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A Priori Definition of Maximal CP Violation

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Abstract

We propose an a priori definition of maximal CP violation. Our definition is that maximal CP violation occurs when a unique convention-independent imaginary parameter t is maximized. This parameter t is a quartic function of the Kobayashi-Maskawa (KM) matrix V. When t vanishes, CP is conserved. The maximum value, $t_{max} = 2\sqrt{3}$, is mucher greater than the experimental upper limit, $t_{obs} \leq 3 \times 10^{-4}$. Thus the observed CP violation is much less than maximal. For $t = t_{max}$, the KM matrix corresponds to maximum mixing of the quark generations, just as maximal parity violation corresponds to maximum mixing of the vector and axial-vector interactions.

Parity violation was discovered in 1957. Soon after, several authors suggested that parity might be violated "maximally". "Maximal" parity violation means that the vector and axial vector currents occur with equal normalizations and equal coupling constants in the fundamental Lagrangian of weak interactions. This suggestion proved to be correct. It is incorporated in the standard model by the fact that only the left-handed current couples to the W boson.

This success led to the suggestion that CP violation, discovered in 1964, might also be maximal. In the Kobayashi-Maskawa (KM) framework,(1) CP violation is associated with imaginary parts of elements of the KM matrix V. Maximal CP violation might then correspond to some element, or some term in some element of V being pure imaginary, or, in other words, to a term having a phase of $\pm \frac{\pi}{2}$. Several authors have considered this point of view.[2]

The situation for this notion of maximal CP violation differs, however, from that for maximal parity violation. Since a left-handed spinor field remains left-handed under chiral transformations, the latter is an invariant concept. The former notion is not invariant, since the phases of elements of V can be changed by changing the phases of the quark fields. Physical states in Hilbert space are rays; thus observables must remain unchanged under such rephasing of quark fields. Roos, and Gronau and Schechter tried to avoid this difficulty by finding a parametrization (the Murnaghan construction[3]) in which a certain sum of phases, the "invariant" phase, remains invariant under most (the similarity transformations) rephasings of the quark fields. Unfortunately, whether or not present data allows the invariant phase to be $\pm \pi/2$ depends not only on the convention of adopting the Murnaghan construction, but, in addition, on a further convention, the order in which the matrices in the Murnaghan construction are multiplied. Thus the statement that the invariant phase is $\pm \pi/2$ is not convention independent.

One could adopt, as an alternate definition of maximal CP violation, that choice of V which maximizes CP violation in a specific process, for example the choice of V which maximizes ε_K or ε_{B_d} (the ε parameter in the K^o-$\bar{K}^o$ system, where K^o is $d\bar{s}$, or in the B_d^o-$\bar{B}_d^o$ system, where B^o is $d\bar{b}$. Such a definition is process-dependent, and is not analogous to the universal definition of maximal parity violation.

We record the widely-held view that whether or not CP violation is maximal can only be decided when one knows the fundamental origin of CP violation. We share the view that the KM framework is not a fundamental theory of CP violation; rather the KM framework provides a description of CP violation. Nonetheless an a priori definition of maximal CP violation at the KM level may be useful.

A definition of maximal CP violation should be convention-independent and universal, i.e., process-independent. We propose such a definition which uses a single parameter, t, defined below. When t vanishes, CP is conserved in all processes. When t attains its maximum value (allowing an arbitrary three-generation KM matrix V) CP is violated maximally. We find below that present data shows that CP-violation is much less than maximal.

To give our definition of maximal CP violation, we must first review the convention-independent formulation of CP violation and of weak interactions generally.[4] The convention independent functions of V which occur in weak interaction rates are $|V_{ij}|^2$. Not all N^2 of these (for the case of N generations) are independent. The 2N-1 independent conditions from the diagonal elements of the unitarity equations $V^\dagger V = 1$ and $VV^\dagger = 1$ reduce the number of independent quadratic parameters to $(N-1)^2$. These can be chosen to be

$$T^{i\alpha} = \mathrm{Tr} V^\dagger \lambda^i V \lambda^\alpha, \quad i,\alpha = 3,8,15,\ldots N^2-1.$$

This is a complete set of convention-independent functions of V. Although these parameters are all real, they may implicitly require CP violation. Nonetheless, we would like to find an imaginary parameter which explicitly requires CP violation.

For this purpose, we consider quartic functions of V. In the three-generation case, CP violation can be parametrized in terms of the nine convention-independent complex quantities

$$\Delta_{i\alpha} = V_{j\beta}V_{k\gamma}\, V^*_{j\gamma}V^*_{k\beta}, \ i,j,k \text{ and } \alpha,\beta,\gamma \text{ cyclic.}$$

One can equally well use the convention-independent quantities

$$T^{i\alpha j\beta} = \mathrm{Tr}V^\dagger\lambda^i V\lambda^\alpha V^\dagger\lambda^j V\lambda^\beta, i,\alpha,j,\beta = 3,8,15,\ldots\ N^2-1$$

for N generations. These obey $T^{i\alpha j\beta} = T^{j\beta i\alpha}$ and $T^{i\alpha j\beta *} = T^{j\alpha i\beta}$. For three generations, there are 10 of these, namely T^{3333}, T^{3338}, T^{3383}, T^{3388}, T^{3883}, T^{3838}, T^{8383}, T^{8838}, T^{8883}, and T^{8888}. All of these are real, except $T^{3388*} = T^{3883}$. Thus there are ten parameters, nine real and one imaginary, associated with these quartic T's. For three generations, the relation between the Δ's and the traces is $\Delta_{i\alpha} = \mathrm{Tr}V^\dagger\Lambda_j V\Lambda_\beta V^\dagger\Lambda_k V\Lambda_\gamma$, i,j,k and α,β,γ cyclic, where the Λ's are projection operators in generation space; for example, $\Lambda_2 = \mathrm{diag}\ (0\ 1\ 0)$. The projection operators are sums of the diagonal matrices [5]. Using this relation, we showed that all the Δ's have the same imaginary part,

$$t \equiv \mathrm{Im}\Delta = \frac{1}{12}\ \mathrm{Im}\ T^{3388} = \quad c_1c_2c_3s_1^2s_2s_3s_\delta,$$

using the KM parametrization. Thus there are also ten parameters, nine real and one imaginary, associated with the Δ's. The convention-independent parameter t controls all CP violation in the KM framework. When t vanishes, CP is conserved in all processes.[6]

We propose using t as the parameter which characterizes maximal CP violation. We define maximal CP violation to occur when t assumes its maximum value, given any KM matrix V. This definition of maximal CP violation is universal, and, like the usual definition of maximal parity violation, it is an a priori definition, independent of the experimental situation. Experimental information is not used to formulate this definition of maximal CP violation, but rather to determine whether or not maximal CP violation is realized in nature.

We now calculate the maximum value of t and the form of V at the maximum. Since t is convention-independent, we can use any parametrization of V to calculate its maximum value. Using the KM parametrization, we find

$$t = c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta .$$

The maximum value of t occurs at $c_1=1/\sqrt{3}$, $c_2=1/\sqrt{2}$, $c_3=1/\sqrt{2}$, $s_\delta=1$. The value is

$$t_{max} = 1/6\sqrt{3}.$$

This value is much less than the observed upper limit

$$t_{obs} < 3\times 10^{-4}.$$

Thus the observed CP violation is much less than maximal. The KM matrix for the maximal case is

$$V_{max} = 1/\sqrt{3} \begin{pmatrix} 1 & -1 & -1 \\ 1 & -x & -x^2 \\ 1 & -x^2 & -x \end{pmatrix} , \text{ where } x = e^{2\pi i/3}.$$

This matrix, which was discussed by Wolfenstein[7] in the context of a model with three neutrinos, corresponds to maximum mixing of the d quark weak eigenstates in terms of the mass eigenstates. Thus maximum mixing of the quark generations corresponds to maximal CP violation. We find this result of our a priori criterion for maximal CP violation satisfying: maximum mixing of the quark generations is analogous for CP violation to maximum mixing of V and A for parity violation.

It remains to assess the significance of models such as that of Gronau and Schechter. Our view is that such models are interesting, but that they should not be called models with maximal CP violation. There is no convention-independent separation between the mixing angles and the phase in the KM matrix V. Whether or not a phase is $\pm\pi/2$ depends on the conventions used to parametrize V, therefore the fact that the phase can have such a value in some parametrization does not have physical significance.

Finally, we emphasize that the fact that present data on weak interaction rates constrain $|V_{ij}|$ so severely that the observed CP-violation parameter ε_K can only be fit with the CP-violating phase set to its maximum value $\pm\pi/2$ should be regarded as showing that the KM model with three generations is on the edge of being ruled out by experiment, rather than being regarded as evidence for "maximal" CP violation.

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References and footnotes

1. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

2. D. Hochberg and R.G. Sachs, Phys. Rev. D27, 606 (1983); B. Stech, Phys. Lett. B130, 189 (1983); L. Wolfenstein, Phys. Lett. B144, 425 (1984); M. Gronan and J. Schechter, Phys. Rev. Lett. 54, 385, 1209 (E) (1985); M. Roos, Univ. of Helsinki report Hu-TFT-84-38 (1984).

3. The Murnaghan construction of a matrix in SU(N) is

$$V = D \prod_{i<j} V^{(ij)},$$

where D is a diagonal SU(N) matrix and $V^{(ij)}$ is an SU(2) matrix of the form

$$V^{(ij)} = \begin{pmatrix} c_{ij} & s_{ij} e^{i\phi_{ij}} \\ -s_{ij} e^{-\phi_{ij}} & c_{ij} \end{pmatrix}$$

acting on generations i and j. The form of V depends on the order in which the $V^{(ij)}$ are multiplied. Throughout this article, we abbreviate $\cos\theta_i$ or $\sin\theta_i$ by c_i or s_i. Certain sums of the ϕ's are invariant under simularity rephasings of the KM matrix. In particular,

$$\Phi = \phi_{12} + \phi_{23} + \phi_{31}$$

which Gronau and Schechter call the "invariant phase", has this property for the case of three generations.

4. O.W. Greenberg, FERMILAB-Pub-85/44-T, to appear in Phys. Rev. D; D.-d. Wu, Enrico Fermi Institute report 85-35.

5. For the three-generation case, the relation is

$$\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2 & 3 & 1 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda^3 \\ \sqrt{3}\ \lambda^8 \end{pmatrix}$$

6. Both first- and second-order CP violation can be expressed in terms of the quartic Δ's or T's: first- (second-) order CP violation is linear (quadratic) in these quantities.

7. L. Wolfenstein, Phys. Rev. D18, 958 (1978).