them up. For Δ an error of 0.2% may be achievable with ACOL, and certainly should be within reach for LEPI/II. $\Gamma(Z \to \ell^+\ell^-)$ is expected to be determined at LEPI/SLC with an error of $\delta\Gamma/\Gamma = 2\%$. Finally, for the asymmetries at the Z-peak and at LEPII energies ($\sqrt{s} = 190 GeV$) the estimated accuracies are: $\delta A_{FB}(M_Z) = 0.01$, $\delta A_{LR}(M_Z) = 0.02$, $\delta A_{FB}(190\ GeV) = 0.03$, and $\delta A_{LR}(190\ GeV) = 0.02$. In case of the asymmetries at $\sqrt{s} = M_Z$ the experimental errors would in principle be smaller but the standard model prediction is not known more accurately due to the as yet undetermined mass of the t-quark and the Higgs-boson, which enter the calculation at the one loop level.

From Fig. 1d it is clear that the width $\Gamma(Z \to \ell^+\ell^-)$ is practically unable to yield a bound on the W'-parameters. Δ at the level of 0.2% accuracy does give bounds significantly better than the ones from $|\ \Delta\hat{s}^2\ | \leq 0.005$ which however go away when the error increases to 1% (see Fig. 1b). The most stringent limits, finally, may result from the asymmetries and we give the bounds on $g_{W'}$ versus $M_{W'}$ in Fig. 3a and b where in a only the forward-backward asymmetry at $\sqrt{s} = M_Z$ is used and in b all four. If only $A_{LR}(M_Z)$ is added to $A_{FB}(M_Z)$ practically the same bounds as in b would apply whereas adding only $A_{FB}(190\ GeV)$ to $A_{FB}(M_Z)$ would only slightly improve the bounds in a. The bounds depend on the duality parameter n which is exhibited in Fig. 3. This means that in contrast to an isoscalar weak vector boson Y coupled to the weak hypercharge current [10] the higher energy of LEPII is of much less advantage than longitudinal polarization of the incident electron beam at LEPI/SLC. The limits derived from the ratio Δ would lie between the ones of Fig. 3a and 3b. We have shown the bounds for a mass region inaccessible to LEPII – otherwise LEPII could directly produce a Z'-boson. From Fig. 3 we observe that if experiments at LEP and SLC agree with the standard model within the possible experimental accuracies, a W'-boson must be very weakly coupled to fermion pairs if it is allowed to exist. Contrary to the Y-boson case no lower bound on a Z' mass in excess of the LEPII *cms* energy can be reached here as $|\ g_{W'}\ |$ is not bounded from below whereas for the isoscalar Y, $g_Y > e$ must hold for all masses M_Y.

In summary we have shown how excited weak vector bosons would fare at

LEPI/II and SLC. If experiments there show no deviations from the standard model strong bounds on the coupling constant of a W'-boson can be derived. In particular, a longitudinally polarized e^--beam at LEPI/SLC could greatly strengthen these limits and thus our belief in the standard model or – thinking more courageously – open our eyes to new physics beyond the standard model.

ACKNOWLEDGEMENTS

We would like to thank the members of the working group on compositeness of the LEP200 Workshop, in particular A. Blondel, B. Schrempp, F. Schrempp and D. Treille for useful and stimulating discussions. One of us (U.B.) would like to acknowledge financial support of the Max-Kade-Foundation, New York.

REFERENCES

[1] S.L. Glashow, *Nucl. Phys.* **22** (1961) 579;
S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, ed. N. Svartholm, Almquist and Wiksells, Stockholm 1968, p. 367.

[2] G. Barbiellini and C. Santoni, CERN-EP/85-117 (1985) preprint.

[3] UA1 collaboration, G. Arnison et al., CERN-EP/85-185 (1985) preprint;
UA2 collaboration, J.A. Appel et al., *Z. Phys.* **C30** (1986) 1.

[4] M. Dine et al., *Nucl. Phys.* **B259** (1985) 549;
J.P. Derendinger, L.E. Ibañez and H.P. Nilles, *Nucl. Phys.* **B267** (1986) 365.

[5] For a review see e.g. W. Buchmüller, *Acta Physica Austriaca, Suppl.* **XXVII** (1985) 517.

[6] M. Kuroda and D. Schildknecht, *Phys. Lett.* **121B** (1983) 173;
U. Baur, D. Schildknecht and K.H. Schwarzer, *Phys. Rev.* **D35** (1987) 297.

[7] R. Kögerler and D. Schildknecht, CERN-TH 3231 (1982) (unpublished);
H. Fritzsch, R. Kögerler and D. Schildknecht, *Phys. Lett.* **114B** (1982) 157;
B. Schrempp and F. Schrempp, DESY 84-055 (1984) (unpublished).

[8] U. Baur, D. Schildknecht and K.H. Schwarzer, *Phys. Rev.* **D35** (1987) 297.

[9] G. Barbiellini et al., in "Physics at LEP", CERN Yellow Report 86/02 (1986), Vol. 2, ed. J. Ellis and R.D. Peccei.

[10] U.Baur, M. Lindner and K.H. Schwarzer, MPI-PAE/PTh 74/86, FERMILAB-PUB-86/163-T preprint, to be published in Nucl. Phys. B.

[11] L. Hall, *Nucl. Phys.* **B178** (1981) 75;
S. Dawson, J.S. Hagelin and L. Hall, *Phys. Rev.* **D23** (1981) 2666.

[12] G. Altarelli, in: "Physics at LEP", CERN Yellow Report 86/02 (1986), Vol. 1, ed. J. Ellis and R.D. Peccei.

[13] S.L. Wu, *Phys. Rep.* **107** (1984) 59.

FIGURE CAPTIONS

1. The standard model and excited weak vector boson predictions for
   a) the ratio $\Delta = m_W/M_Z$ and
   c) the width $\Gamma(Z \to \ell^+\ell^-)$
   versus M_Z for various values of $\Delta\hat{s}^2 = " \sin^2\theta_W" - \hat{s}^2 = 0$ (dotted), $\pm$ 0.002 (dashed), $\pm$ 0.005 (dash - dotted) and the standard model (solid curve). The sign of $\Delta\hat{s}^2$ is indicated in the plots, the mass of the W' is taken as 500 GeV and the parameter n is chosen to be 1. In b) and d) we show how the deviations from the standard model develop with $\Delta\hat{s}^2$ and how they are essentially independent of $M_{W'}$ and the duality parameter n (solid line : $M_{W'} = 500\ GeV$, $n = 1.0$; dashed curve : $M_{W'} = 500\ GeV$, $n = 1.5$; dash - dotted curve : $M_{W'} = 1000\ GeV$, $n = 1.0$); M_Z in b) and d) was chosen as 92 GeV.

2. a) The forward-backward asymmetry $A_{FB}^{e^+e^-\to\mu^+\mu^-}$ for unpolarized e^--beam and ideal detector acceptance (x=1) and
   b) the left-right asymmetry $A_{LR}^{e^+e^-\to\mu^+\mu^-}/P_e$
   for the standard model (dashed) and an excited vector boson of mass 300 and 500 GeV respectively with a $\Delta\hat{s}^2$ as defined in Fig. 1 of +0.005 (solid line for each $M_{W'}$) and -0.005 (dotted curve for each $M_{W'}$). Note that the shapes of the variant curves need not be symmetric around the standard model curves with variation of $\Delta\hat{s}^2$. M_Z was chosen as 92 GeV and $n = 1$.

3. Limits on the coupling strength $g_{W'}$ of an excited weak vector boson of mass $M_{W'}$ for the case of the duality parameter $n = 1$ (solid lines) and $n = \frac{3}{2}$ (dash - dotted curves) coming from
   a) the asymmetry $A_{FB}^{e^+e^-\to\mu^+\mu^-}$ at the Z-pole only and
   b) all four asymmetries $A_{FB}^{e^+e^-\to\mu^+\mu^-}$ and $A_{FB}^{e^+e^-\to\mu^+\mu^-}/P_e$ at the Z-pole and $\sqrt{s} = 190\ GeV$ respectively.
   The allowed region of $g_{W'}$ lies between the upper and the corresponding lower curve. For the Z-mass we used 92 GeV. Other cases are discussed in the text.

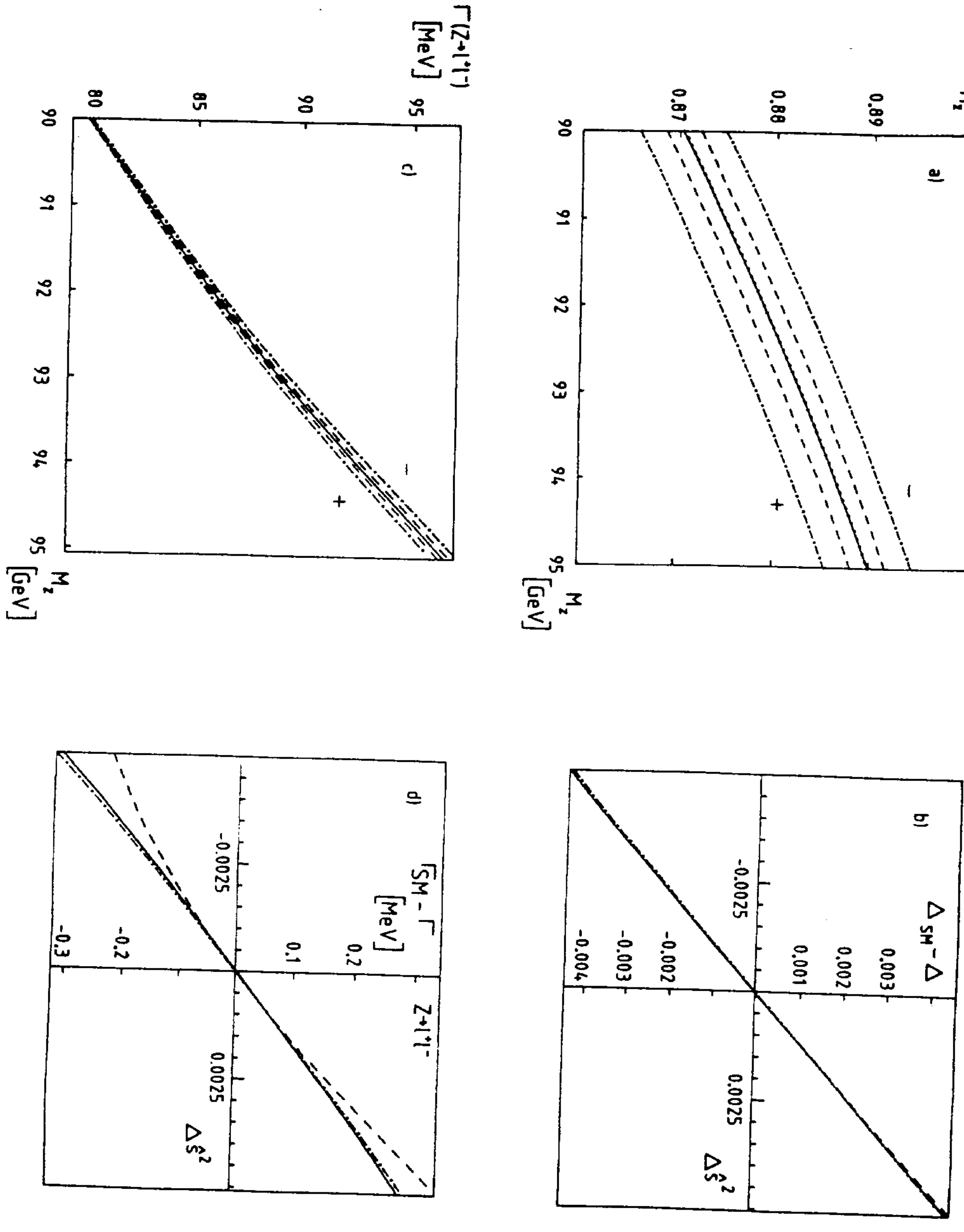
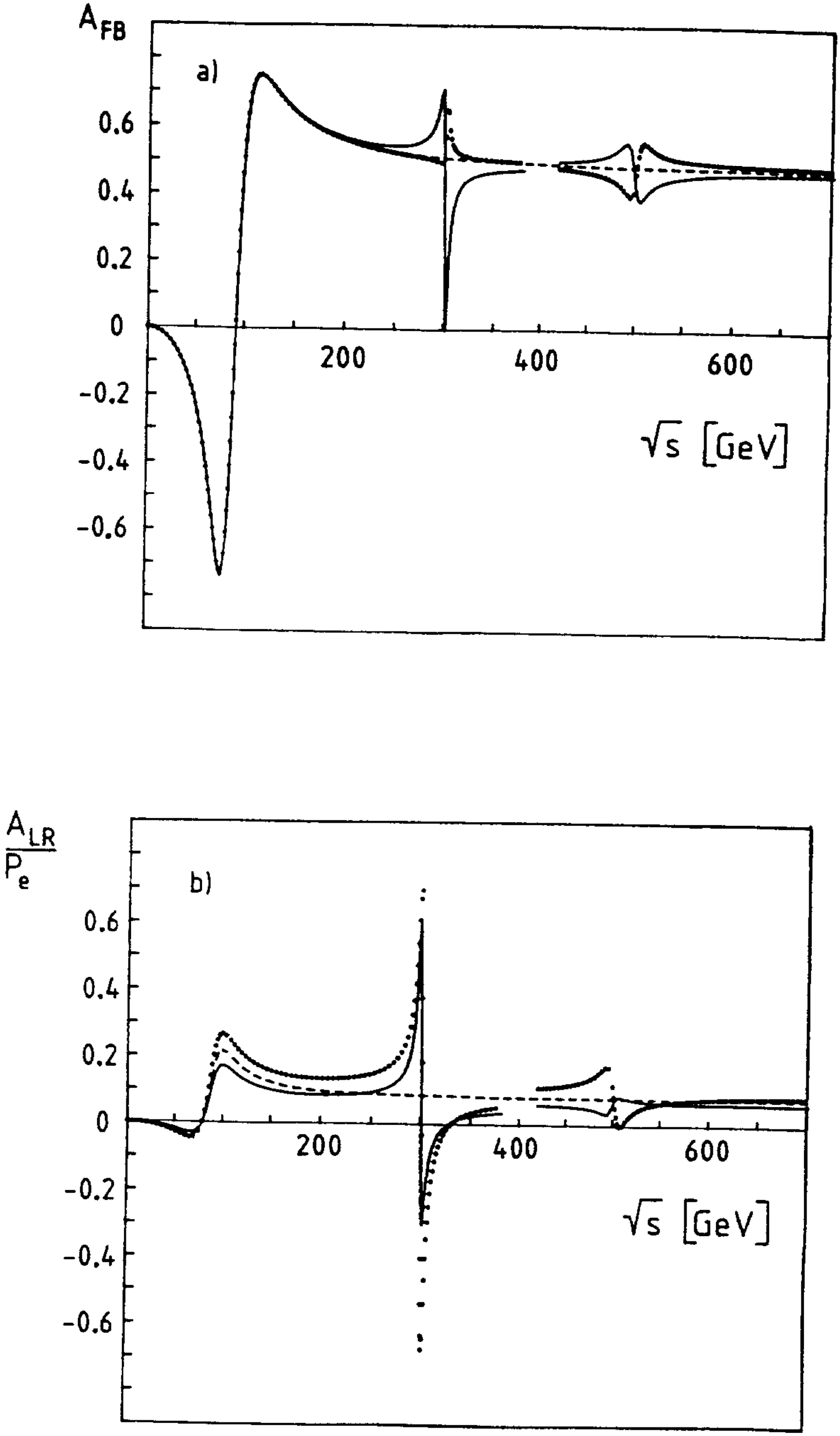
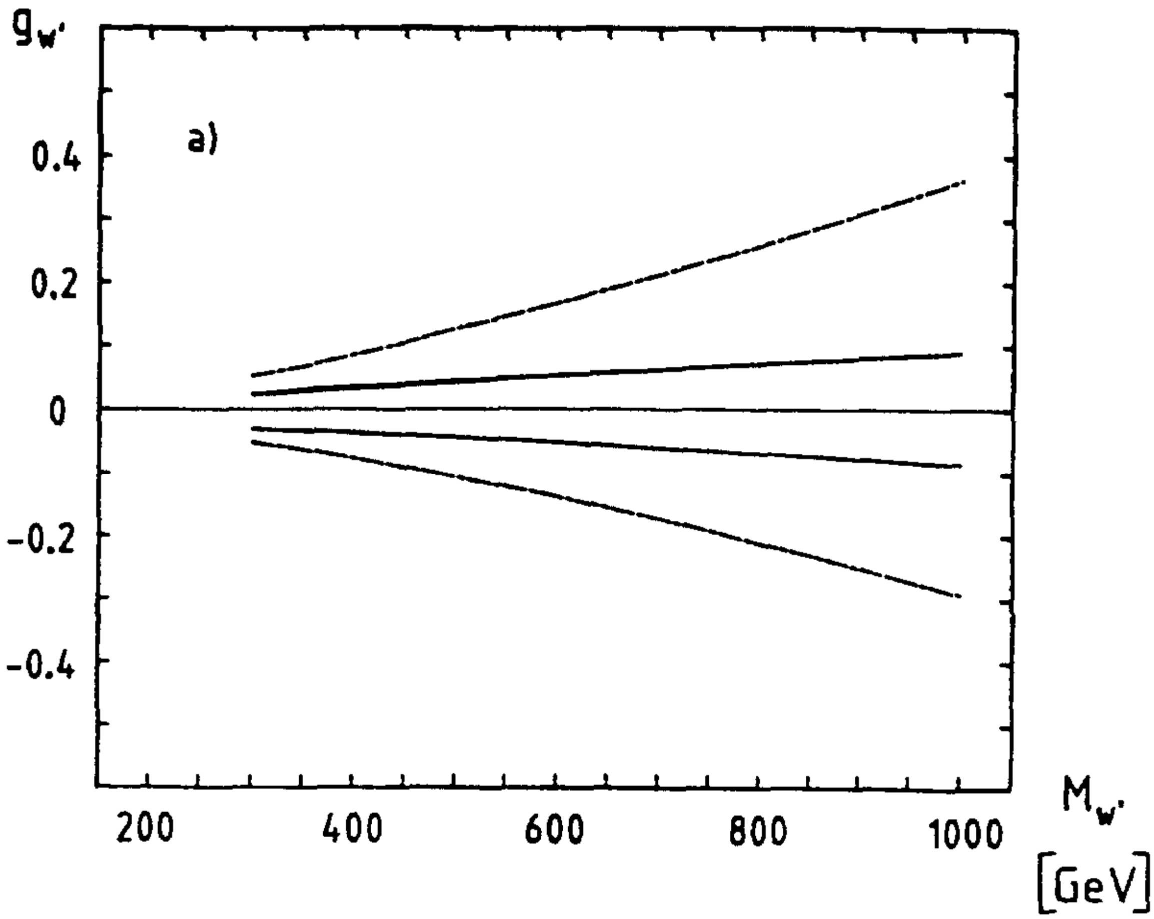

a)
b)
c)
d)
Δ = m_W/M_Z
0.87
0.88
0.89
90
91
92
93
94
95
M_Z [GeV]
Γ(Z→l⁺l⁻) [MeV]
80
85
90
95
Δ_SM − Δ
0.003
0.002
0.001
-0.002
-0.003
-0.004
-0.0025
0.0025
Δŝ²
Γ_SM − Γ [MeV]
Z→l⁺l⁻
0.2
0.1
-0.2
-0.3
+
−

Fig. 1

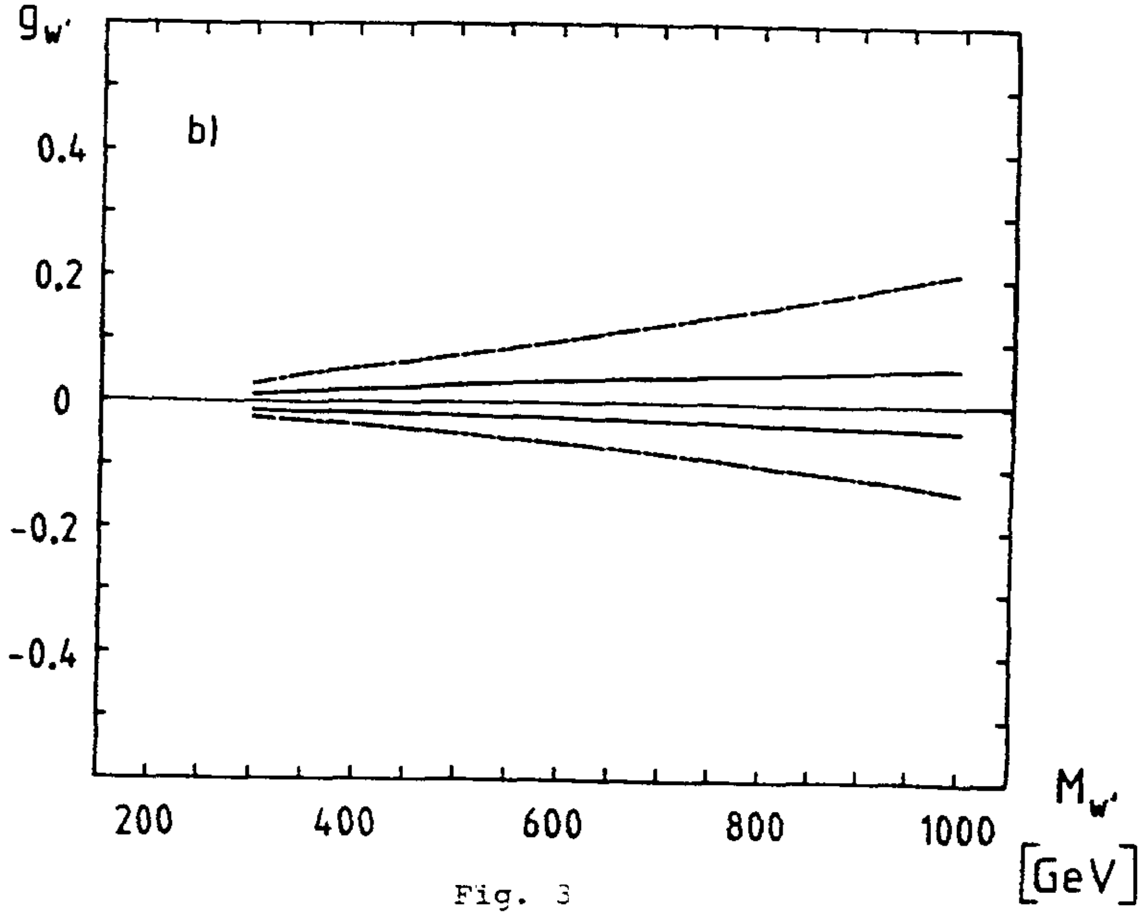


A_{FB}
a)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
200
400
600
$\sqrt{s}$ [GeV]
$\frac{A_{LR}}{P_e}$
b)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
200
400
600
$\sqrt{s}$ [GeV]

Fig 2



g_{w'}
a)
0.4
0.2
0
-0.2
-0.4
200
400
600
800
1000
M_{w'}
[GeV]



g_{w'}
b)
0.4
0.2
0
-0.2
-0.4
200
400
600
800
1000
M_{w'}
[GeV]

Fig. 3