



## Decaying Particles Do Not 'Heat Up' the Universe

Robert J. SCHERRER

Astronomy and Astrophysics Center  
The University of Chicago  
Chicago, Illinois 60637

Michael S. TURNER

Theoretical Astrophysics  
Fermilab National Accelerator Laboratory  
Batavia, Illinois 60510  
and  
Enrico Fermi Institute  
The University of Chicago  
Chicago, Illinois 60637

### Abstract

It is usually assumed that a massive relic species, which comes to dominate the mass density of the Universe and later decays, 'heats up' the Universe when the age of the Universe  $\simeq$  its lifetime. We show that if its decay follows the usual exponential decay law, then the Universe is never reheated, rather it just cools more slowly. We calculate the evolution of the temperature and entropy, and find that to within numerical factors of order unity, the usual estimates for the entropy increase are found to be correct. Our results have implications for primordial nucleosynthesis in scenarios where a massive relic with lifetime  $\simeq 10^{-2} - 10^3$  sec is present, and for baryogenesis in the new inflationary Universe scenario.



## I. Introduction

In the standard, hot big bang cosmology (for a review see ref. 1), the energy density contributed by a massive particle species (denoted by 'X') becomes negligible  $\left[ \frac{\rho_x}{\rho_\gamma} \simeq \left( \frac{m_x}{T} \right)^{\frac{5}{2}} \exp\left( \frac{-m_x}{T} \right) \right]$  when the temperature falls below its mass, unless that species possesses a non-zero chemical potential (e.g. baryons), or it drops out of equilibrium and its abundance 'freezes out.' In the case of  $\mu \neq 0$  or 'freeze out', the relic X abundance relative to photons  $\left( \frac{n_x}{n_\gamma} \right)$  remains approximately constant, and  $\frac{\rho_x}{\rho_\gamma}$  grows as  $T^{-1}$  (or  $R(t) =$  cosmic scale factor). Eventually, the energy density of the X particles dominates that of the photons--and the total energy density, if  $t \lesssim 10^{10}$  sec. [The present energy densities of matter (baryons and other non-relativistic (NR) matter) and radiation (3K background and  $\nu\bar{\nu}$  backgrounds) are such that earlier than about  $10^{10}$  sec after the bang  $\rho_{\text{rad}} > \rho_{\text{matter}}$ ]. If the relic X particles subsequently decay into light (i.e., relativistic) particles which thermalize, then the entropy of the Universe and radiation content will be increased.

The usual approximation made is to assume that the decays happen at time  $t \simeq \tau_x$  (= lifetime of the X), over a short time interval ( $\Delta t \ll \tau_x$ ).<sup>2</sup> In this case it is straightforward to compute the increase in entropy per comoving volume, and the increase in temperature. [In doing this simple calculation we will assume that at the time of decay,  $\rho_x \gg \rho_r$ .] Let the relic abundance of Xs be  $r \equiv \frac{n_x}{s}$  (after freeze out and before decay), where  $s = \frac{2\pi^2 g_* T^3}{45}$  is the entropy density and  $g_*$  counts the effective number of relativistic degrees of freedom. The energy density in radiation is  $\rho_r = \left( \frac{\pi^2 g_*}{30} \right) T^4$  and the energy density in Xs is

$\rho_x = (rm_x)s$ . The entropy per comoving volume is  $S = sR^3$ . In the simultaneous decay approximation (SDA) it follows immediately from energy conservation (at the decay epoch  $t \simeq \Gamma^{-1}$ ) that:

$$T_{\text{before}} \simeq 0.65 g_*^{-\frac{1}{3}} \left( \Gamma^2 \frac{m_{\text{pl}}^2}{rm_x} \right)^{\frac{1}{3}} \quad (1a)$$

$$T_{\text{after}} \simeq 0.78 g_*^{-\frac{1}{4}} (\Gamma m_{\text{pl}})^{\frac{1}{2}} \quad (1b)$$

$$\frac{T_{\text{after}}}{T_{\text{before}}} \simeq 1.2 g_*^{\frac{1}{12}} \frac{(rm_x)^{\frac{1}{3}}}{(\Gamma m_{\text{pl}})^{\frac{1}{6}}} \quad (1c)$$

$$\frac{S_{\text{after}}}{S_{\text{before}}} \simeq 1.7 g_*^{\frac{1}{4}} \frac{rm_x}{(\Gamma m_{\text{pl}})^{\frac{1}{2}}} \quad (1d)$$

where 'before' refers to just before the decays ( $t \simeq \tau_x^-$ ), 'after' refers to just after the decays ( $t \simeq \tau_x^+$ ), and  $\Gamma \equiv \tau_x^{-1}$  is the X decay rate. Here and throughout we use units in which  $\hbar = k_B = c = 1$ , and Newton's constant  $G \equiv m_{\text{pl}}^{-2}$ , where  $m_{\text{pl}} = 1.22 \times 10^{19} \text{GeV}$ .

In brief, what we find is that when one takes into account the fact that the decays are not simultaneous, but instead follow an exponential decay law ( $dN_x = -N_x \Gamma dt$ ), the temperature of the Universe never increases, i.e.,  $\frac{dT}{dt}$  is always  $< 0$ . What happens instead is that the temperature falls more slowly,  $T \propto R^{\frac{-3}{8}}$  rather than  $R^{-1}$  (see Fig. 1), due to the entropy release from decays. Up to numerical factors of order unity, the 'reheat temperature', more precisely the temperature just after the entropy release ( $t \simeq \Gamma^{-1}$ ), and the entropy increase are given by Eqn.(1). In the next section, we carefully set up the problem of

decaying relic particles and solve it; in this section we also briefly discuss some applications of our results. In Section III we summarize our results and discuss some possible implications.

## II. Entropy Release by Decaying Relics in the Standard Cosmology

### A. Equations for decaying particles

With some loss of generality, we make the following physically reasonable assumptions:

(1) At all times the microscopic entropy of the Universe is dominated by relativistic particles.

(2) The entropy released by the decaying X particles is rapidly ( $\Delta t \ll H^{-1} \simeq$  expansion time) thermalized.

(3) We restrict the problem to Friedmann-Robertson-Walker (FRW) cosmological models (i.e. the standard cosmology).

The first assumption implies that  $S$  ( $\equiv$  the entropy per comoving volume) can be expressed as

$$S = R(t)^3 s(T) \tag{2}$$

where  $s(T)$  is the entropy density,

$$s(T) = \frac{(\rho + p)}{T} \tag{3a}$$

$$= \left( \frac{2\pi^2 g_*}{45} \right) T^3 \tag{3b}$$

The quantity  $g_*(T)$  as usual counts the effective number of relativistic degrees of freedom:

$$g_* = \sum_{\text{Bose}} g_B + \left( \frac{7}{8} \right) \sum_{\text{Fermi}} g_F \tag{4}$$

where the sum runs over all species with  $m \ll T$  which are in thermal equilibrium. Note that the energy density in relativistic particles ( $\equiv \rho_r$ ) and temperature ( $= T$ ) are related to  $s$  and  $S$  by,

$$\rho_r = \left( \frac{3}{4} \right) T s(T) \quad (5a)$$

$$= 0.75 \frac{a^{-\frac{1}{3}} S^{\frac{4}{3}}}{R^4} \quad (5b)$$

$$T = a^{\frac{-1}{3}} s^{\frac{1}{3}} \quad (5c)$$

$$= \frac{a^{-\frac{1}{3}} S^{\frac{1}{3}}}{R} \quad (5d)$$

where  $a \equiv \frac{2\pi^2 g_*}{45}$ . [ It should be recognized that Eqns.(3b, 4) are rigorously valid only at temperatures for which there are no particle species with mass  $\approx$  temperature. At temperatures  $T$  for which there is one (or more) particle species with  $m \approx T$  we will use the entropy density,  $s \equiv \frac{(\rho + p)}{T}$ , to define  $g_*(T)$ , via Eqn.(3b). In practice this technical point creates no difficulties.]

Choose an initial epoch  $t_0$  such that: (1)  $t_0 \ll \Gamma^{-1}$  (i.e., X particles have not yet begun to decay); (2) the X particles are NR, i.e.,  $T_0 \ll m_x$ , and their abundance is 'frozen out' (no X particles are being created or destroyed, i.e., constant number per comoving volume). Now specify their initial abundance by the ratio of their number density to the entropy density ( $\equiv r$ ),

$$r \equiv \left( \frac{n_x}{s} \right) \Big|_{T_0} \quad (6)$$

Using the fact that the number density of photons  $n_\gamma = \left( \frac{2 \zeta(3)}{\pi^2} \right) T^3$  and

Eqn.(3b) for  $s(T)$ , it follows that

$$s = \left( \frac{\pi^4 g_*}{45(3)} \right) n_\gamma \quad (7a)$$

$$\simeq 1.80 g_* n_\gamma \quad (7b)$$

$$\left( \frac{n_x}{n_\gamma} \right) |_{T_0} \simeq 1.80 g_* r \quad (8)$$

The energy density in X particles at this epoch is just

$$\rho_{x0} \equiv r m_x s(T_0) \quad (9)$$

The evolution of  $\rho_x$  is governed by

$$\dot{\rho}_x = -3H\rho_x - \Gamma\rho_x \quad (10)$$

where  $H \equiv \frac{\dot{R}}{R}$ ,  $R =$  cosmic scale factor, and overdot signifies time derivative.

The first term on the rhs of Eqn.(10) represents the dilution of  $\rho_x$  due to the expansion of the Universe, while the second represents the decays. Eqn.(10) can be written in a more suggestive form,

$$\frac{d ( R^3 \rho_x )}{dt} = -\Gamma ( R^3 \rho_x )$$

where the basic physics is clear: the number of Xs per comoving volume

$\left( N_x = \frac{R^3 \rho_x}{m_x} \right)$  follows an exponential decay law. The solution to Eqn.(10) is

just

$$\rho_x = \rho_{x0} \left( \frac{R}{R_0} \right)^{-3} e^{-\Gamma t} \quad (11)$$

The change in entropy per comoving volume is given by

$$dS = \frac{dQ}{T}$$

where  $dQ$  (= the heat added per comoving volume) is due to the decays,

$$dQ = \Gamma R^3 \rho_x dt$$

Thus the evolution of the entropy per comoving volume is given by

$$\begin{aligned} \dot{S} &= \frac{\Gamma R^3 \rho_x}{T} \\ &= a^{\frac{1}{3}} \Gamma R^4 \rho_x S^{-\frac{1}{3}} \end{aligned} \quad (12)$$

Eqn.(12) can be integrated (at least formally):

$$S^{\frac{4}{3}} = S_0^{\frac{4}{3}} + \left( \frac{4}{3} \right) \rho_{x0} R_0^4 \int_{t_0}^t a^{\frac{1}{3}} \left( \frac{R(t')}{R_0} \right) e^{-\Gamma t'} d\Gamma t' \quad (13)$$

where the subscript 'zero' denotes the value of that quantity at the initial epoch ( $t = t_0$ ).

The evolution of the cosmic scale factor  $R(t)$  is governed by the usual Friedmann equation,

$$H^2 \equiv \left( \frac{\dot{R}}{R} \right)^2 = \left( \frac{8\pi G}{3} \right) (\rho_x + \rho_r + \rho_o) \quad (14)$$

where  $\rho_o$  is the effective energy density of everything besides relativistic particles in thermal equilibrium and Xs. For example, the other forms of energy density might be vacuum energy ( $\Rightarrow \rho_o = \text{constant}$ ), a stable, NR particle species ( $\Rightarrow \rho_o \propto R^{-3}$ ), a relativistic particle species which is decoupled, e.g., neutrinos when  $T \lesssim 2\text{MeV}$  ( $\Rightarrow \rho_o \propto R^{-4}$ ), or the 'effective energy density' of the curvature term  $\left( = \frac{k}{R^2} \right)$  which we left out of Eqn.(14) ( $\Rightarrow \rho_o \propto \pm R^{-2}$ ).

The relevant equations then are

$$\rho_x = \rho_{x0} \left( \frac{R}{R_0} \right)^{-3} e^{-\Gamma t} \quad (15a)$$

$$S^{\frac{4}{3}} = S_0^{\frac{4}{3}} \left[ 1 + \left( \frac{4}{3} \right) \left( \frac{r m_x}{a_0^{\frac{1}{3}} T_0} \right) \int_{t_0}^t a^{\frac{1}{3}} \left( \frac{R(t')}{R_0} \right) e^{-\Gamma t'} d\Gamma t' \right] \quad (15b)$$

$$H^2 = \left( \frac{8\pi G}{3} \right) (\rho_x + \rho_r + \rho_0) \quad (15c)$$

supplemented by  $\rho_r = 0.75 a^{-\frac{1}{3}} S^{\frac{4}{3}} R^{-4}$ . These equations can be cast in a more useful form by introducing the following dimensionless variables:

$$x = \Gamma t$$

$$x_H = \Gamma \left( \frac{3}{8\pi G \rho_{x0}} \right)^{\frac{1}{2}}$$

$$z \equiv \frac{R}{R_0}$$

$$f_x \equiv \frac{\rho_x}{\rho_{x0}}$$

$$f_r \equiv \frac{\rho_r}{\rho_{x0}}$$

$$f_0 \equiv \frac{\rho_0}{\rho_{x0}}$$

Eqns.(15a-15c) can then be written as

$$f_x = z^{-3} e^{-x} \quad (16a)$$

$$S^{\frac{4}{3}} = S_0^{\frac{4}{3}} \left[ 1 + \left( \frac{4}{3} \right) \left( \frac{r m_x}{a_0^{\frac{1}{3}} T_0} \right) \int_0^x a^{\frac{1}{3}} z(x') n e^{-x'} dx' \right] \quad (16b)$$



$$\left(\frac{z'}{z}\right) = x_H^{-1} (f_x + f_r + f_o)^{\frac{1}{2}} \quad (16c)$$

where prime denotes  $\frac{d}{dx}$ . Since by choice  $x_0 = \Gamma t_0 \ll 1$ , we have set the lower limit in the integral in Eqn.(16b) equal to zero.

Eqn.(16b) has an obvious interpretation in terms of the evolution of the energy density in radiation. Recall that  $\rho_r = 0.75a^{-\frac{1}{3}}S^{\frac{4}{3}}R^{-4}$  and suppose that  $a(T) = \text{constant}$ ; then multiplying (16b) by  $0.75a^{-\frac{1}{3}}R^{-4}$  leads to an equation for  $\rho_r$ :

$$\rho_r = \rho_{r0}z^{-4} + \left[ \rho_{x0} \int_0^x z(x') e^{-x'} dx' \right] z^{-4} \quad (17)$$

The first term on the rhs of Eqn.(17) is just the 'original' energy density in radiation, redshifted  $\propto R^{-4}$ ; the second term is the energy density in radiation due to the X decays.

Now let's discuss the qualitative behavior of the solutions to Eqns.(16b-17). First consider the early epoch  $x \lesssim 1$ ; suppose that  $z(x) = \left(\frac{x}{x_0}\right)^n$  (note:  $n = \frac{1}{2}$  if the Universe is radiation-dominated;  $n = \frac{2}{3}$  if it is matter-dominated). Using the approximations  $a^{\frac{1}{3}} \simeq \text{constant}$  and  $e^{-x} \approx 1$ , together with  $z = \left(\frac{x}{x_0}\right)^n$ , Eqns.(16b-17) can be easily integrated,

$$S^{\frac{4}{3}} \simeq S_0^{\frac{4}{3}} \left[ 1 + \left(\frac{4}{3}\right) \left(\frac{rm_x}{a_0^{\frac{1}{3}}T_0}\right) a^{\frac{1}{3}x} \frac{z(x)}{(1+n)} \right] \quad (18a)$$

$$\rho_r \simeq \rho_{r0}z^{-4} + \rho_{x0}z^{-3} \left(\frac{x}{(1+n)}\right) \quad (18b)$$

Referring to the first and second terms on the rhs of Eqn.(18a,b) as the 'old entropy' ('old radiation') and the 'new entropy' ('new radiation') respectively, we can see that the 'new entropy' term increases  $\propto (x \text{ or } t)^{\frac{3(1+n)}{4}} \propto R^{\frac{3(1+n)}{4n}}$  while the 'new radiation' term increases  $\propto R^{-3+\frac{1}{n}} \propto R^{-1}$  for  $n = \frac{1}{2}$ ;  $\propto R^{-\frac{3}{2}}$  for  $n = \frac{2}{3}$ ). For  $x \lesssim 1$ ,  $\rho_{x_0} z^{-3}$  is just the energy density in X particles; according to Eqn.(18), the energy density in radiation 'dumped in' by decays is just the fraction  $\frac{\Gamma t}{(1+n)}$  of the energy density in X particles.

An important epoch occurs when the 'old' and 'new' parts of the rhs of Eqn.(18a,b) become comparable; denote this epoch by  $x_{=}$  and  $z_{=} = z(x_{=})$ . Physically this corresponds to the time after which most of the entropy and radiation content are due to the X decays. In terms of  $x_0$ ,  $\rho_{r_0}$  and  $\rho_{x_0}$ ,  $x_{=}$  and  $z_{=}$  are given by:

$$\frac{x_{=}}{x_0} = \left[ \frac{(1+n)\rho_{r_0}}{x_0\rho_{x_0}} \right]^{\frac{1}{(1+n)}} \quad (19a)$$

$$z_{=} = \left[ \frac{(1+n)\rho_{r_0}}{x_0\rho_{x_0}} \right]^{\frac{n}{(1+n)}} \quad (19b)$$

where we have assumed that  $x_0 \lesssim x_{=} \lesssim 1$ . If  $\frac{\rho_{r_0}}{\rho_{x_0}}$  is sufficiently large, then Eqn.(19a) gives a value for  $x_{=} > 1$ , meaning that  $\rho_x$  never dominates  $\rho_r$  and X decays never produce a significant amount of entropy (i.e., relative to the initial entropy).

For  $x \gg 1$  the integral in Eqn.(16b) converges because of the  $e^{-x'}$  in the integrand. Physically this corresponds to the entropy per comoving volume reaching its asymptotic value after all the decays have occurred. The ratio of the

final entropy per comoving volume to the initial value is

$$\frac{S_f}{S_0} = \left[ 1 + \left( \frac{4}{3} \right) \left( \frac{rm_x}{a_0^{\frac{1}{3}} T_0} \right) I \right]^{\frac{3}{4}} \quad (20)$$

$$I = \int_0^{\infty} a^{\frac{1}{3}} z(x) e^{-x} dx \quad (21)$$

If  $z = \left( \frac{x}{x_0} \right)^n$  for  $x$  near 1, say  $10^{-3} < x < 10$ , when almost all of the decays occur, then  $I$  is very easy to evaluate,

$$I = \bar{a}^{\frac{1}{3}} x_0^{-n} n! \quad (22a)$$

where  $\bar{a}^{\frac{1}{3}}$  is a weighted average of  $a^{\frac{1}{3}}$  (if  $a$  is not constant) in the interval near  $x \approx 1$ . [ One would expect  $z = \left( \frac{x}{x_0} \right)^n$  for  $x \lesssim 10$  if the energy density of the original radiation, or the unspecified components represented by  $\rho_0$  dominated the total energy density during the entire decay epoch. ]

If, on the other hand, the energy density of the X particles and their decay products dominate the total energy density during the decay epoch, then  $z$  will not be represented by a single power law; rather  $n$  will change from  $\frac{2}{3}$  to  $\frac{1}{2}$  as the X particles decay and the entropy they release begins to dominate the energy density of the Universe. This is the case of greatest interest, since it corresponds to the situation where the entropy production is significant. Without loss of generality we can choose  $t_0$  such, that in addition to  $x_0$  being  $\ll 1$ ,  $\rho_x$  dominates the energy density. It is then straightforward to show that in this limit ( $\rho_{x_0} \gg \rho_{r_0} + \rho_{o_0}$ )

$$I = 1.09 x_H^{\frac{-2}{3}} \bar{a}^{\frac{1}{3}} \quad (22b)$$

where the 1.09 is the result of numerically integrating Eqns.(16a-c). [The details of this are given in the appendix.]

Combining Eqn.(20) with either Eqn.(22a) or (22b) as appropriate, we have for  $\frac{S_f}{S_o}$

$$\frac{S_f}{S_o} \simeq \left[ 1 + \left(\frac{4}{3}\right)^{\frac{3}{4}} \left(\frac{8\pi}{3}\right)^{\frac{n}{2}} \left(\frac{n! \bar{a}^{\frac{1}{3}}}{n^n a_o^{\frac{1}{12}}}\right) \left(\frac{\rho_{oo}^{\frac{n}{2}}}{\rho_{ro}^{\frac{1}{4}}}\right) \frac{rm_x}{(m_{pl}\Gamma)^n} \right]^{\frac{3}{4}} \quad (23a)$$

$(\rho_o \gg \rho_r, \rho_x)$

$$\frac{S_f}{S_o} = \left[ 1 + 2.65 \left(\frac{\frac{1}{\bar{a}^{\frac{1}{3}}}}{a_o^{\frac{1}{12}}}\right) \frac{rm_x}{(m_{pl}\Gamma)^{\frac{1}{2}}} \right]^{\frac{3}{4}} \quad (23b)$$

$$\simeq 1 + 1.98 \left(\frac{\frac{1}{\bar{a}^{\frac{1}{3}}}}{a_o^{\frac{1}{12}}}\right) \frac{rm_x}{(m_{pl}\Gamma)^{\frac{1}{2}}}$$

$(\rho_r \gg \rho_o, \rho_x)$

$$\frac{S_f}{S_o} = \left[ 1 + 2.95 \bar{a}^{\frac{1}{3}} \frac{(rm_x)^{\frac{4}{3}}}{(m_{pl}\Gamma)^{\frac{2}{3}}} \right]^{\frac{3}{4}} \quad (23c)$$

$$\simeq 2.25 \bar{a}^{\frac{1}{4}} \frac{rm_x}{(\Gamma m_{pl})^{\frac{1}{2}}}$$

$(\rho_x \gg \rho_r, \rho_o)$

for reference  $\frac{rm_x}{(m_p \Gamma)^2} = 0.353 \left( \frac{rm_x}{\text{MeV}} \right) \left( \frac{\tau}{\text{sec}} \right)^{\frac{1}{2}}$ . Note in the first case, Eqn.(23a),

the result seems to depend upon the choice of initial epoch through the  $\frac{\rho_{oo}^{\frac{n}{2}}}{\rho_{ro}^{\frac{1}{4}}}$

factor. However,  $z \propto x^{-n}$  implies that  $\rho_o \propto z^{\frac{-2}{n}}$ ; for  $x \ll 1$ ,  $\rho_r \propto z^{-4}$ . These two

facts imply that  $\frac{\rho_o^{\frac{n}{2}}}{\rho_r^{\frac{1}{4}}}$  is independent of  $x$  for  $x < 1$ . Eqn.(23b,c) represent the

cases of greatest interest: Eqn.(23b) the case where the initial radiation always dominates the energy (small entropy production; note, Eqn.(23b) follows from Eqn.(23a) by setting  $n = \frac{1}{2}$ ), and Eqn.(23c) the case where the X particles dominate the energy density during the decay epoch (large entropy production). Note that Eqn.(23c) is, up to numerical factors of order unity, identical to the usual estimate, Eqn.(1d).

Let us end this section by summarizing our results. Consider the two cases where the entropy production is significant, i.e.,  $\frac{S_r}{S_o} \gg 1$ : (a) the case where during the decay epoch the energy density is dominated by the X and its decay products; (b) the case where during the decay epoch the energy density is dominated by something other than  $\rho_r$  or  $\rho_x$ , i.e.,  $\rho_o$  dominates the energy density, and  $z = \left( \frac{x}{x_o} \right)^n$ .

Case(a): Assume that by  $t = t_o$ ,  $\rho_x \gg \rho_r$ ; then  $z = \left( \frac{x}{x_o} \right)^{\frac{2}{3}}$ . From  $x = x_o$  to  $x = x_*$  the entropy per comoving volume and radiation energy density are

dominated by the 'old' or original components. During this period the 'new radiation' energy density increases  $\propto R^{-\frac{3}{2}}$ ;  $\rho_r R^4$  and  $S$  remain nearly constant. From  $x = x_{\underline{}}$  to  $x \simeq 1$ , the 'new' contributions to  $S$  and  $\rho_r$  are dominant, and  $\rho_r \propto R^{-\frac{3}{2}} \propto t^{-1}$ ,  $T \propto \rho_r^{\frac{1}{4}} \propto R^{-\frac{3}{8}}$ , and  $S \propto R^{\frac{15}{8}}$ . For  $x > 1$ ,  $S$  levels off at the value  $S_f$  [given by Eqn.(23c)], and  $\rho_r$  begins to decrease  $\propto R^{-4}$  ( $T \propto R^{-1}$ ). Note that the temperature is always decreasing, albeit, at a rate slower than the usual  $R^{-1}$ , for  $x_{\underline{}} < x \lesssim 1$ . Using Eqn.(23c), and taking  $z(1) \simeq x_o^{-\frac{2}{3}}$ , it follows that the temperature when the entropy levels off ( $x \simeq 1$ ) is

$$T_f \simeq \left(\frac{1}{2}\right) \bar{a}^{-\frac{1}{4}} (\Gamma m_p)^{\frac{1}{2}} \quad (24)$$

which only differs numerically from the usual estimate, Eqn.(1b), by factors of order unity, but which has a very different interpretation. The Universe is not heated up to this temperature at  $t \simeq \Gamma^{-1}$ , but rather has cooled down to this temperature. The entropy increase is

$$\begin{aligned} \frac{S_f}{S_o} &\simeq 2.25 \bar{a}^{\frac{1}{4}} \frac{r m_x}{(m_p \Gamma)^{\frac{1}{2}}} \\ &\simeq 1.83 \bar{g}_*^{\frac{1}{4}} \frac{r m_x}{(m_p \Gamma)^{\frac{1}{2}}} \end{aligned}$$

which differs from the 'usual estimate', i.e., Eqn.(1d), by only about 10%.

Case(b): Assume that due to the unspecified source of energy density  $\rho_o$ ,  $z = \left(\frac{x}{x_o}\right)^n$  during the decay period. From  $x = x_o$  to  $x = x_{\underline{}}$  the entropy per comoving volume and radiation energy density are dominated by the so-called 'old' or original components, and  $S \propto \text{constant}$ ,  $\rho_r \propto R^{-4}$ . The 'new' contribution

to  $\rho_r$  is increasing as  $R^{-3+\frac{1}{n}}$ . At  $x = x_*$  the 'new' contributions to S and  $\rho_r$  begin to dominate the 'old' or original contributions and  $\rho_r \propto R^{-3+\frac{1}{n}}$ ,  $T \propto R^{\frac{-3}{4}+\frac{1}{4n}}$ , and  $S \propto R^{\frac{3(1+n)}{4n}}$ . Note that unless  $n \leq \frac{1}{3}$  (corresponding to  $\rho_o$  decreasing as  $R^{-6}$  or faster), the temperature is always decreasing. At  $x \simeq 1$ , S levels off to the value given in Eqn.(23a), and subsequently  $S = \text{constant}$  and  $\rho_r \propto R^{-4}$ . The temperature when the entropy levels off ( $x \simeq 1$ ) is

$$T_f \simeq \left(\frac{4}{3}\right)^{\frac{7}{16}} \left(\frac{3}{8\pi}\right)^{\frac{3n}{8}} \left(\frac{n^{3n} n! a_o^{\frac{1}{4}}}{\bar{a}}\right)^{\frac{1}{4}} \left(\frac{\rho_{ro}^{\frac{1}{2}}}{\rho_{oo}^n}\right)^{\frac{3}{8}} (r m_x)^{\frac{1}{4}} (\Gamma m_{pl})^{\frac{3n}{4}} \quad (25)$$

The evolution of  $\rho_r$  and S for these two cases is shown in Figs. 1 and 2.

### B. Applications

In the new inflationary Universe scenario there can be an epoch where the energy density of the Universe is dominated by coherent scalar field oscillations<sup>3</sup>. The energy density in these scalar field oscillations behaves just like NR matter (a cold condensate of NR Higgs particles). Eventually these oscillations decay into radiation (i.e., relativistic particles). The decay of the coherent field oscillations is equivalent to the decay of NR Higgs particles and is described by an equation which is identical to Eqn.(10). The reheating of the inflationary Universe in this case is just the situation we have just considered in case(a), namely  $\rho_{x0} \gg \rho_{r0} + \rho_{o0}$ . Thus the evolution of the radiation energy density is as shown in Fig.1, and the radiation temperature at the time when the entropy levels off (usually referred to as the 'reheat temperature') is just given by Eqn.(24),

$$T_{RH} \simeq \left(\frac{1}{2}\right) \bar{a}^{-\frac{1}{4}} (\Gamma m_{pl})^{\frac{1}{2}}$$

In the inflationary cosmology the baryon asymmetry of the Universe  $\left(\frac{n_B}{s} \simeq 10^{-10}\right)$  must be produced after inflation (any initial baryon asymmetry is exponentially diluted by the tremendous entropy production associated with inflation). If the baryon asymmetry is produced in the standard way, the out-of-equilibrium decay of superheavy bosons produced in the reheating process<sup>4</sup>, then the time-temperature relationship for the Universe and the evolution of the entropy per comoving volume are both crucial for calculating the baryon asymmetry which evolves. In that regard, the results of this paper are of some importance -- as the baryon asymmetry is being produced, the entropy per comoving volume is increasing and the time-temperature relationship is not the usual one. We are currently investigating these effects.

An alternate to the standard method of baryogenesis is direct production of a baryon asymmetry by the decays of the Higgs particles themselves<sup>5</sup>. Suppose that the decay of each Higgs particle on average produces a net baryon number of  $\epsilon$ . The quantity  $\epsilon$  is related to the C, CP violation in the decay of the Higgs particle. [ Note that the baryon number per decay  $\epsilon$  need not be directly produced in the Higgs particle decay, but could just as well be the result of a chain of decays, e.g. Higgs particle  $\rightarrow$  other particles  $\rightarrow$  quarks and leptons, with net baryon number  $\epsilon$ . ] The baryon number-to-entropy ratio ( $\equiv \frac{n_B}{s}$ ) produced this way depends upon  $\epsilon$ , the number density of Higgs particles ( $\equiv n_x$ ), and the entropy produced. The baryon number produced per comoving volume is just  $\epsilon N_{x_0}$ , where  $N_{x_0}$  is the initial (i.e., for  $x \ll 1$ ) number of Higgs particles per comoving volume. In terms of  $r$  and  $S_0$  (the initial entropy per comoving volume)

$$\epsilon N_{x_0} = \epsilon r S_0$$

The baryon asymmetry produced this way is just



$$\begin{aligned}
 \frac{n_B}{s} &= \frac{\epsilon N_{x_0}}{S_f} & (26) \\
 &= \epsilon r \left( \frac{S_0}{S_f} \right) \\
 &= 0.44 \bar{\alpha}^{-4} \epsilon \frac{(m_{pl}\Gamma)^{\frac{1}{2}}}{m_x}
 \end{aligned}$$

where we have used Eqn.(23c) for  $\frac{S_0}{S_f}$  (in the limit of  $S_f \gg S_0$ , which is clearly the regime of interest for the inflationary Universe.) The usual estimate for  $\frac{n_B}{s}$  is<sup>5</sup>

$$\begin{aligned}
 \frac{n_B}{s} &\simeq \frac{0.75 \epsilon T_{RH}}{m_x} \\
 &\simeq 0.48 \bar{\alpha}^{-4} \epsilon \frac{(m_{pl}\Gamma)^{\frac{1}{2}}}{m_x}
 \end{aligned}$$

which as it turns out is a remarkably accurate estimate. Of course, the baryon asymmetry directly produced by the decay of the Higgs field oscillations may subsequently be damped by baryon number nonconserving processes (e.g.  $2 \leftrightarrow 2$  scattering processes.)

[Throughout this discussion of the reheating of the inflationary cosmology we have assumed the existence of an initial entropy per comoving volume ( $= S_0$ ). In the inflationary scenario this quantity is expected to be very small and highly model dependent. As is apparent, though, none of our results depend upon  $S_0$  -- it is merely used as a fiducial. We could just as well have used  $N_{x_0}$  as our fiducial.]

Next consider the so-called decaying particle cosmology.<sup>6</sup> This is a scenario

in which until rather recently ( $\frac{R}{R_{\text{today}}} \gtrsim 10^{-2}$  or so) the energy density of the Universe was dominated by unstable, NR relic X particles, which subsequently decay into particles which today are still relativistic and which make a considerable contribution to the present energy density of the Universe. [ The purpose of this scenario is to solve 'the  $\Omega$ -problem'; i.e. to reconcile the inflationary prediction of  $\Omega = 1$  with the observational data  $\Omega_{\text{obs}} \simeq 0.1 - 0.3$ , by producing a smooth component of energy density with  $\Omega \simeq 0.9 - 0.7$ , which by virtue of its uniformity would not have been detected. ]

In this scenario the decay products do not thermalize because they are effectively interactionless. The equations we have derived in this section are still applicable though, when we realize that  $\left(\frac{3}{4}\right)a^{-\frac{1}{3}}S^{\frac{4}{3}}$  is the energy density in relativistic decay products times  $R^4$  ( $\equiv R^4\rho_r$ ,  $\rho_r$  = energy density of the R decay products), and take  $S_0$ , the fiducial, to be the entropy per comoving volume in photons. With this identification  $r = \frac{n_x}{s_\gamma}$ ; using the fact that  $s_\gamma = 3.60n_\gamma$ , it follows that  $r = 0.278\eta_x$  ( $\eta_x \equiv \frac{n_x}{n_\gamma}$ ). In this scenario the energy density of the X and its decay products dominate the total energy density during the decay epoch so that for I we can use the expression in Eqn.(22b). Using our previous results we then find that after the decays ( $x \gtrsim 10$ ),

$$\frac{\rho_r}{\rho_\gamma} \simeq 2.82(rm_x)^{\frac{4}{3}}(\Gamma m_{pl})^{-\frac{2}{3}} \quad (27a)$$

Since both  $\rho_r$  and  $\rho_\gamma \propto R^{-4}$  (after the decay epoch),  $\frac{\rho_r}{\rho_\gamma}$  remains constant; using the fact that the present fraction of critical density contributed by photons ( $\equiv \Omega_\gamma$ ) is

$$\Omega_{\gamma} h^2 = 2.36 \times 10^{-5} \theta^4$$

where  $\theta = \frac{T_{\gamma}}{2.7K}$  and  $h = \frac{H_0}{100 \text{kms}^{-1} \text{Mpc}^{-1}}$  ( the present value of the Hubble parameter )

it follows that

$$\begin{aligned} \frac{\Omega_R h^2}{\theta^4} &= 1.21 \times 10^{-5} (\eta_x m_x)^{\frac{4}{3}} (\Gamma m_{pl})^{\frac{-2}{3}} \\ &= 1.40 \left( \frac{\eta_x m_x}{100 \text{eV}} \right)^{\frac{4}{3}} \left( \frac{\tau}{10^9 \text{yrs}} \right)^{\frac{2}{3}} \end{aligned}$$

For a more detailed discussion of the decaying particle cosmology see ref.6.

### III. Concluding Remarks

The conventional lore of the early Universe has it that a NR, relic species which comes to dominate the energy density of the Universe, and then subsequently decays, 'heats up' the Universe when it decays. We have shown that if the decays of the species follow the usual exponential decay law this is not the case. Instead, due to the heating effect of the decays, Universe cools more slowly ( $T \propto R^{\frac{-3}{8}}$ , instead of the usual  $T \propto R^{-1}$ ) and the entropy per comoving volume increases ( $\propto R^{\frac{15}{8}}$ ). Up to numerical factors of order unity the usual estimate for the increase in entropy per comoving volume is found to be correct. What in the usual analysis is called the 'reheat temperature' is instead the temperature of the Universe when the entropy per comoving volume levels off (at the time  $t \simeq \Gamma^{-1}$ ).

If during the decay epoch the energy density of the Universe is dominated by some other form of energy density and  $R$  increases more slowly than  $t^{\frac{1}{3}}$  (corresponding to a source of energy density which decreases more rapidly than  $R^{-6}$ ), then the temperature of the Universe does increase during the decay epoch

( $T \sim R^{\frac{1-3n}{4n}}$  for  $t < \Gamma^{-1}$ , where  $R \propto t^n$ ).

Our results may be of some importance when considering the effect of relic particle species which decay and release entropy around the epoch of primordial nucleosynthesis (temperatures 10 MeV – 0.1 MeV) -- the gravitino being such a particle<sup>2</sup>, and when considering baryogenesis in the new inflationary Universe scenario. In the former example, it is usually assumed that the Universe goes through primordial nucleosynthesis twice (before and after the decay epoch). As we have shown, it goes through the relevant temperature range for nucleosynthesis only once, but with a different time-temperature relationship (with the NR particles present,  $H(T) > H_0(T)$  = the expansion rate without the relic particles present). In addition, before the high energy particles produced by the decays thermalize, they may produce or destroy various nuclei which are being synthesized.<sup>7</sup> In the case of baryogenesis, the Universe will evolve through the epoch of baryogenesis with a nonstandard time-temperature relationship, and the details of baryogenesis depend crucially upon this relationship. Both topics are currently under investigation.

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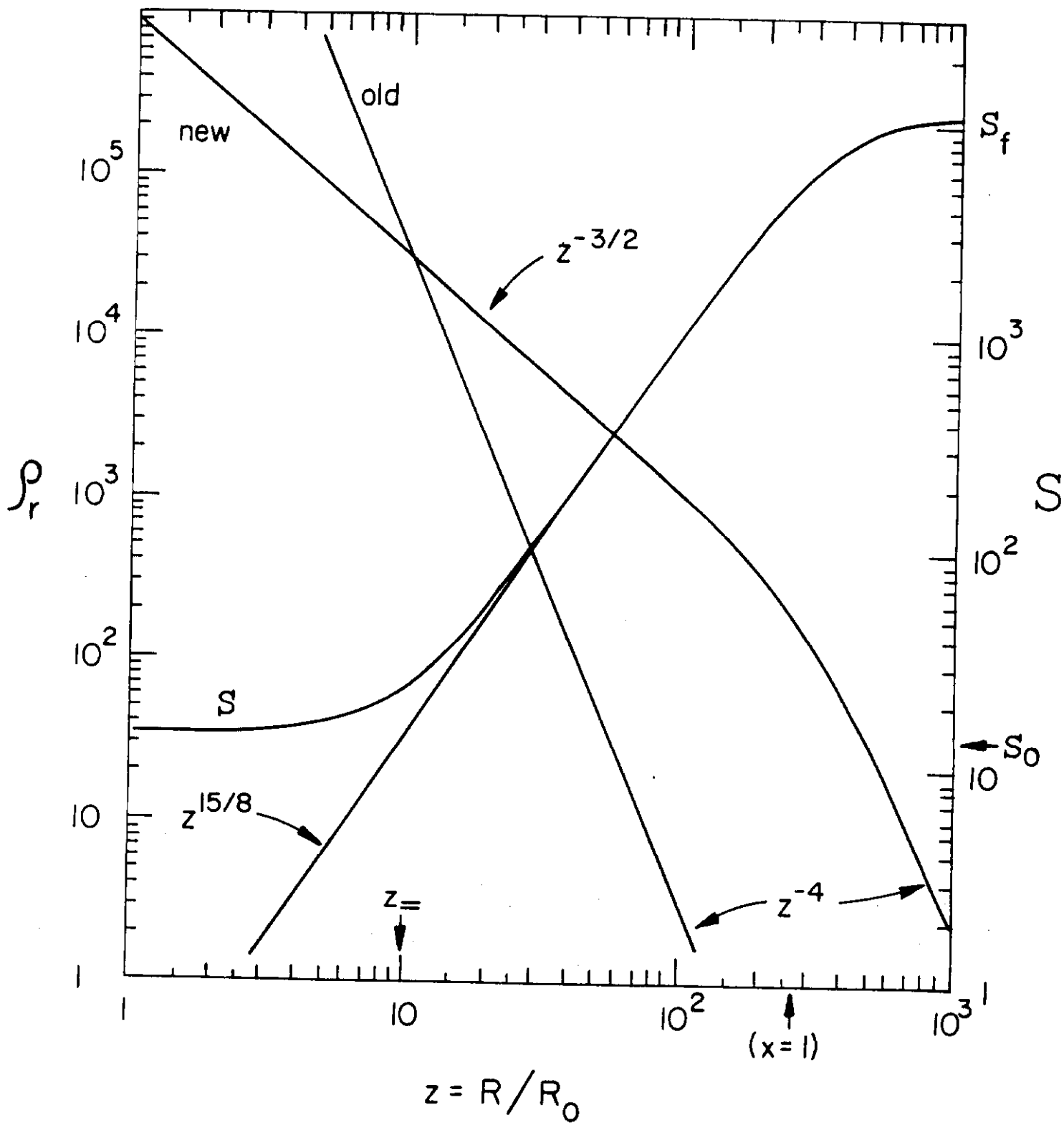
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Figure Captions

Figure 1: The evolution of  $\rho_r$  and  $S$  as a function of the cosmic scale factor  $z \equiv \frac{R}{R_0}$  in the case where the energy density of the X particles dominates the energy density of the Universe during the decay epoch. The line labeled 'old' shows the energy density in the original radiation component; the line labeled 'new' shows the contribution to the radiation energy density from X decays and is proportional to  $R^{-\frac{3}{2}}$ . From  $z = z_{\underline{}}$  (when the 'old' and 'new' contributions are equal) until  $x \simeq 1$ ,  $\rho_r \propto R^{-\frac{3}{2}}$ , implying that  $T \propto R^{-\frac{3}{8}}$  -- decays do not heat the Universe, they just cause it to cool more slowly. From  $z = z_{\underline{}}$  to  $x \simeq 1$  the entropy per comoving volume  $S$  increases proportional to  $R^{\frac{15}{8}}$ ; at  $x \simeq 1$ ,  $S$  levels off. For  $x \gg 1$ ,  $S \simeq \text{constant}$  and  $\rho_r \propto R^{-4}$ . Although the axes are labeled with arbitrary units, data for the curves was generated by numerically integrating Eqns.(16a-c).

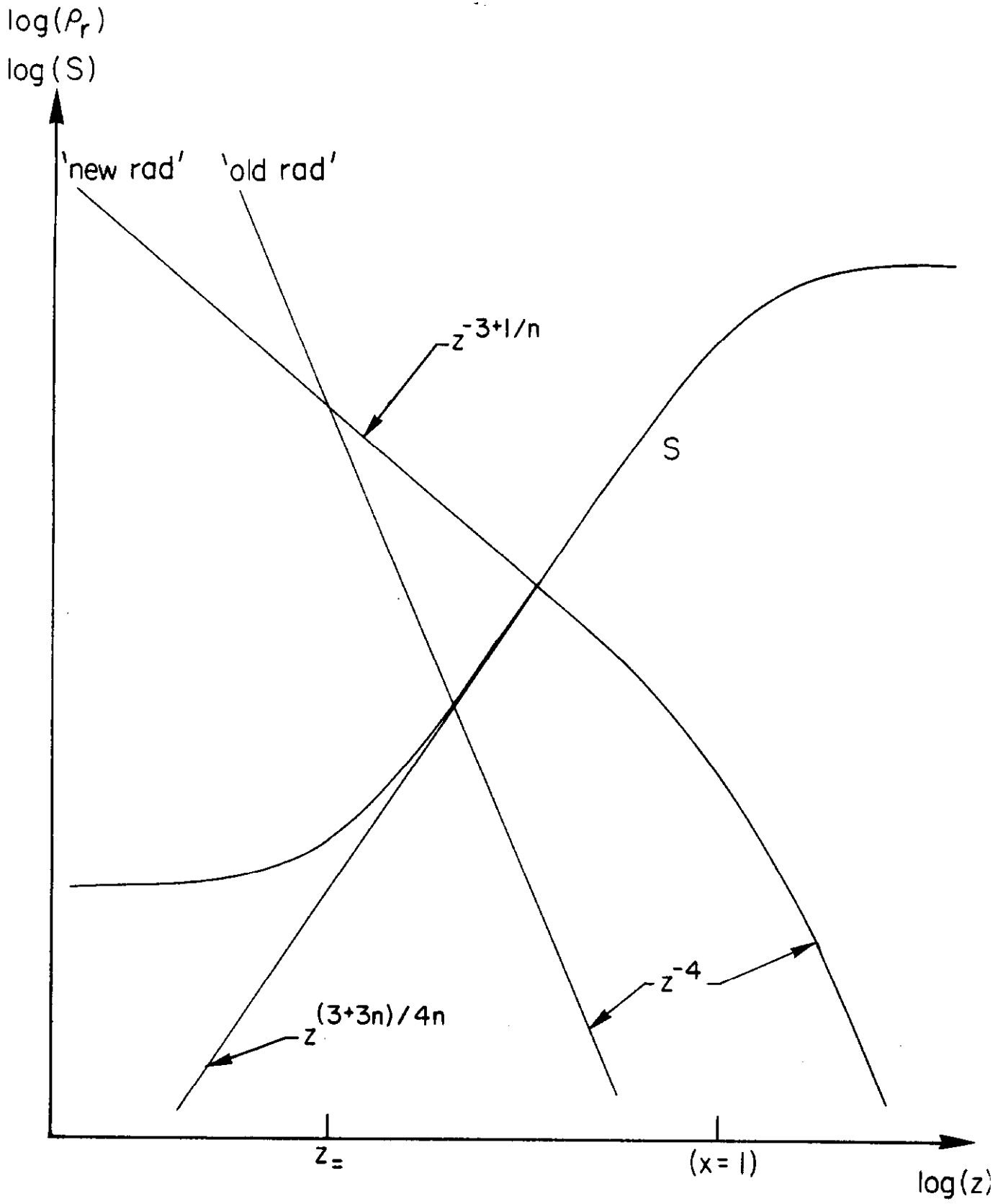
Figure 2: Schematic representation of the evolution of  $\rho_r$  and  $S$  as a function of  $z \equiv \frac{R}{R_0}$  in the case where the energy density of the Universe is dominated by an unspecified component ( $\rho_0 \propto R^{-\frac{2}{n}}$ ), such that  $R \propto t^n$  during the decay epoch. The lines labeled 'old' and 'new' show the original radiation component and the contribution from X decays respectively. The 'new' component evolves  $\propto R^{-3 + \frac{1}{n}}$ , and from  $z = z_{\underline{}}$  onward dominates  $\rho_r$ . From  $z = z_{\underline{}}$  until  $x \simeq 1$ ,  $T \propto R^{\frac{(1-3n)}{4n}}$ , i.e., only increases if  $n \leq \frac{1}{3}$ .

From  $z = z_{\text{=}}$  until the end of the decay epoch ( $x \simeq 1$ )  $S \propto P^{\frac{3(1+n)}{4n}}$ . After the decays ( $x \gg 1$ ),  $S \simeq \text{constant}$  and  $\rho_r \propto R^{-4}$ .



- FIG. 1 -





- FIG. 2 -

Appendix: Evaluation of  $I = \int_0^{\infty} z(x) a^{\frac{1}{3}} e^{-x} dx$

We are interested in the case where the X particles and the entropy produced by their decays dominates the energy density of the Universe during the decay epoch ( $x_0 \lesssim x \lesssim 10$ ). In this limit we have:

$$\frac{z'}{z} = x_H^{-1} (f_x + f_{\text{new}})^{\frac{1}{2}} \quad (\text{A1})$$

$$\left(\frac{S}{S_0}\right)^{\frac{4}{3}} = \left(\frac{4}{3}\right) \left(\frac{r m_x}{a_0^{\frac{1}{3}} T_0}\right) \int_0^x a^{\frac{1}{3}} z(x') e^{-x'} dx' \quad (\text{A2})$$

$$f_{\text{new}} \equiv \frac{\left(\frac{3}{4}\right) a^{-\frac{1}{3}} S^{\frac{4}{3}} R_0^{-4} z^{-4}}{\rho_{x_0}} \quad \left( = \frac{\rho_{\text{newrad}}}{\rho_{x_0}} \right) \quad (\text{A3})$$

$$= a^{-\frac{1}{3}} z^{-4} \int_0^x a^{\frac{1}{3}} z(x') e^{-x'} dx'$$

$$f_x = z^{-3} e^{-x} \quad (\text{A4})$$

By using Eqns.(A2-A4), Eqn.(A1) for  $z(x)$  can be rewritten as:

$$\frac{z'}{z} = x_H^{-1} \left[ z^{-3} e^{-x} + a^{-\frac{1}{3}} z^{-4} \int_0^x a^{\frac{1}{3}} z(x') e^{-x'} dx' \right]^{\frac{1}{2}} \quad (\text{A5})$$

By introducing the new variable  $y \equiv x_H^{\frac{2}{3}} z$  the  $x_H$  dependence in Eqn.(A5) can be eliminated:

$$\frac{y'}{y} = \left[ y^{-3} e^{-x} + y^{-4} a^{-\frac{1}{3}} \int_0^x a^{\frac{1}{3}} y(x') e^{-x'} dx' \right]^{\frac{1}{2}} \quad (\text{A6})$$

If we assume that during the decay epoch  $a \approx$  constant or that its dependence can be expressed as  $a \approx a(x)$ , then it follows that the solution to Eqn.(A6) must be of the form:  $y = y(x)$ , supplemented by two boundary conditions [ Eqn.(A6) is a second order differential equation. ] The two boundary conditions can be taken to be

$$y(x_0) = x_H^{\frac{2}{3}} z(x_0) = x_H^{\frac{2}{3}}$$

$$\frac{y'(x_0)}{y(x_0)} = \frac{z'(x_0)}{z(x_0)} = x_H^{-1}$$

For  $x_0 \lesssim x \ll 1$ , the solution must be that of a matter-dominated cosmology:  $y \propto x^{\frac{2}{3}}$ ; the solution which has this behavior and satisfies the boundary conditions discussed above is

$$y(x) = \left( \frac{3x}{2} \right)^{\frac{2}{3}} \tag{A7}$$

(recall that for  $y$  or  $z \propto x^{\frac{2}{3}}$ ,  $x_0 = \frac{2x_H}{3}$ ).

The existence of the solution  $y(x)$  for  $x \ll 1$  which satisfies the appropriate boundary conditions, and the fact that Eqn.(A6) admits solutions which are independent of  $x_H$  demonstrates that the solution we seek has the form:

$z(x) = x_H^{-\frac{2}{3}} y(x)$ . The evaluation of the integral I is now straightforward:

$$I = \int_0^{\infty} a^{\frac{1}{3}} z(x) e^{-x} dx \tag{A8}$$

$$= x_H^{-\frac{2}{3}} \int_0^{\infty} a^{\frac{1}{3}} y(x) e^{-x} dx$$

$$\simeq 1.09 \bar{a}^{\frac{1}{3}} x_H^{-\frac{2}{3}}$$

the 1.09 comes from a direct numerical integration of Eqns.(A1-A4) (where  $a$  was assumed to be constant). As before  $\bar{a}$  is a weighted average of  $a$  during the decay epoch ( $x \approx 1$ ). Note that throughout this paper we have written  $z(x)$  as if it were only a function of  $x$ ; this is somewhat misleading. As we have just shown, it is  $y(x)$  that is a function of a single variable, and  $z = z(x_H, x) = x_H^{-\frac{2}{3}} y(x)$ .

[Note, by recognizing that  $\frac{S_f}{S_o}$  must be independent of our choice of the initial epoch, one could have argued that  $I$  must be  $\propto x_H^{-\frac{2}{3}}$  by insisting that the expression for  $\frac{S_f}{S_o}$  in Eqn.(20) be independent of  $x_H$ .]