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Supersymmetry Restoration in the Compactified $O(16) \otimes O(16)'$ Heterotic String Theory

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We consider twisted compactifications of the $O(16)\otimes O(16)'$ heterotic string from ten to nine dimensions. We construct a model that interpolates between the $O(16) \otimes O(16)'$ model at large radii of the compact dimension, and the spontaneously broken $E_8 \otimes E_8'$ heterotic superstring at small radii. Further compactification to five dimensions leads to a model with an exponentially suppressed one-loop cosmological constant at small radii. The notion of duality between strings at small and large radii of the compact dimensions is clarified.

Space-time supersymmetry of the compactified $E_8 \otimes E_8'$ heterotic superstring [1] provides a potential solution to the naturalness problem in elementary particle physics. On the other hand, nature is not supersymmetric, at least at the energies accessible to the existing accelerators. A satisfactory mechanism for this low-energy supersymmetry breaking is still missing. Recently a new, ten-dimensional $O(16) \otimes O(16)'$ string theory without space-time supersymmetry has been constructed [2,3], by "twisting" the heterotic superstring, that is, by modifying its boundary conditions in a way consistent with the requirement of modular invariance [2-6]. The $O(16) \otimes O(16)'$ model is modular invariant, tachyon-free and has a positive one-loop cosmological constant. It is an interesting question to see how compactification to dimensions less than ten affects the spectrum and the cosmological constant. In this article, we study in detail a simple example of a non-trivial compactification of the $O(16) \otimes O(16)'$ heterotic string, from ten to nine dimensions. We consider compactification on a twisted torus and find a case where supersymmetry is asymptotically restored as the radius of the compact dimension becomes small, in units of $\sqrt{2\pi\alpha'}$, the square root of the inverse string tension. Further compactification to five dimensions leads to a model with an exponentially suppressed one-loop cosmological constant at small radii.

The $O(16) \otimes O(16)'$ model is most easily constructed [2] by using the fermionic formulation of the heterotic superstring, with the right-moving Green-Schwarz fermions S^a, and the left-moving Ramond-Neveu-Schwarz fermions χ^I and χ'^I transforming as $(\mathbf{16}, \mathbf{1}') \oplus (\mathbf{1}, \mathbf{16}')$ under $O(16) \otimes O(16)'$. The left-moving fermions belong to four sectors: (NS,NS), (NS,R), (R,NS) and (R,R), corresponding to different σ boundary conditions for χ^I and χ'^I on the two-dimensional world-sheet (σ, t). In order to obtain the $O(16) \otimes O(16)'$ model, one projects onto the eigenvalue $R = 1$

eigenstates of the Z_2 twist operator

$$R = (-1)^{F_S} e^{2\pi i j_{12}} e^{2\pi i j'_{12}}, \tag{1}$$

where F_S is the space-time fermion number and j_{12} and j'_{12} are generators of rotations in $O(16)$ and $O(16)'$, respectively. The massless spectrum of this model contains the following particles. In the untwisted sector, with the periodic right-moving fermions, the projection onto $R = 1$ gives $\mathbf{8_v}$ gauge bosons $(\mathbf{120}, \mathbf{1'}) \oplus (\mathbf{1}, \mathbf{120'})$, $\mathbf{8_s}$ fermions $(\mathbf{128}, \mathbf{1'}) \oplus (\mathbf{1}, \mathbf{128'})$, and the bosonic part of the supergravity multiplet, $(g_{\mu\nu}, B_{\mu\nu}, \phi)$. From the twisted sector, with the antiperiodic right-moving fermions, one obtains $\mathbf{8_{s'}}$ fermions in the representation $(\mathbf{16}, \mathbf{16'})$.

We are interested in constructing a nine-dimensional string theory which, in the limit of an infinite radius of the compact dimension, reproduces the ten-dimensional $O(16) \otimes O(16)'$ model. Compactification on a torus [7] of radius r results in a model with the spectrum consisting of Kaluza-Klein and topological excitations of dimensionally reduced ten-dimensional states. The $U(1)$ isometry group of the torus gets enlarged to $SU(2)$ at $r = \sqrt{\alpha'}$; this so-called Frenkel-Kač point corresponds to a minimal value [of order $(2\pi\alpha')^{-9/2}$] of the one-loop cosmological constant. A way to construct some other models is to introduce an extra twist in the compact dimension [8].

The requirement of modular invariance of the compactified string forces the left-moving and the right-moving compact momenta $\vec{p} = (p_L, p_R)$ to lie on an even, self-dual Lorentzian lattice with the signature $(+,-)$ [9]; the total string momentum coming from the compact dimension corresponds to the sum $p_L + p_R$, whereas the winding number to the difference $p_L - p_R$. Any two-dimensional lattice with such a property can be obtained by an $SO(1,1)$ boost of σ_1, a Lorentzian lattice in $\mathbf{R}^{1,1}$, generated by the light-like vectors $\frac{1}{\sqrt{2}}(\vec{x} \pm \vec{t}\,)$ [9]. We begin our construction at the radius $r = \sqrt{\alpha'}$, and, for the moment, set $\alpha' = \frac{1}{2}$; we will restore dimensionfull constants when considering compactification at an arbitrary radius r. We parametrize momenta on the lattice σ_1 as

$$\vec{p} = m\tfrac{1}{\sqrt{2}}(\vec{x} + \vec{t}\,) + n\tfrac{1}{\sqrt{2}}(\vec{x} - \vec{t}\,) = \tfrac{1}{\sqrt{2}}(m + n, m - n), \tag{2}$$

where m and n are integers corresponding to the string momenta and the winding numbers, respectively. We introduce an extra twist [8] by modifying the operator R of eq.(1) to

$$R' = e^{2\pi i \vec{\delta}\cdot\vec{p}}\, R\,, \tag{3}$$

where $\vec{\delta}$ is constrained to be half of a lattice vector, in order to insure $R'^2 = 1$.

In the untwisted sector, the right and left moving mass operators are respectively given by

$$\tfrac{1}{4}M_R^2 \;=\; N_R + \tfrac{1}{2}p_R^2 \tag{4}$$

$$\tfrac{1}{4}M_L^2 \;=\; N_L + c + \tfrac{1}{2}p_L^2, \tag{5}$$

where N_R (N_L) are normal-ordered number operators for right (left) movers and c is the normal ordering constant for the left movers; $c = -1$ for (NS,NS), $c = 0$ for (NS,R) and (R,NS), and $c = 1$ for (R,R). In the twisted sector

$$\tfrac{1}{4}M_R^2 \;=\; N_R - \tfrac{1}{2} + \tfrac{1}{2}(p_R + \delta_R)^2 \tag{6}$$

$$\tfrac{1}{4}M_L^2 \;=\; N_L + c + \tfrac{1}{2}(p_L + \delta_L)^2\,. \tag{7}$$

The square of the vector $\vec{\delta}$ is constrained to be an integer, in order to have matched right and left mass levels in the twisted sector, eqs.(6-7). It is easy to show that any vector $\vec{\delta}$ with $\vec{\delta}^2$ integer, equal to half of a lattice vector on σ_1, can be obtained from one of the light-like vectors, $\vec{\delta}_I$ or $\vec{\delta}_{II}$,

$$\vec{\delta}_I = \tfrac{1}{2\sqrt{2}}(\vec{x}+\vec{t}\,) \quad \Rightarrow \quad e^{2\pi i \vec{\delta}_I \cdot \vec{p}} = (-1)^n, \tag{8}$$

$$\vec{\delta}_{II} = \tfrac{1}{2\sqrt{2}}(\vec{x}-\vec{t}\,) \quad \Rightarrow \quad e^{2\pi i \vec{\delta}_{II} \cdot \vec{p}} = (-1)^m, \tag{9}$$

by shifting with some lattice vectors. These two vectors define two separate theories, which we call Twist I and Twist II, respectively.

Repeating the arguments along the lines of [2] leads to the conclusion that the only effect of modifying the twist operator from R to R' is to project onto the states with the opposite sign of $(-1)^{F_S}$, for odd winding numbers n (Twist I) or odd momentum numbers m (Twist II). This procedure is reminiscent of the type of compactification considered previously in [10] for superstrings. For the special value of the radius $r = \sqrt{\alpha'}$ considered so far, Twist I and Twist II theories are identical, due to complete symmetry between momenta and winding numbers. The massless spectrum comes entirely from the untwisted sector; besides the usual $\mathbf{8_v}$ gauge bosons $(\mathbf{120},\mathbf{1}')\oplus(\mathbf{1},\mathbf{120}')$, $\mathbf{8_s}$ fermions $(\mathbf{128},\mathbf{1}')\oplus(\mathbf{1},\mathbf{128}')$, and the bosonic part of the supergravity multiplet, it contains two $\mathbf{8_s}$ gauge-singlet fermions with $m = n = \pm 1$.

In order to construct Twist I and Twist II theories for an arbitrary value of the radius r of the torus, let us boost the lattice σ_1 with a rapidity $y = \log a$. Under this transformation

$$\vec{p} = \frac{1}{2\sqrt{\alpha'}}(m+n, m-n) \quad \rightarrow \quad \vec{p}_y = \frac{1}{2\sqrt{\alpha'}}(ma+\frac{n}{a}, ma-\frac{n}{a}), \tag{10}$$

which leads to the identification $a = \sqrt{\alpha'}/r$. This Lorentz transformation also acts on the twist vectors $\vec{\delta}_I$ and $\vec{\delta}_{II}$, rescaling them by the factors a and a^{-1}, respectively; twist operators remain unchanged.

Let us first discuss the light particle spectrum in the limit of an infinite radius, that is, $a \to 0$. Twist I and Twist II theories behave differently in this limit. The untwisted sector of the Twist I model contains massless particles identical to the dimensionally reduced $O(16) \otimes O(16)'$ untwisted string excitations, and a tower of their doubly-degenerate Kaluza-Klein excitations with the same quantum numbers, starting at the mass level $M^2 = a^2/\alpha'$. The spectrum of the twisted sector begins at the mass level $M^2 = a^2/4\alpha'$ with two $\mathbf{8_{s'}}$ fermions in the representation $(\mathbf{16}, \mathbf{16'})$. The doubling of the massive spectrum is due to the discrete symmetry that reverses the sign of the momentum number m. The reason why Twist I model behaves at large radius like the dimensionally reduced $O(16) \otimes O(16)'$ string theory is that in this limit all light states have zero winding number n, therefore the twist operator acts in the same way as the original operator R of eq.(1). Twist II model behaves in a different way, since the corresponding twist operator gets modified for odd momentum numbers m. As a result of this modification, all light particles are removed from the twisted sector, and the space-time quantum numbers of the Kaluza-Klein excitations are changed in the untwisted sector. The first massive level $M^2 = a^2/\alpha'$ in the latter sector contains two $\mathbf{8_s}$ fermions $(\mathbf{120}, \mathbf{1'}) \oplus (\mathbf{1}, \mathbf{120'})$, two $\mathbf{8_v}$ vector bosons $(\mathbf{128}, \mathbf{1'}) \oplus (\mathbf{1}, \mathbf{128'})$, and also, doubled fermionic part of the supergravity multiplet. The next level contains supersymmetric partners of these particles, and so on. Such a model corresponds to the dimensionally reduced $E_8 \otimes E_8'$ heterotic superstring, with supersymmetry broken spontaneously and the gauge group broken to $O(16) \otimes O(16)'$.

In the limit of a small radius of the compact dimension, Twist I (Twist II) model behaves exactly like Twist II (Twist I) model at large radius, with the parameter a replaced by a^{-1}. For small radii, topological excitations with non-zero winding number n and zero momentum m become light, playing exactly the same rôle as Kaluza-Klein excitations at large radii. This duality between Kaluza-Klein and topological excitations has been observed before [7,8], but in the case of twisted strings considered here its consequences are different than in the case of untwisted strings. It is clear from our analysis, that neither Twist I nor Twist II models are self-dual. Under the duality transformation, $a \leftrightarrow a^{-1}$, Twist I and Twist II models are interchanged. Therefore, for example, there is no reason to expect that the Frenkel-Kač point $a = 1$ extremises the cosmological constant.

Let us now summarize the asymptotic behaviour of the nine-dimensional twisted string models constructed here. Twist I interpolates between the $O(16) \otimes O(16)'$ heterotic string at large radii and the spontaneously broken $E_8 \otimes E_8'$ heterotic superstring at small radii. Twist II interpolates between the same, however in the opposite limits of the radii. We conclude that only the Twist I model can be regarded as a compactification of the ten-dimensional $O(16) \otimes O(16)'$ heterotic string. In this model, supersymmetry is asymptotically restored at small radii of the compact dimension.

It is worth mentioning that besides the Frenkel-Kač point, $a = 1$, considered earlier, there exists another point, $a = \sqrt{2}$ ($a = \frac{1}{\sqrt{2}}$) for Twist I (Twist II), at which there are some extra particles appearing in the massless spectrum. At this point, the twisted sector contains two massless scalars in the representation $(\mathbf{16}, \mathbf{16}') \in SO(32)/O(16) \otimes O(16)'$. This suggests that this point may correspond to a compactification [11] of the $SO(32)$ model, which is tachyonic in ten dimensions

[2,4].

We proceed now to the evaluation of the one-loop cosmological constant, first for Twist I model. In order to make it possible to compare ten-dimensional and nine-dimensional theories, we define

$$\Lambda_{10} \equiv \frac{1}{2\pi r}\Lambda_9\,, \tag{11}$$

where Λ_9 is the nine-dimensional cosmological constant. We obtain

$$\Lambda_{10} = \frac{1}{(2\pi\alpha')^5}\int_F \frac{d^2\tau}{4\pi(\tau_2)^2}\frac{a\sqrt{\tau_2}}{(2\pi\tau_2)^4}[P_F(a,\tau) - P_B(a,\tau)], \tag{12}$$

where the integration is over the fundamental region of the modular parameter $\tau = \tau_1 + i\tau_2$ [10,12]. The difference between the fermionic and bosonic partition functions is given by

$$\begin{aligned} P_F(a,\tau) \; - \; P_B(a,\tau) \; = \; & \frac{2^{10}}{\theta_1'^4|\theta_1'|^8}\,\{\,[\mathcal{E}_0 + \mathcal{E}_{1/2}][\theta_2^8\bar\theta_3^4\theta_4^8 - \theta_4^8\bar\theta_2^4\theta_3^8 - \theta_3^8\bar\theta_4^4\theta_2^8] \\ & +[\mathcal{E}_0 - \mathcal{E}_{1/2}][-\tfrac{1}{2}\bar\theta_2^4(\theta_2^{16} + \theta_3^{16} + \theta_4^{16}) + \theta_2^8(\theta_3^4|\theta_3|^8 - \theta_4^4|\theta_4|^8)] \\ & +[\mathcal{O}_0 - \mathcal{O}_{1/2}][\tfrac{1}{2}\bar\theta_3^4(\theta_2^{16} + \theta_3^{16} + \theta_4^{16}) - \theta_3^8(\theta_2^4|\theta_2|^8 + \theta_4^4|\theta_4|^8)] \\ & +[\mathcal{O}_0 + \mathcal{O}_{1/2}][-\tfrac{1}{2}\bar\theta_4^4(\theta_2^{16} + \theta_3^{16} + \theta_4^{16}) + \theta_4^8(\theta_3^4|\theta_3|^8 - \theta_2^4|\theta_2|^8)]\,\}\,, \end{aligned} \tag{13}$$

where the Jacobi theta functions are evaluated at the argument $(0|\tau)$, and $\theta_1' \equiv \theta_2\theta_3\theta_4$. The functions $\mathcal{E}(a,\tau)$ and $\mathcal{O}(a,\tau)$ are given by the sums [7,10]

$$\sum_{n,m=-\infty}^{\infty} \exp[-2\pi i\tau_1 nm - \pi\tau_2(m^2a^2 + n^2/a^2)], \tag{14}$$

where the summation goes over n even (odd) for the functions $\mathcal{E}$ $(\mathcal{O})$, and m integer (half-integer) for the subscripts 0 (1/2). Modular invariance of the cosmological constant, eqs.(12-14), can be easily proven by using the standard Poisson resummation techniques [7,10]. The cosmological constant for the Twist II model is obtained by replacing the argument a of the functions $\mathcal{E}$ and $\mathcal{O}$ by a^{-1}.

We numerically evaluated the cosmological constant for Twist I and II theories. The results are plotted on Fig.1. As expected, the cosmological constant of the Twist I model decreases monotonically from the asymptotic $O(16) \otimes O(16)'$ value $\Lambda_{10} \approx 0.0371(2\pi\alpha')^{-5}$ [3] at infinite radius, to the asymptotic zero value at radius zero, reflecting supersymmetry restoration discussed before. The asymptotic behaviour of this cosmological constant can be evaluated analytically, yielding

$$\Lambda_{10} \overset{a\to\infty}{\longrightarrow} (n_F - n_B)\frac{24}{(2\pi^2)^5}\frac{\varsigma(10,1/2)}{(2\pi\alpha')^5}\frac{1}{a^8} + \mathcal{O}(e^{-a^2}) \approx \frac{0.525}{(2\pi\alpha')^5}\frac{1}{a^8}, \qquad (15)$$

where $n_F - n_B = 64$ is the difference between the number of fermionic and bosonic massless degrees of freedom.

From eq.(15) it follows that, in models with an equal number of exactly massless fermionic and bosonic degrees of freedom, the one-loop cosmological constant is exponentially suppressed at small radii, i.e. large a. A model with such a small cosmological constant can be constructed by compactifying the nine-dimensional Twist I model to five dimensions, on four torii with radii $r_9 = r_8 = r_7 = r_6 = \sqrt{\alpha'}$. For these special values of the radii, 8 additional $\mathbf{8_v}$ gauge bosons, corresponding to the non-zero roots of $[SU(2)]^4$, appear in the massless spectrum. Although the mass splitting within the multiplets of broken supersymmetry is of order a^{-1}, the cosmological constant behaves like e^{-a^2} for large a. The one-loop cosmological constant in four-dimensional analogues of such a model would be of order $(2\pi\alpha')^{-2}e^{-1/M_S^2\alpha'}$, where M_S is the supersymmetry breaking scale. The question whether such a huge suppression of the cosmological constant persists to higher string loops is very difficult to answer at the present time. Also, since the two-dimensional sigma model on the world-sheet becomes strongly coupled for small values of the radius of the compact dimension [13], some non-perturbative effects may become important. Hopefully, these effects would create a potential barrier preventing the compact dimension from shrinking to a zero size, without generating a large cosmological constant. It is quite surprising, that the supersymmetry breaking scale need not to be much lower than the string tension, to insure a practically vanishing one-loop cosmological constant.

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Figure Captions

Fig. 1: Cosmological constant Λ_{10} [in units of $0.01(2\pi\alpha')^{-5}$] for Twist I and Twist II models, plotted as a function of the radius r of the compact dimension [in units of $\sqrt{\alpha'}$].