



# Fermi National Accelerator Laboratory

MPI-PAE/PTh 74/86

FERMILAB-PUB-86/163-T

December 1986

## Testing the Standard Model versus a Composite Structure of Weak Interactions in High Precision Experiments at LEP and SLC

U. BAUR\*

*Fermi National Accelerator Lab.  
P.O.Box 500, Batavia, IL 60510 (USA)*

M. LINDNER

*Max-Planck-Institut für Physik und Astrophysik  
- Werner-Heisenberg-Institut für Physik -  
P.O.Box 40 12 12, Munich (Fed. Rep. Germany)*

K.H. SCHWARZER

*Sektion Physik, University of Munich  
Theresienstrasse 37, Munich (Fed. Rep. Germany)*

### ABSTRACT

Models with a composite structure of the weak interactions usually contain an isoscalar weak vector boson  $Y$  or  $Y_L$  which is coupled to the weak hypercharge current, or its lefthanded part. We study the impact of  $Y$  and  $Y_L$  on the  $Z$ -boson parameters and other measurables at future high energy  $e^+e^-$  colliders and discuss the corresponding standard model predictions. An analysis of the forward-backward and left-right asymmetry for the general case of two massive vector bosons with arbitrary vector and axial vector coupling constants is undertaken which may be applied to any other model containing additional weak vector bosons. If experiments at LEPI/SLC and LEP II agree with the standard model predictions within the possible experimental accuracies,  $Y(Y_L)$ -bosons with a mass of up to 1 TeV (0.9 TeV) could be excluded.

---

\* Max-Kade-Fellow



## 1. Introduction

The standard Glashow-Weinberg-Salam (GSW) model [1] of electroweak interactions based on the gauge group  $SU(2)_L \times U(1)_Y$  has achieved important successes in describing neutral and charged current processes and in correctly predicting [2] the mass of the  $W$ - and  $Z$ -bosons. However, essential ingredients of a renormalizable spontaneously broken gauge theory such as the standard model have not been confronted with experiment yet. In particular, the Yang-Mills structure of the self-couplings of the intermediate vector bosons and the Higgs-mechanism have not been tested. Moreover, experiments have not been accurate enough so far to probe the standard model much beyond tree level.

It is therefore still conceivable that the intermediate vector bosons are composite objects rather than the elementary gauge bosons of a spontaneously broken  $SU(2)_L \times U(1)_Y$  gauge theory. Composite  $W$ - and  $Z$ -bosons occur in preon models of quarks and leptons [3] where the substructure scale  $\Lambda_H$  is related to the Fermi scale of the weak interactions,  $\Lambda_H \sim G_F^{-1/2} \approx 300 \text{ GeV}$ . If the preon Lagrangian is invariant under global  $SU(2)_{WI}$  weak isospin transformations and vector dominance is invoked [4] such models yield the standard predictions for the neutral current structure and for the masses of  $W$ - and  $Z$ -bosons. On the other hand a spectrum of additional heavy vector bosons would constitute a main and direct experimental signature for such a richer short distance structure. The exact nature of this spectrum remains model dependant but some general features are nevertheless evident. Independently of the underlying preon model one always expects excited  $W$ 's and  $Z$ 's [5]. Furthermore, if the constituents of the intermediate vector bosons are colored objects, color octet partners of  $W$  and  $Z$  should appear [6]. Finally, many models predict additional isoscalar weak vector bosons with masses in the few hundred  $\text{GeV}$  range (see e.g. refs. [7], [8] and [9]).

While a direct exploration of such particles must, most likely, await the construction of a multi- $\text{TeV}$  hadron-hadron collider [10], virtual effects of the new particles may already show up at much lower energies. In this paper we would like to discuss the effects of isoscalar weak vector bosons at LEPI/SLC and LEP II, and try to assess what the capabilities of these colliders are to set new limits on the parameters characterizing these particles. As a result, we shall conclude that,

provided that LEP II will be built, isoscalar weak vector bosons with a mass of up to 1 TeV can be excluded, or that necessarily deviations from the standard model have to show up.

Isoscalar weak vector bosons manifest themselves in high energy  $e^+e^-$  collisions in two different ways:

i) The physical  $Z$ -boson feels the mixing in of the new vector bosons. As a result, the exact values of the mass and the couplings of the  $Z$  will differ from the standard model predictions.

ii) Due to the exchange of an additional vector boson, the  $e^+e^- \rightarrow \mu^+\mu^-$  cross-section and, even more importantly, the electroweak asymmetries may be significantly changed.

In our paper we shall discuss in depth both issues. In particular, we shall present an analysis of the electroweak asymmetries for the most general case of two massive vector bosons with completely arbitrary vector and axial-vector coupling constants. This analysis can be directly applied to models with composite isoscalar weak vector bosons, as well as to other models with new massive vector bosons like GUT's [11], left-right symmetric [12] or superstring inspired [13] models. Our paper will, therefore, be also useful for studying asymmetries in such models. Given certain experimental accuracies for quantities measured at LEP I/SLC and LEP II, we shall then estimate limits on the parameters characterizing isoscalar weak vector bosons which one should be able to obtain, provided that no deviations from the standard model are observed. This will show us which quantity is most sensitive to the presence of isoscalar weak vector bosons, how useful polarization of the incident electron beam or the higher energy of LEP II are, and how accurately the standard model can be tested at future  $e^+e^-$  colliders.

Two possible scenarios for isoscalar weak vector bosons will be considered. In scenario A we investigate an isoscalar vector boson  $Y$  which couples to the weak hypercharge current. In scenario B, on the other hand, the isoscalar  $Y_L$  couples only to the lefthanded part of the weak hypercharge current. The first scenario corresponds to the situation where the preon Lagrangian has a global symmetry  $G \supseteq SU(2)_{WI} \times U(1)_Y$ , whereas scenario B may be realized if the compositeness scale of the righthanded sector is much larger than the one of the lefthanded sector,

or if righthanded quarks and leptons are elementary. Typical models of this type are the ones analyzed in refs. [7] and [9]. Here, only the lefthanded quarks and leptons are composite states. The  $Y_L$ -boson belongs to a 15-plet or a 143-plet of isoscalar vector bosons associated with a global  $SU(4)$  or  $SU(12)$  symmetry acting on the lefthanded fermions.

Our paper is organized as follows:

In Section 2 we consider the measurable quantities relevant for our purposes and give an overview over the corresponding standard model predictions. In Section 3 we discuss the influence of the  $Y(Y_L)$  on the  $Z$ -mass and couplings and how this is reflected by measurable quantities at LEP and SLC. Section 4 is devoted to a careful analysis of the electroweak asymmetries. In the first part we shall present our general discussion of the asymmetries for two massive vector bosons which in the second part will be applied to the  $Y$  and  $Y_L$ . In Section 5, finally, we estimate those limits on the parameters characterizing the  $Y$  and  $Y_L$  which one should be able to derive from LEPI/SLC and LEP II experiments if no deviations from the standard model were to be observed. Section 6 contains our conclusions.

## 2. Measurables and their Standard Model Predictions

In this Section we would like to discuss the measurable quantities which we will employ in our subsequent analysis, and review the standard model predictions for them. We shall consider five quantities which are sensitive to the presence of additional heavy vector bosons: the masses of  $W$  and  $Z$ ,  $m_W$  and  $M_Z$ , the leptonic width of the  $Z$ -boson and the electroweak asymmetries  $A_{FB}$  and  $A_{LR}$ .

### 2.1. THE $Z$ -MASS $M_Z$

The most precise and most important quantity to be measured at LEPI and SLC is the  $Z$ -mass  $M_Z$  which will set the scale for high-precision tests of the electroweak theory. The errors on  $M_Z$  will be dominated by the uncertainty in determining the absolute energy scale  $E$ . If  $E$  can be measured with an accuracy of  $\delta E/E = \pm 3 \cdot 10^{-4}$ ,  $M_Z$  can be obtained with an error of  $\delta M_Z = 28 \text{ MeV}$  [14].

In the standard model  $M_Z$  is given by [15]

$$M_Z^2 = \frac{\mu^2}{s^2 c^2} \quad (2.1)$$

where  $\hat{s}^2 = \sin^2\theta_W$ ,  $\theta_W$  is the Weinberg-angle,  $\hat{c}^2 = 1 - \hat{s}^2$  and

$$\mu^2 = \frac{\mu_0^2}{1 - \Delta r} \quad (2.2)$$

with

$$\mu_0^2 = \frac{\pi\alpha(0)}{\sqrt{2} G_F} = (37.281 \text{ GeV})^2. \quad (2.3)$$

Here,  $G_F$  denotes the Fermi-constant,  $G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$ , and  $\alpha(0) \approx (137.04)^{-1}$  is the electromagnetic coupling constant. The quantity  $\Delta r$  in Eq. (2.2) represents the radiative corrections to  $M_Z$  at zero momentum transfer and depends on the t-quark mass  $m_t$  and the Higgs-boson mass  $m_H$ . For  $m_t$  smaller than the  $W$ -mass  $m_W$ , and  $m_H$  in the range from 10  $\text{GeV}$  to 1  $\text{TeV}$  one finds  $\Delta r \approx 0.06 \dots 0.08$  [16]. The effects of radiative corrections can to a very good approximation be represented by writing

$$\mu^2 \simeq \mu_0^2 \frac{\alpha(M_Z)}{\alpha(0)} \quad (2.4)$$

where  $\alpha(M_Z)$  is the running electromagnetic coupling constant at the scale  $m = M_Z$  [14]. Since Eq. (2.4) includes only QED contributions it ceases to be valid for heavy top quark masses ( i.e.  $m_{top}$  in excess of  $2m_W$  ).

## 2.2. THE $W$ -MASS $m_W$

Since the  $Z$ -mass by itself alone does not say anything about the validity of the standard model and therefore will be used as a reference scale in experiments at LEPI/SLC, small deviations from the standard model in  $M_Z$  due to the presence of an additional neutral heavy vector boson will reveal themselves only if also the  $W$ -mass  $m_W$  is measured with an accuracy comparable to the one of the  $Z$ -mass measurement. This can be done directly at LEP II using the reaction  $e^+e^- \rightarrow W^+W^-$  [17]. The resulting error in  $m_W$  is expected to be of the order of 100  $\text{MeV}$ . However, a similar precision in the determination of the  $W$ -mass may also be possible by combining the results of LEPI/SLC and the improved CERN  $p\bar{p}$  collider (ACOL) [14]. There, the ratio

$$\Delta = \frac{m_W}{M_Z} \quad (2.5)$$

will eventually provide a more precise test of the electroweak theory than the separate measurements of  $W$ - and  $Z$ -masses since it is free from the systematic error on the energy scale. With a realistic improvement of the detectors, such as that planned by UA1, one can presumably aim at a precision of [14]

$$\delta\left(\frac{m_W}{M_Z}\right) = \pm 2 \cdot 10^{-3} . \quad (2.6)$$

Combining this with the precision measurement of  $M_Z$  at LEPI/SLC yields an error in  $m_W$  of the order of 200 MeV. In the standard model,  $\Delta$  coincides with  $\cos\theta_W$  and is only a function of  $M_Z$ . This can be easily seen by solving Eq.(2.1) for  $\hat{s}^2$ ,

$$\hat{s}^2 = \frac{1}{2} - \left[ \frac{1}{4} - \frac{\mu^2}{M_Z^2} \right]^{1/2} \quad (2.7)$$

which then yields

$$\Delta = \left\{ \frac{1}{2} + \left[ \frac{1}{4} - \frac{\mu^2}{M_Z^2} \right]^{1/2} \right\}^{1/2} . \quad (2.8)$$

### 2.3. THE LEPTONIC WIDTH OF THE $Z$ -BOSON

While the masses of the intermediate vector bosons are directly accessible at LEPI/II and SLC, information on the  $Z$ -couplings can only be gained indirectly from a measurement of either the total  $Z$ -width  $\Gamma_Z$  or the leptonic width  $\Gamma(Z \rightarrow \ell^+\ell^-)$ ;  $\ell = e, \mu, \tau$ .  $\Gamma_Z$  can be directly obtained from the line shape of the  $Z$ -peak with an error expected to be of the order of  $\delta\Gamma_Z/\Gamma_Z \approx 2\%$  [14].  $\Gamma(Z \rightarrow \ell^+\ell^-)$ , on the other hand, can be determined using the luminosity monitor [14]

$$I = \int \sigma(W)_{e^+e^- \rightarrow \ell^+\ell^-} dW = \frac{6\pi^2}{M_Z^2} \frac{\Gamma^2(Z \rightarrow \ell^+\ell^-)}{\Gamma_Z} . \quad (2.9)$$

If  $I$  can be measured with  $\delta I/I \approx 3\%$ , the leptonic  $Z$ -width can as well be obtained with an error of about 2 %.

From the point of view of testing the  $Z$ -couplings  $\Gamma(Z \rightarrow \ell^+\ell^-)$  has to be preferred over  $\Gamma_Z$ . The reason for this is twofold. First of all,  $\Gamma(Z \rightarrow \ell^+\ell^-)$  is a theoretically "clean" quantity which, in the standard model, can be accurately predicted once  $M_Z$  is known. Including electroweak radiative corrections one finds (neglecting the lepton masses)

$$\Gamma(Z \rightarrow \ell^+\ell^-) = \frac{G_F}{12\sqrt{2}\pi} M_Z^3 (1 - 4\hat{s}^2 + 8\hat{s}^4) (1 + \delta_\ell) \quad (2.10)$$

with  $\hat{s}^2$  from Eq.(2.7). The quantity  $\delta_\ell$  represents the electroweak radiative correction and turns out to be very small if the radiatively corrected formula (2.7) is used for  $\hat{s}^2$  [14]:

$$\delta_\ell \leq 0.1\% . \quad (2.11)$$

The total  $Z$ -width, on the other hand, is affected by strong interaction corrections and the uncertainty in the QCD coupling constant  $\alpha_s$  introduces a theoretical error in  $\Gamma_Z$  of the order of 1 %. The second reason for preferring  $\Gamma(Z \rightarrow \ell^+\ell^-)$  is that  $\Gamma_Z$  strongly depends on parameters such as  $m_t$  or the number of massless neutrino species which are presently completely unknown. In our subsequent analysis the total  $Z$ -width will, therefore, not be discussed further.

#### 2.4. THE FORWARD-BACKWARD ASYMMETRY $A_{FB}$

Another observable at  $e^+e^-$  colliders which is particularly sensitive to additional heavy vector bosons and to the Lorentz-structure of their couplings is the integrated forward-backward asymmetry, defined by

$$A_{FB} = \frac{F - B}{F + B} \quad (2.12)$$

with

$$F \pm B = \left[ \int_0^x \pm \int_{-x}^0 \right] dz \frac{d\sigma(e^+e^- \rightarrow ff)}{dz} ; \quad (2.13)$$

$z$  is the cosine of the angle between the fermion  $f$  and the beam direction and  $x$  the detector acceptance ( $x \leq 1$ ). The most precise determination of  $A_{FB}$  can be performed on the  $Z$ -peak and if the final state fermions consist of muons. For an integrated luminosity of  $\int \mathcal{L} dt = 100 \text{ pb}^{-1}$  the experimental error is estimated to be [18]

$$\delta A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}(M_Z) \approx 3 \cdot 10^{-3} . \quad (2.14)$$

For comparison, at the highest projected LEP II energy of  $\sqrt{s} = 190 \text{ GeV}$  and with an integrated luminosity of  $500 \text{ pb}^{-1}$  the experimental error is expected to be about a factor ten larger than at resonance [18], i.e.

$$\delta A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}(190 \text{ GeV}) \approx 0.03 . \quad (2.15)$$

The values for the integrated luminosities used above are typical estimates for a 100 day running period at LEP I/II with unpolarized beams. If the electron beam

at LEPI/II could achieve a 50 % longitudinal polarization, as it will be possible at the SLC, the expected luminosity would be about a factor 2.5 smaller and the errors in  $A_{FB}$  would increase by about a factor 1.6 [18].

In the standard model, the tree-level expression for  $A_{FB}$  is given by

$$A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-} = \frac{x}{1 + \frac{x^2}{3}} \frac{N}{D} \quad (2.16)$$

with

$$N = \frac{e^2}{4s} g_Z^2 \frac{s - M_Z^2}{\rho(s)} (1 + P_e v_e) + \frac{g_Z^4}{64} \frac{v_e}{\rho(s)} [2v_e + P_e(1 + v_e^2)] , \quad (2.17)$$

$$D = 2 \frac{e^4}{s^2} + \frac{e^2}{4s} g_Z^2 \frac{s - M_Z^2}{\rho(s)} v_e (P_e + v_e) + \frac{g_Z^4}{128} \frac{1 + v_e^2}{\rho(s)} (1 + v_e^2 + 2P_e v_e) \quad (2.18)$$

where

$$\rho(s) = (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 ; \quad (2.19)$$

$s$  is the center of mass energy squared,  $g_Z = g_W / \cos\theta_W$ ,  $g_W^2 = 4\sqrt{2}G_F m_W^2$ ,  $P_e$  is the degree of the longitudinal polarization of the  $e^-$ -beam and

$$v_e = 1 - 4\sin^2\theta_W \quad (2.20)$$

is the  $Z$  vector coupling constant for charged leptons. The expression of  $A_{FB}$  for an unpolarized  $e^-$ -beam can be obtained by setting  $P_e = 0$  in Eqs. (2.16) to (2.18). Note that all terms containing  $P_e$  in Eqs. (2.17) and (2.18) are proportional to  $v_e$ , which is small because  $\sin^2\theta_W$  is close to 1/4. Longitudinal polarization will, therefore, only slightly affect the shape of  $A_{FB}$  in the standard model.

The expression for  $A_{FB}$  considerably simplifies on top of the  $Z$ -peak ( $s = M_Z^2$ ). In the narrow width approximation ( $\Gamma_Z \ll M_Z$ ) we find

$$A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-} = 2 \frac{x}{1 + \frac{x^2}{3}} \frac{v_e}{1 + v_e^2} \frac{2v_e + P_e(1 + v_e^2)}{1 + v_e^2 + 2P_e v_e} . \quad (2.21)$$

In Fig. 1 we show the forward-backward asymmetry versus  $\sqrt{s}$  for  $P_e = 0$  and  $x = 1$ . As one can see,  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  varies strongly in the neighbourhood of the  $Z$ -resonance with a zero at ( $P_e = 0$ )

$$s = M_Z^2 \left[ 1 + \frac{v_e^2}{8\sin^2\theta_W \cos^2\theta_W} \right]^{-1} \quad (2.22)$$

and a minimum (maximum) at (for  $\Gamma_Z \ll M_Z, P_e = 0$ )

$$\frac{s}{s - M_Z^2} = \frac{16 \sin^2 \theta_W \cos^2 \theta_W}{(1 + v_e^2)^2} \left[ 2v_e^2 \mp (4v_e^4 + (1 + v_e^2)^2)^{1/2} \right]. \quad (2.23)$$

For  $M_Z = 92 \text{ GeV}$  the numerical values are  $\sqrt{s}(A_{FB} = 0) \approx 91.8 \text{ GeV}$ ,  $\sqrt{s}_{min} \approx 78.9 \text{ GeV}$  and  $\sqrt{s}_{max} \approx 114.3 \text{ GeV}$ .

Electroweak radiative corrections are known to influence  $A_{FB}$  significantly. In general the corrected asymmetry can be written down as a sum of three terms:

$$A_{FB} = A_{FB}^{Born} + \delta A_{FB}^{QED} + \delta A_{FB}^{weak}. \quad (2.24)$$

Some one-loop corrections can already be included by using the radiatively corrected value of  $\sin^2 \theta_W$ .  $\delta A_{FB}^{QED}$  represents the real and virtual photon emission and the one-loop photon vacuum polarization diagrams which contribute to the running  $\alpha$  which is already contained in (2.4). Finally,  $\delta A_{FB}^{weak}$  are the purely weak corrections to  $A_{FB}$  which depend on the unknown quantities  $m_t$  and  $m_H$ . At resonance,  $\delta A_{FB}^{weak}$  is a sizeable fraction of  $A_{FB}^{Born}$  and rather sensitive to  $m_t$  and  $m_H$ . For  $30 \text{ GeV} \leq m_t \leq 180 \text{ GeV}$ ,  $10 \text{ GeV} \leq m_H \leq 1 \text{ TeV}$  and  $90 \text{ GeV} \leq M_Z \leq 98 \text{ GeV}$ ,  $\delta A_{FB}^{weak}$  can vary by as much as  $\pm 0.01$  [19].

## 2.5. THE LEFT-RIGHT ASYMMETRY $A_{LR}$

The final quantity we want to discuss is the left-right asymmetry  $A_{LR}$ . It requires a longitudinal polarization for the  $e^-$ -beam and is given by

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \quad (2.25)$$

where  $\sigma_{L,R}$  are the cross-sections for  $e_{L,R}^- + e^+ \rightarrow X$ , and  $X$  can be any channel. The most precise determination of  $A_{LR}$  will be possible on top of the  $Z$ -resonance: for an integrated luminosity of  $40 \text{ pb}^{-1}$  with a 50 % longitudinally polarized  $e^-$ -beam the error is estimated to be [18]

$$\delta A_{LR}(M_Z) \approx 1.5 \cdot 10^{-3} \quad (2.26)$$

whereas for e.g.  $\sqrt{s} = 190 \text{ GeV}$  and  $\int \mathcal{L} dt = 200 \text{ pb}^{-1}$  [18]

$$\delta A_{LR}(190 \text{ GeV}) \approx 0.02. \quad (2.27)$$

In the standard model, the tree-level expression for  $A_{LR}$  reads

$$A_{LR} = P_e \frac{N'}{D'} \quad (2.28)$$

with

$$N' = \frac{e^2}{2s} |e_f| g_Z^2 v_f \frac{s - M_Z^2}{\rho(s)} + \frac{g_Z^4}{32} v_e \frac{1 + v_f^2}{\rho(s)}, \quad (2.29)$$

$$D' = 4 \frac{e^4}{s^2} e_f^2 + \frac{e^2}{2s} |e_f| g_Z^2 v_e v_f \frac{s - M_Z^2}{\rho(s)} + \frac{g_Z^4}{64} (1 + v_f^2) \frac{1 + v_e^2}{\rho(s)} \quad (2.30)$$

and where  $e_f$  is the electric charge (in units of  $e$ ) and

$$v_f = 1 - 4 |e_f| \sin^2 \theta_W \quad (2.31)$$

is the vector coupling constant of the final state fermion  $f$  which we supposed to be massless. The above expression for  $A_{LR}$  considerably simplifies on the  $Z$ -peak if the narrow width approximation is used. One finds

$$A_{LR}(M_Z) = 2P_e \frac{v_e}{1 + v_e^2}. \quad (2.32)$$

Note that  $A_{LR}(M_Z)$  is *independent* from the vector coupling constants of the final state fermions and, therefore, the same for all fermions with masses much smaller than  $M_Z$ , except  $e$  and  $\nu_e$ . In Section 4 we shall show that this is a unique feature of the standard model which is destroyed if a second massive vector boson is present. Comparing  $A_{LR}$  in different channels on top of the  $Z$ -resonance will, therefore, be another important test for the standard model at LEPI/SLC.

In Fig.1 we show  $A_{LR}/P_e$  versus  $\sqrt{s}$  for the channel  $e^+e^- \rightarrow \mu^+\mu^-$ . Since  $A_{LR}^{e^+e^- \rightarrow \mu^+\mu^-}$  is proportional to the vector coupling constant  $v_e$ , which itself is small, the left-right asymmetry does not vary very much. Similar to  $A_{FB}, A_{LR}$  has one zero which is located at

$$s = M_Z^2 \left[ 1 + \frac{1 + v_e^2}{16 \sin^2 \theta_W \cos^2 \theta_W} \right]^{-1} \quad (2.33)$$

and one minimum (maximum) at ( $\Gamma_Z \ll M_Z$ )

$$\frac{s}{s - M_Z^2} = \frac{16 \sin^2 \theta_W \cos^2 \theta_W}{1 - v_e^4} \left[ 1 + v_e^2 \mp (2(1 + v_e^2))^{1/2} \right] \quad (2.34)$$

in the standard model. The numerical values for  $M_Z = 92 \text{ GeV}$  are  $\sqrt{s}(A_{LR} = 0) \approx 79 \text{ GeV}$ ,  $\sqrt{s}_{min} \approx 67 \text{ GeV}$  and  $\sqrt{s}_{max} \approx 100 \text{ GeV}$ .

Finally, we briefly discuss electroweak radiative corrections to  $A_{LR}$ . Some of them can be included by using the radiatively corrected value of  $\sin^2\theta_W$ . Similar to the case of the forward-backward asymmetry, the purely weak corrections turn out to be a sizeable fraction of the Born term on top of the  $Z$ -resonance and are sensitive to  $m_t$  and  $m_H$ . They can vary by as much as  $\pm 0.02$  for  $30 \text{ GeV} \leq m_t \leq 180 \text{ GeV}$ ,  $10 \text{ GeV} \leq m_H \leq 1 \text{ TeV}$  and  $90 \text{ GeV} \leq M_Z \leq 98 \text{ GeV}$  [19].

### 3. Isoscalar Weak Vector Bosons and $Z$ -Boson Parameters

We now turn to a more detailed discussion of the composite weak boson scenarios which we have briefly described in the introduction. In the first part of this Section we develop the effective Lagrangian formalism used to describe composite weak boson interactions and derive the Lagrangian for interactions of the  $Z$ -boson with quarks and leptons as well as the expression for the  $Z$ -mass in the presence of isoscalar weak vector bosons. In the second part we shall analyze how  $Y$  and  $Y_L$  affect the observable quantities  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$ .

#### 3.1. EFFECTIVE LAGRANGIAN FORMALISM

To describe the interactions of fermions with the isotriplet of weak bosons,  $\mathbf{W}$ , and the isoscalar  $Y(Y_L)$  we use an effective Lagrangian. The weak isospin  $SU(2)_{WI}$  symmetry is supposed to be a global symmetry of the underlying preon model in the limit of a vanishing electromagnetic coupling constant  $e = \sqrt{4\pi\alpha}$ . The effective Lagrangian for the interaction of  $\mathbf{W}$  and  $Y(Y_L)$  with quarks and leptons in the limit  $\alpha \rightarrow 0$  is given by

$$\mathcal{L} = g_W \mathbf{j}_{\mu L} \mathbf{W}_\mu + g_Y j_\mu^{Y(L)} \tilde{Y}_{(L)\mu} \quad (3.1)$$

where  $\mathbf{W}_\mu$  and  $\tilde{Y}_{(L)\mu}$  describe the  $\mathbf{W}$ - and  $Y(Y_L)$ -field and

$$\begin{aligned} \mathbf{j}_{\mu L} &= \sum_L \bar{L} \gamma_\mu \frac{\boldsymbol{\tau}}{2} L, \\ j_\mu^Y &= j_\mu^{Y_L} + \sum_R \bar{R} \gamma_\mu Y_R R, \end{aligned} \quad (3.2)$$

$$j_\mu^{Y_L} \equiv j_{\mu L}^Y = \sum_L \bar{L} \gamma_\mu y_L L$$

are the weak isospin and hypercharge currents;  $j_\mu^{Y_L}$  is the lefthanded part of  $j_\mu^Y$ ,  $L(R)$  runs over all lefthanded (righthanded) fermion  $SU(2)_{WI}$  doublets (singlets),  $\tau$  are the Pauli-matrices and  $y_L = Q - T_3$  and  $y_R = Q$  are the respective hypercharge quantum numbers.  $Q$  and  $T_3$ , finally, denote the electric charge and the third component of the weak isospin, respectively.

For  $\alpha \neq 0$ , the result of Eq. (3.1) is modified by the mixing of  $Y(Y_L)$  and  $W^3$  with the photon [20] ( $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ ;  $V = W^3, \tilde{Y}_{(L)}$ ;  $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ )

$$\mathcal{L}_{mix} = -\frac{1}{4} \lambda_W (\tilde{F}_{\mu\nu} W^{3\mu\nu} + \tilde{F}^{\mu\nu} W_{\mu\nu}^3) - \frac{1}{4} \lambda_Y (\tilde{F}_{\mu\nu} \tilde{Y}_{(L)}^{\mu\nu} + \tilde{F}^{\mu\nu} \tilde{Y}_{(L)\mu\nu}) \quad (3.3)$$

where  $\tilde{A}_\mu$  denotes the photon field. This mixing and the saturation of the electromagnetic form factors by massive vector bosons ( $W^3$ - and  $Y(Y_L)$ -dominance [4]) relates the mixing strengths  $\lambda_W$  and  $\lambda_Y$  to the electric charge unit  $e$  via

$$g_W \lambda_W = g_Y \lambda_Y = e. \quad (3.4)$$

The  $W^3$ - and  $Y(Y_L)$ -vector boson dominance thus implies a universal coupling of the fermions to the respective vector bosons, namely  $g_W$  and  $g_Y$ .

The physical content of the model can easily be obtained after removing the mixing term by the nonunitary transformation

$$\begin{pmatrix} \tilde{A}_\mu \\ W_\mu^3 \\ \tilde{Y}_{(L)\mu} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{b_Z} & -\frac{1}{b_Y} \\ 0 & -\frac{\lambda_W}{b_Z} \frac{M_Z^2}{m_W^2 - M_Z^2} & -\frac{\lambda_W}{b_Y} \frac{M_Y^2}{m_W^2 - M_Y^2} \\ 0 & -\frac{\lambda_Y}{b_Z} \frac{M_Z^2}{m_Y^2 - M_Z^2} & -\frac{\lambda_Y}{b_Y} \frac{M_Y^2}{m_Y^2 - M_Y^2} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Y_{(L)\mu} \end{pmatrix}. \quad (3.5)$$

Here,  $A_\mu$ ,  $Z_\mu$  and  $Y_{(L)\mu}$  denote the physical photon,  $Z$ - and  $Y(Y_L)$ -fields,

$$b_i^2 = 1 - \lambda_W^2 - \lambda_Y^2 + \lambda_W^2 \frac{m_W^4}{(m_W^2 - M_i^2)^2} + \lambda_Y^2 \frac{m_Y^4}{(m_Y^2 - M_i^2)^2} \quad (3.6)$$

with  $i = Z, Y$  and  $M_Y$  is the physical  $Y(Y_L)$ -mass. The unobservable "bare" mass  $m_Y$  (before mixing with the photon) of the  $Y(Y_L)$ -boson and  $\lambda_W$  can be related to  $M_Y$  and  $G_F$  by [10]

$$m_Y^2 = M_Y^2 \left( 1 - \lambda_Y^2 \frac{M_Y^2 - m_W^2}{(1 - \lambda_W^2) M_Y^2 - m_W^2} \right), \quad (3.7)$$

$$\lambda_W^2 = \frac{e^2}{m_W^2} \frac{\sqrt{2}}{8} G_F^{-1} . \quad (3.8)$$

According to Eqs. (3.4) - (3.7), the effects of  $Y(Y_L)$  are characterized by the two parameters  $\lambda_Y$  and  $M_Y$ . Since  $m_Y^2$  has to be nonnegative,  $\lambda_Y^2$  is bounded from above by [10]

$$\lambda_Y^2 \leq \lambda_{Y_{max}}^2 = 1 - \lambda_W^2 \frac{M_Y^2}{M_Y^2 - m_W^2} < 1 - \lambda_W^2 . \quad (3.9)$$

Using the transformation Eq. (3.5) we can now determine the  $Z$ -mass and the effective Lagrangian for the interactions of the  $Z$ -boson with quarks and leptons. We find:

$$M_Z^2 = \frac{m_W^2}{1 - \lambda_W^2 - \lambda_Y^2} \frac{m_Y^2}{M_Y^2} \quad (3.10)$$

and

$$\mathcal{L}_{Zff} = \tilde{g}_Z [j_{\mu L}^3 + \delta j_{\mu}^{Y(L)} - s_W^2 j_{\mu}^{em}] Z_{\mu} , \quad (3.11)$$

where  $j_{\mu L}^3$  denotes the third component of the weak isospin current,  $j_{\mu}^{em}$  the electromagnetic current,

$$\tilde{g}_Z = \frac{e}{b_Z} \frac{M_Z^2}{M_Z^2 - m_W^2} \quad (3.12)$$

$$\delta = \frac{M_Z^2 - m_W^2}{M_Z^2 - m_Y^2} \quad (3.13)$$

and, finally,

$$s_W^2 = 1 - \frac{m_W^2}{M_Z^2} . \quad (3.14)$$

Note that  $s_W^2$  formally agrees with  $\sin^2 \theta_W$  which multiplies the  $j_{\mu}^{em}$  term in the standard model.

Eqs. (3.10) - (3.14) show that in presence of  $Y(Y_L)$  the  $Z$ -mass and couplings deviate from the standard model predictions. In addition to the weak isospin and the electromagnetic current, the  $Z$ -boson couples also to  $j_{\mu}^Y (j_{\mu}^{Y_L})$  with relative strength  $\delta$  which for  $M_Y^2 \gg m_W^2$  can approximately be written as

$$\delta \approx - \frac{\lambda_W^2}{1 - \lambda_W^2 - \lambda_Y^2} \frac{m_W^2}{M_Y^2} . \quad (3.15)$$

If  $M_Y$  lies in the few hundred  $GeV$  range, as one should naively expect in a preon model with a scale  $\Lambda_H \sim O(G_F^{-1/2})$ , the absolute value of  $\delta$  is of the order of a few percent.

For the subsequent discussion of the leptonic width of the  $Z$ -boson and the electroweak asymmetries, it is convenient to express  $\mathcal{L}_{Zff}$  in terms of vector and axial vector coupling constants  $V_Z^Y (V_Z^{YL})$  and  $A_Z^Y (A_Z^{YL})$  respectively. One obtains

$$\mathcal{L}_{Zff} = Z_\mu \left[ \sum_f \bar{f} \gamma_\mu \frac{1}{2} (V_Z^{Y(L)} + A_Z^{Y(L)} \gamma_5) f \right] \quad (3.16)$$

where in the  $Y$ -case

$$A_Z^Y = \frac{e}{b_Z} \frac{M_Z^2}{M_Z^2 - m_W^2} \frac{m_Y^2 - m_W^2}{M_Z^2 - m_Y^2} T_3 \quad (3.17)$$

$$V_Z^Y = 2 \frac{e}{b_Z} \frac{m_Y^2}{M_Z^2 - m_Y^2} Q - A_Z^Y \quad (3.18)$$

and in the  $Y_L$ -case

$$A_Z^{YL} = -\frac{e}{b_Z} \frac{M_Z^2}{M_Z^2 - m_Y^2} Q + A_Z^Y \quad (3.19)$$

$$V_Z^{YL} = -\frac{e}{b_Z} \left( 1 - \frac{m_Y^2}{M_Z^2 - m_Y^2} \right) Q - A_Z^Y. \quad (3.20)$$

In Eqs. (3.17) - (3.20) we have used that  $y_L = Q - T_3$  and  $y_R = Q$ .

In the limit of a vanishing  $Y(Y_L)$ -photon mixing strength  $\lambda_Y$  Eq. (3.10) reduces to the standard model expression for  $M_Z$ . This can be easily seen by observing that  $m_Y^2 \rightarrow M_Y^2$  for  $\lambda_Y^2 \rightarrow 0$  (see Eq. (3.7)) and using Eq. (3.8) for  $\lambda_W^2$ . In this limit also  $\tilde{g}_Z$  and  $s_W^2$  approach their standard model values  $e/\sin\theta_W \cos\theta_W$  and  $\sin^2\theta_W$ , respectively. However, since  $g_Y \lambda_Y = e$  (see Eq. (3.4)) is a constant,  $\delta$  does *not* go to zero for  $\lambda_Y^2 \rightarrow 0$ . For small values of  $\lambda_Y^2$  we find

$$\delta \approx -\frac{\lambda_W^2 m_W^2}{(1 - \lambda_W^2) M_Y^2 - m_W^2} \left( 1 + \lambda_Y^2 \frac{M_Y^2 - m_W^2}{(1 - \lambda_W^2) M_Y^2 - m_W^2} \right). \quad (3.21)$$

$\delta$  vanishes only for  $M_Y \rightarrow \infty$ .

Eqs. (3.7) - (3.14) represent the tree-level results for the mass and the couplings of the  $Z$ -boson. Since we are interested in implications of  $Y(Y_L)$  at LEPI/SLC and

LEP II energies, i.e. at a scale  $\mu^2 \geq M_Z^2$ , we should also take into account radiative corrections. However, the effective theory of composite weak vector bosons we are working with is nonrenormalizable and a full renormalization program, therefore, cannot be carried out. Nevertheless, the evolution of the leading log corrections can be done in the framework of the effective field theory approach [21] and this modifies  $e$  (there are no leading log corrections to  $G_F$ ) in the same way as in the standard model, provided that the particle spectrum below  $m_W$  is the same. Vector dominance guarantees that the corrections to  $\alpha$  are finite to all orders in perturbation theory [22]. Subsequently, we, therefore, shall use the radiatively improved value of  $e$  (i.e.  $\alpha \approx 1/128$ ) in Eqs. (3.4), (3.8), (3.12) and (3.17) - (3.20).

### 3.2. PHENOMENOLOGICAL IMPLICATIONS

Using the expressions for the  $Z$ -mass (Eq. (3.10)) and the vector and axial vector coupling constants (Eq. (3.17) - (3.20)) it is now rather straightforward to study the implications of  $Y(Y_L)$  on measurable quantities. Here, we would like to concentrate on two observables: the ratio  $m_W/M_Z$ , which, as we have discussed in Section 2, can hopefully be measured very accurately at ACOL, and the leptonic width of the  $Z$ -boson. The influence of  $Y(Y_L)$  on electroweak asymmetries will be discussed in detail in Section 4.

In Fig. 2 we present plots for  $m_W/M_Z$  versus  $M_Z$  for various values of  $M_Y$  and  $\lambda_Y^2$ . Fig. 2a shows the variation of  $m_W/M_Z$  with  $\lambda_Y^2$  for  $M_Y = 500 \text{ GeV}$ , whereas Fig. 2b illustrates how  $m_W/M_Z$  versus  $M_Z$  behaves as a function of  $M_Y$  for  $\lambda_Y^2 = 0.2$ . For values of  $M_Y$  and  $\lambda_Y^2$  different from those chosen in Fig. 2a and 2b respectively, curves of a similar shape are obtained. For comparison, we have also included in Fig. 2 the standard model prediction for  $m_W/M_Z$  taking into account leading log corrections (see Eqs. (2.4) and (2.8)) which, as we have seen in Section 2, very accurately represent the complete  $O(\alpha)$  corrections to  $m_W$  and  $M_Z$ .

Before we discuss Fig. 2 in more detail we would like to make a general remark about how one should compare standard model and composite weak boson model results. Since we can only include leading log corrections in the effective theory we are working with, composite weak boson model predictions should be compared with standard model calculations where also only leading log correc-

tions have been taken into account. A comparison with the results of a complete one-loop calculation would make sense only if one could carry out a full renormalization program for the effective theory of composite weak boson interactions. This is, however, not possible since the effective theory is nonrenormalizable and we, therefore, cannot do better than comparing composite  $W$ -boson model and standard model predictions both at the leading log level.

From Fig. 2 we observe that the  $Y$ - and  $Y_L$ -boson affect the ratio  $m_W/M_Z$  in the same way. Furthermore, we see that  $m_W/M_Z$  is always larger than in the standard model and increases (decreases) with increasing values of  $\lambda_Y^2$  ( $M_Y$ ). In the limit  $\lambda_Y^2 \rightarrow 0$  (for fixed  $M_Y$ ) and  $M_Y \rightarrow \infty$  (for fixed  $\lambda_Y^2$ ) we recover the standard model prediction for  $m_W/M_Z$ .

In Fig. 3 we display the leptonic  $Z$ -width (lepton masses are neglected)

$$\Gamma(Z \rightarrow \ell^+ \ell^-) = \frac{M_Z}{48\pi} (V_Z^{Y(L)^2} + A_Z^{Y(L)^2}), \quad (3.22)$$

$\ell = e, \mu, \tau$ , versus  $M_Z$  for various values of  $\lambda_Y^2$  and  $M_Y$ , together with the standard model prediction in the leading log approximation which we obtain by setting  $\delta_\ell = 0$  in Eq. (2.10) and employing Eqs. (2.4) and (2.7). Fig. 3a shows how  $\Gamma(Z \rightarrow \ell^+ \ell^-)$  changes with  $\lambda_Y^2$  for  $M_Y = 500 \text{ GeV}$  while Fig. 3b presents results for  $\lambda_Y^2 = 0.2$  when  $M_Y$  is varied. For values of  $M_Y$  and  $\lambda_Y^2$  different from those in Fig. 3a and 3b respectively, curves of a similar shape are obtained. As one can see,  $Y$  and  $Y_L$  influence the leptonic  $Z$ -width in a completely different way. While  $\Gamma(Z \rightarrow \ell^+ \ell^-)$  is in the  $Y$ -case larger than in the standard model, the contrary is true if the underlying preon model contains a  $Y_L$ -boson. For increasing  $M_Y$  ( $\lambda_Y^2$ ) the deviation of the leptonic  $Z$ -width from its standard model value decreases (increases) and vanishes in the limit  $M_Y \rightarrow \infty$ . On the other hand, since  $\delta$  remains finite for  $\lambda_Y^2 \rightarrow 0$  (see Eq. (3.21)),  $\Gamma(Z \rightarrow \ell^+ \ell^-)$  may differ appreciably from the standard model result even for very small values of  $\lambda_Y^2$ . This is illustrated by the two  $\lambda_Y^2 = 0.0$  curves in Fig. 3a.

From Fig. 2 and Fig. 3 we recognize that both,  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+ \ell^-)$ , may be significantly affected by the  $Y(Y_L)$ -boson. High precision measurements of  $M_Z$ ,  $m_W/M_Z$  and the leptonic  $Z$ -width should, therefore, result in rather stringent bounds on the parameters  $M_Y$  and  $\lambda_Y^2$ , provided that no deviations from the

standard model are observed. We will return to these questions in detail in Section 5.

#### 4. Electroweak Asymmetries

We will now consider the electroweak asymmetries in more detail. In the first part of this Section we shall present the tree-level expressions for  $A_{FB}$  and  $A_{LR}$  for the most general case of two massive vector bosons with completely arbitrary vector and axial vector coupling constants and discuss some of their properties. Special emphasis will be given to the location of the zeros of  $A_{FB}$  and  $A_{LR}$  since this will give us some hints how the shape of the electroweak asymmetries versus  $\sqrt{s}$  in the two vector boson case might look like, and how big the deviations from the standard model predictions might be. In the second part of this Section we study  $A_{FB}$  and  $A_{LR}$  in the  $Y$ - and  $Y_L$ -case, thereby verifying the correctness of the results of the general analysis in the first part.

##### 4.1. GENERAL DISCUSSION

We start with some definitions. We assume that in addition to the photon two massive neutral vector bosons  $V_i$  and  $V_j$ , with masses  $M_i$  and  $M_j$ , contribute to the reaction  $e^+e^- \rightarrow f\bar{f}$  in the  $s$ -channel. The  $V_k f\bar{f}$ ,  $k = i, j$ , vector and axial vector coupling constants are denoted by  $v_{kf}$  and  $a_{kf}$ , respectively. Finally,  $\Gamma_{i,j}$  is the total width of  $V_{i,j}$ .

Using the definitions (2.12), (2.13) and (2.25) it is now straightforward to calculate the tree-level expressions for  $A_{FB}^{e^+e^- \rightarrow f\bar{f}}$  and  $A_{LR}^{e^+e^- \rightarrow f\bar{f}}$ ,  $f \neq e, \nu_e$ . One obtains

$$A_{FB}^{e^+e^- \rightarrow f\bar{f}} = \frac{x}{1 + \frac{x^2}{3}} \frac{(1 - P_e) \sum_{h_f, h_e} h_f h_e |F(h_f, h_e)|^2 + 2P_e \sum_{h_f} h_f |F(h_f, 1)|^2}{(1 - P_e) \sum_{h_f, h_e} |F(h_f, h_e)|^2 + 2P_e \sum_{h_f} |F(h_f, 1)|^2} \quad (4.1)$$

and

$$A_{LR}^{e^+e^- \rightarrow f\bar{f}} = P_e \frac{\sum_{h_f, h_e} h_e |F(h_f, h_e)|^2}{\sum_{h_f, h_e} |F(h_f, h_e)|^2} \quad (4.2)$$

where

$$F(h_f, h_e) =$$

$$-\frac{e^2}{s}e_f + \frac{1}{4} \frac{(v_{if} - h_f a_{if})(v_{ie} - h_e a_{ie})}{s - M_i^2 + iM_i \Gamma_i} + \frac{1}{4} \frac{(v_{jf} - h_f a_{jf})(v_{je} - h_e a_{je})}{s - M_j^2 + iM_j \Gamma_j} \quad (4.3)$$

and  $h_{f,e} = \pm 1$  are the helicities of  $f$  and  $e$ , respectively.

While Eqs. (4.1) - (4.3) are in a form convenient for numerical evaluation, they offer little insight into the general properties of  $A_{FB}$  and  $A_{LR}$ . However, if  $V_i$  and  $V_j$  couple universally to charged leptons, some information on the electroweak asymmetries for the most important channel  $e^+e^- \rightarrow \mu^+\mu^-$  may be gained by performing the sums in Eqs. (4.1) and (4.2). It turns out that

i) if one of the resonances couples in a purely righthanded way to charged leptons it has no influence on  $A_{FB}$  if the incident electron beam is fully lefthandedly polarized, i.e. if  $P_e = 1$ .

ii) if both,  $V_i$  and  $V_j$  couple purely vector like, or if one vector boson couples vector like and the other one axial vector like to charged leptons,  $A_{FB}$  is independent of  $P_e$  and  $A_{LR}$  vanishes.

iii)  $A_{LR}(M_Z)$  depends in general on the final state fermion vector and axial vector coupling constants, contrary to the one boson case (see Eq. (2.32)).

In general, useful information on the shape of  $A_{FB}$  and  $A_{LR}$  versus  $\sqrt{s}$  can be obtained from the positions of the maxima, minima and zeros of the electroweak asymmetries. While we were not able to find analytical solutions for the minima and maxima, it is easy to see that requiring  $A_{FB} = 0$  and  $A_{LR} = 0$ , respectively, leads in both cases to equations cubic in  $s$ . Therefore, there are *at most* three zeros (apart from a trivial one at  $s = 0$ ) in  $A_{FB}$  and  $A_{LR}$ .

In view of practical applications where one of the massive vector bosons has to be identified with the  $Z$ -boson, let us study in somewhat greater detail the zeros in the case of  $e^+e^- \rightarrow \mu^+\mu^-$  where

a)  $V_i$  and  $V_j$  couple universally to charged leptons and where

b) we assume

$$\Gamma_i \ll M_i \text{ and } v_{ie} \approx 0 \quad (4.4)$$

Under these simplifying assumptions we find for the zeros in  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$

$$s_1 = M_i^2 \quad (4.5)$$

$$s_{2,3} = M_j^2 + \frac{1}{2A} (-B \pm \sqrt{B^2 - 4AC}) \quad (4.6)$$

with

$$A = 2e^2(a_{ie}^2 + \gamma_{je}a_{je}^2) + \frac{1}{2}v_{je}^2a_{je}^2(\gamma_{je} + \delta_{je}) + \frac{1}{2}v_{je}^2a_{ie}^2\delta_{je}, \quad (4.7)$$

$$B = 2e^2a_{je}^2\gamma_{je}(M_j^2 - M_i^2) + \frac{1}{2}v_{je}^2a_{ie}^2\delta_{je}M_j^2 + \frac{1}{2}v_{je}^2a_{je}^2(\gamma_{je} + \delta_{je})(2M_j^2 - M_i^2), \quad (4.8)$$

$$C = \frac{1}{2}v_{je}^2a_{je}^2M_j^2(M_j^2 - M_i^2)(\gamma_{je} + \delta_{je}) + 2e^2a_{ie}^2M_j^2\Gamma_j^2, \quad (4.9)$$

$$\gamma_{je} = 1 - P_e \frac{v_{je}}{a_{je}}, \quad (4.10)$$

$$\delta_{je} = 1 - P_e \frac{a_{je}}{v_{je}}, \quad (4.11)$$

and for the zeros in  $A_{LR}^{e^+e^- \rightarrow \mu^+\mu^-}$  we derive

$$S_1 = M_i^2, \quad (4.12)$$

$$S_{2,3} = \frac{1}{2A} (-B \pm \sqrt{B^2 - 4AC}), \quad (4.13)$$

with

$$A = 2e^2 + \frac{1}{2}(a_{ie}^2 + v_{je}^2 + a_{je}^2), \quad (4.14)$$

$$B = -(2e^2 + \frac{1}{2}a_{ie}^2)M_j^2 - (2e^2 + \frac{1}{2}v_{je}^2 + \frac{1}{2}a_{je}^2)M_i^2, \quad (4.15)$$

$$C = 2e^2M_i^2M_j^2. \quad (4.16)$$

From Eqs. (4.5) - (4.16) a number of useful observations can be made:

1) Comparing Eq. (4.5) with Eq. (2.22) (for  $v_e = 0$ ) shows that the first zero in  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  is (almost) unaffected by the existence of a second massive vector boson if the first one couples (almost) purely axial vector like.

2) The second and third zeros,  $s_{2,3}$  and  $S_{2,3}$ , exist if  $B^2 - 4AC \geq 0$  and  $B^2 - 4AC \geq 0$ , respectively.

3) One can show that for  $M_j > M_i$  and unpolarized incident electron beam ( $P_e = 0$ )  $s_{2,3}$  fulfill the inequalities

$$M_j^2 - \frac{B}{A} < s_{2,3} < M_j^2. \quad (4.17)$$

4) Similarly,  $S_{2,3}$  fulfill the inequality

$$S_{2,3} < -\frac{B}{A} < M_i^2 + M_j^2. \quad (4.18)$$

5) (4.17) and (4.18) indicate that the region in  $\sqrt{s}$  where the electroweak asymmetries show most structure lies essentially *below* the mass of the second vector boson.

6) The expressions for A, B and C greatly simplify if also  $V_j$  couples purely axial vector like to charged leptons and, moreover, is a narrow resonance ( $\Gamma_j \ll M_j$ ). This, for example, happens sometimes for additional weak vector bosons in GUT's [23] or, as we shall see below, for the  $Y_L$ -boson if  $\lambda_Y^2$  takes on a certain value. In this case both  $s_2$  and  $s_3$  exist and one finds:

$$s_2 = \frac{a_{ie}^2}{a_{ie}^2 + a_{je}^2} M_j^2 + \frac{a_{je}^2}{a_{ie}^2 + a_{je}^2} M_i^2, \quad (4.19)$$

$$s_3 = M_j^2. \quad (4.20)$$

If we identify  $V_i$  with the  $Z$ -boson, Eqs. (4.5) to (4.20) should hold to a very good approximation as long as the vector and/or the axial vector coupling constants of the additional massive vector boson are larger than  $v_e = 1 - 4\sin^2\theta_W \approx 0.08$  (for  $\sin^2\theta_W = 0.23$  corresponding to  $M_Z = 92 \text{ GeV}$ ). This, for example, is the case in all GUT's, left-right symmetric or superstring inspired models where the coupling constants of the second vector bosons are typically of the order of the electromagnetic coupling constant  $e$ , or larger. It also holds true in the composite weak boson models we are considering. Since  $\lambda_Y^2$  is bounded from above (see Eq. (3.9)),  $g_Y$  according to Eq. (3.4) has to be larger than about 0.35.

Comparing the number and the positions of the zeros in  $A_{FB}$  and  $A_{LR}$  in models with an additional massive vector boson  $V_j$  with the ones in the standard model (see Fig. 1 and Eqs. (2.22) and (2.33)), we observe that at least above the  $Z$ -pole and below  $\sqrt{M_Z^2 + M_j^2}$  rather large deviations from the standard model may occur. This means that by measuring electroweak asymmetries at LEP II energies ( $\sqrt{s} = 190 \text{ GeV}$ ) one actually may be sensitive to  $V_j$ -masses much larger than  $\sqrt{s}$ . Subsequently, we shall demonstrate this for the  $Y$ - and the  $Y_L$ -boson. In Section 5 we shall estimate lower limits on the  $Y$ - and  $Y_L$ -masses one should

be able to obtain from experiments at LEPI/SLC and LEP II, provided that no differences from the standard model asymmetries are observed.

#### 4.2. IMPLICATIONS OF $Y$ AND $Y_L$ ON ELECTROWEAK ASYMMETRIES

Using Eqs. (4.1) - (4.3) it is now a straightforward exercise to calculate the electroweak asymmetries in the  $Y$ - and  $Y_L$ -case. The vector and axial vector coupling constants of the  $Z$ -boson, including the mixing effects with the  $Y(Y_L)$ -boson, are given in Eqs. (3.17) - (3.20). The coupling constants  $V_{Y(L)}$  and  $A_{Y(L)}$  of  $Y(Y_L)$  were discussed in detail in [10]. For completeness, we recall here their form and summarize those properties of  $V_{Y(L)}$  and  $A_{Y(L)}$  which are relevant for our subsequent discussion.

The vector and axial vector coupling constants  $V_Y$  ( $V_{Y_L}$ ) and  $A_Y$  ( $A_{Y_L}$ ) of the  $Y(Y_L)$ -boson are given by

$$A_Y = \frac{e}{b_Y} \frac{M_Y^2}{M_Y^2 - m_Y^2} \frac{m_Y^2 - m_W^2}{M_Y^2 - m_W^2} T_3, \quad (4.21)$$

$$V_Y = 2 \frac{e}{b_Y} \frac{m_Y^2}{M_Y^2 - m_Y^2} Q - A_Y, \quad (4.22)$$

and

$$A_{Y_L} = -\frac{e}{b_Y} \frac{M_Y^2}{M_Y^2 - m_Y^2} Q + A_Y, \quad (4.23)$$

$$V_{Y_L} = -\frac{e}{b_Y} \left(1 - \frac{m_Y^2}{M_Y^2 - m_Y^2}\right) Q - A_Y. \quad (4.24)$$

As shown in [10],  $V_{Y(L)}$  and  $A_{Y(L)}$  are rather sensitive to  $\lambda_Y^2$ . Inserting the expressions for  $b_Y$  (Eq.(3.6)) and  $m_Y^2$  (Eq. (3.7)) into Eqs. (4.21) - (4.24) one finds that

i) for *small* values of  $\lambda_Y^2$ ,  $Y(Y_L)$  couples almost exclusively to  $j_\mu^Y$  ( $j_\mu^{Y_L}$ ), and due to the vector dominance relation Eq. (3.4) the  $V$ 's and  $A$ 's become large for small  $\lambda_Y^2$ ,

ii) for *large*  $\lambda_Y^2$  the  $Y_L$ -boson couples almost purely *righthandedly* to fermions,

iii) for

$$\lambda_Y^2 = \frac{(1 - \lambda_W^2)M_Y^2 - m_W^2}{3M_Y^2 - 4m_W^2} \approx 0.26 \quad (4.25)$$

the  $Y_L$ -boson couples purely axial vector like to charged leptons (i.e.  $V_{Y_L} = 0$ ). The numerical value in Eq. (4.25) applies for  $M_Y \geq 300 \text{ GeV}$ . In this region the zero in  $V_{Y_L}$  is almost independent of  $M_Y$ .

Using Eqs. (3.17), (3.18), (4.21) and (4.22) for the  $v$ 's and  $a$ 's in Eq. (4.3) in the  $Y$ -case, and (3.19), (3.20), (4.23) and (4.24) in the  $Y_L$ -case we have computed the electroweak asymmetries as a function of  $\sqrt{s}$  for a variety of values for the two parameters  $M_Y$  and  $\lambda_Y^2$  characterizing  $Y$  and  $Y_L$ . Since  $e^+e^- \rightarrow \mu^+\mu^-$  is the most important channel for LEPI/II and SLC experiments, we have restricted ourselves to the asymmetries in this particular reaction. The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ , which is very close to the central value measured by UA2 [24]. The shape of the asymmetry curves does not change if  $M_Z$  is varied by a few  $\text{GeV}$ . To compute  $\Gamma_Z$  and  $\Gamma_{Y(L)}$  we assumed that the  $t$ -quark mass fulfils the inequality  $M_Z/2 < m_t \ll M_Y/2$ , but our results are insensitive to the choice of this parameter. Finally, the detector acceptance  $x$  was fixed to be  $x = 1$ . For values  $x < 1$  one has to rescale  $A_{FB}$  by a factor  $4x/3 + x^2$  ( $A_{LR}$  is independent of  $x$ ).

Our results are presented in Figs. 4 to 8. To illustrate the implications of  $Y$  and  $Y_L$  on the electroweak asymmetries, we have included the standard model tree-level prediction in each figure. Fig. 4 shows how the forward-backward asymmetry for an unpolarized electron beam ( $P_e = 0$ ) and  $M_Y = 500 \text{ GeV}$  changes with  $\lambda_Y^2$ . Curves of a similar shape are also obtained for different values of  $M_Y$ . It turns out that  $Y$  and  $Y_L$  affect  $A_{FB}$  in a rather different way. While the  $Y_L$ -boson reveals itself through a typical resonance structure, the presence of the  $Y$ -boson is signalled by a rather marked step which moves with increasing  $\lambda_Y^2$  to higher values of  $\sqrt{s}$ . For  $\sqrt{s} \leq M_Z$  the deviations from the standard model prediction are rather small. Above the  $Z$ -pole, however, the  $Y(Y_L)$ -influence is quite dramatic and large deviations from the standard model may occur already for  $\sqrt{s} \ll M_Y$ . For small (large) values of  $\lambda_Y^2$  and  $\sqrt{s} \rightarrow \infty$  the forward-backward asymmetry approaches values close to  $3/4$  in the  $Y_L$ -case, thereby reflecting the approximately lefthanded (righthanded) coupling of the  $Y_L$ . Fig. 4 also demonstrates the correctness of the general results derived in Section 4.1. For all three values of  $\lambda_Y^2$  shown,  $A_{FB}$  has one zero in the  $Y$ - , and three zeros in the  $Y_L$ -case. Furthermore, one observes that the inequality (4.17) is satisfied and that  $Y$  and  $Y_L$  practically do not affect

the first zero in  $A_{FB}$ . In Fig. 4b the parameter  $\lambda_Y^2$  has been tuned to the value in Eq. (4.25) where  $V_{Y_L}$  vanishes for charged leptons. In this case the resonance structure of  $Y_L$  in  $A_{FB}$  is maximally enhanced. Identifying the  $Y_L$ -boson with  $V_j$  from Section 4.1, Eqs. (4.19) and (4.20) are seen to be very well satisfied.

Fig. 5 illustrates the variation of  $A_{FB}$  with  $M_Y$  for fixed  $\lambda_Y^2 = 0.2$  and  $P_e = 0$ . While the shape of the curves remain qualitatively the same, the values of  $\sqrt{s}$  where  $A_{FB}$  is minimal, maximal or zero increase almost proportional with  $M_Y$ . The same effect also occurs for different values of  $\lambda_Y^2$ .

In Fig. 6 we compare the forward-backward asymmetry in the  $Y$ - and  $Y_L$ -case with  $M_Y = 500 \text{ GeV}$  and  $\lambda_Y^2 = 0.1$  for different values of  $P_e$ . If  $\sqrt{s} \leq 150 \text{ GeV}$ ,  $A_{FB}$  is almost independent of  $P_e$  since  $Y(Y_L)$ - contributions are suppressed and, as we have noticed in Section 2,  $P_e$ -dependent  $Z$ -boson contributions are proportional to  $v_e = 1 - 4\sin^2\theta_W \approx 0.08$ , and therefore small. At larger values of  $\sqrt{s}$ , however, the forward-backward asymmetry depends considerably on  $P_e$ . For values of  $M_Y$  and  $\lambda_Y^2$  different from the ones chosen in Fig. 6 one finds in general a variation of  $A_{FB}$  with  $P_e$  similar to the one shown there. Only in the  $Y_L$ -case for  $\lambda_Y^2 \approx 0.26$ , the value where  $V_{Y_L}$  vanishes for charged leptons,  $A_{FB}$  is independent of  $P_e$ .

In Figs. 7 and 8 we present our results for  $A_{LR}/P_e$ . Fig. 7 illustrates how  $A_{LR}/P_e$  versus  $\sqrt{s}$  behaves as a function of  $\lambda_Y^2$ . Again the  $Y(Y_L)$ -mass is  $500 \text{ GeV}$ , but similar curves are also obtained for other values of  $M_Y$ . For  $\sqrt{s} > M_Z$ , the left-right asymmetry in the  $Y$ - and  $Y_L$ -case in general differs substantially from the standard model prediction of  $A_{LR}$ . The presence of the  $Y$ -boson is signalled by a rather pronounced peak which with increasing  $\lambda_Y^2$  is shifted towards higher values of  $\sqrt{s}$ . The  $Y_L$ -boson, on the other hand, affects  $A_{LR}$  in a completely different way. For small or large values of  $\lambda_Y^2$  the appearance of the  $Y_L$  is characterized by a step where  $A_{LR}$  in general changes its sign. Due to the approximately lefthanded (righthanded) coupling of  $Y_L$ ,  $A_{LR}$  approaches values close to  $+1$  ( $-1$ ) for small (large)  $\lambda_Y^2$  and  $\sqrt{s} \gg M_Y$ . If the  $Y_L$ -boson couples purely axial vector like as in Fig. 7b ( $\lambda_Y^2 \approx 0.26$ ), its influence on the left-right asymmetry is minimized and deviations from the standard model prediction are quite small. Identifying  $Y_L = V_i$  we immediately see that Eq. (4.12) and the inequality (4.18) are satisfied.

Fig. 8, finally, shows the variation of  $A_{LR}/P_e$  with  $M_Y$ . The parameter  $\lambda_Y^2$  was chosen to be 0.2, but for different values the results are quite similar. As one can see, increasing  $M_Y$  results in effects analogous to the ones observed for  $A_{FB}$  in Fig. 5. While the shape of the curves remains unchanged, the minima, maxima and zeros move almost proportional with  $M_Y$  to larger values of  $\sqrt{s}$ .

From Figs. 4 to 8 we recognize that in preon models containing an isoscalar weak vector boson  $Y(Y_L)$  the electroweak asymmetries at LEP II energies may be significantly affected, even for values of  $M_Y$  as large as 1 TeV. In general, the influence of  $Y$  and  $Y_L$  will be quite different. By measuring  $A_{FB}$  and  $A_{LR}$  at LEP II (provided that the incident electron beam can be longitudinally polarized) one, therefore, will be sensitive to  $Y(Y_L)$ -masses much larger than  $\sqrt{s}$ . On the other hand, near the  $Z$ -peak the deviations are normally rather small. Since the  $Z$ -contribution is maximally enhanced at  $\sqrt{s} \simeq M_Z$ ,  $Y(Y_L)$ -effects there arise predominantly from the influence of  $Y(Y_L)$  on the  $Z$ -boson vector and axial vector coupling constants via its mixing with the photon (see Section 3.1). Nevertheless, due to the large counting rate at the  $Z$ -pole they should be observable if  $M_Y$  is in the few hundred GeV range. Alternatively, if no deviations from the standard model are seen at LEPI/SLC and LEP II, measurements of  $A_{FB}$  and  $A_{LR}$  in addition to  $M_Z$ ,  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  will further constrain the  $(\lambda_Y^2, M_Y)$  parameter space. Since  $Y$  and  $Y_L$  affect electroweak asymmetries at LEP II energies to a much larger extent than near the  $Z$ -pole, the most stringent bounds on  $M_Y$  and  $\lambda_Y^2$  are expected to come from  $A_{FB}$  and  $A_{LR}$  measurements at LEP II. This will be discussed in more detail in the next Section.

## 5. Searching for $Y$ and $Y_L$ at LEP and SLC

We will now combine the results of the previous Sections in order to obtain restrictions on the  $(\lambda_Y^2, M_Y)$  parameter space which one can expect to result from experiments at LEPI/SLC and LEP II, provided that no deviations from the standard model are observed. Rather than performing a full numerical simulation, we shall carry out a somewhat simplified analysis and focus on the question which quantity may constrain  $\lambda_Y^2$  and  $M_Y$  most efficiently. We start by briefly recalling present experimental limits on  $(\lambda_Y^2, M_Y)$ . In the second part we shall discuss to

what extent  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  measurements may reduce the allowed  $\lambda_Y^2$  and  $M_Y$  range. In Section 5.3 we consider bounds resulting from electroweak asymmetries and in Section 5.4, finally, we combine all constraints and derive lower limits on the  $Y(Y_L)$ -masses.

### 5.1. CONSTRAINTS ON $(\lambda_Y^2, M_Y)$ FROM PRESENT EXPERIMENTS

Limits on  $\lambda_Y^2$  and  $M_Y$  presently come from measurements of  $m_W$  and  $M_Z$  at the CERN  $p\bar{p}$  collider [24] and from low energy lepton-hadron scattering experiments [25]. Data from neutrino-lepton scattering [25] and  $A_{FB}$  measurements at PEP and PETRA [26] are not yet accurate enough to yield sensible bounds.

According to Eqs. (3.7) and (3.10) the  $Z$ -boson mass is affected by the  $Y(Y_L)$  photon mixing. Since  $M_Z$  decreases with increasing  $\lambda_Y^2$ , an upper limit on the  $Y(Y_L)$  photon mixing parameter can be obtained from the present experimental values of the  $W$ - and  $Z$ -mass. Using Eqs. (3.7) and (3.10) together with the results of Ref. [24] we find  $\lambda_Y^2 < 0.67$  (0.75) for  $M_Y = 0.5$  TeV (1 TeV) which is slightly more restrictive than the bound (3.9) from the consistency condition  $m_Y^2 \geq 0$ .

Constraints far better than the ones from  $m_W$  and  $M_Z$  are provided by present low energy lepton-hadron scattering data [25]. Since the contributions of  $Y$  and  $Y_L$  to the effective low energy Lagrangian cannot be parametrized by  $\rho$  and  $\sin^2\theta_W$  (see e.g. [8]), we have to use the individual couplings  $\epsilon(u_L)$  ( $\epsilon(d_L)$ ) and  $\epsilon(u_R)$  ( $\epsilon(d_R)$ ) of left- and righthanded  $u(d)$ -quarks in order to obtain information about the allowed values of  $\lambda_Y^2$  and  $M_Y$ . From the expressions derived for the  $\epsilon$ 's in Ref. [8] we observe that the  $Y(Y_L)$ -contributions to the couplings of the  $u$ - and  $d$ -quarks are either proportional to  $1/m_Y^2$  or to  $1/\lambda_Y^2 m_Y^2$ . These contributions become consequently obviously large for a small value of  $\lambda_Y^2$  for the latter term whereas for both terms a large value of  $\lambda_Y^2$  as well implies a bigger contribution to the couplings as  $m_Y^2 \rightarrow 0$  for  $\lambda_Y^2 \rightarrow \lambda_{Y,max}^2$  (cf. Eq. (3.7)). Therefore, for given  $M_Y$  the experimental results [25] for the couplings

$$\epsilon(u_L) = 0.344 \pm 0.026 \quad \epsilon(d_L) = -0.419 \pm 0.022 \quad (5.1a)$$

$$\epsilon(u_R) = -0.153 \pm 0.022 \quad \epsilon(d_R) = 0.076 \pm 0.041, \quad (5.1b)$$

lead to lower and upper bounds on  $\lambda_Y^2$ . Curves (1) and (2) in Fig. 9 (10) represent those bounds for the  $Y(Y_L)$ -boson and a  $Z$ -mass of 92 GeV. For  $M_Y$

less than about 430  $GeV$  no solution for  $\lambda_Y^2$  compatible with (5.1) exists, but for  $Y(Y_L)$ -masses exceeding 500  $GeV$  the allowed  $\lambda_Y^2$  range is still very large (e.g. for  $M_Y = 800 GeV$  :  $0.035 \leq \lambda_Y^2 \leq 0.702$ ). Since  $\epsilon(d_L)$  gives in general the most stringent bounds and the formulas for this quantity coincide in the  $Y$ - and  $Y_L$ - case, restrictions on  $\lambda_Y^2$  from low energy lepton-nucleon scattering are rather similar for the  $Y$ - and the  $Y_L$ -boson. The limiting contour resulting from (5.1) depends, however, somewhat on the  $Z$ -mass chosen. Varying  $M_Z$  within the range of present experimental data [24] we obtain an absolute lower limit of about

$$M_Y > 370 GeV \quad (5.2)$$

for the  $Y(Y_L)$ -mass which is somewhat more than what was found in Ref. [8]. The deviation can essentially be ascribed to the different experimental values of the  $\epsilon$ 's used here and in Ref. [8].

## 5.2. RESTRICTIONS FROM $m_W/M_Z$ AND $\Gamma(Z \rightarrow \ell^+\ell^-)$ MEASUREMENTS

Once the  $Z$ -mass has been determined at LEPI and SLC, measurements of  $m_W/M_Z$  and the leptonic  $Z$ -width will provide new limits on  $M_Y$  and  $\lambda_Y^2$  (or deviations from the standard model will show up). In order to estimate these limits we shall assume that the leptonic  $Z$ -width can be determined at LEPI/SLC with an accuracy of [14]

$$\frac{\delta\Gamma(Z \rightarrow \ell^+\ell^-)}{\Gamma(Z \rightarrow \ell^+\ell^-)} = 2 \% \quad (5.3)$$

(see Section 2.3). For the experimental error in the ratio of  $W$ - and  $Z$ -masses we consider two different values. To demonstrate the importance of a precise measurement of  $m_W/M_Z$  we calculate limits resulting from this ratio for the error expected for ACOL (see Eq. (2.6)) as well as for the less optimistic one of

$$\delta\left(\frac{m_W}{M_Z}\right) = 1 \% . \quad (5.4)$$

We identify the errors given in Eqs. (5.3), (5.4) and (2.6) with

$$\delta R = | R_{SM} - R_{Y(L)} | \quad (5.5)$$

where  $R$  denotes either  $m_W/M_Z$  or  $\Gamma(Z \rightarrow \ell^+\ell^-)$ , and  $R_{SM}$  and  $R_{Y(L)}$  represent the corresponding predictions of the standard and the composite  $W$ -boson model

including leading log corrections, respectively. Assuming that no differences from the standard model appear in the  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  measurements and using Eqs. (2.4), (2.8) and (2.10) (with  $\delta_\ell = 0$ ) we now convert the results of Figs. 2 and 3 into contours limiting the allowed range in the  $(\lambda_Y^2, M_Y)$  plane. Since deviations from the standard model grow for  $m_W/M_Z$  and the leptonic  $Z$ -width with increasing  $\lambda_Y^2$  (see Section 3.2), we find from both quantities upper limits for the  $Y(Y_L)$  photon mixing parameter. Curve (3a) and (3b) in Fig. 9 represent the upper bound on  $\lambda_Y^2$  from  $m_W/M_Z$  in the  $Y$ -case for the error given in Eq. (2.6) and (5.4), respectively. The corresponding limits for the  $Y_L$ -boson are shown in Fig. 10. Curve (4) in Fig. 9 (10) gives the upper limit resulting from the leptonic  $Z$ -width for the  $Y(Y_L)$ . In each case a  $Z$ -mass of 92 GeV was employed, but our results do not change if  $M_Z$  is varied within the range allowed by present experimental data.

From Fig. 9 (10) we observe that if  $m_W/M_Z$  can be measured at ACOL or LEPI/II with an accuracy of 0.2 % it will give for  $M_Y > 450$  GeV (550 GeV) the most stringent upper limit on  $\lambda_Y^2$ . The present upper limit (curve (2)) will be reduced by about a factor two. On the other hand, if the ratio of  $W$ - and  $Z$ -masses can be obtained only up to 1 %, a measurement of  $m_W/M_Z$  will only marginally improve the upper bound from present low energy lepton-hadron experiments. In this case a confirmation of the standard model prediction for  $\Gamma(Z \rightarrow \ell^+\ell^-)$  within 2 % will already be more restrictive for  $\lambda_Y^2$  (see Fig. 9 and 10). The upper limit resulting from the leptonic  $Z$ -width varies significantly with  $M_Y$  and for  $Y(Y_L)$ -masses less than 400 GeV (450 GeV) the deviation from the standard model is bigger than 2 % even for vanishing  $\lambda_Y^2$ . This is a reflection of the finiteness of  $\delta$  (cf. Eq. (3.21)) in the limit  $\lambda_Y^2 \rightarrow 0$ .

### 5.3. LIMITS FROM ELECTROWEAK ASYMMETRIES

Although  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  measurements at ACOL and LEPI/SLC may considerably reduce the allowed  $\lambda_Y^2$  range, there will remain a substantial part of the  $(\lambda_Y^2, M_Y)$  plane which is still compatible with the data. High precision experiments at LEPI/SLC and LEP II determining the electroweak asymmetries may provide a further reduction of the allowed values of  $\lambda_Y^2$  and  $M_Y$ . In order to see how useful the higher energy of LEP II and a longitudinally polarized  $e^-$ -beam

will be, we shall now analyze for four different possible situations the constraints on the  $(\lambda_Y^2, M_Y)$  parameter space which would result if no deviations from the standard model forward-backward and left-right asymmetry are observed in the reaction  $e^+e^- \rightarrow \mu^+\mu^-$ :

a) Only  $A_{FB}(\sqrt{s} = M_Z)$  with  $P_e = 0$  can be measured. This corresponds to the situation where LEP II is not built and a longitudinally polarized  $e^-$ -beam is not available at LEP I/SLC.

b) Both,  $A_{FB}(\sqrt{s} = M_Z)$  and  $A_{FB}(\sqrt{s} = 190 \text{ GeV})$  with  $P_e = 0$  are determined. This possibility would e.g. be realized if LEP II is built, but polarized beams are not available.

c)  $A_{FB}$  and  $A_{LR}$  can be measured at  $\sqrt{s} = M_Z$  but not at higher values of  $\sqrt{s}$ . This would be the case if LEP II is not built, but a longitudinally polarized  $e^-$ -beam can be achieved at LEP I/SLC.

d)  $A_{FB}$  and  $A_{LR}$  can both be determined at  $\sqrt{s} = M_Z$  and  $\sqrt{s} = 190 \text{ GeV}$ . This could only be realized if LEP II is built and a polarized  $e^-$ -beam would be available both at LEP I/SLC and LEP II.

Before we estimate the limits on  $(\lambda_Y^2, M_Y)$  in each of the four situations, we would like to discuss briefly radiative corrections to the electroweak asymmetries. The biggest contribution to these corrections will come from initial and final state bremsstrahlung radiation. Since this is a pure QED effect we expect that it will affect the electroweak asymmetries in the standard and the composite  $W$ -boson model in the same way. Moreover, these corrections turn out to be detector dependent [27] and we, therefore, shall assume for our subsequent analysis that the experimentalists will correct for this effect.

In Section 2.4 and 2.5 we have noted that in the standard model the purely weak corrections to  $A_{FB}$  and  $A_{LR}$  at the  $Z$ -peak constitute a sizeable fraction of the corresponding tree-level result and depend on the unknown quantities  $m_t$  and  $m_H$ . Varying  $m_t$  between  $30 \text{ GeV}$  and  $180 \text{ GeV}$  and  $m_H$  between  $10 \text{ GeV}$  and  $1 \text{ TeV}$ ,  $A_{FB}$  and  $A_{LR}$  at  $\sqrt{s} = M_Z$  change by as much as  $\pm 0.01$  and  $\pm 0.02$  depending somewhat on the  $Z$ -mass. Comparison with the expected experimental errors (cf. Eqs. (2.14) and (2.26)) shows that the variation of the weak corrections is significantly larger than  $\delta A_{FB}(M_Z)$  and  $\delta A_{LR}(M_Z)$ . Measuring the forward-

backward and left-right asymmetry at the  $Z$ -pole one, therefore, is in the standard model sensitive to the  $t$ -quark and the Higgs-boson mass. From the point of view of testing the standard model versus a composite structure of the weak interactions this sensitivity, however, is not an advantage. Since the weak corrections to  $A_{FB}$  and  $A_{LR}$  arise beyond the leading log level and the effective Lagrangian theory we use to describe composite  $W$ -boson interactions is nonrenormalizable, we have no way of properly treating these corrections in the preon model.

To estimate limits on  $\lambda_Y^2$  and  $M_Y$  from the electroweak asymmetries at  $\sqrt{s} = M_Z$  we, therefore, adopt the following procedure.  $A_{FB}$  and  $A_{LR}$  are both, in the standard and in the composite  $W$ -boson model, treated at the leading log level. The standard model prediction for the electroweak asymmetries is not fully determined due to the variation of the purely weak corrections with  $m_t$  and  $m_H$ . This leads to a "theoretical" uncertainty which should be superimposed on the possible experimental errors. Since the uncertainty is much larger than the expected experimental errors (cf. Eqs. (2.14) and (2.26)), the combined errors will essentially coincide with the "theoretical" ones and consequently we shall use

$$\delta A_{FB}(M_Z) = 0.01 \quad , \quad \delta A_{LR}(M_Z) = 0.02 \quad (5.6)$$

in our subsequent analysis. At LEP II energies the purely weak corrections to  $A_{FB}$  and  $A_{LR}$  are less important than at  $\sqrt{s} = M_Z$ . This can be traced to the fact that the zeroth-order contribution at  $\sqrt{s} = 190 \text{ GeV}$  is, in contrast to  $\sqrt{s} = M_Z$ , not suppressed by the small factor  $v_e = 1 - 4\sin^2\theta_W$  (see Eqs. (2.21) and (2.32)), whilst the radiative corrections contain in both cases contributions of  $O(\alpha/(\pi\sin^2\theta_W))$  not inhibited by  $v_e$ . For the errors in  $A_{FB}$  and  $A_{LR}$  at LEP II energies we, therefore, shall use the values given in Eqs. (2.15) and (2.27).

Identifying  $R$  in Eq. (5.5) with any of the asymmetries and assuming that no deviations from the standard model appear in asymmetry measurements at LEP I/SLC and LEP II we now obtain with Eqs. (2.16 - 18), (2.28 - 30) and the results from Section 3 and 4 the boundaries limiting the allowed range of the  $(\lambda_Y^2, M_Y)$  plane. Our results for the  $Y(Y_L)$ -boson are displayed in Fig. 11 (12). The labels attached to the plots refer to the corresponding situation a) to d) and the detector acceptance was always assumed to be complete ( $x = 1$ ).

From Fig. 11a and 12a we observe that a measurement of  $A_{FB}(\sqrt{s} = M_Z)$

with unpolarized beams in general results in a lower and an upper limit on  $\lambda_Y^2$ . The lower bound can be easily understood by remembering that, according to Eq. (3.4), the  $Y(Y_L)$ -boson coupling constant  $g_Y$  sharply increases with decreasing  $\lambda_Y^2$ . If the  $Y(Y_L)$  photon mixing parameter is too small the  $Y(Y_L)$ -contribution to  $A_{FB}(\sqrt{s} = M_Z)$  dominates over the  $Z$ -term and the difference from the standard model value becomes larger than 0.01. The upper limit originates mainly from the increasing deviation of the vector and the axial vector coupling constant of the  $Z$ -boson with growing  $\lambda_Y^2$  from the corresponding standard model prediction. Comparing Fig. 11a and 12a with 11b and 12b, respectively, we see that the region in the  $(\lambda_Y^2, M_Y)$  plane which is not excluded shrinks considerably if the forward-backward asymmetry is also measured at  $\sqrt{s} = 190 \text{ GeV}$ . This is just a reflection of the sensitivity of  $A_{FB}$  above the  $Z$ -pole to values of  $M_Y$  much larger than  $\sqrt{s}$ . The narrow strips of allowed  $\lambda_Y^2$  values in Fig. 11b and 12b result from an accidental coincidence of the standard and the composite  $W$ -boson model values of  $A_{FB}$  at  $\sqrt{s} = M_Z$  and  $\sqrt{s} = 190 \text{ GeV}$ . Going from Fig. 11a and 12a to 11c and 12c one recognizes that the restrictions on  $\lambda_Y^2$  become also more pronounced if in addition to  $A_{FB}(\sqrt{s} = M_Z)$   $A_{LR}$  can be measured at the  $Z$ -peak. However, the improvement is clearly less significant than for the case displayed in Fig. 11b (12b). While the upper bound is significantly reduced over the whole  $M_Y$  range in the  $Y_L$ -case, the lower limit is considerably strengthened for  $M_Y < 650 \text{ GeV}$  in the  $Y$ -case. From Fig. 11d and 12d we, finally, observe that the allowed region in the  $(\lambda_Y^2, M_Y)$  plane can be most efficiently restricted if both,  $A_{FB}$  and  $A_{LR}$ , are determined at the  $Z$ -pole and at LEP II energies. Comparison of Fig. 11d (12d) with 11c (12c) demonstrates the usefulness of the higher LEP II energy in reducing the  $(\lambda_Y^2, M_Y)$  region not excluded by experimental data, whereas from Fig. 11b (12b) and 11d (12d) the advantage of a longitudinally polarized  $e^-$ -beam, especially at LEP II energies, becomes apparent. Due to the sensitivity of  $A_{LR}$  above the  $Z$ -peak to values of  $M_Y$  much bigger than  $\sqrt{s}$  the allowed region of the  $(\lambda_Y^2, M_Y)$  plane shrinks considerably if a polarized  $e^-$ -beam can be achieved for LEP II and the left-right asymmetry can be measured.

For our analysis we have assumed that the electroweak asymmetries will be determined either at  $\sqrt{s} = M_Z$  or at  $\sqrt{s} = 190 \text{ GeV}$ . At LEP I/SLC, however, a measurement of  $A_{FB}$  and/or  $A_{LR}$  in a whole range of a few  $\text{GeV}$  around the  $Z$ -

pole will be possible and one might wonder whether this could lead to considerably more stringent bounds than a determination of the electroweak asymmetries just at the  $Z$ -peak. It turns out that this is not the case. Since the deviations in  $A_{FB}$  and  $A_{LR}$  from the standard model are rather small around  $\sqrt{s} = M_Z$  (see Figs. 4 to 8 and Section 4.2) and as the counting rate sharply drops once one is moving away from the  $Z$ -pole, one does not gain very much from such a measurement. The situation may, however, be completely different in the LEP II case. If LEP II could be operated also at  $\sqrt{s}$  values sufficiently different from the designed one of 190 GeV and  $A_{FB}$  and  $A_{LR}$  could be determined, the sensitivity of the electroweak asymmetries above the  $Z$ -pole to values of  $M_Y$  much larger than  $\sqrt{s}$  may lead to a further significant improvement of the bounds on  $\lambda_Y^2$  and  $M_Y$ .

#### 5.4. LOWER LIMITS ON $M_Y$ FROM LEP AND SLC EXPERIMENTS

Finally, we may combine the bounds from present low energy lepton-hadron experiments, from  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  measurements and from electroweak asymmetries. The resulting lower limits on  $M_Y$  for each of the four situations discussed in Section 5.3 are presented in Table 1.

In situation a), where  $A_{FB}(\sqrt{s} = M_Z)$  is the only asymmetry which is determined, we find that the bounds from electroweak asymmetries are in general *weaker* than the ones resulting from  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$ . The lower limits on  $M_Y$ , therefore, can be directly read off from Figs. 9 and 10. If we compare them with the present lower bound of  $M_Y > 370$  GeV (cf. (5.2)) we see that if LEP II is not built and a longitudinally polarized  $e^-$ -beam is not available at LEP I/SLC the lower limit can only slightly be improved at future  $e^+e^-$  colliders. Furthermore, we observe that the bound on  $M_Y$  in this case is almost independent from the accuracy of the  $m_W/M_Z$  measurement.

The lower limits on  $M_Y$  can be significantly strengthened if LEP II is built and, consequently,  $A_{FB}(\sqrt{s} = 190$  GeV) can be determined (situation b)). For the  $Y$ -boson the most restrictive upper and lower bound on  $\lambda_Y^2$  then results from  $m_W/M_Z$  or the leptonic  $Z$ -width, and the forward-backward asymmetry, respectively. In the  $Y_L$ -case, on the other hand, the contour limiting the allowed region of the  $(\lambda_Y^2, M_Y)$  plane almost exclusively comes from  $A_{FB}$  measurements at  $\sqrt{s} = M_Z$  and  $\sqrt{s} = 190$  GeV, and the lower limit on  $M_Y$ , therefore, is independent of the

precision achieved for  $m_W/M_Z$ .

If instead of  $A_{FB}(\sqrt{s} = 190 \text{ GeV})$  the left-right asymmetry at the  $Z$ -peak can be determined, i.e. LEP II is not built, but a polarized  $e^-$ -beam can be achieved for LEP I/SLC (situation c) ), the bounds obtained for situation a) can only in the  $Y$ -case be improved somewhat. The allowed region of the  $(\lambda_Y^2, M_Y)$  plane is only slightly reduced by  $A_{FB}$  and  $A_{LR}$  measurements at  $\sqrt{s} = M_Z$ . For the  $Y_L$ -boson the upper limit from the electroweak asymmetries almost coincides with the one from  $m_W/M_Z$  if this ratio can be measured with an accuracy of 0.2 %.

The most restrictive lower bounds on  $M_Y$  can be found if  $A_{FB}$  and  $A_{LR}$  both can be measured at the  $Z$ -pole and at LEP II energies (situation d) ). If  $m_W/M_Z$  can be determined with an accuracy of 0.2 %  $Y(Y_L)$ -masses less than 1.0  $TeV$  (0.9  $TeV$ ) can be excluded. The allowed areas in the  $(\lambda_Y^2, M_Y)$  plane for this case are shown in Fig. 13. From Fig. 13a we observe that for the  $Y$ -boson the upper and lower limit on  $\lambda_Y^2$  result from  $m_W/M_Z$  and electroweak asymmetry measurements, respectively. The bound on  $M_Y$ , therefore, depends somewhat on the precision which can be achieved for the ratio of  $W$ - and  $Z$ -masses. If  $m_W/M_Z$  can be obtained only with 1 % error, the lower limit on  $M_Y$  is reduced to 830  $GeV$ . For the  $Y_L$ -boson, on the other hand, the limiting contour exclusively results from the electroweak asymmetries (see Fig. 13b) and the bound on  $M_Y$  is independent from the accuracy in the  $m_W/M_Z$  determination.

Comparing the limits on  $M_Y$  for situation a) to d) (cf. Table 1) we see that at LEP I/SLC even with a longitudinally polarized  $e^-$ -beam *at best*  $Y(Y_L)$ -masses less than 600  $GeV$  (500  $GeV$ ) can be excluded. These values could only then be significantly improved if the errors (5.6) for the forward-backward and the left-right asymmetry at the  $Z$ -peak, which are mainly due to the theoretical uncertainty of the standard model prediction for  $A_{FB}(\sqrt{s} = M_Z)$  and  $A_{LR}(\sqrt{s} = M_Z)$ , could be considerably reduced.

If LEP II is built, the lower bound on  $M_Y$  can be improved up to 1  $TeV$  and the most severe restrictions on  $\lambda_Y^2$  arise in general from measurements of  $A_{FB}$  and  $A_{LR}$  at LEP II energies.

## 6. Conclusions

In this paper we have analyzed the implications of isoscalar weak vector bosons  $Y$  and  $Y_L$  on measurable quantities at future  $e^+e^-$  colliders. The  $Y$  and  $Y_L$  are coupled to the weak hypercharge current and its lefthanded part, respectively, and such particles typically appear in preon models with a composite structure of the weak interactions.

After a discussion of the relevant observables and their standard model predictions, we have developed an effective Lagrangian formalism to describe composite weak vector boson interactions. We also derived the expression for the  $Z$ -mass and the Lagrangian for interactions of the  $Z$ -boson with quarks and leptons in the presence of  $Y$  and  $Y_L$ , and analyzed how they affect the observable quantities  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$ .

In Section 4 the electroweak asymmetries were considered in detail. In the first part we presented the tree-level expressions for  $A_{FB}$  and  $A_{LR}$  for the general case of two massive vector bosons with arbitrary vector and axial vector coupling constants and discussed the location of zeros of the electroweak asymmetries which are useful in order to gain information about the shape of  $A_{FB}$  and  $A_{LR}$  versus  $\sqrt{s}$  in the two vector boson case. Our results can be applied to any model with additional weak vector bosons, e.g. GUT's, left-right symmetric or superstring inspired models.

In the second part we studied the asymmetries in the  $Y$ - and  $Y_L$ -case, and illustrated the results of the general analysis in the first part.

Finally, in Section 5 we obtained restrictions on the  $(\lambda_Y^2, M_Y)$  parameter space which one can expect to result if experiments at LEP and SLC agree with the standard model predictions within the possible experimental accuracies.

Our results can be summarized as follows:

- i) The ratio  $m_W/M_Z$  is affected in the same way by  $Y$  and  $Y_L$  and is always larger than in the standard model (see Fig. 2). Deviations from the standard model increase (decrease) with increasing  $\lambda_Y^2$  ( $M_Y$ ).
- ii) On the other hand,  $Y$  and  $Y_L$  influence the leptonic  $Z$ -width in a completely different way: While  $\Gamma(Z \rightarrow \ell^+\ell^-)$  in the  $Y$ -case is larger than in the stan-

standard model, the contrary is true for the  $Y_L$ -boson (see Fig. 3)

- iii) For any model containing two massive neutral vector bosons there are at most three zeros in  $A_{FB}$  and  $A_{LR}$ . If one of the vector bosons couples (almost) axial vector like to charged leptons, as it is the case for the  $Z$ -boson, the first zero is (almost) unaffected by the existence of the second massive vector boson. The two other zeros, if they exist, are bounded from above in this case (see (4.17-18)).
- iv) At  $\sqrt{s}$  values below and around the  $Z$ -mass  $Y$  and  $Y_L$  affect  $A_{FB}$  and  $A_{LR}$  only slightly. Above the  $Z$ -peak, however, the electroweak asymmetries are rather sensitive to the presence of isoscalar weak vector bosons. At LEP II energies large deviations from the standard model predictions may occur even for  $Y(Y_L)$ -masses much larger than  $\sqrt{s}$  (see Figs. 4 to 8).
- v) If experiments at LEP and SLC agree with the standard model within the possible experimental accuracies, the region of the  $(\lambda_Y^2, M_Y)$  plane which is compatible with present data will be strongly reduced (see Figs. 9 to 13). With LEPI/SLC  $Y(Y_L)$ -masses up to 600 GeV (500 GeV) can be excluded. Due to the sensitivity of the electroweak asymmetries above the  $Z$ -pole to values of  $M_Y$  much larger than  $\sqrt{s}$ , the lower limit on  $M_Y$  may be pushed up to 1 TeV at LEP II (if a longitudinally polarized  $e^-$ -beam can be realized at LEP II).

Our results clearly demonstrate the usefulness of the higher LEP II energy and of a polarized incident electron beam in searching for  $Y$  and  $Y_L$ . Since isoscalar weak vector bosons in preon models with composite  $W$ - and  $Z$ -bosons are expected to have a mass of the order of a few hundred GeV, many models of this type would be excluded if one could place a lower bound of 1 TeV on the  $Y(Y_L)$ -masses. Our analysis shows that this limit can be reached with LEP II but not with LEPI/SLC unless the errors  $\delta A_{FB}(\sqrt{s} = M_Z)$  and  $\delta A_{LR}(\sqrt{s} = M_Z)$  can be significantly reduced. This, however, will be difficult since as we have seen in Section 5.3, the dominant uncertainty arises from the conceptual difficulty of comparing the fully renormalizable standard model with an effective theory in which only leading log corrections can be included.

The analysis performed in this paper can easily be repeated for any other

model containing a second massive weak vector boson. The main difference to the  $Y(Y_L)$ -case are the different vector and axial vector coupling constants. In view of the results for the general two vector boson case obtained in Section 4.1 for the electroweak asymmetries, we in general expect large deviations in  $A_{FB}$  and  $A_{LR}$  from the standard model at LEP II energies. If asymmetry measurements agree with the standard model predictions at LEP II strong limits on the mass and/or the coupling constant will not only be obtainable for  $Y$  and  $Y_L$  but also for excited  $Z$ -bosons [28] in composite weak boson models, or additional massive neutral vector bosons appearing in GUT's, left-right symmetric models, superstring inspired models etc. Our results may, therefore, at least qualitatively be valid in a more general context.

#### ACKNOWLEDGEMENTS

We would like to thank the members of the working group on compositeness of the LEP200 Workshop, in particular A. Blondel, B. Schrempp, F. Schrempp and D. Treille for useful and stimulating discussions. We are also grateful to A. Sirlin for helpful comments. One of us (U.B.) would like to acknowledge financial support of the Max-Kade-Foundation, New York.

## REFERENCES

- [1] S.L. Glashow, *Nucl. Phys.* **22** (1961) 579;  
S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264;  
A. Salam, Proc. 8th Nobel Symposium, ed. N. Svartholm, Almquist and Wiksells, Stockholm 1968, p. 367.
- [2] UA1 collaboration, G. Arnison et al., CERN-EP/85-185 (1985) preprint;  
UA2 collaboration, J.A. Appel et al., *Z. Phys.* **C30** (1986) 1.
- [3] For a review see e.g. W. Buchmüller, *Acta Physica Austriaca, Suppl.* **XXVII** (1985) 517.
- [4] R. Kögerler and D. Schildknecht, CERN-TH 3231 (1982) (unpublished);  
H. Fritzsche, R. Kögerler and D. Schildknecht, *Phys. Lett.* **114B** (1982) 157;  
B. Schrempp and F. Schrempp, DESY 84-055 (1984) (unpublished).
- [5] M. Kuroda and D. Schildknecht, *Phys. Lett.* **121B** (1983) 173;  
U. Baur, D. Schildknecht and K.H. Schwarzer, MPI-PAE/PTh 29/85 (1985), to appear in *Phys. Rev. D* (Jan. 1987).
- [6] S.F. King and S.R. Sharpe, *Nucl. Phys.* **B253** (1985) 1;  
U. Baur and K.H. Streng, *Z. Phys.* **C30** (1986) 325.
- [7] W. Buchmüller, *Phys. Lett.* **145B** (1984) 151.
- [8] M. Kuroda, D. Schildknecht and K.H. Schwarzer, *Nucl. Phys.* **B261** (1985) 432.
- [9] C. Korpa and Z. Ryzak, *Phys. Rev.* **D34** (1986) 2139.
- [10] U. Baur and K.H. Schwarzer, *Phys. Lett.* **180B** (1986) 163.
- [11] R.W. Robinett and J.L. Rosner, *Phys. Rev.* **D25** (1982) 3036.
- [12] M. Cvetič and B.W. Lynn, SLAC-PUB-3900 (1986) preprint.

- [13] J.P. Ader, S. Narison and J.C. Wallet, *Phys. Lett.* **176B** (1986) 215;  
P.J. Franzini and F.J. Gilman, SLAC-PUB-3932 (1986) preprint.
- [14] G. Altarelli, in: "Physics at LEP", CERN Yellow Report 86/02 (1986),  
Vol. 1, ed. J. Ellis and R.D. Peccei.
- [15] See e.g. W.J. Marciano and A. Sirlin, *Phys. Rev.* **D29** (1984) 945 and  
references therein.
- [16] See e.g. D. Yu. Bardin, S. Riemann and T. Riemann, *Z. Phys.* **C32**  
(1986) 121 and references therein.
- [17] G. Barbiellini et al., in "Physics at LEP", CERN Yellow Report 86/02  
(1986), Vol. 2, ed. J. Ellis and R.D. Peccei.
- [18] A. Blondel, private communication.
- [19] B.W. Lynn and R.G. Stuart, *Nucl. Phys.* **B253** (1985) 216;  
B.W. Lynn, M.E. Peskin and R.G. Stuart, in "Physics at LEP", CERN  
Yellow Report 86/02 (1986), ed. J. Ellis and R.D. Peccei.
- [20] P.Q. Hung and J.J. Sakurai, *Nucl. Phys.* **B143** (1978) 81;  
J.D. Bjorken, *Phys. Rev.* **D19** (1979) 335.
- [21] L. Hall, *Nucl. Phys.* **B178** (1981) 75;  
S. Dawson, J.S. Hagelin and L. Hall, *Phys. Rev.* **D23** (1981) 2666.
- [22] T.D. Lee, *Phys. Rev.* **168** (1968) 1714.
- [23] P. Langacker, R.W. Robinett and J.L. Rosner, *Phys. Rev.* **D30** (1984)  
1470.
- [24] G. Altarelli, talk given at the 23rd Int. Conf. on High Energy Physics,  
Berkeley, USA, July 16 - 23, 1986 and Rome preprint n. 529 (1986).
- [25] G. Barbiellini and C. Santoni, CERN-EP/85-117 (1985) preprint.
- [26] B. Naroska, talk given at the 21st. Recontré de Moriond, Les Arcs,  
France, March 9-16, 1986 and DESY 86-051 preprint.

- [27] F.A. Berends and R. Kleiss, *Nucl. Phys.* **B177** (1981) 237;  
F.A. Berends, R. Kleiss and S. Jadach, *Nucl. Phys.* **B202** (1982) 63;  
M. Böhm and W. Hollik, *Nucl. Phys.* **B204** (1982) 45.
- [28] U. Baur, M. Lindner and K.H. Schwarzer, in preparation

**Table 1**

Situation	a	b	c	d
$Y$ -boson	460 (450)	720 (560)	600 (530)	1010 (830)
$Y_L$ -boson	500 (500)	880 (880)	500 (500)	880 (880)

Lower bounds on  $M_Y$  (in  $GeV$ ) resulting from a combination of the constraints from present low energy lepton-nucleon data, a measurement of  $m_W/M_Z$  with 0.2 % error and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  with 2 % error, and from electroweak asymmetries. Numbers in parentheses apply if the ratio of  $W$ - and  $Z$ - masses can be obtained only with a precision of 1.0 %.

## FIGURE CAPTIONS

1. The standard model forward-backward asymmetry  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  for unpolarized beams and ideal detector acceptance ( $x = 1$ ) and the standard model left-right asymmetry  $A_{LR}^{e^+e^- \rightarrow \mu^+\mu^-} / P_e$  ( $P_e$ : degree of longitudinal polarization of the incident electron beam) versus  $\sqrt{s}$ . The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ . In the lower right corner a close-up of the region near the  $Z$ -pole is shown.
  
2. The ratio  $m_W/M_Z$  versus  $M_Z$  in presence of the isoscalar boson  $Y$  or  $Y_L$  (solid lines)
  - a) for  $M_Y = 500 \text{ GeV}$ ,  $\lambda_Y^2 = 0.2, 0.4, 0.6$  ;
  - b) for  $\lambda_Y^2 = 0.2$ ,  $M_Y = 300, 500, 800, 1000 \text{ GeV}$  .

The numbers attached to the curves denote the values of  $\lambda_Y^2$  and  $M_Y$  (in  $\text{GeV}$ ), respectively. The dashed line represents the standard model prediction.
  
3. The leptonic  $Z$ -width  $\Gamma(Z \rightarrow \ell^+\ell^-)$  versus  $M_Z$  in presence of the isoscalar boson  $Y$  (solid lines) and  $Y_L$  (dotted lines)
  - a) for  $M_Y = 500 \text{ GeV}$ ,  $\lambda_Y^2 = 0.0, 0.2, 0.6$  ;
  - b) for  $\lambda_Y^2 = 0.2$ ,  $M_Y = 500, 800, 1000 \text{ GeV}$  .

On the curves the corresponding values of  $\lambda_Y^2$ , respectively  $M_Y$  (in  $\text{GeV}$ ) are shown. The dashed line represents the standard model prediction.
  
4. The forward-backward asymmetry  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  for unpolarized electron beam ( $P_e = 0$ ) and ideal detector acceptance ( $x = 1$ ) in the  $Y(Y_L)$ -case (dashed (dotted) line) for  $M_Y = 500 \text{ GeV}$  and
  - a)  $\lambda_Y^2 = 0.1$ ,
  - b)  $\lambda_Y^2 = 0.26$ ,
  - c)  $\lambda_Y^2 = 0.5$ .

The solid line represents the standard model prediction. The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ .
  
5. The forward-backward asymmetry  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  for unpolarized electron beam ( $P_e = 0$ ) and ideal detector acceptance ( $x = 1$ ) in the  $Y(Y_L)$ -

case (dashed (dotted) lines) for  $\lambda_Y^2 = 0.2$  and  $M_Y = 500 \text{ GeV}$  and  $M_Y = 1 \text{ TeV}$ , respectively. The numbers attached to the curves denote the  $Y(Y_L)$ -mass in  $\text{GeV}$ . The solid line represents the standard model prediction. The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ .

6. The forward-backward asymmetry  $A_{FB}^{e^+e^- \rightarrow \mu^+\mu^-}$  for ideal detector acceptance ( $x = 1$ ),  $\lambda_Y^2 = 0.1$ ,  $M_Y = 500 \text{ GeV}$  and  $P_e = 0$  (solid line),  $P_e = 0.5$  (dashed line) and  $P_e = 1$  (dotted line)
- in the  $Y$ -case,
  - in the  $Y_L$ -case.
- For the  $Z$ -mass we used  $92 \text{ GeV}$ .

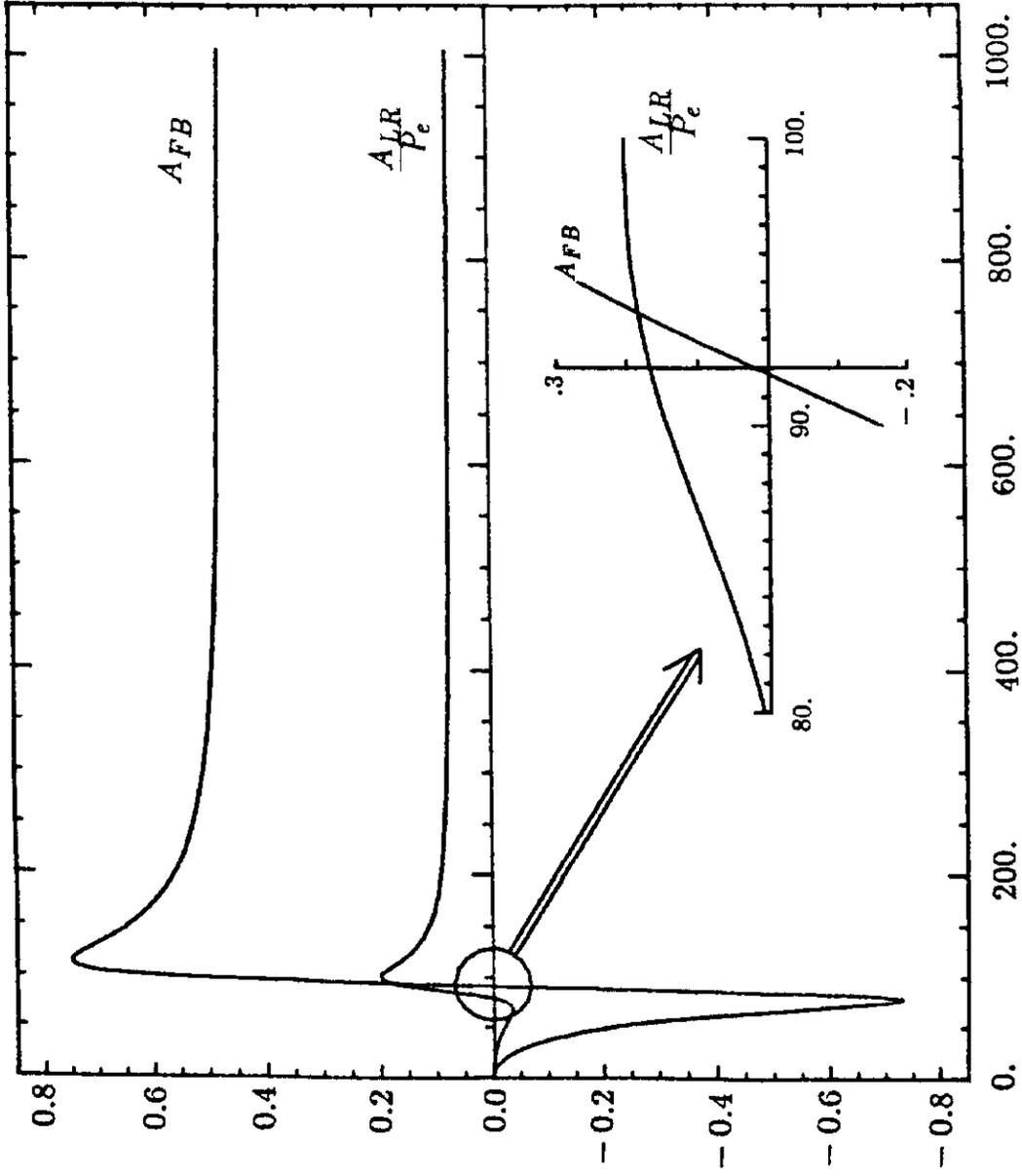
7. The left-right asymmetry  $A_{LR}^{e^+e^- \rightarrow \mu^+\mu^-} / P_e$  in the  $Y(Y_L)$ -case (dashed (dotted) line) for  $M_Y = 500 \text{ GeV}$  and
- $\lambda_Y^2 = 0.1$ ,
  - $\lambda_Y^2 = 0.26$ ,
  - $\lambda_Y^2 = 0.5$ .

The solid line represents the standard model prediction. The  $Z$ - mass is  $92 \text{ GeV}$ .

8. The left-right asymmetry  $A_{LR}^{e^+e^- \rightarrow \mu^+\mu^-} / P_e$  in the  $Y(Y_L)$ -case (dashed (dotted) lines) for  $\lambda_Y^2 = 0.2$  and  $M_Y = 500 \text{ GeV}$  and  $M_Y = 1 \text{ TeV}$ , respectively. On the curves the corresponding values of  $M_Y$  (in  $\text{GeV}$ ) are shown. The solid line gives the standard model prediction. For the  $Z$ - mass we used  $92 \text{ GeV}$ .

9. Lower and upper limits on  $\lambda_Y^2$  versus  $M_Y$  in the  $Y$ -case. Curves (1) and (2) represent the lower and upper limit from present low energy lepton-nucleon experiments, respectively. Curves (3) and (4) show the upper limits which would result from measurements of  $m_W/M_Z$  and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  with
- 0.2% error (curve (3a))
  - 1.0% error (curve (3b))
- in  $m_W/M_Z$  and 2% error in the leptonic  $Z$ -width. The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ .

10. The same as in Fig. 9 for the  $Y_L$ -case.
11. Contours limiting the allowed  $\lambda_Y^2$  and  $M_Y$  range which result from a measurement of
- a)  $A_{FB}(\sqrt{s} = M_Z)$  with 1 % error,
  - b)  $A_{FB}(\sqrt{s} = M_Z)$  and  $A_{FB}(\sqrt{s} = 190 \text{ GeV})$  with 1 % and 3 % error, respectively,
  - c)  $A_{FB}(\sqrt{s} = M_Z)$  and  $A_{LR}(\sqrt{s} = M_Z)$  with 1 % and 2 % error, respectively,
  - d)  $A_{FB}$  and  $A_{LR}$  at both  $\sqrt{s} = M_Z$  and  $\sqrt{s} = 190 \text{ GeV}$  with the corresponding errors from a), b) or c)
- in the  $Y$ -case. The hatched side of the boundary curves indicates the allowed region. For the  $Z$ -mass we used  $92 \text{ GeV}$ .
12. The same as in Fig. 11 for the  $Y_L$ -case.
13. Allowed regions in the  $(\lambda_Y^2, M_Y)$  plane (hatched areas) resulting from a combination of the lower (curve(1)) and upper limits (curve(2)) from present lepton-hadron scattering data, the upper bounds from a measurement of  $m_W/M_Z$  with 0.2 % error (curve (3)) and  $\Gamma(Z \rightarrow \ell^+\ell^-)$  with 2 % error (curve (4)), and the limiting contour from  $A_{FB}$  and  $A_{LR}$  measurements at  $\sqrt{s} = M_Z$  and  $\sqrt{s} = 190 \text{ GeV}$  (curve (5))
- a) for the  $Y$ -boson,
  - b) for the  $Y_L$ -boson.
- The  $Z$ -mass was chosen to be  $92 \text{ GeV}$ .



$\sqrt{s}$  [GeV]

Fig. 1

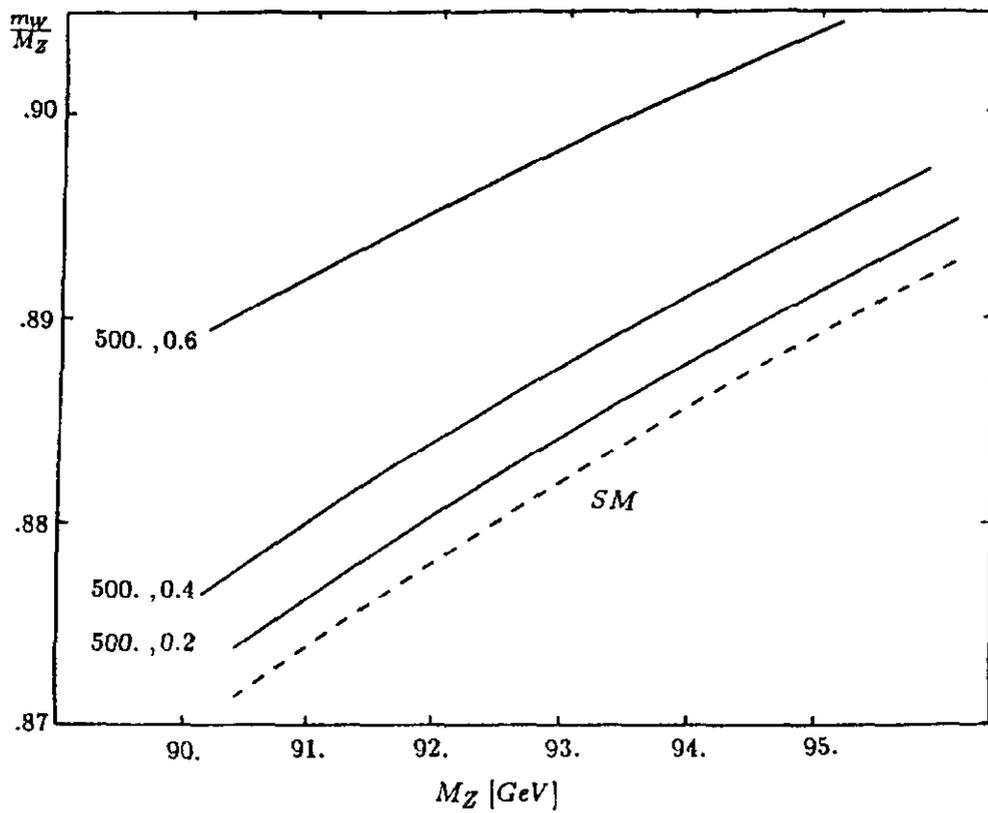


Fig. 2a

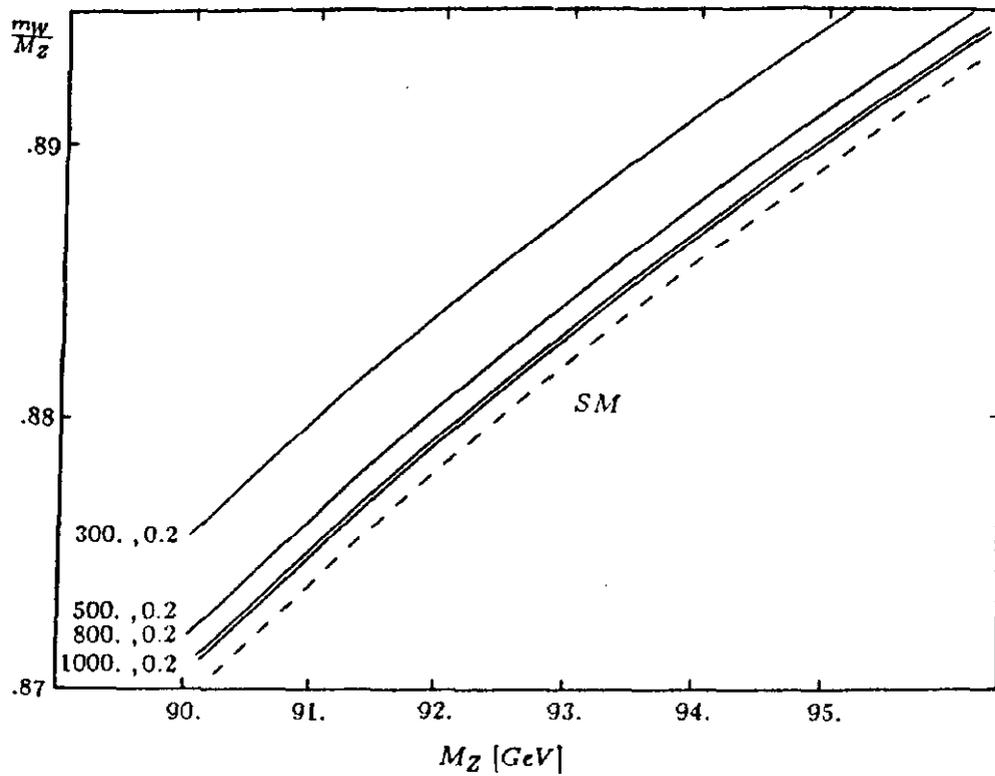
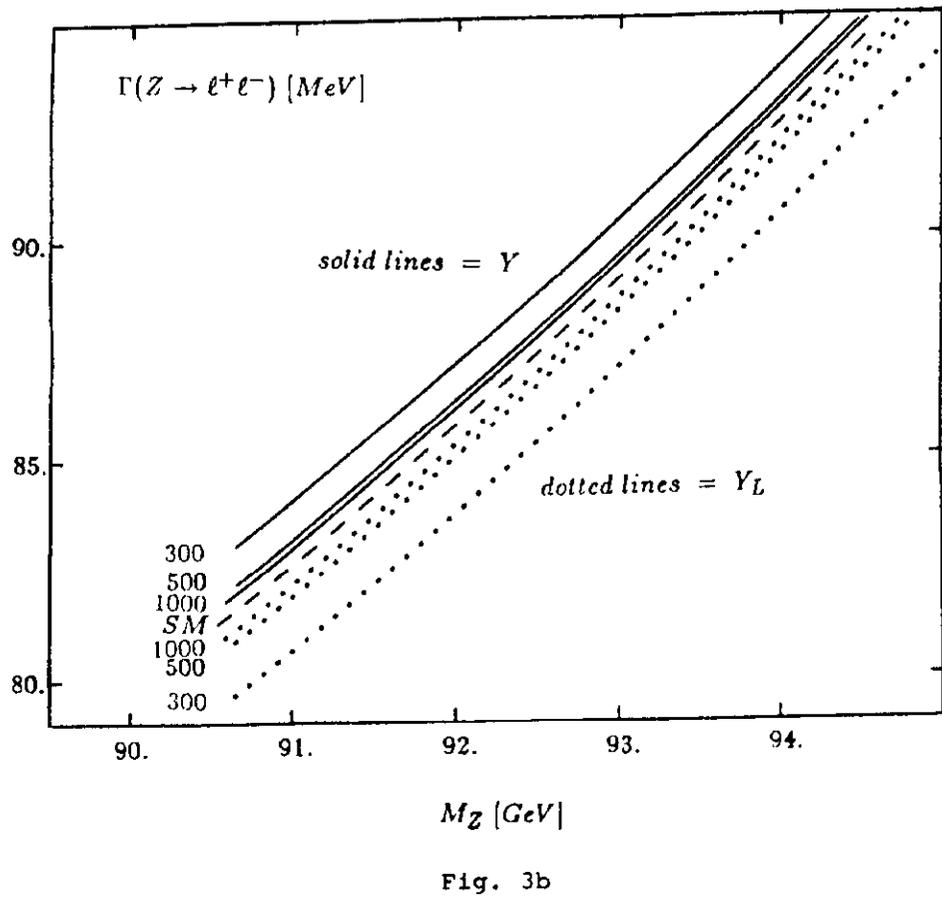
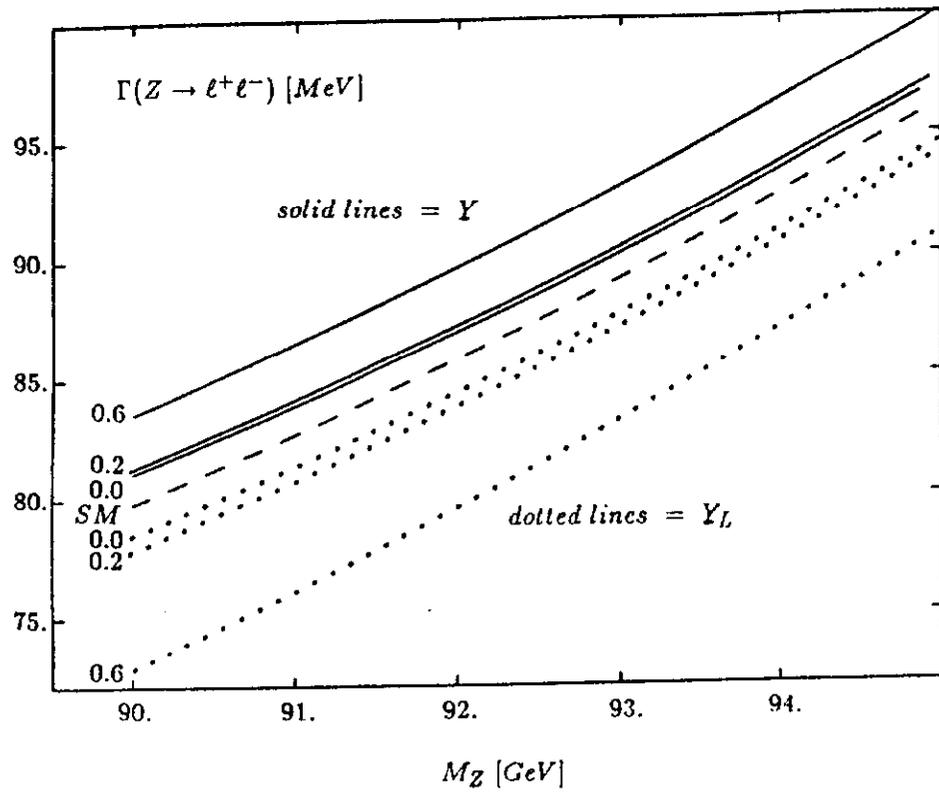


Fig. 2b



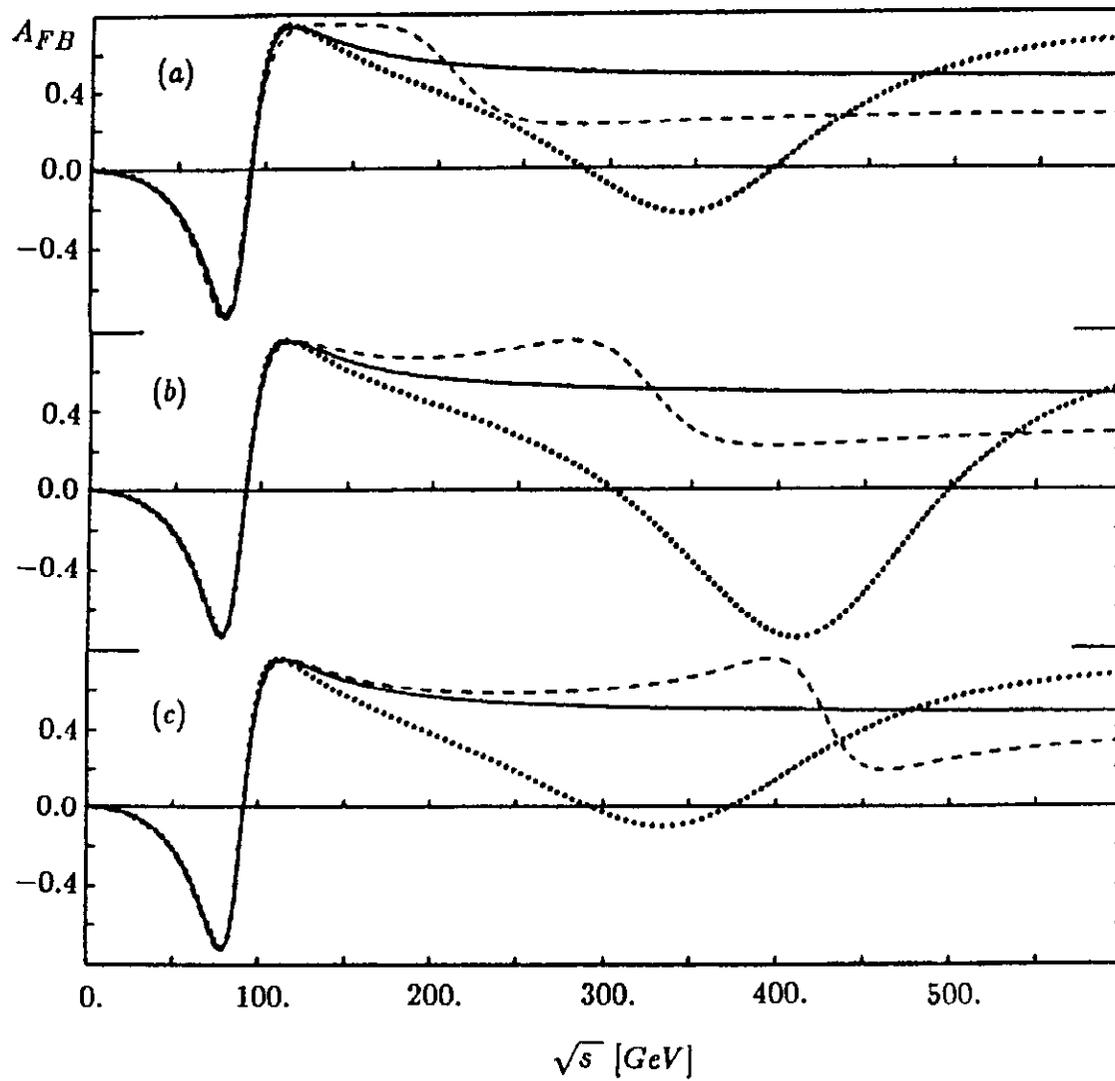


Fig. 4

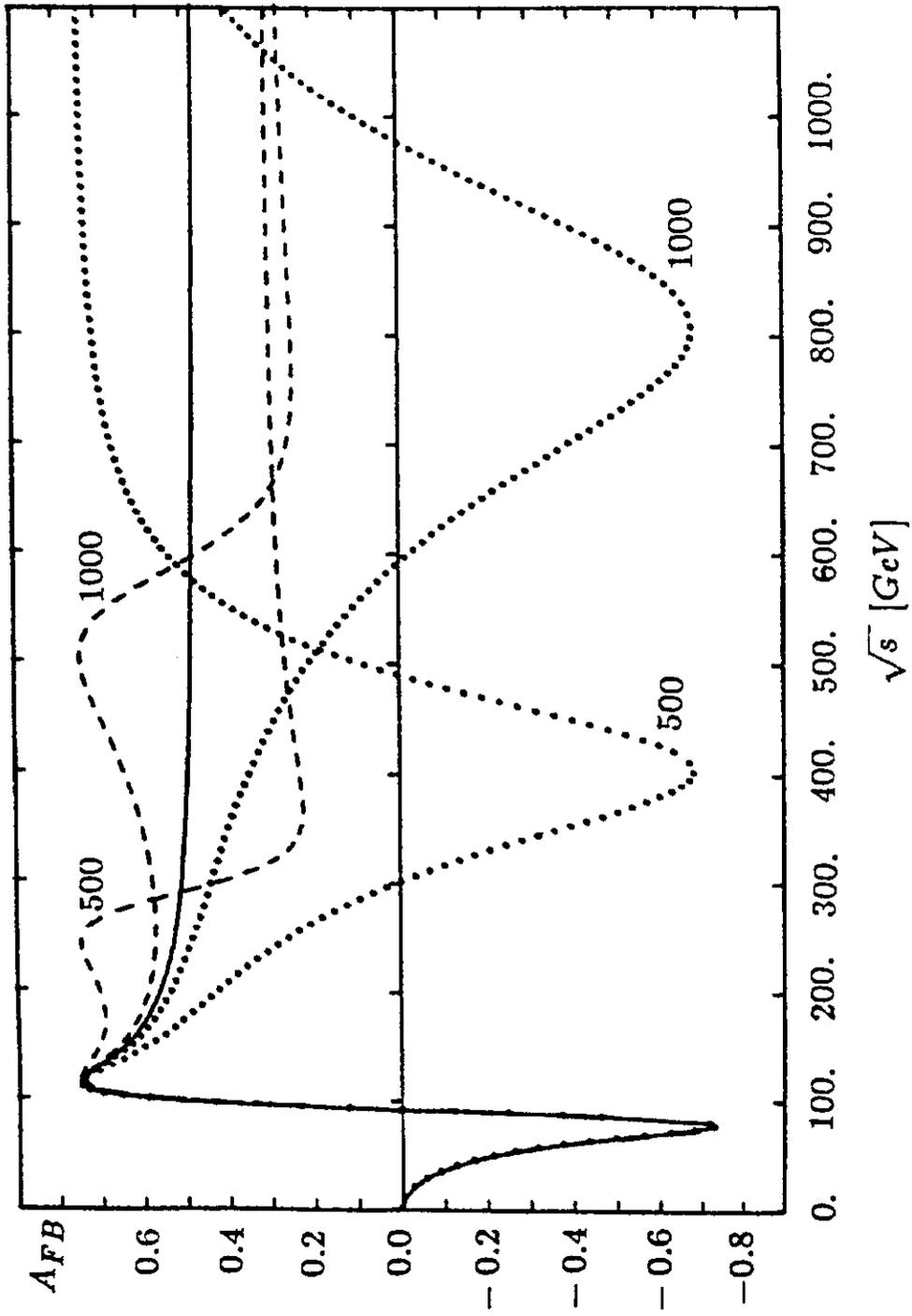


Fig. 5

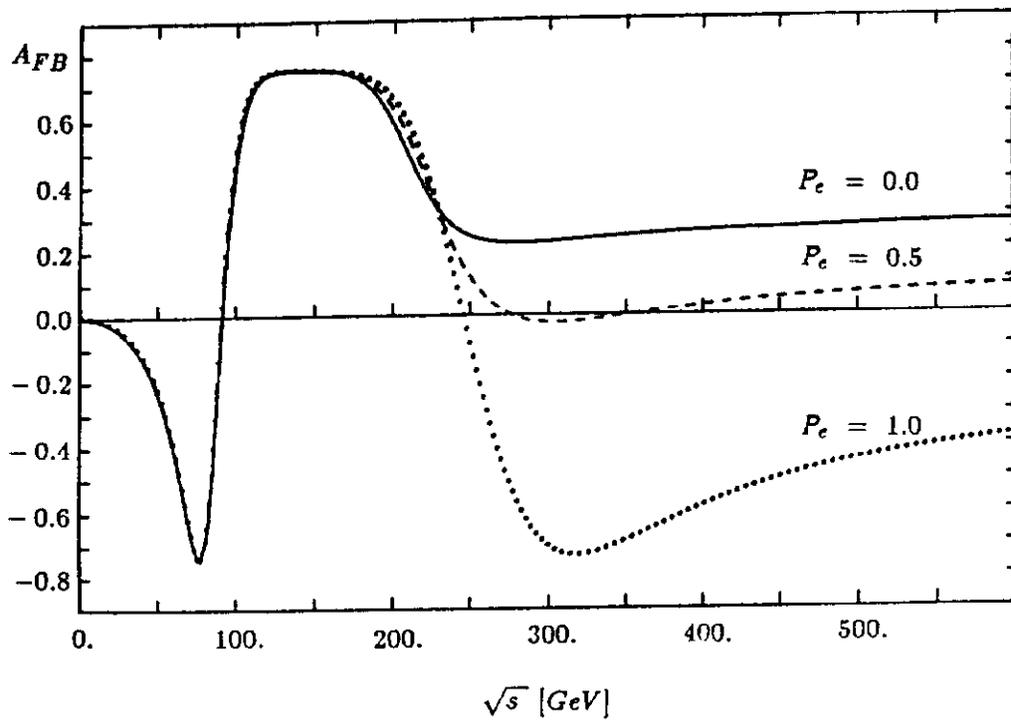


Fig. 6a

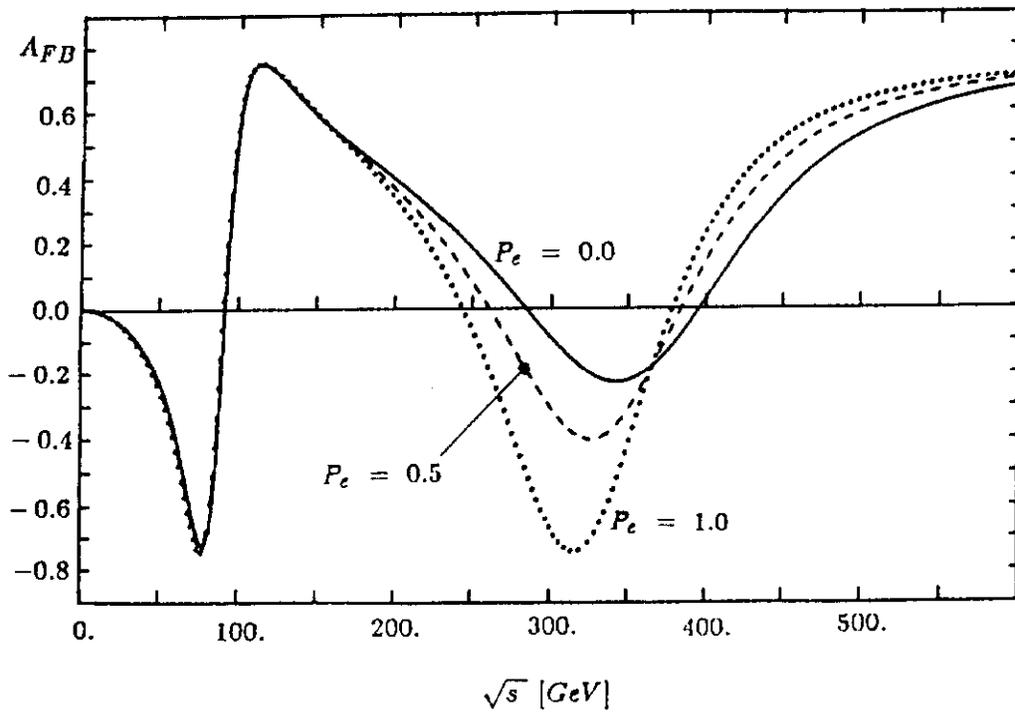


Fig. 6b

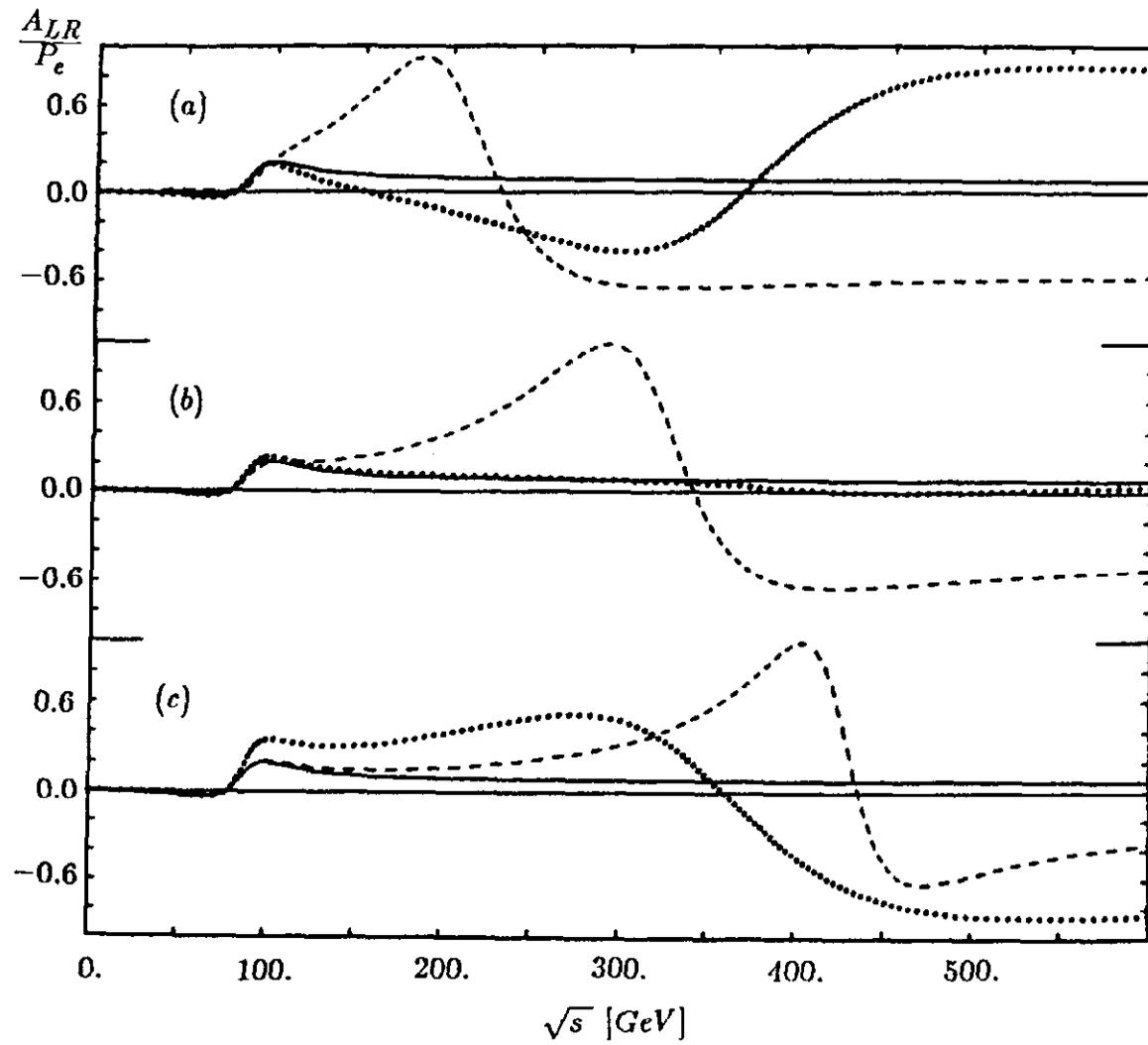


Fig. 7

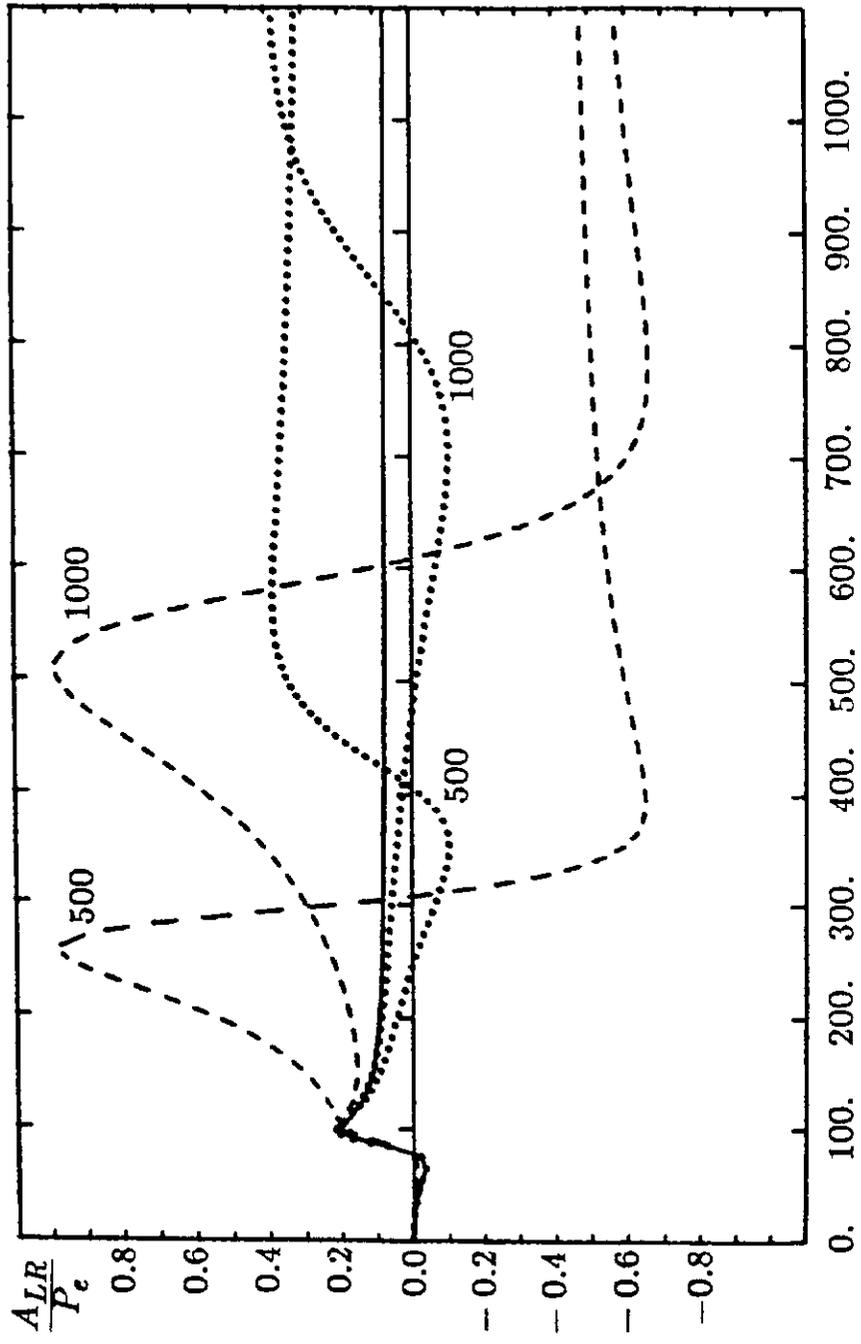


Fig. 8

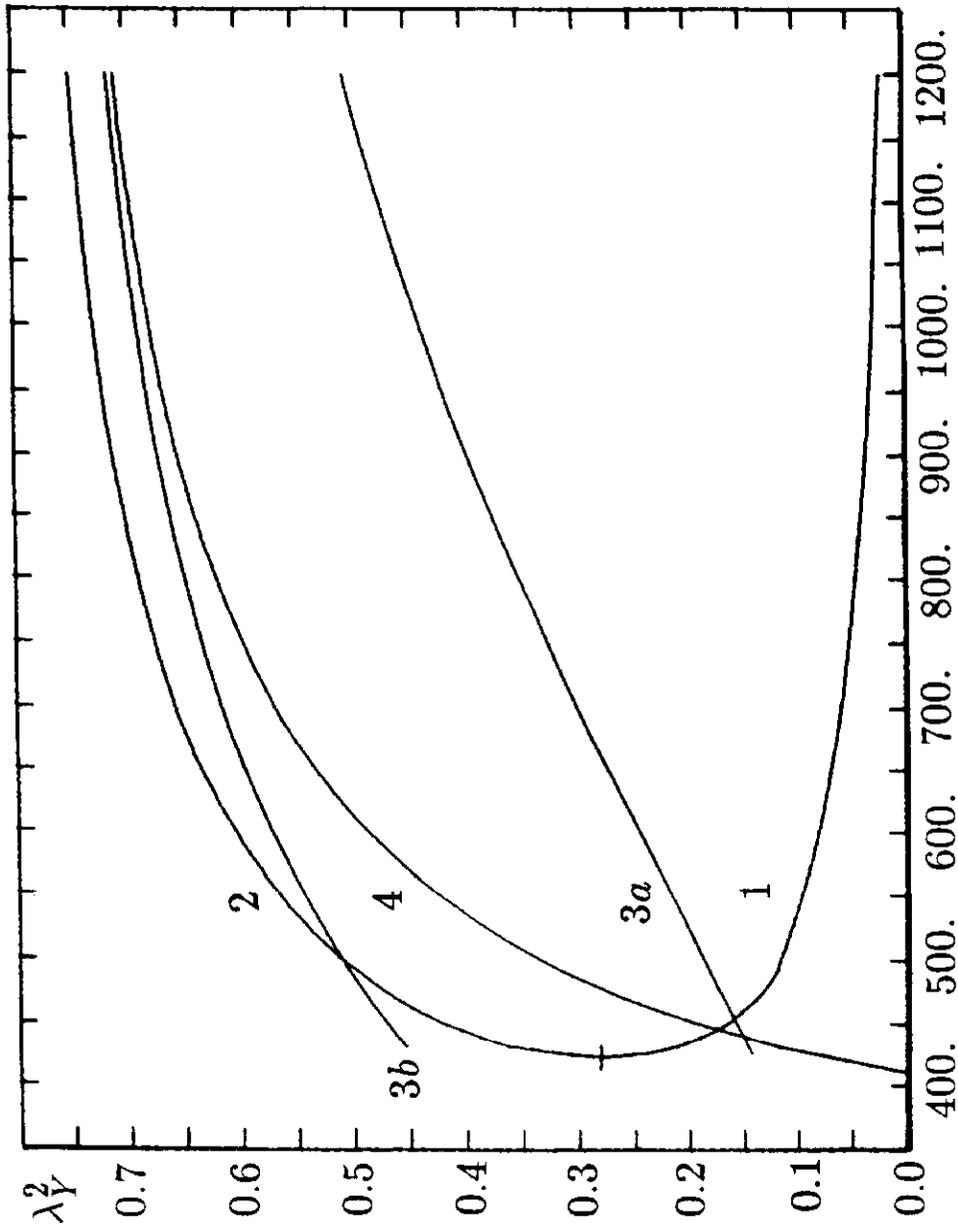


Fig. 9

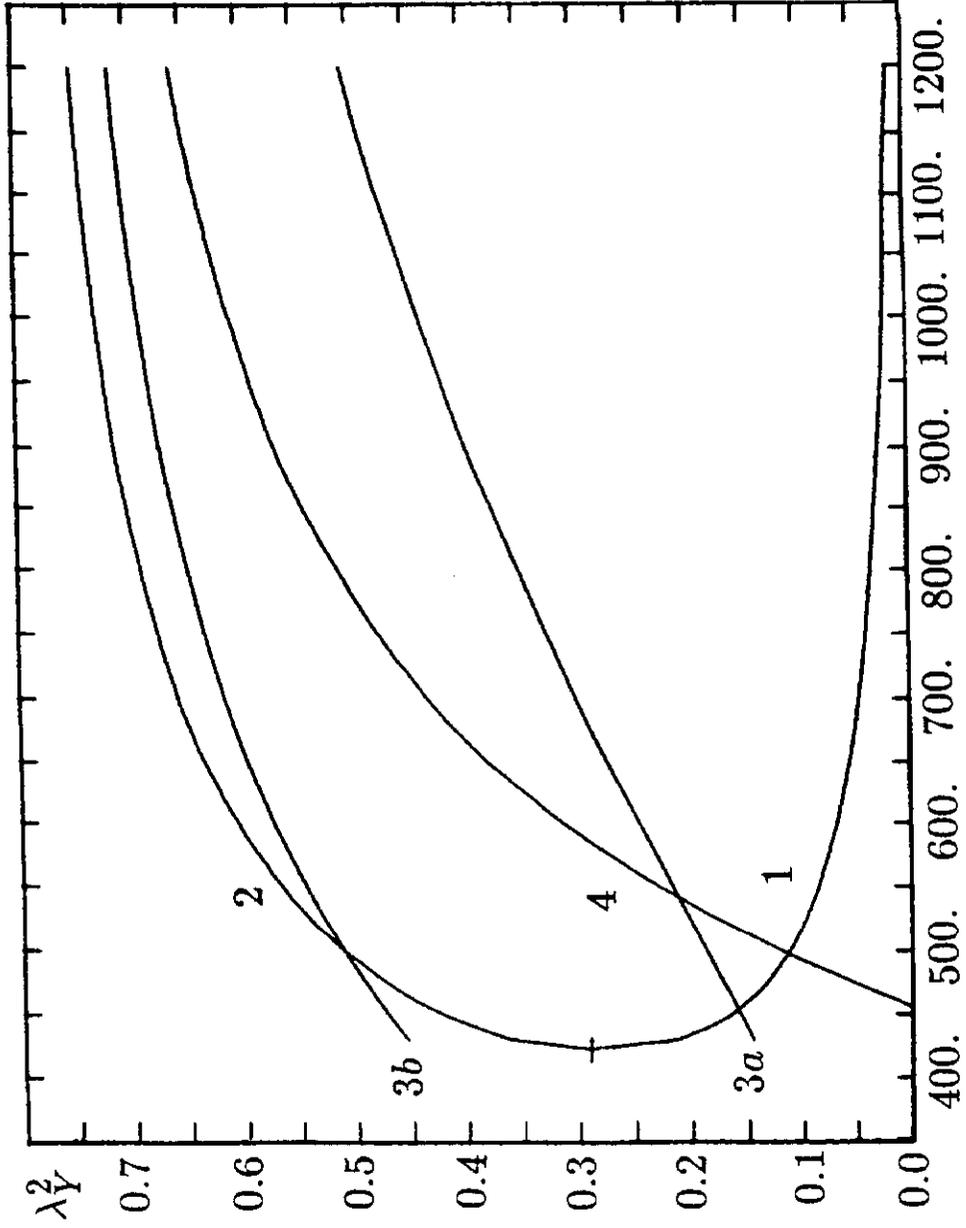
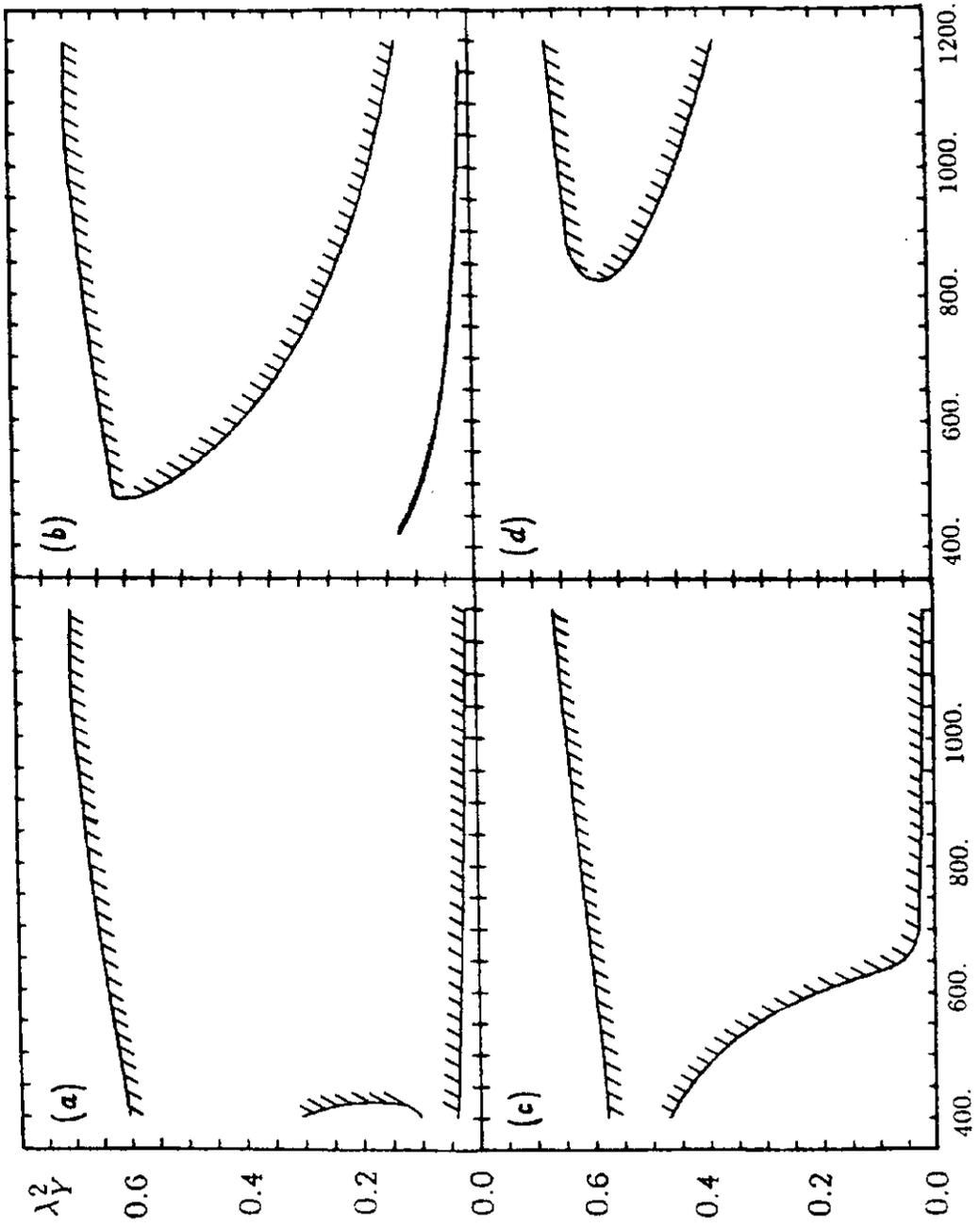
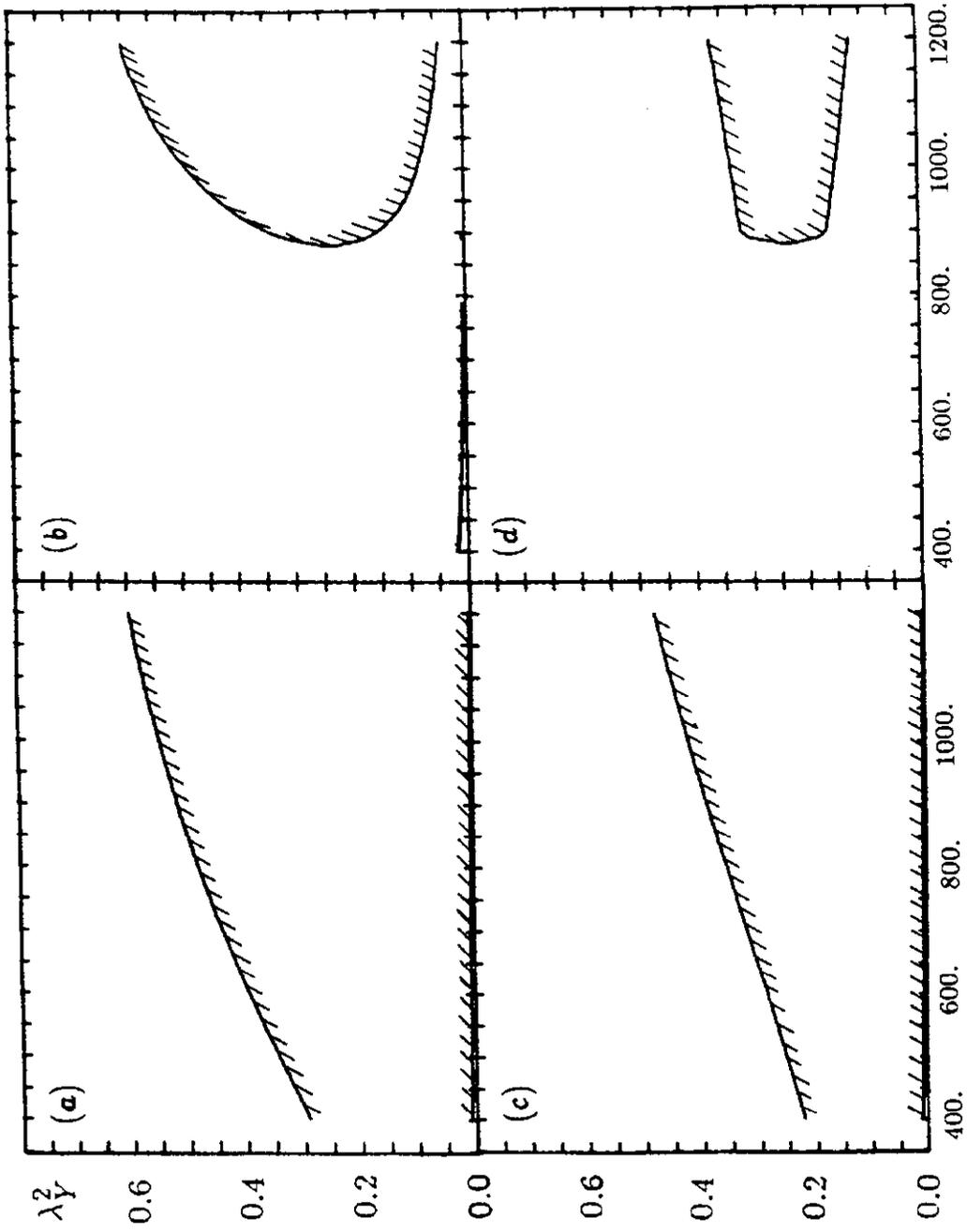


Fig. 10



My [GeV]

Fig. 11



My [GeV]

Fig. 12

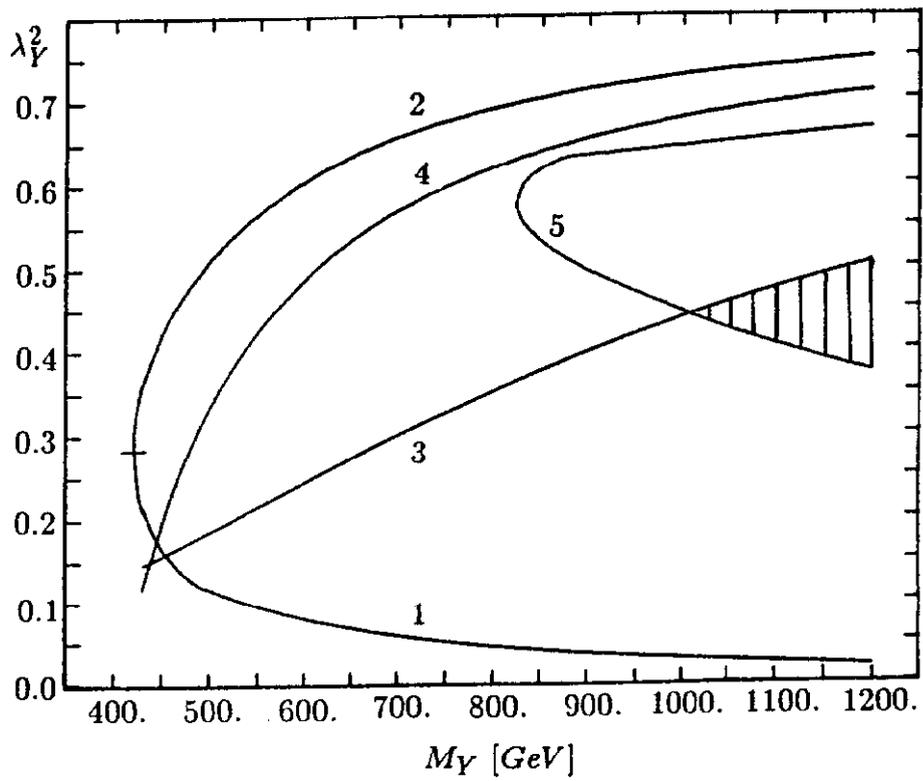


Fig. 13a

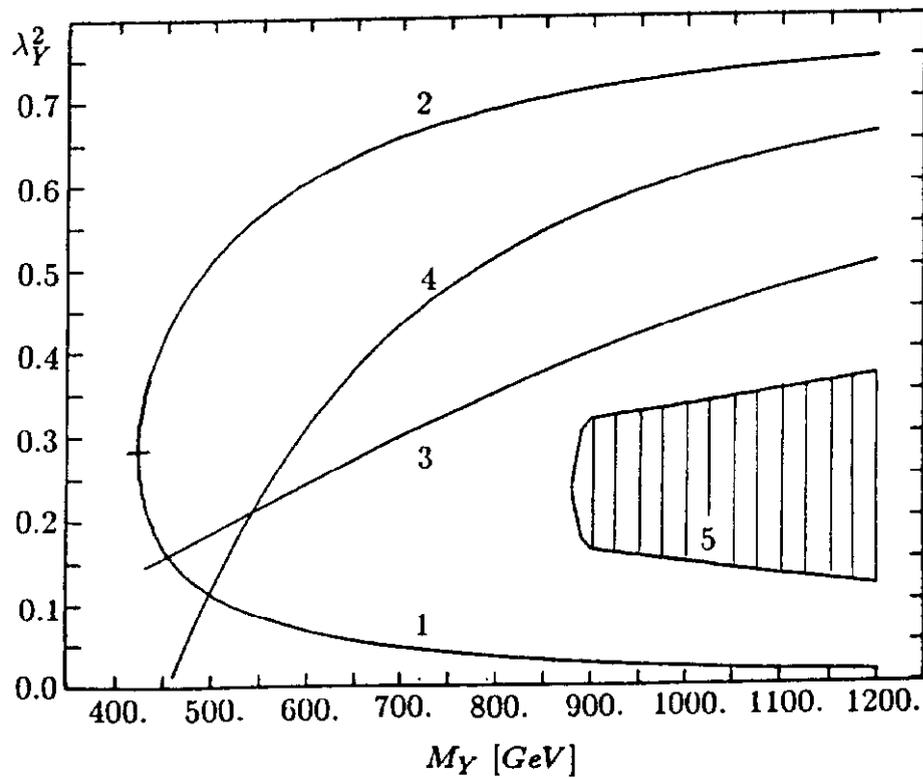


Fig. 13b