



Fermi National Accelerator Laboratory

FERMILAB-Pub-86/40-A
UMN-TH-551/86
March 1986

Microwave Background Anisotropy and Decaying Cold Particle Scenarios

Edward W. Kolb
Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
Batavia, IL 60510

Keith A. Olive
Department of Physics
University of Minnesota
Minneapolis, MN 55455

Nicola Vittorio
Department of Astronomy
University of California
Berkeley, CA 94720

Abstract

Decaying particle scenarios for galaxy formation are subject to constraints from limits on the anisotropy of the microwave background radiation. In this paper we obtain limits on the properties of decaying cold dark matter.



There is a strong theoretical prejudice that the Universe should be very nearly spatially flat. If the cosmological constant is zero, the Universe should then have a mass density very close to the critical density $\rho_c = 1.88 \times 10^{-29} h^2 \text{ g cm}^{-3}$ [$h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$]. Primordial nucleosynthesis calculations can successfully predict light-element abundances only if the mass density of baryons in the Universe today is less than the critical density.¹ Even if one assumes that there is a non-baryonic component to the mass density of the Universe, observations of galaxy distributions consistently yield values for the present mass density less than 20% of the critical density.² The present mass density deduced in this manner includes all non-baryonic components that are massive.

Turner, Steigman, and Krauss³ pointed out that if the dark matter is relativistic, it would not participate in gravitational clustering, and would not be included in mass density estimates from large-scale structure. If the dark matter is non-baryonic it is not limited by considerations of primordial nucleosynthesis. However if the Universe was always dominated by relativistic particles (radiation dominated) there would be insufficient time for galaxies to form.⁴

A cosmological scenario that results in a Universe presently dominated by massless particles was proposed by Dicus, Kolb, and Teplitz.⁵ They proposed a massive neutrino in the range $100 \text{ eV} \leq M \leq 10 \text{ GeV}$ that froze out from thermal equilibrium at $T \gtrsim 1 \text{ MeV}$, dominated the mass density of the Universe at some later time, then decayed to the massless particles that dominate the Universe today. They also calculated the neutrino lifetime necessary to give an energy density in the neutrino decay products equal to the critical density. They noted

two fundamental problems with the scenario. First, a radiation-dominated Universe is younger than a matter-dominated Universe with the same density. Second, galaxy formation during the radiation dominated era is greatly suppressed. Davis, LeCar, Pryor and Witten first considered galaxy formation with decaying particles.⁶ With recent emphasis on the " Ω problem", the decaying particle scenario has been a subject of keen interest.^{3,7-17} It has been realized that if the decaying particle scenario is to be a solution to the Ω problem, it must pass observational tests such as the anisotropy limits of the cosmic microwave background (CMB). Although the original scenario involved massive neutrinos, it can work with any particle that interacts weakly enough to decouple before decay.

Attempts to model galaxy formation distinguish between three types of matter: hot, warm and cold.¹⁸ These three types are distinguished by their velocity at the time of matter domination. Cold matter is extremely non-relativistic, hot matter is semi-relativistic, and warm matter somewhere in between. Previous work on galaxy formation in decaying particle scenarios have for the most part assumed warm or hot dark matter. For instance, Vittorio and Silk concluded that the allowed region of parameter space for light neutrinos is¹⁴

$$2 < z_D < 4 \quad (1a)$$

$$0.4 < h < 0.5 \quad (1b)$$

$$10 \times 10^9 < t_0 \leq 12 \times 10^9 \text{ yr} \quad (1c)$$

$$40 < m_\nu < 110 \text{ eV} , \quad (1d)$$

where z_D is the redshift at decay (assuming instantaneous decay), t_0 is

the present age of the Universe, and m_ν is the neutrino mass. In this paper we derive limits similar to (1a) - (1d) for very massive neutrinos as an example of cold dark matter, and generalize the results to apply to gravitinos.

The decay products of a particle denoted by H, that decayed at a temperature T_D into massless particles, will have a present energy density of

$$\rho_D = m_H Y n_{\gamma 0} (T_0/T_D) \quad (2)$$

where $Y = n_H/n_\gamma$ at decay, $n_{\gamma 0}$ is the present photon density ($n_{\gamma 0} = 399 \text{ cm}^{-3}$) and T_0 is the present temperature ($T_0 = 2.7 \text{ K}$). It is useful to express present energy densities in terms of the critical density, $\Omega_i = \rho_i/\rho_c$. The decay products of the H will contribute to Ω an amount

$$\Omega_D = 9 \times 10^{-6} Y \left(\frac{m_H}{T_D} \right) h^{-2} . \quad (3)$$

A reasonable guess for Ω_D might be $\Omega_D = 0.9$, since baryons can today give a contribution of $\Omega_B = 0.1$.¹ We shall adopt the value $\Omega_D = 0.9$ throughout, but our results are not sensitive to this choice. The choice $h = 1$ leads to an embarrassingly young Universe, so we adopt $h = 1/2$.

If neutrinos have a mass less than 1 MeV, they will be relativistic at freeze out, and Y is $3/11$. If, however, the neutrinos have a mass greater than 1 MeV, the neutrinos will be non-relativistic at freeze out, and Y will depend upon the mass, and will be much smaller than

one.¹⁹ In Figure 1 we show Y as a function of mass for both Dirac and Majorana neutrinos.²⁰ In Figure 2 we show the decay redshift, $z_D + 1 = T_D/T_0$, as a function of mass that will result in $\Omega_D = 0.9$.

The temperature of the Universe when the H energy density dominates the energy density is found by equating the energy density in relativistic particles, $\rho_R = g_*(\pi^2/30)T^4$, where $g_* = 3.36$ at $T < 1$ MeV for three light neutrinos, and the H energy density, $\rho_H = m_H n_H = m_H Y n_\gamma = m_H Y 2\zeta(3)T^3/\pi^2$. Equating the two energy densities defines a temperature, T_{eq} , given by

$$\frac{m_H}{T_{eq}} = 4.5 \text{ } Y^{-1} . \quad (4)$$

Note that if Y is much less than one, m_H/T_{eq} is large, and H is "cold" matter. This is the case we will consider in this paper.

Our task now is to use the limits on small-scale CMB anisotropies to place limits on the temperature of decay T_D , or equivalently the redshift of decay, $z_D + 1 = T_D/T_0$. The calculational methods are described in detail in the paper of Vittorio and Silk.¹⁴

Here we will just review some qualitative points. We consider a cold dark matter (CDM) dominated universe and, as usual, assume density perturbations to be adiabatic, gaussian, and with a scale-invariant power spectrum $|\delta_k|^2 = Ak$. Despite the initial scale invariance, the fluctuation spectrum at later times exhibits a characteristic length L_{eq} , the horizon size at matter-radiation equality.¹⁸ This is a reflection of the different growth rates experienced by a density fluctuation in the radiation- and matter- dominated eras. Scales

smaller than L_{eq} enter the horizon when the universe is still radiation dominated: in this period there is no appreciable growth. Scales larger than L_{eq} enter the horizon in the matter dominated era: in this period there is growth independent of the perturbation scale, and the primordial slope in the density fluctuation is preserved. Since in a decaying particle cosmology, $L_{eq} \propto z_D^{-1}$, at high z_D L_{eq} will be negligibly small: in practice the spectrum at later time is still the primordial one, $|\delta_k|^2 = Ak$. This spectrum has the problem of being unable to reproduce the observed large scale structure. To minimize this problem we choose to normalize the spectrum by fixing the amplitude of density fluctuations on $30h^{-1}\text{Mpc}$ as observed from the J_3 integral over the correlation function.¹⁴ Also, if $z_D \gg 1$, the fluctuation spectrum looks the same, regardless of the nature of the particle considered (hot, warm, or cold), since we are effectively looking at the primordial slope. So, we can anticipate that the limit from the CMB should be the same as the limit found previously.¹⁴ Indeed, detailed calculations of the growth of fluctuations prior, during, and after recombination, and of the induced fluctuation in the background radiation on small angular scales confirm this simple prediction. The observed upper limit on the small scale anisotropy of the CMB implies $z_D < 4$. As in the neutrino case, we still have a lower limit on the redshift of decay, which describes the time necessary to transfer the decay energy from the non-relativistic to the relativistic component. So, eq.(1a) still holds also in the CDM decaying scenario. Our choice of the scale for normalization of the spectrum ($30h^{-1}\text{Mpc}$) results in a different limit than obtained in ref. 15.

The limits $3 \leq 1+z_D \leq 5$, result in $\Omega_D = 0.9$ if the product $m_H Y$ is in the range

$$17 < m_H Y < 29 \text{ eV} . \quad (5)$$

Using the results from the figures, the limit on $m_H Y$ can be converted into a limit on m_H . Therefore, for very massive neutrinos, the limits corresponding to Eq. (1) are

$$3 < 1+z_D < 5 \quad (6a)$$

$$1.8 < m_H < 2.2 \text{ GeV (Dirac neutrino)} \quad (6b)$$

$$3.6 < m_H < 4.5 \text{ GeV (Majorana neutrino)} . \quad (6c)$$

The limits $3 < 1+z_D < 5$ obtain for all cold particle decaying scenarios. However, the resulting limits on the mass of the particle depends upon the details of the particle involved. For neutrinos, the abundance Y was well determined by the weak interaction rates. However there is no reliable model to estimate the neutrino lifetime. For gravitinos, the situation is reversed. Gravitinos essentially decouple at the Planck epoch. Only entropy creation, e.g., inflation²¹, or gravitino regeneration during reheating²²⁻²⁴ can affect their relative abundance. This introduces uncertainty in the gravitino-photon ratio. For gravitinos, the decay width is well determined by the gravitino mass, $m_{3/2}$, and the Planck mass, m_p :

$$\Gamma_{3/2} = 8\pi\alpha m_{3/2}^3 / m_p^2 \quad (7)$$

where α is a coupling constant with tree-level value²⁴ $\alpha = 1/32\pi$. Of course, we must require that the decay of the gravitino be into an "invisible" mode, such as axion plus axino.¹⁰

We may find the temperature of the Universe at gravitino decay by equating the decay lifetime, $\tau_{3/2}^{-1}$, and the age of a gravitino dominated Universe, $t = (2/3)\rho_{3/2}^{-1/2} m_p$. The energy density at decay is given by $\rho_{3/2} = m_{3/2} Y T_D^3$. Therefore T_D is given by

$$T_D = \left[\frac{16\pi\alpha m_{3/2}^{5/2}}{3m_p Y^{1/2}} \right]^{2/3} \quad (8)$$

$$= 2.2 \times 10^{-1} \alpha^{1/2} m_{3/2}^{3/2} m_p^{-1/2} (\Omega_D h^2)^{-1/4}$$

where the last equality in Eq. (8) follows from Eq. (3). The microwave background anisotropy limits ($3 \leq 1+z_D \leq 5$) then place a limit on $m_{3/2}$:

$$1.8 \times 10^{-1} < m_{3/2} < 2.5 \times 10^{-1} \text{ GeV} , \quad (9)$$

for $\Omega_D = 0.9$, $h = 1/2$, and $\alpha = 1/32\pi$. The range in Eq. (9) is remarkably narrow.

Using Eqs. (2) and (9), we can also place a limit on Y

$$9.8 \times 10^{-8} < Y < 1.2 \times 10^{-7} . \quad (10)$$

Finally, it is possible to relate the abundance of gravitinos to the reheat temperature after inflation. The gravitinos are regenerated after inflation to an abundance²²⁻²⁴

$$Y \approx 10^{-3} T_R / m_p . \quad (11)$$

This requirement given in Eq. (10) implies

$$T_R \approx 10^{15} \text{ GeV} \quad (12)$$

Previous limits²²⁻²⁵ on the reheat temperature do not apply for the invisible decay modes considered here.

With the assumption of cold dark matter decaying particle scenarios giving $\Omega_D = 0.9$, our conclusions are (1) $3 \leq 1+z_D \leq 5$, (2) if the parent particle is a massive neutrino, then

$$1.8 < m_H < 2.2 \text{ GeV (Dirac)}$$

$$3.6 < m_H < 4.5 \text{ GeV (Majorana),}$$

(3) if the parent particle is a gravitino

$$1.8 \times 10^{-1} < m_{3/2} < 2.5 \times 10^{-1} \text{ GeV}$$

$$9.8 \times 10^{-8} < Y < 1.2 \times 10^{-7}$$

$$T_R \approx 10^{15} \text{ GeV} .$$

This work was supported in part by NASA and DOE at Fermilab, and by the DOE under contract number DE-AC02-93ER-40105 at the University of Minnesota.

References

1. J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. A. Olive, *Astrophys. J.* 281, 493 (1984).
2. S. M. Faber and J. Gallagher, *Annu. Rev. Astron. Astrophys.* 17, 135 (1979); M. Davis and P. J. E. Peebles, *Annu. Rev. Astron. Astrophys.* 21, 109 (1983).
3. M. S. Turner, G. Steigman, and L. Krauss, *Phys. Rev. Lett.* 52, 2090 (1984).
4. Y. Hoffman and S. A. Bludman, *Phys. Rev. Lett.* 52, 2087 (1984).
5. D. A. Dicus, E. W. Kolb, and V. L. Teplitz, *Phys. Rev. Lett.* 39, 168 (1977); *Astrophys. J.* 221, 327 (1978).
6. M. Davis, M. LeCar, C. Pryor, and E. Witten, *Astrophys. J.* 250, 423 (1981).
7. P. Hut and S. D. M. White, *Nature* 310, 637 (1984).
8. G. Gelmini, D. N. Schramm, and J. W. F. Valle, *Phys. Lett.* 146B, 311 (1984).
9. K. A. Olive, D. Seckel, and E. Vishniac, *Astrophys. J.* 292, 1 (1985).
10. K. A. Olive, D. N. Schramm, and M. Srednicki, *Nucl. Phys.* B255, 495 (1985).
11. M. Fukugita and T. Yanagida, *Phys. Lett.* 144B, 386 (1984).
12. Y. Suto, H. Kodoma, and K. Sato, *Phys. Lett.* 157B, 259 (1985).
13. Y. Suto and M. Noguchi, Univ. of Tokyo preprint UTAP-27 (1985).
14. N. Vittorio and J. Silk, *Phys. Rev. Lett.* 54, 2269 (1985).
15. M. S. Turner, *Phys. Rev. Lett.* 55, 544 (1985).
16. M. S. Turner, *Phys. Rev.* D31, 1212 (1985).
17. D. A. Dicus and V. L. Teplitz, Univ. Texas preprint ORO (1985).
18. J. R. Bond and A. Szalay, *Astrophys. J.* 274, 443 (1983); G. Blumenthal, *Nature* 311, 517 (1984).
19. B. W. Lee and S. Weinberg, *Phys. Rev. Lett.* 39, 165 (1977); P. Hut, *Phys. Lett.* 69B, 85 (1977).
20. E. W. Kolb and K. A. Olive, *Phys. Rev. D* 33, 1202 (1986).

21. J. Ellis, A. D. Linde, and D. V. Nanopoulos, Phys. Lett. 118B, 59 (1982).
22. D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Phys. Lett. 127B, 30 (1983).
23. J. Ellis, J. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. B238, 453 (1984).
24. J. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. 145B, 181 (1984).
25. J. Ellis, D. V. Nanopoulos, and S. Sarkar, Nucl. Phys. B259, 175 (1985). R. Juskiewicz, J. Silk and A. Stebbins, Phys. Lett. 158B, 463 (1985).

Figure Captions

Figure 1: The ratio of neutrinos to photons, Y , as a function of the mass of the neutrino for Majorana and Dirac neutrinos.

Figure 2: The decay redshift necessary for the massless decay products of the neutrino to give $\Omega = 0.9$ (with $h = 1/2$) as a function of the mass of the neutrino. The results for Majorana and Dirac neutrinos are given.

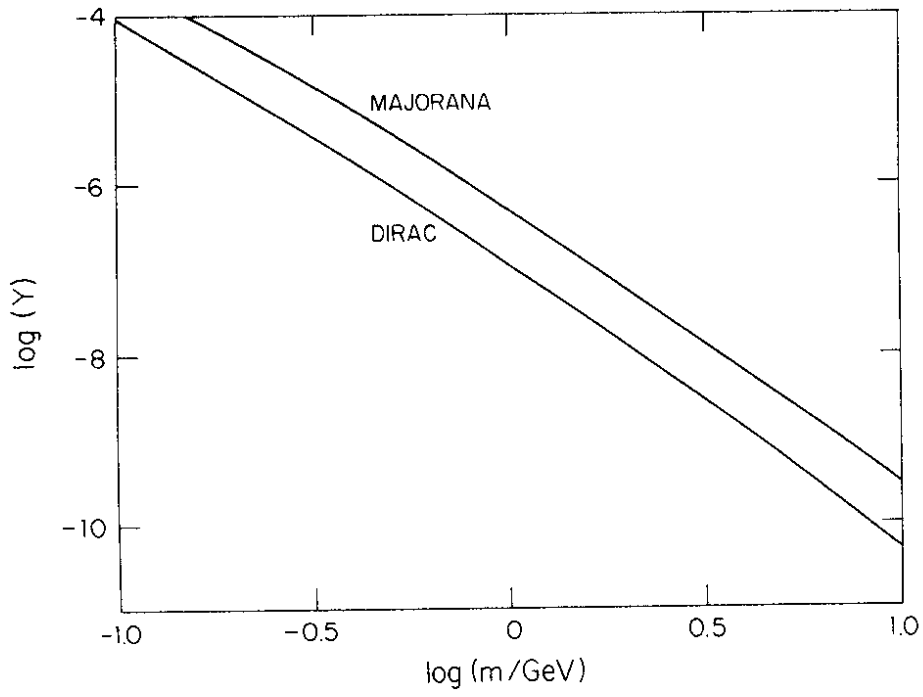


Figure 1: The ratio of neutrinos to photons, Y , as a function of the mass of the neutrino for Majorana and Dirac neutrinos.

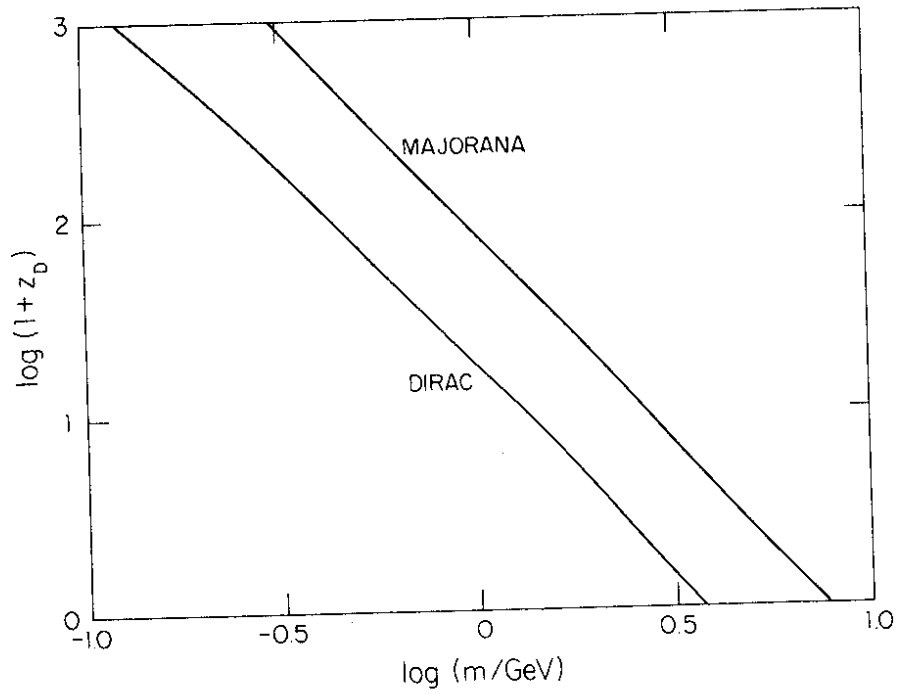


Figure 2: The decay redshift necessary for the massless decay products of the neutrino to give $\Omega = 0.9$ (with $h = 1/2$) as a function of the mass of the neutrino. The results for Majorana and Dirac neutrinos are given.