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Two-loop correction to weak-interaction parameters due to a heavy fermion-doublet

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Abstract

The two-loop corrections to the ρ-parameter and the vector boson mass-shifts due to a heavy fermion doublet were calculated. For very heavy ($\approx TeV$) quarks we find improved limits on the allowed mass difference within a doublet. The limit on the top quark mass ($300GeV$) is unaffected. For a degenerate doublet there is a Higgs mass dependent limit on the quark mass of 3–6 TeV for a Higgs mass of 0–3 TeV.

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I. Introduction

It is well-known that the parameters of the standard $SU_L(2)\times U(1)$ gauge model of the weak interactions are affected by the presence of heavy particles in the theory. This is in marked contrast with theories like QED and QCD where the Appelquist-Carazzone decoupling theorem[1] ensures that the effects of heavy particles are suppressed by a power of their inverse masses. The reason for the difference is that in the standard model particles get their masses from the Higgs-mechanism and therefore masses and coupling constants are related. This violates the assumptions underlying the decoupling theorem.

This failure of decoupling implies that some radiative corrections due to heavy fermions may actually grow with the fermion masses. For instance the ρ-parameter, the ratio of neutral to charged current strength, receives at the one-loop level a correction dependent on the mass difference within a fermion doublet[2]. The experimental information on ρ puts a limit on the possible mass difference . For a quark doublet coming in three colors this limit is approximately 300 GeV. Unfortunately one finds no correction to ρ in the case of a mass-degenerate multiplet. The reason is that the correction to ρ comes from a breaking of the $SU_R(2)$ symmetry of the Higgs-sector. In the standard model this symmetry is only broken by the difference of Yukawa couplings in a doublet and by the hypercharge-coupling.

Also in other processes the contribution from a degenerate multiplet does not grow with the mass at the one-loop level. There are however finite corrections that are independent of the mass and therefore limits can be put on the number of such multiplets that can be present. The present limits on the mass of a degenerate fermion multiplet come from some more indirect arguments. It has been argued[3] that the large Yukawa couplings make the one-loop effective Higgs-potential unstable for fermion masses larger than about $200-600$ GeV depending on the Higgs-mass; for a Higgs-mass in the TeV range there is no more a reliable limit. Another bound comes from the breakdown of perturbation theory[4,5]. If the Yukawa-couplings are large enough the two-loop correction to some process will become equal to the one-loop correction. So far this has only been calculated completely for the gluon-fusion mechanism of Higgs-production. The effects of a degenerate heavy multiplet have been considered from a more general point of view at the one-loop level in ref.'s [6,7], with emphasis on the appearance of anomalies if one removes the multiplet from the low-energy theory by making the mass infinite. In ref.[7] the effect of a heavy doublet has been summarized by an effective interaction consisting of an anomaly term and a corresponding interaction of the Wess-Zumino type. It has been suggested in this paper that the theory without the heavy fermions but with the Wess-Zumino term is renormalizable. We consider this to be extremely unlikely. The effective interaction is not obviously renormalizable and inserting the one-loop effective interaction inside a loop diagram one expects divergencies to appear. In the full theory with the heavy doublet these divergencies can only be cut off by the mass of this doublet. The presence of effects growing with the heavy doublet mass would therefore be a signal of the non-renormalizability of the effective model with the Wess-Zumino term.

Considering the above, an explicit two-loop calculation of heavy-fermion effects for a number of processes might be of interest. In this paper we consider such corrections for the ρ-parameter and for the mass-shift of the W and Z bosons. The outline of this paper is as follows: In chapter two we define the renormalization scheme. In chapter three the weak interaction parameters involved in the calculation are defined. In chapter four we present the calculation of $\delta\rho$ for unequal masses of the doublet. In chapter five we do the calculation of $\delta\rho$ for equal masses. In chapter six the mass-shifts of W and Z are given for an equal mass doublet. In chapter seven we discuss the results. The appendix explains the notation and some technical details of the calculation.

II. Renormalization

In performing higher order calculations in gauge theories the complexities of renormalization theory have to be taken into account. It is not enough just to calculate the radiative corrections to the process one is interested in. The expressions obtained will often contain infinities. Meaningful answers can only be obtained if one expresses everything in terms of physical parameters. The relation between the parameters of the Lagrangian and the physical parameters in which all corrections are to be expressed can be quite complex. In order to get the correct results at the two-loop level at least all one-loop renormalizations must be performed. In general this involves a large number of graphs.

In this paper we are only interested in the effects due to a heavy fermion doublet and we limit ourselves to the leading effects in terms of the heavy fermion masses.

In this limit the situation is simplified considerably. The main simplification comes from the fact that only self-energy graphs need to be considered. There are basically three sectors of the theory to be considered : the fermion, Higgs-boson and vector boson sector. The contributions of the QCD sector do not grow fast enough with the fermion mass to be taken into account. In the fermion sector one should discriminate between the light fermions, like the standard generations of quarks and leptons, and the heavy fermions whose effects we want to calculate. The light fermions do not receive a mass correction from the heavy fermion sector as long as mixing can be ignored, which we assume to be the case. The light fermions appear only as a source for the vector bosons, whose physics we want to study. The heavy fermions receive a mass renormalization coming from the diagrams of fig.(1). For a quark doublet one gets the following relation between the bare mass parameters m_t, m_b and the physical masses of the quarks:

$$(m_t)_{phys} = m_t + \frac{g^2 m_t}{128\pi^2 M_W^2}\Big(+\frac{6m_t^2}{\varepsilon} - \frac{6m_b^2}{\varepsilon} - 8m_t^2 + 5m_b^2 - \frac{m_b^4}{m_t^2} + 2m_t^2\ln(m_t^2)$$
$$+ (-3\frac{m_b^4}{m_t^2} + \frac{m_b^6}{m_t^4})\ln(m_b^2) + (m_t^2 - 3m_b^2 + 3\frac{m_b^4}{m_t^2} - \frac{m_b^6}{m_t^4})\ln(|m_t^2 - m_b^2|)\Big) \tag{2.1}$$

$$(m_b)_{phys} = m_b + \frac{g^2 m_b}{128\pi^2 M_W^2}\Big(+\frac{6m_b^2}{\varepsilon} - \frac{6m_t^2}{\varepsilon} - 8m_b^2 + 5m_t^2 - \frac{m_t^4}{m_b^2} + 2m_b^2\ln(m_b^2)$$
$$+ (-3\frac{m_t^4}{m_b^2} + \frac{m_t^6}{m_b^4})\ln(m_t^2) + (m_b^2 - 3m_t^2 + 3\frac{m_t^4}{m_b^2} - \frac{m_t^6}{m_b^4})\ln(|m_t^2 - m_b^2|)\Big). \tag{2.2}$$

The particles t and b have the quantum numbers of an ordinary quark doublet, but are assumed to be very heavy. Note that only the mass renormalization is important; the wave function renormalization does not contribute, since we consider only processes where the heavy quarks appear inside a loop. For the Higgs sector and the vector boson sector we use the method of ref.[8]. We rescale fields and parameters in the original Lagrangian by factors containing besides the usual $1/\varepsilon$ pole terms also finite factors. These factors are chosen in a way, so that at the one-loop level the Lagrangian parameters become equal to the physical parameters as far as the leading behaviour of the heavy fermion masses is concerned. We need rescalings of the following form:

$$\phi \rightarrow \phi(1+\delta_z) \qquad Z \rightarrow Z(1+\delta_z) + \frac{M}{g}\delta_t$$

$$M \rightarrow M(1+\delta M) \qquad m \rightarrow m(1-\delta m). \tag{2.3}$$

Here ϕ is the Higgs ghost, Z the Higgs-field, M the vector boson mass and m the Higgs-mass, δ_t is the tadpole contribution. We also define quantities δ_1,δ_2 by $\delta_1 \equiv \delta_M + \frac{1}{2}\delta_t$ and $\delta_2 \equiv \delta_M - \delta_m$. In order to satisfy Ward identities we have to take δ_z equal to δ_1. This introduces a wavefunction renormalization for the physical Higgs field ,which is of no consequence since the Higgs only appears inside loops.

For the case of unequal quark masses, where we are only interested in contributions behaving as the fourth power of the quark masses m_t and m_b we have :

$$\delta_1 = \delta_z = 0$$

$$(2\pi)^4 i\delta_t = \frac{2g^2}{m^2M^2}(m_t^2(1,m_t) + m_b^2(1,m_b))$$

$$(2\pi)^4 i\delta_2 = \frac{g^2}{m^2M^2}(m_t^4(2,m_t) + m_b^4(2,m_b)) \tag{2.4}$$

where we used the notation of the appendix. In the case of equal quark masses also the terms behaving as the square of the quark masses are needed. This yields:

$$(2\pi)^4 i\delta_1 = (2\pi)^4 i\delta_z = -\frac{1}{2}g^2\frac{m_t^2}{M^2}(2,m_t)$$

$$(2\pi)^4 i\delta_2 = \frac{2g^2m_t^4}{m^2M^2}(2,m_t) + (-\frac{1}{3} - \frac{n}{6})g^2\frac{m_t^2}{M^2}(2,m_t). \tag{2.5}$$

III. Definition of weak interaction parameters

In the previous chapter we fixed the fermion- and Higgs mass parameters by comparing them with the physical masses (pole of the propagator) of these particles. This still leaves us with three free parameters to be determined by experiment: g, s=sinθ_w and M . These parameters will be determined from a comparison with low energy processes . For these processes we take Coulomb scattering, μ-decay and $\nu_\mu e$, $\bar{\nu}_\mu e$ scattering. On the basis of these processes we determine a number of experimental parameters $g_{exp}, s_{exp}, M_{exp}$. From the fine-structure constant we determine $g_{exp}^2 s_{exp}^2 = 4\pi\alpha$.

$$G_F^+ = \frac{g_{exp}^2}{8M_{exp}^2} \approx \frac{1.01\ 10^{-5}}{\sqrt{2}m_p^2} \qquad m_p = proton\ mass$$

G_F^+ is the Fermi constant determined from μ-decay. It is to be emphasized that M_{exp} is a purely low-energy parameter and is not directly related to the W-mass beyond the tree-level. The sine of the weak mixing angle s_{exp} is determined from the ratio of cross-sections of $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. At low energy these cross-sections are described by the four-fermion interaction :

$$\mathcal{L}_{int} = \frac{G_F^0}{2}(\bar{\nu}_\mu\gamma^\alpha(1+\gamma_5)\nu_\mu)(\bar{e}\gamma_\alpha(1-4s_{exp}^2+\gamma_5)e) \tag{3.1}$$

s_{exp}^2 can be determined from the ratio of cross-sections :

$$\frac{\sigma^{\bar{\nu}e}}{\sigma^{\nu e}} = \frac{1-4s_{exp}^2+16s_{exp}^4}{3-12s_{exp}^2+16s_{exp}^4} \tag{3.2}$$

G_F^0 is the Fermi constant for neutral current processes. The corrections due to heavy fermions on these parameters come from the propagator correction of the vector bosons. We have the following two-point vertex corrections in an expansion around $k^2 = 0$:

$$\begin{aligned} W_\mu^+ W_\nu^- &= (2\pi)^4 i\delta^+\ \delta_{\mu\nu} + (2\pi)^4 i f^+(k^2\delta_{\mu\nu} + A^+ k_\mu k_\nu) \\ W_\mu^0 W_\nu^0 &= (2\pi)^4 i\delta^0\ \delta_{\mu\nu} + (2\pi)^4 i f^0(k^2\delta_{\mu\nu} + A^0 k_\mu k_\nu) \\ A_\mu W_\nu^0 &= (2\pi)^4 i f_{AW}(k^2\delta_{\mu\nu} - k_\mu k_\nu) \\ A_\mu A_\nu &= (2\pi)^4 i f_{AA}(k^2\delta_{\mu\nu} - k_\mu k_\nu). \end{aligned} \tag{3.3}$$

Higher order terms in k^2 can be ignored, since they do not grow with the heavy fermion mass fast enough. Also the $k_\mu k_\nu$- pieces will only give terms proportional to the light-fermion masses in any cross section involving only light external fermions and they will be ignored. Using these definitions we get the following relations (see also ref.[2]) :

$$
\begin{aligned}
s^2_{exp} &= \frac{s^2}{1-f_{AA}}(1-f_{AA}-c/sf_{AW}) \\
g^2_{exp} &= \frac{g^2}{1-f_{AA}-c/sf_{AW}} \\
M^2_{exp} &= \frac{M^2-\delta^+}{1-f_{AA}-c/sf_{AW}} \\
G^+_F &= \frac{g^2/8}{M^2-\delta^+} \qquad G^0_F = \frac{g^2/8}{M^2-c^2\delta^0}
\end{aligned}
\tag{3.4}
$$

with $c = cos\theta_w$. The definition of the ρ-parameter is now simply $\rho = G^0_F/G^+_F$ and it is a finite function of $g_{exp}, s_{exp}, M_{exp}, (m_t)_{phys}, (m_b)_{phys}, (m_H)_{phys}$. At the tree-level $\rho = 1$. The correction $\delta\rho = \rho - 1$ is the subject of the next two chapters. We define another low energy parameter

$$
M^0_{exp} = \Big(\frac{g^2_{exp}}{8c^2_{exp}G^0_F}\Big)^{\frac{1}{2}}. \tag{3.5}
$$

At the tree-level one has the relations $M_{W^+} = M_{exp}$ and $M_{W^0} = M^0_{exp}$ where M_{W^+} and M_{W^0} are the physical masses of the charged and neutral vector bosons. The corrections $M_{W^+} - M_{exp}$ and $M_{W^0} - M^0_{exp}$ are discussed in chapter 6.

IV. $\delta\rho$ for unequal masses

In this section we calculate the correction to ρ due to a doublet with unequal masses. We will only be concerned with the leading effects for large quark masses. At the two-loop order the leading effects behave like the fourth power of the quark masses. To this order only propagator corrections are important. There are a number of such corrections. We have for the Fermi-constant:

$$
\begin{aligned}
g^2(G^+_F)^{-1}/8 &= M^2 - \delta^- = M^2_{exp} \\
g^2(G^0_F)^{-1}/8 &= c^2(M^2_0 - \delta^0) = M^2_{exp} + \delta^+ - c^2\delta^0.
\end{aligned}
\tag{4.1}
$$

Therefore ρ is given by:

$$
\rho = G^0_F/G^-_F = \frac{M^2_{exp}}{M^2_{exp} - \delta^- - c^2\delta^0} = 1 + \delta\rho_1 - \delta\rho_1^2 - \delta\rho_2 + O(g^6) \tag{4.2}
$$

where $\delta\rho_1$ is the one-loop correction to ρ. $\delta\rho_2$ is the contribution coming from the diagrams of fig.(2) . $\delta\rho_1$ consists of two pieces, a fermionic contribution $\delta\rho_1^f$ and a bosonic one $\delta\rho_1^b$. The fermionic piece is given by :

$$\begin{aligned}
\delta\rho_1^f = & \frac{g^2}{64\pi^2 M_{exp}^2}\Big(m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln(m_t^2/m_b^2)\Big) \\
& + \frac{g^2\varepsilon}{256\pi^2 M_{exp}^2}\Big(-m_b^2 - m_t^2 + \frac{2m_t^4}{m_t^2 - m_b^2}\ln(m_t^2) - \frac{2m_b^4}{m_t^2 - m_b^2}\ln(m_b^2) \\
& - \frac{2m_b^2 m_t^2}{m_t^2 - m_b^2}(\ln^2(m_t^2) - \ln^2(m_b^2))\Big) + O(\varepsilon^2).
\end{aligned} \tag{4.3}$$

In this expression m_t and m_b are still the bare parameters in the Lagrangian. In order to get the correct result to order g^4 we must use the relations (2.1) and (2.2) and substitute these in (4.3) . The expression for $\delta\rho_1^b$ is a rather complicated function of m_H, M, g, s . In order to get the correct expression to order g^4 here one should also substitute the relations between bare and dressed parameters. This gives a contribution to ρ :

$$\frac{\partial\rho_1^b}{\partial m_H}\delta m_H + \frac{\partial\rho_1^b}{\partial M}\delta M + \frac{\partial\rho_1^b}{\partial g}\delta g + \frac{\partial\rho_1^b}{\partial s}\delta s. \tag{4.4}$$

This contribution was found by calculating the counterterm diagrams of fig.(3) . The counterterms are the terms generated by the rescalings of formulae (2.3) and

(2.4) . Adding all contributions we find for the correction to ρ :

$$\delta\rho = \frac{g^2_{exp}}{64\pi^2 M^2_{exp}}(m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln(m_t^2/m_b^2))$$
$$+ \frac{g^4_{exp}}{4096\pi^4 M^4_{exp}}\Big(-12(m_t^4 Sp(1 - m_b^2/m_t^2) + m_b^4 Sp(1 - m_t^2/m_b^2))$$
$$- (2m_t^4 - 2m_t^6/m_b^2)\ln(m_t^2/m_b^2)\ln(|1 - m_b^2/m_t^2|)$$
$$+ (2m_b^6/m_t^2 - 2m_b^4)\ln(m_t^2/m_b^2)\ln(|1 - m_t^2/m_b^2|)$$
$$- (3m_t^6/m_b^2 - 7m_t^4 + 5m_t^2 m_b^2 - m_b^4)\ln(|1 - m_b^2/m_t^2|)$$
$$- (3m_b^6/m_t^2 - 7m_b^4 + 5m_b^2 m_t^2 - m_t^4)\ln(|1 - m_t^2/m_b^2|)$$
$$+ (m_b^4 - m_t^4 - 12\frac{(m_b^2 + m_t^2)m_b^2 m_t^2}{m_t^2 - m_b^2})\ln(m_t^2/m_b^2) + 23m_t^4 - 22m_t^2 m_b^2 + 23m_b^4\Big). \tag{4.5}$$

In this expression m_t and m_b are the physical ones as defined in chapter 2. For a small mass difference $m_t - m_b = \varepsilon \ll m_t$ this expression reduces to :

$$\delta\rho = \frac{g^2}{48\pi^2}\frac{\varepsilon^2}{M^2} + \frac{g^4}{1024\pi^4}\frac{m_t^2\varepsilon^2}{M^4}. \tag{4.6}$$

If one of the fermions is massless, i. e. if we have a heavy lepton with mass m_ℓ with its massless neutrino, the contribution to ρ is:

$$\delta\rho = \frac{g^2}{64\pi^2}\frac{m_\ell^2}{M^2} + \frac{g^4}{2048\pi^4}\frac{m_\ell^4}{M^4}(10 - \pi^2) \tag{4.7}$$

These results do not depend on the hypercharge assignments of the fermions in the doublet, because in order to get a maximal dependence on the masses all the $SU_R(2)$ breaking must come from the difference in Yukawa-couplings . For a quark doublet coming in three colors, the expression should be tripled.

V. $\delta\rho$ for equal masses

If the masses of the quarks are taken to be equal , all the breaking of the $SU_R(2)$ invariance of the theory comes from the hypercharge coupling. Therefore the ρ - parameter will receive no correction if $\tan\theta_w$ is taken to be zero. Because hypercharge terms must be present the dependence of ρ on the mass of the quarks is only quadratic as compared to the quartic dependence for different mass quarks. At first sight this seems to imply that also vertex corrections like fig. 4a have to be taken into account. However, these cancel against the counter diagram of fig. 4b where the counterterm comes from the rescalings of formula (2.5). In this case the rescalings (2.5) also introduce vertex counterterms which contribute to the vector boson two point functions. A complication is the need to introduce order g^4 counterterms from double rescalings. These are of the form δ_1^2 for the W^+W^- propagator and δ_1^2/c^2 for the W^0W^0 propagator.Therefore they cancel in the expression for the ρ parameter. Adding all contributions we arrive at the following expression for $\delta\rho$:

$$\delta\rho = \frac{g^4}{256\pi^4}\frac{m_t^2}{M^2}\Big(-\frac{11}{12}\tan^2\theta_w - \tan^2\theta_w \ln(m_t^2/m_H^2) + \frac{M^2}{m_H^2 - M^2}\ln(M^2/m_H^2) - \frac{M^2}{c^4 m_H^2 - c^2 M^2}\ln(M^2/c^2 m_H^2)\Big) \tag{5.1}$$

with m_H the Higgs-mass and $c = \cos\theta_w$. The answer depends on the Higgs mass in an essential way. Some limiting cases are

$$\delta\rho = \frac{g^4}{256\pi^4}\frac{m_t^2}{M^2}\left(-\frac{11}{12}\tan^2\theta_w - \tan^2\theta_w \ln(m_t^2/M^2) - \frac{\ln c^2}{c^2}\right) \tag{5.2}$$

for $m_H \to 0$,

$$\delta\rho = \frac{g^4}{256\pi^4}\frac{m_t^2}{M^2}\left(-\frac{11}{12}\tan^2\theta_w - \tan^2\theta_w \ln(m_t^2/M^2) - 1 - \frac{\ln c^2}{s^2c^2}\right) \tag{5.3}$$

for $m_H = M$,

$$\delta\rho = \frac{g^4}{256\pi^4}\frac{m_t^2}{M^2}\left(-\frac{11}{12}\tan^2\theta_w - \tan^2\theta_w \ln(m_t^2/M_0^2) - \frac{\ln c^2}{\tan^2\theta_w} + \frac{1}{c^2}\right) \tag{5.4}$$

for $m_H = M_0$ and

$$\delta\rho = \frac{g^4}{256\pi^4}\frac{m_t^2}{M^2}\left(-\frac{11}{12}\tan^2\theta_w - \tan^2\theta_w \ln(m_t^2/m_H^2)\right) \tag{5.5}$$

for $m_H \gg M_0, M$.

For a quark doublet coming in three colors these results should be tripled.

VI. Mass shift of vector bosons

In this chapter we calculate the difference between the actual masses of the vector bosons and the results one would get using tree level predictions from low energy data. We only consider the case were the masses of the quarks are equal. The results can be expressed in terms of f_{AA}, f_{AW}, f^+, f^0 as defined in chapter 3. First consider a process where a charged vector boson is produced , for instance $\bar{u} + d \to W^- \to e^- + \bar{\nu}_e$. The amplitude for this process is determined by the factor

$$\frac{g^2}{k^2(1-f^+) + M^2 - \delta^+} = \frac{g^2_{exp}}{\left(\frac{k^2(1-f^+)}{1-f_{AA}-c/sf_{AW}}\right) + M^2_{exp}}. \tag{6.1}$$

Thus we see that the mass M_{W^+} is given by

$$M^2_{W^+} = M^2_{exp}\left(\frac{1 - f_{AA} - c/sf_{AW}}{1-f^+}\right). \tag{6.2}$$

The mass shift that we are interested in is given by

$$\frac{\delta M}{M} \equiv \frac{M_W - M_{exp}}{M_{exp}} = \frac{1}{2}(f^+ - f_{AA} - c/sf_{AW}) + O(g^6). \tag{6.3}$$

In an analogous way we can consider a process where a neutral vector boson is exchanged , for instance $e^+e^- \to W^0 \to \mu^+\mu^-$. Here the amplitude is determined by the factor

$$\frac{g^2/c^2}{k^2(1-f_0) + M^2/c^2 - \delta^0} = \frac{g^2_{exp}}{c^2_{exp}}\left(\frac{(1-f_{AA})(1-f_0)k^2}{(1-f_{AA}-c/sf_{AW})(1-f_{AA}+s/cf_{AW})} + M^2_{0,exp}\right)^{-1} \tag{6.4}$$

with $M_{0,exp}$ as defined in (3.5). The corresponding mass shift becomes

$$\frac{\delta M_0}{M_0} \equiv \frac{M_{W^0} - M_{0,exp}}{M_{0,exp}} = \frac{1}{2}(f_0 - f_{AA} - c/sf_{AW} + s/cf_{AW}) + O(g^6). \tag{6.5}$$

The calculation of $\delta M/M$ and $\delta M_0/M_0$ is now straightforward. The resulting expression is a bit more complicated than for the ρ- parameter , because the expansion in the external momenta has to be pushed one order further. Keeping also the one-loop contribution we find the following :

$$\frac{\delta M}{M} = \frac{g^2}{192\pi^2} + \frac{g^4}{4608\pi^4}\frac{m_t^2}{M^2}\Big(\frac{37}{24} + 4D + 10D^2 + \ln(m_t^2/m_H^2) + 9D^2\ln(M^2/m_H^2) + 10D^3\ln(M^2/m_H^2)\Big) \tag{6.6}$$

where we defined

$$D \equiv \frac{M^2}{m_H^2 - M^2}$$

and

$$\frac{\delta M_0}{M_0} = \frac{g^2}{192\pi^2 c^2} + \frac{g^4}{4608\pi^4 c^2}\frac{m_t^2}{M^2}\Big(\frac{37}{24} + 4D_0 - 10D_0^2 + \ln(m_t^2/m_H^2) + 9D_0^2\ln(M_0^2/m_H^2) - 10D_0^3\ln(M_0^2/m_H^2)\Big) \qquad (6.7)$$

where we defined

$$D_0 \equiv \frac{M^2}{c^2 m_H^2 - M^2}.$$

In the limit $\sin^2\theta_w \to 0$ the expressions (6.6) and (6.7) become equal , as they should. Some special cases in this limit are

$$\frac{\delta M}{M} = \frac{g^2}{192\pi^2} + \frac{g^4}{4608\pi^4}\frac{m_t^2}{M^2}\left(\frac{181}{24} - \ln(m_t^2/M^2)\right) \qquad (6.8)$$

when $\sin\theta_w \to 0, m_H \to 0$,

$$\frac{\delta M}{M} = \frac{g^2}{192\pi^2} + \frac{g^4}{4608\pi^4}\frac{m_t^2}{M^2}\left(\frac{65}{24} + \ln(m_t^2/M^2)\right) \qquad (6.9)$$

when $\sin\theta_w \to 0, m_H = M$,

$$\frac{\delta M}{M} = \frac{g^2}{192\pi^2} + \frac{g^4}{4608\pi^4}\frac{m_t^2}{M^2}\left(\frac{37}{24} + \ln(m_t^2/m_H^2)\right) \qquad (6.10)$$

when $\sin\theta_w \to 0, m_H \to \infty$. For a quark doublet coming in three colors these results should be tripled.

VII. Discussion of the results

The results of the previous chapter can be compared to the presently known values of the weak interaction parameters. For the ρ-parameter[9] we get the following bounds , taking into account a -1% correction due to known effects[10] :

$$-0.02 \lesssim \delta\rho \text{ (heavy fermions)} \lesssim 0.03.$$

The mass shift of the vector bosons agrees with the predictions within an accuracy of about 3% , so we have $|\delta M/M|$, $|\delta M_0/M_0| \lesssim 0.03$.

We will assume that a heavy quark doublet comes in three colors. In the case of different quark masses both the one-loop and the two-loop correction to ρ are positive. As a consequence one gets tighter constraints on the allowed mass differences inside a multiplet then the 300 GeV from the one-loop correction. Significant changes ($\gtrsim$ 50 GeV) appear for masses larger than about 1 TeV ,so that the present limit on the mass of the top-quark is not affected. The allowed regions in the $m_b, m_t - m_b$ plane are depicted in fig. (5) . For large enough values of the quark masses perturbation theory breaks down. This is most clearly seen in formula (4.6) for small mass differences, where the two-loop correction equals the one-loop correction for a mass

$$m_t = \frac{8\pi M}{g\sqrt{3}} \simeq 1.8\ TeV.$$

In the case of a degenerate quark multiplet the results depend in an essential way on the Higgs mass . Limits on the quark mass can be obtained from the mass shift and the ρ-parameter. The correction to ρ is negative in this case. The allowed regions in the m_t, m_H plane are given in fig. (6). Apparently the ρ-parameter is more sensitive to the heavy quark effects then the mass-shifts. For large Higgs-mass the limit on the quark mass gets weaker , because the factor $m_t^2/M^2 \ln(m_t^2/m_H^2)$ is becoming important. In order to get a bound on the quark mass one therefore needs a limit on m_H. For a Higgs mass of about 3 TeV perturbation theory in the Higgs-sector itself breaks down and this provides a natural cut-off for the validity of the calculation. For $m_H \simeq 3\ TeV$ the limit on m_q is about 4 TeV and for this ratio one also starts leaving the area where the approximation $m_t \gg m_H$is valid. For the ρ -parameter one might be worried that a small mass difference can upset the calculation. In this case it is probably better to take the limit from the mass shift of the vector bosons. This gives a limit of about 6 TeV on the quark mass. One must bear in mind that in this mass range the use of perturbation theory has become questionable.

If such heavy quarks are really present, they would give rise to some interesting physics in future colliders. As an example consider a 2 TeV quark and assume a Higgs-mass of about 200 GeV . The quarks could be pair-produced and subsequently decay via emission of a W-boson. Because of the strong coupling of the Higgs to the heavy quark a number of Higgses might appear in the decay-products. The Higgses themselves will decay predominantly in W and Z bosons. This way one would have an event with a quark jet and for instance seven vector-bosons as outgoing particles. Even more exotic possibilities might arise, if the quarks form bound states due to the strong force coming from the exchange of a Higgs particle.

Finally we want to make some remarks on the non-decoupling of the very heavy fermions. The presence of effects growing like m_f^2 indicate that the standard Lagrangian plus Wess-Zumino term is not renormalizable. What is apparently happening is that the low energy effective interaction when inserted in a loop does not correctly reproduce the correct results of the full theory. In particular the approximation that the energy flowing through a Higgs line is smaller than the quark mass cannot be used any more inside a diagram containing Higgs lines. However , we have seen that in the limit $m_H \to \infty$ some of the corrections become smaller . In this context the combined limits $m_q \to \infty, m_H \to \infty$ may be of interest. In general we do not expect any cancellations to appear because the Higgs mass and the fermion mass are unrelated . In supersymmetric models these masses are related however and one might expect some sort of decoupling to happen. Some results in this direction have already been obtained at the one-loop level[11]. The two-loop level is under investigation.

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Appendix

Using the techniques of refs. [5,8] it is possible to express all corrections in terms of a few basic integrals . We use the following notation for these integrals :

$$(n_1M_1|n_2M_2|n_3M_3) = \int d^npd^nq\frac{1}{(p^2-M_1^2)^{n_1}(q^2+M_2^2)^{n_2}(r^2+M_3^2)^{n_3}} \qquad (A.1)$$

where $r = p + q$,

$$(n_1, M) = \int d^np\frac{1}{(p^2+M^2)^{n_1}} \qquad (A.2)$$

Inside these integrals some of the masses will tend to infinity and we need an expansion of the integrals in inverse powers of these masses. Most of the expansions needed are given in refs. [5 , 8]. The new integrals needed here are :

$$(2m_t|m_b|M), (3m_t|m_b|M), (2m_t|2m_b|M), (2m_t|m_b|2M), (m_t|m_b|2M), (m_t|m_b|3M)$$

in the limit $m_t, m_b \to \infty$. It is sufficient to know

$$
\begin{aligned}
(2m_t|m_b|M) =&\pi^4 \left(-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(1 - 2\ln m_t^2) - \frac{1}{2} - \frac{\pi^2}{12} + \ln m_t^2 - \ln^2 m_t^2 - Sp\left(1 - \frac{m_b^2}{m_t^2}\right)\right) \\
&+\pi^4 \frac{M^2}{m_t^2}\left(\ln(M^2/m_t^2)\left(\frac{-m_t^2}{m_t^2 - m_b^2} - \frac{m_b^2 m_t^2}{(m_t^2-m_b^2)^2}\ln(m_b^2/m_t^2)\right)\right. \\
&\left. - \frac{2m_b^2 m_t^2}{(m_t^2-m_b^2)^2} Sp(1 - \frac{m_b^2}{m_t^2}) - \frac{m_b^2 m_t^2}{(m_t^2-m_b^2)^2}\ln(m_b^2/m_t^2) + \frac{m_t^2}{m_t^2 - m_b^2}\right) \\
&+O(\varepsilon, \frac{1}{m_t^4}, \frac{1}{m_t^2 m_b^2}, \frac{1}{m_b^4})
\end{aligned}
\tag{A.3}
$$

$\varepsilon = n - 4$ in dimensional regularization. Sp is the Spence function defined by

$$Sp(x) = -\int_0^x dy \frac{\ln(1-y)}{y} \tag{A.4}$$

The other functions are straightforwardly derived by differentiating with respect to masses , e.g.

$$(3m_t|m_b|M) = -\frac{1}{2}\frac{\partial}{\partial m_t^2}(2m_t|m_b|M) \tag{A.5}$$

and the use of the relation

$$
\begin{aligned}
(2M|m_t|m_b) =&(3-n)(2m_t|m_b|M) - 2m_b^2(2m_t|2m_b|M) \\
&+ (2m_t)(2M) + (m_t^2 - m_b^2 - M^2)(2m_t|m_b|2M)
\end{aligned}
\tag{A.6}
$$

Since we use dimensional regularization we need a prescription for γ_5 outside four dimensions. There is no unique way to continue γ_5 to n dimensions. In our calculation we used the rules : $\{\gamma_5, \gamma_\mu\} = 0$, Tr $\gamma_5\gamma_\mu\gamma_\nu = 0$. This scheme ,with an anti- commuting γ_5, has the advantage of being computationally simple. Since we only have one external momentum and two Lorentz indices in our calculation we never have to consider traces of γ_5 with more than two other gamma matrices. The problems with an anticommuting γ_5 do not arise until one has to take the trace of γ_5 with four or more gamma matrices. Using an anticommuting γ_5 it is e.g. impossible to reproduce the anomaly correctly. The standard model however is anomaly free. Using the anticommuting γ_5 we checked the Ward identities for the photon-photon , photon-vectorboson and Higgs-ghost two point functions and found that they were satisfied .

Because there were several hundred diagrams to be calculated, most of the calculation has been done with the help of the computer program SCHOONSCHIP[12].