Fermi National Accelerator Laboratory

FERMILAB-PUB-84/47-T
April,1984

SU(∞) Gauge Theories on Asymmetric lattices

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ABSTRACT

We study SU(∞) gauge theory on an asymmetric lattice using a generalisation of the Twisted Eguchi Kawai model. We show that it is possible to remove the large N bulk transition by choosing a sufficiently asymmetric lattice and hence to study the physical deconfinement transition without being affected by the former. Results for N=16 indicate a strong first order deconfinement transition.

The phase structure of lattice gauge theories provides valuable information about their continuum limits. While the four dimensional SU(2) and SU(3) theories with the standard Wilson action appear to be a single phase systems, SU(N) theories with $N \geq 4$ with the same action possess a first order transition [1]. This is a bulk transition : it does not imply deconfinement, but does involve a discontinutity in the value of the string tension. To understand the nature of this transition it is useful to study a simple generalisation of the Wilson action, viz.:

$$S = \sum_{P} \{ -\beta \, tr \, U_P - \beta_A \, tr_A \, U_P \} \qquad --- (1)$$

where U_p is the standard plaquette variable and tr_A denotes the trace in the adjoint representation. For N=2 and N=3 one finds [2] lines of first order transitions starting from the $\beta=0$ and $\beta_A=\infty$ axes, meeting at a tricritical point and continuing towards the pure Wilson axis ($\beta_A=0$) but terminating at a critical point for some $\beta_A>0$. An extrapolation of this line brings one to the crossover region of the pure Wilson theory; thus the latter feels the effect of the phase structure of the mixed action theory in the form of a rapid crossover. For $N \geq 4$, however, the line of first order transitions crosses the Wilson axis and terminates at some $\beta_A<0$. This shows that the bulk transition is an artifact of the Wilson action. Under a renormalisation group transformation couplings starting from the vicinity of the Wilson axis would presumably flow around the critical endpoint and approach the infrared fixed point at $\beta=\beta_A=0$. Within the framework of Migdal-Kadanoff approximation this has indeed been observed for SU(2) and SU(3) [3].

At $N=\infty$ the situation becomes complicated. This is because a theory defined with the mixed action of eqn.(1) is essentially equivalent to the pure Wilson theory with a redefined coupling, as the following simple argument indicates:

Consider the Dyson-Schwinger equations for Wilson loops in the theory defined by eqn.(1). For a simple loop the equations may be diagramatically written in the standard fashion in Figure (1). Here $\langle P \rangle$ denotes the average plaquette expectation value. In deriving eqn.(2) we have assumed that the factorisation property of gauge invariant variables is exact. Comparing eqn.(2) with the standard Dyson-Schwinger equations for the Wilson theory [4] it is clear that the model defined by (1) is equivalent to the Wilson model with a coupling β':

$$\beta' = \beta + 2\beta_A \langle P \rangle$$

Equation (3) has been also derived in Ref.[5].

The above feature of the large N limit is consistent with the observation of Bachas and Dashen [6]. These authors argue that the first order transitions in the β-β_A

plane reflect the presence of non-trivial stable minima of the action. Below the line :

$$\beta_A + \frac{N^2-1}{2N^2} \beta \cos\left(\frac{2\pi}{N}\right) = 0 \qquad --- \quad (4)$$

there are no such stable minima and hence the line of transitions must end. It is clear from the above equation that the line of transitions receeds to infinity in the $(\beta/N)-\beta_A$ plane as N goes to infinity.

Since the simple extension of the Wilson model given in eqn.(1) fails to remove the bulk transition for $N=\infty$ it is important to know whether there are other parametrisations which can do the job. The question becomes crucial in the study of the deconfinement transition at finite temperature [7]. SU(2) [8] and SU(3) [9] are known to have second and first order deconfining transitions respectively. Recently some evidence for a first order transition in SU(4) has been reported [10,11]. Several arguments in favor of a first order transition have been presented [12]; the status of these arguments ,however, is not very clear. In a previous communication [13] we studied the finite temperature $SU(\infty)$ theory using Twisted Eguchi-Kawai methods [14]. We used a generalisation of the TEK model to finite temperatures proposed in Ref.[15].(This involves asymmetric twists to mimic the effects of an asymmetric box). We found that for practical values of N_o, the temporal extent of the box, the bulk transition interferes with the deconfinement transition. Similar behaviour has been found in SU(4) [10]. At the bulk transition the string tension jumps discontinuously, making the confinement length larger than N_o, (for small N_o). This induces a spurious deconfinement transition [10]. There has been several other studies of the hot TEK model [16] : we believe all of these are plagued by the same difficulty. For finite N the bulk transition may be avoided by using a mixed action- as done in Ref.[10]. As explained above, for $N=\infty$ this is not possible. To extract any physical information about deconfinement it is absolutely essential to decouple the two transitions. In principle this can be achieved by using a very large N_o, thereby pushing the deconfinement transition deep into the weak coupling region, while the bulk transition remains at intermediate coupling. This, however, appears to be totally unpractical.

In this letter we consider a simple two-parameter generalisation of the Wilson action involving different couplings for the spatial and temporal plaquettes. This is equivalent to a gauge theory on a lattice with different lattice spacings in the temporal and spatial directions. If the asymmetry parameter ξ (the ratio of spacelike to timelike lattice spacings) is large enough one has a box whose physical length in the time direction is small compared to that in the spatial directions - thus simulating finite temperature effects. We construct a hot TEK model which is equivalent to the above theory at $N=\infty$ and study it for N=16 by Monte Carlo simulations. We indeed find that for $\xi \geq 1.75$ the bulk transition completely disappears. The Wilson line, however, continues to show a discontinuous jump, indicating a first order deconfinement transition. The bulk transition is thus indeed a lattice artifact : it is possible to find an "analyticity strip" by going to a larger parameter space involving asymmetric couplings.

The TEK model is defined by the partition function :

$$Z = \int \prod_{\mu} dU_{\mu} \exp(-\beta S_{TEK})$$
$$S_{TEK} = -\sum_{\mu>\nu} Z_{\mu\nu} \,\mathrm{tr}\,(U_{\mu} U_{\nu} U_{\mu}^{\dagger} U_{\nu}^{\dagger}) + h.c. \quad \ldots (5)$$

where the U_{μ}'s are SU(N) matrices and $Z_{\mu\nu}$ is a constant element in Z_N :

$$Z_{\mu\nu} = \exp\left(\frac{2\pi i}{N} n_{\mu\nu}\right)$$
$$n_{\mu\nu} = \text{integers } (\mathrm{mod}\ N) \quad \ldots (6)$$

For a symmetric twist,i.e.

$$N = L^2$$
$$n_{\mu\nu} = L \quad \text{for all } \nu > \mu \quad \ldots (7)$$

the above model is equivalent , at $L=\infty$, to the zero temperature SU(N) gauge theory defined in a symmetric periodic box of size L [14]. The link variables of the field theory, $U_{\mu}(x)$ are related to the reduced variables U_{μ}

by:

$$U_\mu(x) \rightarrow \mathcal{D}(x) U_\mu \mathcal{D}^\dagger(x)$$
$$\mathcal{D}(x) \equiv \prod_\mu (\Gamma_\mu)^{x_\mu} \qquad \ldots\ldots(8)$$

where Γ_μ's are traceless SU(N) matrices satisfying the 't Hooft algebra :

$$\Gamma_\mu \Gamma_\nu = Z_{\nu\mu} \Gamma_\nu \Gamma_\mu \qquad \ldots(9)$$

For any gauge invariant quantity $f(U_\mu(x))$ the following equivalence holds :

$$\langle f(U_\mu(x))\rangle_{\text{FIELD THEORY}} = \langle f(\mathcal{D}(x) U_\mu \mathcal{D}^\dagger(x))\rangle_{\text{TEK}} \qquad \ldots(10)$$

One way to construct a hot TEK model is to consider asymmetric twists [15] which enables one to go to the $N=\infty$ limit by keeping the temporal extent of the box fixed while letting the spatial extents to go to infinity. This type of model has been studied in Ref.[13],[15]and [16].

Another way to have a finite temperature is to have a symmetric box but asymmetric couplings in the spatial and temporal directions [17]. Let a and a_τ denote the spacelike and timelike lattice spacings respectively. The asymmetry parameter is given by :

$$\xi = a/a_\tau \qquad \ldots(11)$$

In order to regain euclidean invariance in the continuum limit one now requires different couplings for the spatial and temporal plaquettes. The action is given by :

$$S = -\sum_x \left\{ \beta_\sigma \sum_{i>j=1}^{3} P_{ij} + \beta_\tau \sum_{i=1}^{3} P_{0i} + h.c. \right\} \quad \cdots (12)$$

where P_{ij} and P_{oi} denote the standard spacelike and timelike plaquettes respectively. The continuum limit is now defined by:

$$a \to 0 \ , \quad \xi \ \text{FIXED}$$

The absence of renormalisation of the velocity of light in the extreme scaling region imposes a relationship between the two bare couplings $\beta_\sigma(a,\xi)$ and $\beta_\tau(a,\xi)$. In the weak coupling limit one obtains :

$$\begin{aligned} \beta_\sigma(a,\xi) &= \frac{1}{\xi g_E^2(a)} + \frac{1}{\xi} c_\sigma(\xi) + O(g_E^2) \\ \beta_\tau(a,\xi) &= \frac{\xi}{g_E^2(a)} + \xi c_\tau(\xi) + O(g_E^2) \end{aligned} \quad \cdots (13)$$

$g^2_E(a)$ is the "euclidean" bare coupling, i.e. the coupling on a symmetric lattice. The functions $c_\sigma(\xi)$ and $c_\tau(\xi)$ have been calculated in weak coupling perturbation theory in Ref.[17].

Consider the above theory defined in a periodic box with L lattice sites in each direction. The physical spatial and temporal sizes of the box are La and La_τ respectively. Thus, for sufficiently large ξ the time extent is much smaller the spatial extent. In the limit $a \to 0$, $L \to \infty$, with ξ and La_τ fixed this describes a finite temperature theory with the physical temperature given by :

$$T = \frac{1}{La_\tau} = \frac{\xi}{La} \quad \ldots (14)$$

It is easy to write down a hot TEK model which is equivalent to the above theory at $N=\infty$. This is simply descibed by the partition function:

$$Z = \int \prod_\mu dU_\mu \exp\left[-(\beta_\sigma S_\sigma + \beta_\tau S_\tau)\right]$$

where

$$S_\sigma = -\sum_{i>j=1}^{3} Z_{ij} \operatorname{tr}(U_i U_j U_i^\dagger U_j^\dagger) + h.c. \quad \ldots (15)$$

$$S_\tau = -\sum_{i=1}^{3} Z_{0i} \operatorname{tr}(U_0 U_i U_0^\dagger U_i^\dagger) + h.c.$$

The $Z_{\mu\nu}$'s are the same as in equations (6) and (7). The equivalence of the above model with the field theory is established in a manner entirely analogous to that of Ref.[14].

The correspondence between the variables of the field theory and the reduced model are the same as in eqn.(8). In particular, the thermal Wilson line is given by :

$$\langle WL \rangle = \frac{1}{N} \mathrm{Re} \langle \mathrm{tr}\, U_0{}^L \rangle_{TEK} \quad \ldots(16)$$

By construction, L is the smallest integer for which $\mathrm{Tr}(U_0)^L$ is nonzero. The energy density may be also obtained by applying the reduction prescription to the corresponding expression in the field theory [18]. In the weak coupling limit one has : $(\beta_E = 1/g_E^2)$

$$\varepsilon = \beta_E \Big\{ (1-\xi^2) - \frac{1}{3N} \Big[\sum_{i>j} Z_{ij}\, \mathrm{tr}(U_i U_j U_i^+ U_j^+) - \xi^2 \sum_i Z_{0i}\, \mathrm{tr}(U_0 U_i U_0^+ U_i^+) \Big] \Big\} \quad \ldots(17)$$

We have performed Monte Carlo simulations of the hot TEK model with asymmetric couplings for N=16 and ξ=1.5,1.75,2.0,3.0 and 4.0. For a given ξ we scan over various values of g^2_E. The couplings β_σ and β_τ are calculated from β_E using equation (13) above. This ensures that in the scaling limit one is describing the continum physics of gauge fields with the physical temperature given by eqn.(14) (for $L=\infty$). In practice, only the leading terms have been retained in eqn.(13). The functions $c_\sigma(\xi)$ and $c_\tau(\xi)$ were obtained from the results of Ref.[17] for each value of ξ separately.

The Metropolis updating procedure is described in Ref.[12]. Figure (2) shows the total action defined by :

$$\langle S\rangle = \frac{1}{N} \mathrm{Re} \left\langle \sum_{\mu\neq\nu} Z_{\mu\nu} \mathrm{Tr}\left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right)\right\rangle \quad \cdots(18)$$

and the Wilson line averages for N=16 and ξ=1.5 for various values of β_E/N between 0.25 and 0.45. (Runs at much smaller and larger couplings were also made : they demonstrated good agreement with the results of lowest order strong and weak coupling expansions at the respective ends.) The data shows that there is a sharp discontinuity in the action at β_E/N = 0.35. This is the bulk transition. The amount of discontinuity is about half that at ξ=1 [14] - indicating that the bulk transition gets weaker for larger ξ. The Wilson line jumps from 0 to about 0.47 at the same value of the coupling, showing that the bulk transition is inducing spurious deconfinement. All the points are averages over typically 1000 sweeps starting from an ordered configuration. In Figure (3) we show the same quantities for ξ=1.75, with runs starting from both ordered and disordered configurations. The discontinuity of the action has now turned into a crossover - in all cases the action converged to the same value for both hot and cold runs. The Wilson line, however, jumps from zero to about 0.45 at β_E/N=0.36 indicating a sharp first order transition. Figure (4) we show the action and Wilson line for N=16, ξ=2. The action is now smooth - the Wilson line jumps at β_E/N=0.325.

In Figure (5) we show the history of the Wilson line at this coupling. Each point is a block average over five sweeps. There is a clear two-state signal with dramatic tunnelling between the ordered and discordered states. Such a behaviour is typical of a first order transition. The absence of a bulk transition has been checked by making hot and cold runs - no hysteresis was observed. Figures (6) and (7) show the action and Wilson lines for $\xi=3$ and $\xi=4$ respectively. In both cases there is no evidence for a first order bulk transition. The Wilson line continues to show a discontinuous jump at $\beta_E/N=0.26$ for $\xi=3$ and $\beta_E/N=0.21$ for $\xi=4$.

The central result of this paper is that by a different parametrisation of the action, viz. by having a sufficiently asymmetric lattice, it is possible to get rid of the unphysical first order bulk transition. It is not clear how the value of ξ above which there is no bulk transition depends on N. We are now studying the N=25,36 and 49 models to determine this. It is possible that as we go higher up in N one needs a higher value of ξ to avoid the bulk transition. Even if that is true,we have a better chance of pushing the critical coupling for deconfinement into the scaling region simply because ξ is a continuously adjustable parameter, and that N is much less restricted in the symmetric twist TEK than in the asymmetric twist TEK. We do not know whether the critical coupling we obtained from $\xi=1.75$ runs is in the scaling region; we hope,however,that the higher N runs we are doing shall provide a more definitive answer to the question of scaling.

Nevertheless, the fact that the Wilson line jumps without being affected by any bulk transition for values of ξ greater than 1.75 strongly indicates that the deconfinement transition at $N=\infty$ is first order. It remains to be seen whether this conclusion is valid when we can work deep into the scaling region. These questions shall be addressed, fortified with larger N data, in a forthcoming communication.

The disappearance of the bulk transition in an asymmetric lattice by itself is an interesting phenomenon and deserves further study even for regular small N theories. It might throw some light on the way couplings flow under a renormalisation group transformation (in particular, do these flows avoid the bulk transition by going up along the ξ axis ?) and hence on the nature of the continum limit of gauge theories.

ACKNOWLEDGEMENTS

J.B.K. would like to thank the Research Board of the University of Illinois for large blocks of computing time on the research Cyber-175. S.R.D would like to thank

W.Bardeen, F.Klinkhamer, P.Mackenzie and C.Zachos for useful discussions. S.R.D also thanks the Computer Department at Fermilab for use of their VAX. The research of J.B.K. was partially supported by the NSF under grant No. NSF-PHY82-01948.

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FIGURE CAPTIONS

Fig. 1: Dyson-Schwinger equations for the mixed action theory.

Fig. 2: Total action and Wilson line for $N=16, \xi=1.5$

Fig. 3: Total action and Wilson line for $N=16, \xi=1.75$

Fig. 4: Total action and Wilson line for $N=16, \xi=2.0$. Crosses are runs with ordered starts, dots with disordered starts. The dashed lines represent results from lowest order strong and weak coupling expansions.

Fig. 5: History of the Wilson line for $N=16$, $\xi=2$ at $\beta_E/N=0.325$. Each point is an average over five sweeps.

Fig. 6: Total action and Wilson line for $N=16$, $\xi=3$.

Fig. 7: Total action and Wilson line for $N=16$, $\xi=4$.

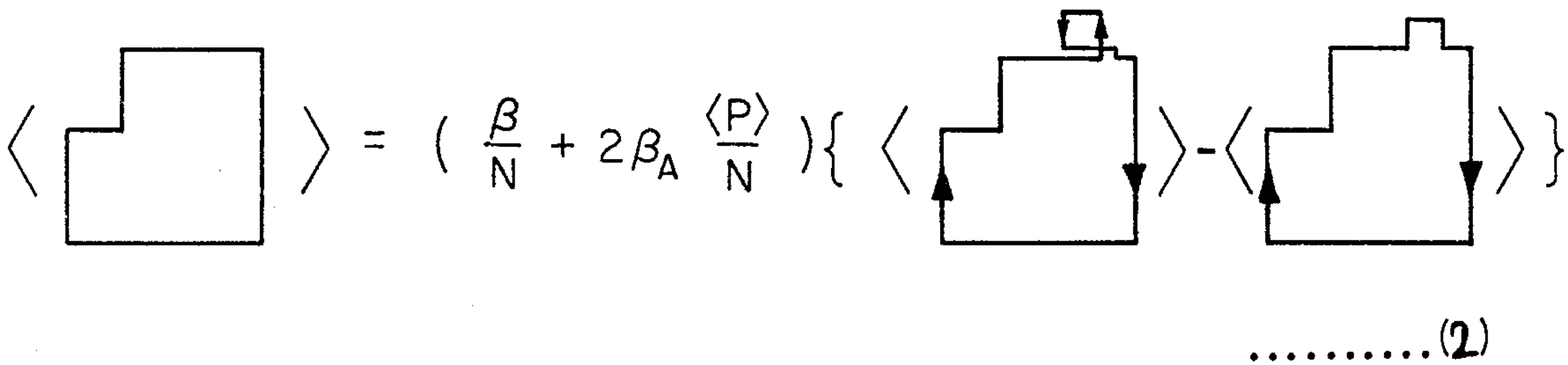


Fig.1.

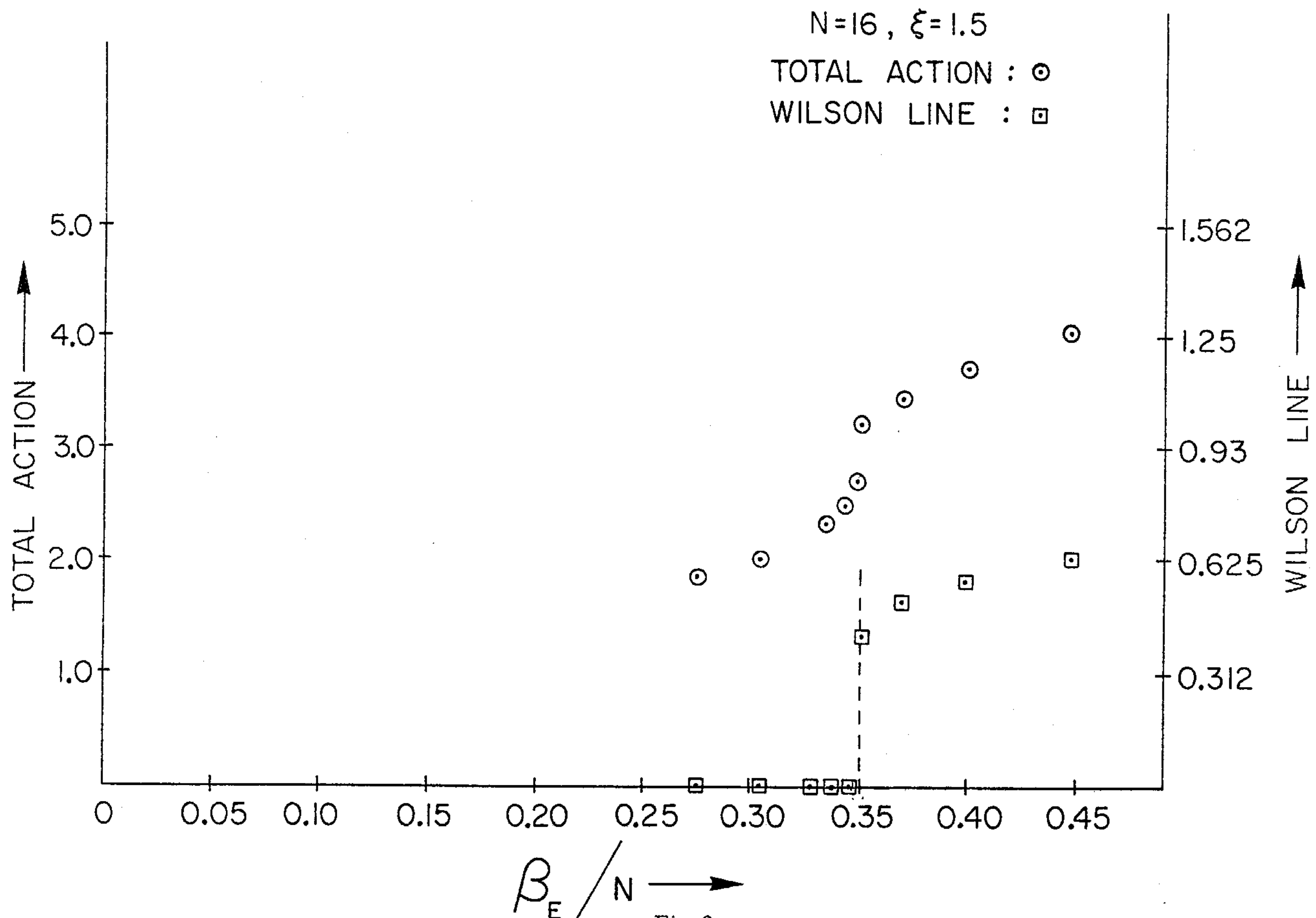

N=16, ξ=1.5
TOTAL ACTION : ⊙
WILSON LINE : ⊡
TOTAL ACTION
WILSON LINE
5.0
4.0
3.0
2.0
1.0
1.562
1.25
0.93
0.625
0.312
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
β_E / N

Fig. 2

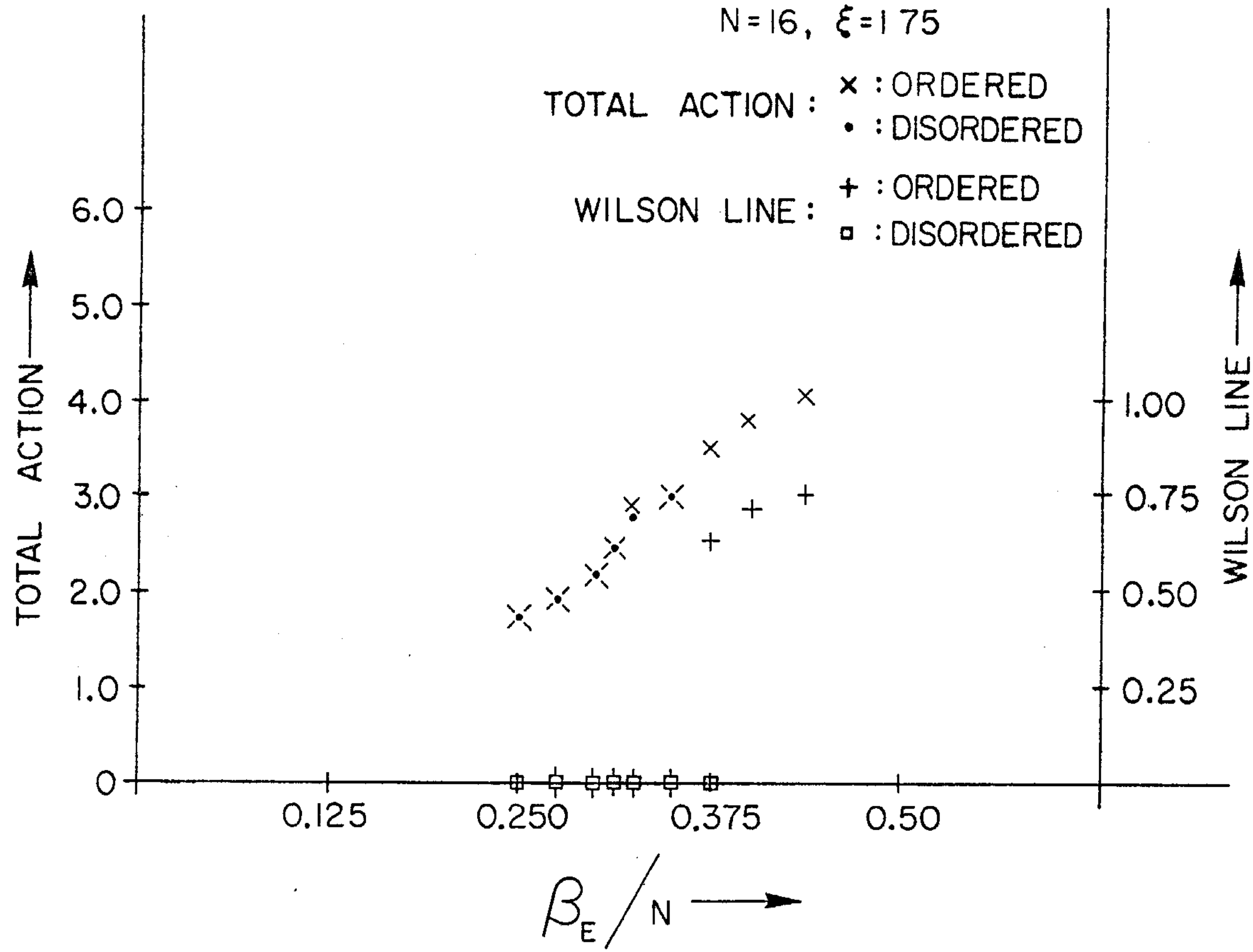

N=16, ξ=1.75
TOTAL ACTION :
× : ORDERED
• : DISORDERED
WILSON LINE :
+ : ORDERED
▫ : DISORDERED
TOTAL ACTION
6.0
5.0
4.0
3.0
2.0
1.0
0
WILSON LINE
1.00
0.75
0.50
0.25
0.125
0.250
0.375
0.50
β_E / N

Fig. 3.

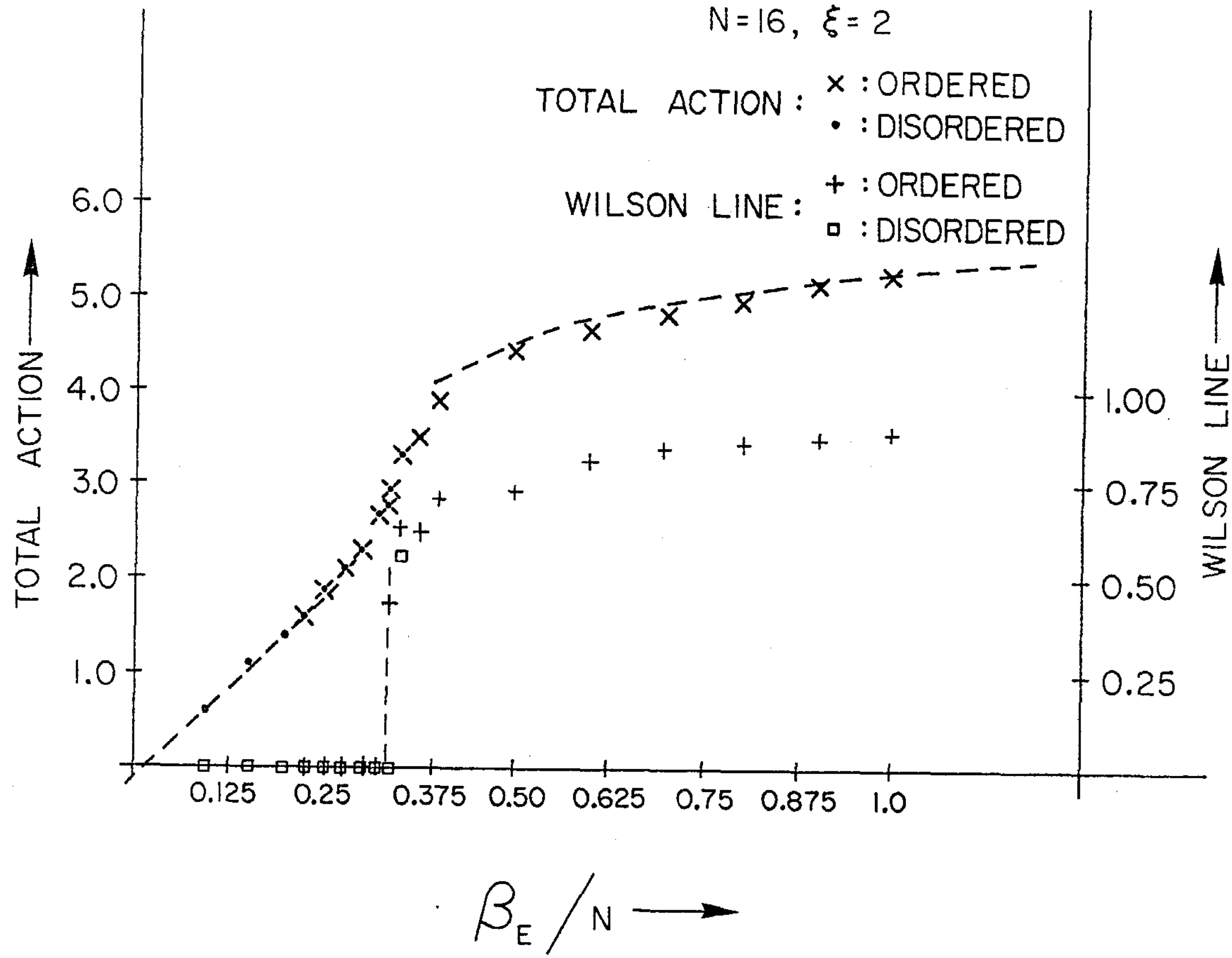
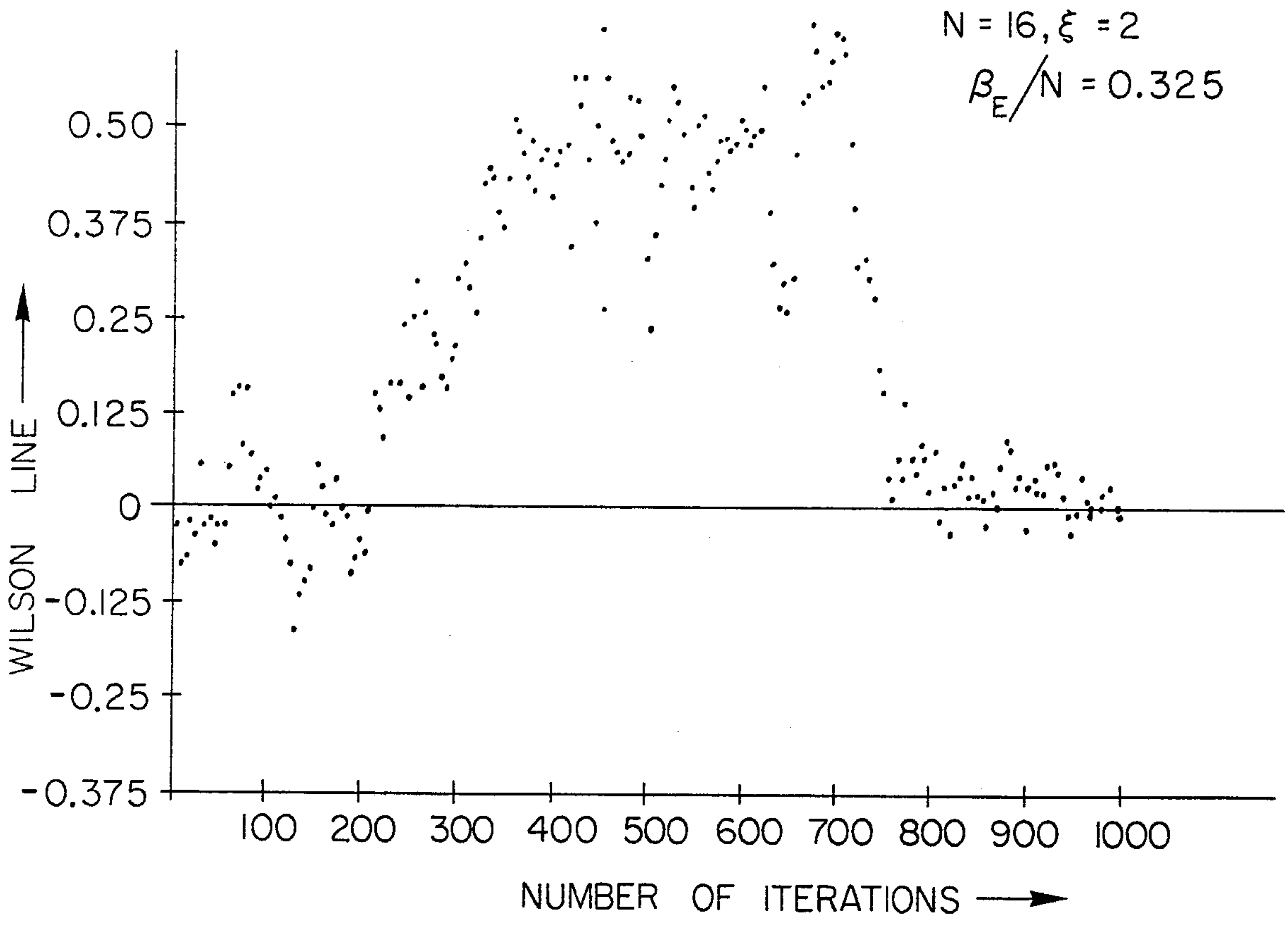

N=16, ξ=2
TOTAL ACTION : × : ORDERED
• : DISORDERED
WILSON LINE : + : ORDERED
□ : DISORDERED
TOTAL ACTION
6.0
5.0
4.0
3.0
2.0
1.0
WILSON LINE
1.00
0.75
0.50
0.25
0.125
0.25
0.375
0.50
0.625
0.75
0.875
1.0
βE / N

Fig. 4.



N = 16, ξ = 2
β_E/N = 0.325
0.50
0.375
0.25
0.125
0
-0.125
-0.25
-0.375
WILSON LINE
100
200
300
400
500
600
700
800
900
1000
NUMBER OF ITERATIONS

Fig. 5.

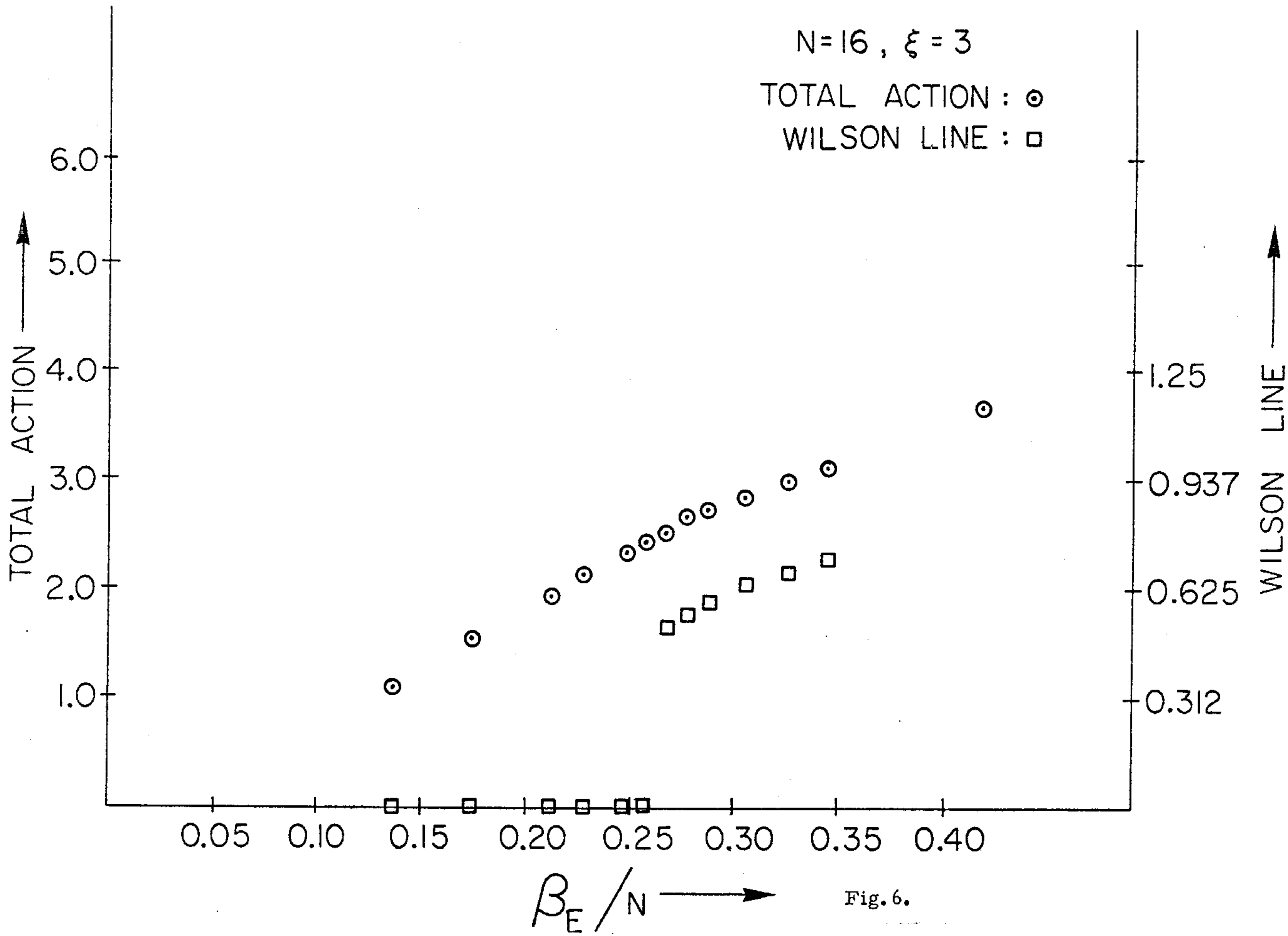
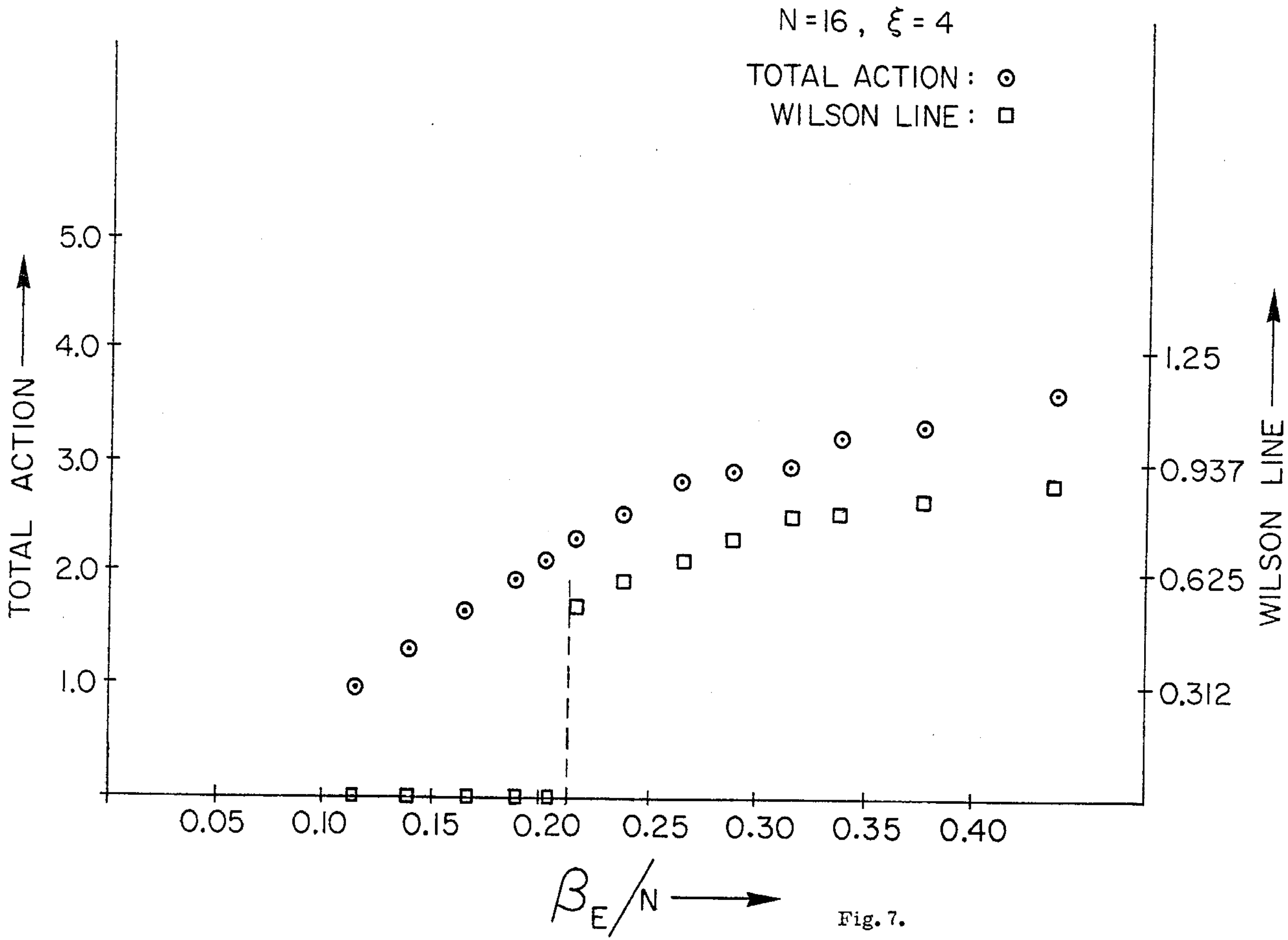

N=16 , ξ = 3
TOTAL ACTION : ⊙
WILSON LINE : □
TOTAL ACTION
6.0
5.0
4.0
3.0
2.0
1.0
WILSON LINE
1.25
0.937
0.625
0.312
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
β_E / N

Fig. 6.



N = 16 , ξ = 4
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WILSON LINE : □
TOTAL ACTION
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4.0
3.0
2.0
1.0
WILSON LINE
1.25
0.937
0.625
0.312
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
β_E/N

Fig. 7.