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A Model for Initial State Parton Showers

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ABSTRACT

We present a detailed model for exclusive properties of initial state parton showers. A numerically efficient algorithm is obtained by tracing the parton showers backwards, i.e. start with the hard scattering partons and then successively reconstruct preceding branchings in falling sequence of spacelike virtualities Q^2 and rising sequence of parton energies. We show how the Altarelli-Parisi equations can be recast in a form suitable for this, and also discuss the kinematics of the branchings. The complete model is implemented in a Monte Carlo program, and some first results are presented.

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A model for exclusive properties of high-p_T events in hadron-hadron interactions requires a number of separate components [1]: QCD hard scattering matrix elements, structure functions, initial state (spacelike) parton evolution, final state (timelike) parton showers, and jet fragmentation. Of these, the initial state parton showers probably are the least well studied. In the present paper we will therefore develop a detailed model for this component, using the backwards evolution formalism, an approach orthogonal to presently available models. In particular, this allows a quite efficient implementation in terms of computer algorithms for event generation. Together with the other components above, this model has been implemented within the framework of the Lund Monte Carlo [2,3]. We present some first results here, to illustrate the methods and problems.

A fast hadron may be viewed as a cloud of quasireal partons. At each instant, an individual parton can initate a cascade, branching into a number of partons. These partons do not have enough energy to be on mass-shell ($M^2<0$), and thus only live for a finite time before reassembling. In a hard interaction between two incoming hadrons, when two partons scatter to high p_T, also the other partons in the two related cascades are provided with the necessary energy to live indefinitely. The correct description for this transfer of energy is obviously given by the various $2\to N$ hard scattering matrix elements, where 2 stands for the two initiators of the cascades and N for the final parton multiplicity. In practice, matrix elements can only be calculated for small values of N, and one has to resort to approximate schemes, such as the leading logarithmic approximation (see e.g. [4]). In particular, for subsequent Monte Carlo applications, it is convenient to imagine that the partons on the two branches which leads from the two initiators to the hard scattering ($7\to3\to1$ and $5\to2$ in Fig. 1) have increasing spacelike virtualities, $Q^2=-M^2>0$, adjusted such that the partons on all other branches (8, 4 and 6 in Fig. 1) may have $M^2\geq 0$; these latter partons are in the following referred to as the timelike ones. Then the momentum transfer given by the central $2\to2$ hard scattering subprocess is enough to ensure that all partons may end up on mass shell. Except for the two hard scatterers, the partons continue essentially along the direction of the respective hadron they belonged to, although occasionally they may have large transverse momenta and give rise to separately visible jets of their own. Other cascades within the two interacting hadrons remain unaffected, i. e. do not receive any energy transfers, and disappear unnoticed into the low-p_T "beam jet" background.

Current programs for initial state parton showers, COJETS by Odorico [5] and the CIT-Florida ones by Field, Fox, Kelly and Shatz [6], use a forward evolution in physical time. This means that the parton structure functions are sampled at some low scale $Q_0^2\approx4$ GeV^2. The two partons chosen, one from each incoming hadron, are then evolved up to the hard scattering Q^2 scale by successive branchings $a\to bc$ according to the Altarelli-Parisi (AP) equations [7]. Finally, the event is accepted with a probability proportional to the $d\sigma/d\hat{t}$ of the hard scattering subprocess. There are two major problems with this approach. Firstly, in each branching $a\to bc$, it is not known whether the hard scattering parton will be on the b or c branch. Secondly, the hard scattering scale Q^2, which sets the upper limit for the spacelike evolution, is not known beforehand. In the CIT-Florida approach, these problems are overcome by brute force, which leads to an extremely inefficient Monte Carlo implementation. By introducing a pretabulation step, Odorico is able to make the generation more efficient, although not entirely transparent.

In the present approach, the starting point is the hard scattering subprocess. An inclusive summation over all initial state showers is equivalent to using Q^2-evolved structure functions, e. g. the parametrizations in [8]. The efficient choice of reaction channel and kinematical variables (x_1, x_2 and $\hat{t}$) for this hard scattering is a standard task, already solved in the Lund and other Monte Carlos. The exclusive parton showers must now be reconstructed step by step, i. e. for a parton b one must find which branching $a \to bc$ gave rise to it, alternatively that b was present already at the cutoff scale Q_0^2.

For this, consider the AP equations

$$\frac{df_b(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \sum_a \int \frac{dx'}{x'} f_a(x',t) \, P_{a\to bc}\left(\frac{x}{x'}\right) \qquad (1)$$

where $f_i(x,t)$ is the structure function for flavour i, $t = \ln Q^2$ is the evolution parameter, $\alpha_s(t)$ the running strong coupling constant and $P_{a\to bc}(z)$ the AP splitting functions. The normal use of these equations is to assume $f_i(x, t_0)$ known at the cutoff $t_0 = \ln Q_0^2$, and then follow the influx of partons b at x from the branching of partons a at $x'>x$ as t increases, to find $f_i(x, t)$ for $t>t_0$. However, let us assume that $f_i(x,t)$ has already been obtained this way [8]. The probability that a parton b disappears from x during a small decrease dt is then given by

$$dP_b = \frac{df_b(x,t)}{f_b(x,t)} = |dt| \frac{\alpha_s(t)}{2\pi} \sum_a \int \frac{dx'}{x'} \frac{f_a(x',t)}{f_b(x,t)} P_{a\to bc}(\frac{x}{x'}) \qquad (2)$$

This probability exponentiates, so that one may define a form factor

$$S_b(x,t_1;t) = \exp \{-\int_t^{t_1} dt' \frac{\alpha_s(t')}{2\pi} \sum_a \int \frac{dx'}{x'} \frac{f_a(x',t')}{f_b(x,t')} P_{a\to bc}(\frac{x}{x'})\} \qquad (3)$$

giving the probability that a parton b remains at x from t_1 to $t<t_1$. Note that the t' dependence of f_a and f_b implies that the influx of partons b from x"<x and outflow of partons a to x">x' with decreasing t' is implicitly accounted for.

The practical interpretation of eq. (3) is

(i) the t value at which the branching $a\to bc$ takes place, i. e. the virtuality of b, is given by putting $S_b(x,t_1;t)$ equal to a random number between 0 and 1 and solving for t; if the random number is smaller than $S_b(x,t_1;t_0)$ the parton b existed at Q_0^2 and there is no more branching;

(ii) The relative probabilities for the different possible branchings $a\to bc$ are given by the integrals

$$\int \frac{dx'}{x'} \frac{f_a(x',t)}{f_b(x,t)} P_{a\to bc}(\frac{x}{x'}) \qquad (4)$$

with t the value chosen in (i);

(iii) the probability distribution in x' is given by the integrand in eq. (4) for the process $a\to bc$ chosen in (ii).

In practice, a numerical integration over x' and t' would be too time-consuming. The convenient Monte Carlo procedure is to find a simple function that is everywhere larger than the integrand in eq. (3) and is analytically integrable, choose t and x' according to this function, and accept the choice with a probability the ratio of the correct integrand to the simple function in this (t, x') point. In case of rejection, the upper limit t_1 of eq. (3) is put equal to the rejected t value and the procedure is iterated.

Two important consequences of eq. (3) should be noted. Firstly, the fact that the x' value is chosen proportional to $f_a(x',t)$ ensures not only that x' values larger than 1 are excluded, but also that the proper limiting behaviour for $x'\to 1$ is obtained. Secondly, although the AP splitting functions are flavour symmetric (up to mass effects), the structure functions are generally not. At the low Q_0^2 scale, a proton contains more u than d (or $\bar{u}$). When a gluon is chosen at the hard scattering and evolved backwards, the different numerators $f_a(x',t')$ indeed ensure that this gluon is more likely to have been emitted by a u than a d. Similarly, if a heavy flavour is chosen at the hard scattering, the denominator $f_b(x,t')$ will vanish at the Q^2 threshold of the heavy flavour production, which means that the x' integral diverges and S_b itself vanishes, so that no heavy flavours remain below threshold.

In passing, we note that these two points distinguish our model from the scheme recently outlined by Gottschalk [9]: he proposes to use z=x/x' values chosen from $P_{a\to bc}(z)$ alone, which means that the x' distribution does not a priori vanish for $x'\to 1$ (indeed, the x distribution thus obtained at Q_0^2 tends to be too large at small and large x and too small at intermediate ones). Also, choosing the different branchings $a\to bc$ just from $P_{a\to bc}(z)$ would lead to a g in a proton equally often coming from a u as from a d.

The procedure outlined above for one single branching can obviously be iterated, to yield two sequences (one for each incoming hadron) of decreasing spacelike virtualities Q^2 (i. e. where the Q^2 of one branching is taken as the upper limit (t_1 in eq. (3)) for the preceding one), increasing x values and specified flavours, stretching backwards in time from the hard scattering to the cascade initiators. The kinematical interpretation of the x variable is not unique, however. As in [9] we choose to use an implementation in terms of invariant masses, where $\hat{s}=x_1x_2s$ both at the hard scattering and at each preceding step, i.e. where the $z=x/x'$ value at each branching corresponds to an increase (moving backwards in time) by a factor $1/z$ in the total $\hat{s}$. The advantage of this choice over using e.g. lightcone variables is that the hard scattering $\hat{s}$ is defined uniquely by the x_1 and x_2 values, without any reference to virtualities or transverse momenta.

We briefly outline the reconstruction of the kinematics, Fig. 1. Start by defining the two hard scatterers 1 and 2 in their CM frame, coming in with momenta along the $\pm z$ axis, with $\hat{s}=(p_1+p_2)^2=x_1x_2s$ (p_i denoting four-vectors). By backwards evolution from the hard scattering Q^2, a branching $3\to1+4$ is found, which defines the virtuality Q_1^2 and the $z=x_1/x_3$ variable. Correspondingly, the branching $5\to2+6$ defines Q_2^2 and $7\to3+8$ Q_3^2. The construction of the four-momentum p_3 now contains two degrees of freedom, apart from an arbitrary azimuthal angle. These may be chosen as the transverse momentum $p_{T3}(=p_{T4})$ and the mass-squared m_4^2 of the associated timelike parton. One constraint is given by our interpretation of z: $(p_3+p_2)^2=\hat{s}/z$. The maximum possible value for m_4^2 is found for $p_{T3}=0$:

$$(m_4^2)_{max} = -Q_1^2 - Q_3^2 + \frac{1}{2Q_2^2} \{(\hat{s}+Q_2^2+Q_1^2)(\frac{\hat{s}}{z}+Q_2^2+Q_3^2) - [((\hat{s}+Q_2^2+Q_1^2)^2-4Q_2^2Q_1^2][(\frac{\hat{s}}{z}+Q_2^2+Q_3^2)^2-4Q_2^2Q_3^2]]^{1/2}\} \quad (5)$$

which, for the special case of $Q_2{}^2=0$, reduces to

$$(m_4^2)_{max} = (\frac{Q_1{}^2}{z} - Q_3^2)(\frac{\hat{s}}{\hat{s}+Q_1^2} - \frac{\hat{s}}{\hat{s}/z+Q_3^2}) \quad (6)$$

Parton 4 may initiate a timelike parton shower, which can be constructed using standard methods (see e.g. [10]). A mass $m_4{}^2$ between $(m_4{}^2)_{max}$ and 0 (or $m_q{}^2$ for a quark) is then obtained, whereafter the kinematics of the branching 3→1+4 is completely specified. The system can be boosted to the CM frame of 3 and 2 and rotated to put their momenta along the ±z axis. The whole procedure may now be repeated for the next branching, 5→2+6 or 7→3+8, etc. Branchings on the two spacelike lines are interleaved to form a single, monotonically falling sequence of virtualities Q^2; obviously this choice is not unique. In the end, the two outermost partons (the cascade initiators) have virtualities $Q^2<Q_0{}^2$, and one may put these $Q^2=0$. Up to small corrections from the introduction of primordial k_T, a final longitudinal boost will bring the partons from their CM frame to the overall CM frame, where the x values of the outermost partons agree with the customary lightcone definition.

In eqs. (1)-(4), the range of x' integration is nominally $x \le x' \le 1$. The lower limit corresponds to $z=x/x'=1$, where the q→qg and g→gg AP splitting functions are singular. In the same limit, the energy carried away by the timelike parton, $(x'-x)\sqrt{s}/2=(1-z)x'\sqrt{s}/2$, vanishes. We therefore choose to require a minimum energy, typically 2 GeV, for the timelike partons. The energy carried away by gluons below this cutoff may be resummed and included as an effective shift of the z value chosen at each step; this shift turns out to have negligible physical consequences. Another constraint on allowed z values is given by the requirement that $(m_4^2)_{max} \geq 0$ (or $m_q^{\,2}$) in eq. (5). This constraint can not be used at the choice of z values, since $Q_3^{\,2}$ (and often $Q_2^{\,2}$) is not known at that time; rather it is implemented in connection with the reconstruction of the kinematics.

The production of W and Z offers a simple testing ground for initial state radiation. In Fig. 2 we show the p_T^W spectrum dn/dp_T^W obtained for W production at central rapidities ($|Y| \lesssim 1$) compared to UA1 and UA2 measurements. The data are not corrected for detector smearing effects, which tend to deplete low $p_T^{\,W}$ values [13], so within present statistics the agreement is fair. We have used the Eichten-Hinchliffe-Lane-Quigg set 2 structure functions [8] with Λ=290 MeV, which gives $\langle p_T^{\,W}\rangle$=5.3 GeV. If EHLQ set 1 with Λ=200 MeV is used, this is decreased by 0.2 GeV. If the Q^2 scale for W production, which defines the maximum allowed spacelike virtuality, is taken to be $M_W^{\,2}/4$ rather than $M_W^{\,2}$, $\langle p_T^{\,W}\rangle$ is decreased by 1.0 GeV. If emitted timelike partons are not allowed to shower, but rather put on mass shell, $\langle p_T^{\,W}\rangle$ is increased by 0.7 GeV. The results are also in fair agreement with the analytic predictions of [14] (using Glück-Hoffmann-Reya structure functions [8] with Λ=400 MeV and no associated timelike showers, which gives $\langle p_T^{\,W}\rangle$=6.8 GeV), although again the region of low $p_T^{\,W}$ is more depleted in the analytic treatment.

Because of the increase in phase space for shower development, $\langle p_T^W \rangle$ does grow with CM energy, from 2.5 GeV at $\sqrt{s}$ = 200 GeV to 5.9 GeV at 630 GeV, to 9.1 GeV at 2 TeV, to 17.6 GeV at 40 TeV. At the same time, the summed $|p_T|$ of associated (recoil) jets increases from 4.0 GeV to 10.9 GeV, to 18.2 GeV, to 43.6 GeV. Needless to say, the figures at 40 TeV are not particularly relevant: since we explicitly assume that M_W^2 is the hard scattering Q^2 scale, graphs like $u+\bar{d}\rightarrow W+g$ with $p_T > M_W/2$ are not included, and at high energies these dominate the $\langle P_T^W \rangle$ [14]. Correspondingly, our model would not be particularly useful for the (Drell-Yan) production of low-mass lepton pairs even a present energies.

We have also made a number of comparisons between our model and collider jet data. Here all fragmentation parameters are fixed by e^+e^- data, and we have used the EHLQ set 2 structure functions with Λ = 290 MeV, a hard scattering $Q^2=2\hat{s}\hat{t}\hat{u}/(\hat{s}^2+\hat{t}^2+\hat{u}^2)\approx\hat{p}_T^2$ and a maximum parton virtuality of $4Q^2\approx 4\hat{p}_T^{\,2}$ for shower development. The main result of these studies is readily visible in Fig. 3: the core of jets seems reasonably well described when initial and final state showers are included, but the number of particles and the energy flow in the low-p_T background is significantly understimated. This discrepancy is present already in "minimum bias" events without high-p_T jets. If we believe in jet universality, we would then conclude that the underlying process must be more complicated, involving e.g. multiple independent hard scatterings [16] or soft gluon exchanges [17]. This obviously warrants further study.

In conclusion, we have formulated an explicit and detailed model for initial state radiation, which offers a fairly efficient means for event generation: at $\sqrt{s}$=540 GeV, requiring a hard scattering $\hat{p}_T > 10$ GeV, one event takes 0.1 s Cyber 175 time, whereof 1/4 is for the hard scattering and initial and final state radiation, and the rest for the subsequent fragmentation and decay chain.

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FIGURE CAPTIONS

Fig. 1. Schematic picture of spacelike shower evolution, with hard scattering partons 1 and 2 and emitted timelike partons 4, 6 and 8.

Fig. 2. Transverse momentum distribution of W, from UA1 [11] and UA2 [12] compared with Monte Carlo results. The latter curve has been normalized to the same area as UA1.

Fig. 3. Transverse energy flow for jets with E_T>35 GeV, in bins of $\Delta\eta$=0.05 around jet core, integrated over $|\Delta\phi|<\pi/2$. Results from UA1 [15] are compared with Monte Carlo simulation with and without shower developent.

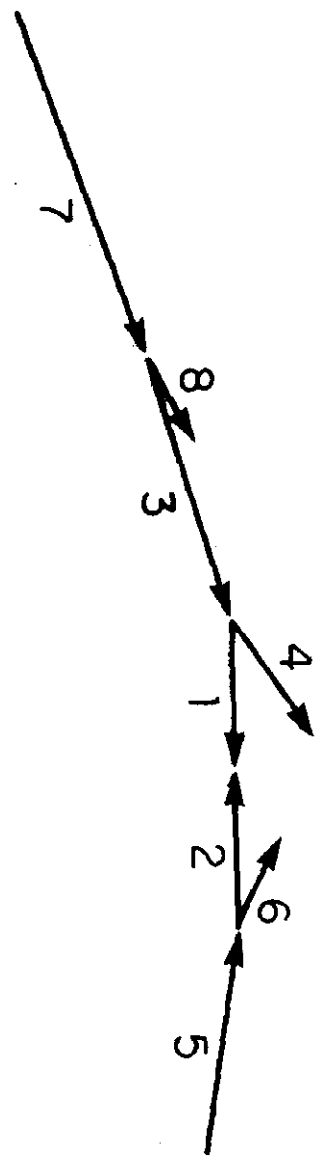
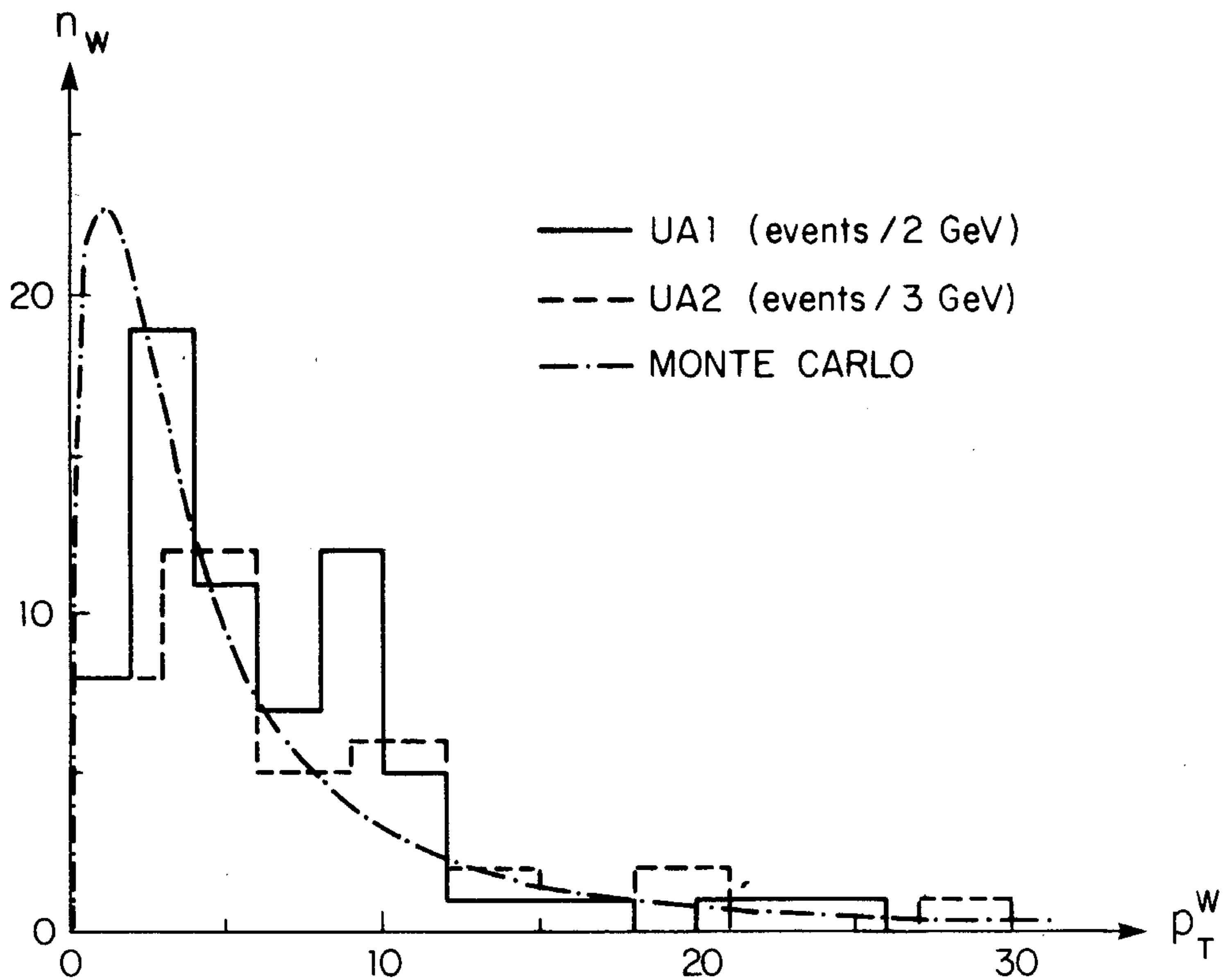
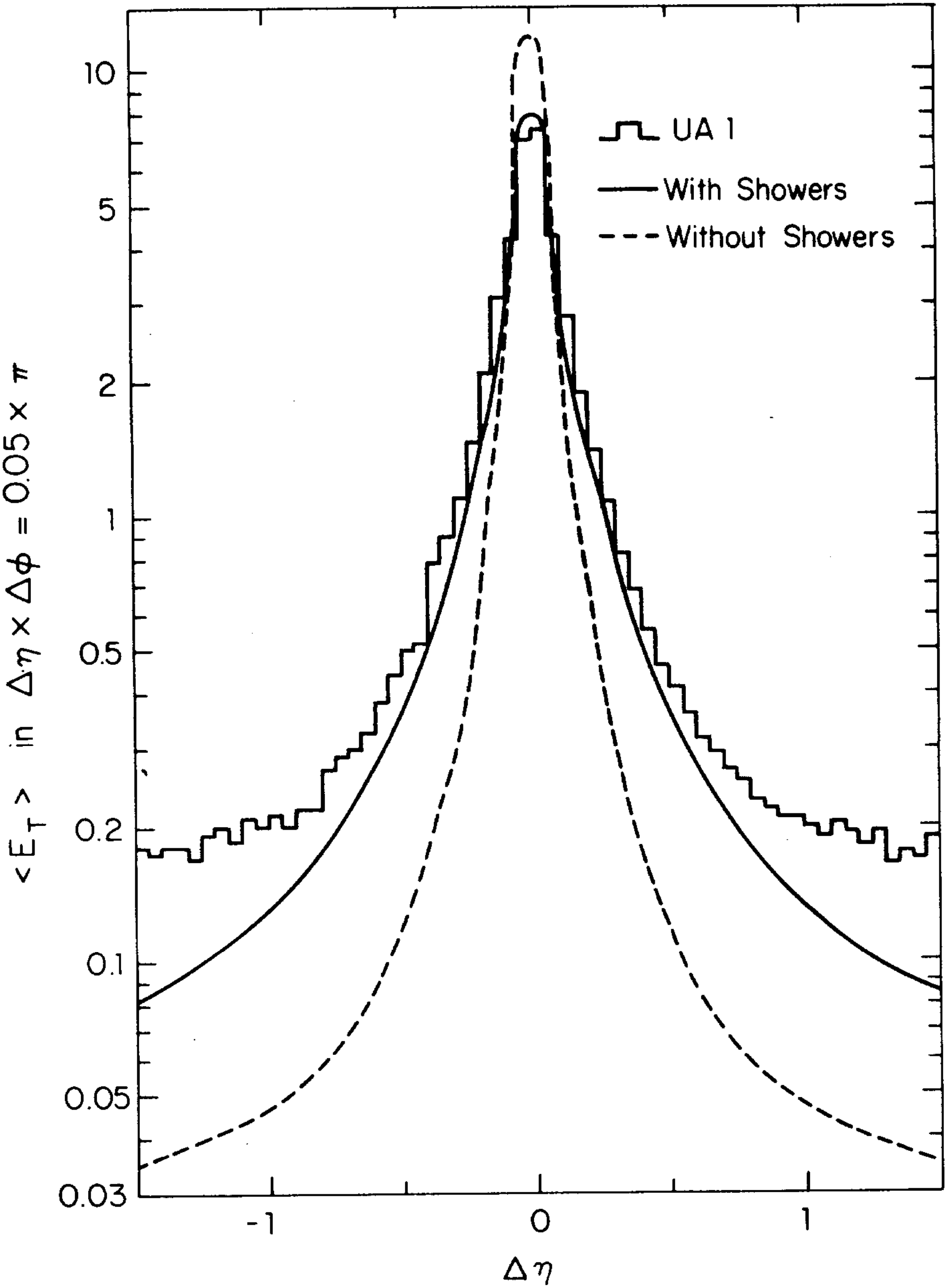


FIG. 1



n_w
20
10
0
0
10
20
30
p_T^W
UA1 (events / 2 GeV)
UA2 (events / 3 GeV)
MONTE CARLO

FIG. 2



UA 1
With Showers
Without Showers
10
5
2
1
0.5
0.2
0.1
0.05
0.03
-1
0
1
Δη
<E_T> in Δη × Δφ = 0.05 × π

FIG. 3