Possible Signature for Production of Majorana Particles in $e^+ - e^-$ and $p - p$ Collisions

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It is shown that the forward-backward asymmetry in the production of Majorana particles in $e^+ - e^-$ and $p - \bar{p}$ collisions is identically equal to zero if the couplings generating the process conserve the CP-parity, and that the asymmetry vanishes to leading order in perturbation theory if the production mechanism is perturbative and the S-matrix is CPT-invariant. In the case of $p - \bar{p}$ interactions the second statement is proved assuming that the strong interactions are described by QCD and that the relevant subprocesses involve in the leading approximation two particles (quarks, gluons or a quark and a gluon) in the initial state. The absence of forward-backward asymmetry is one possible signature for production of Majorana particles (other than light neutrinos) in $e^+ - e^-$ and $p - \bar{p}$ collisions at high energies.
As is well-known, neutral fermions can be of two varieties: they either can be assigned a conserved additive quantum number, i.e. they can be Dirac particles\(^1\), or else be absolutely neutral, i.e. Majorana particles\(^2\). All known neutral fermions are or behave to a very good approximation like Dirac particles and no firm experimental indications confirming or making likely the existence of Majorana fermions have been found to date\(^3\). At the same time Majorana fermions are present in many extensions of the standard SU(3) × SU(2) × U(1) theory. They appear inevitably in the modern supersymmetric (SUSY) theories\(^4\) as superpartners of the neutral gauge and Higgs bosons. In the minimal SUSY version of the standard theory\(^5\) these are the gluinos (\(\tilde{g}\)), photino (\(\tilde{\gamma}\)), zino (\(\tilde{z}\)) and two Higgsinos (\(\tilde{H}^0_i\), \(i = 1, 2\)). Heavy neutral Majorana leptons are often the right-handed counterparts of the ordinary neutrinos in the electroweak theories based on the group SU\(_L\)(2) × SU\(_R\)(2) × U(1)\(^6\). And the ordinary neutrinos themself might be mixtures of Majorana mass eigenstate neutrinos\(^7\) with masses in the few eV region.

Although the properties of neutral Dirac and Majorana particles are very different, in general, no universal and relatively simple solution to the problem of distinguishing experimentally between them has been proposed to date. Our knowledge of the effects unambiguously associated with the existence of Majorana fermions is primarily based on the studies performed for the case of Majorana neutrinos with non-zero but very small masses. Owing to the specific (V-A) structure of the neutrino weak interactions, all effects typical for Majorana neutrinos are very subtle and vanish in the limit of zero neutrino masses\(^8\), when the physical difference between the two possible types of mass eigenstate neutrinos disappears. For Majorana fermions other than neutrinos similar conclusion might not be valid.

In this note we consider one possible signature for production of Majorana fermions / other than light neutrinos/ in high energy \(e^+ - e^-\) and \(p - p\) collisions, based on their specific CP- and C\(_T\)-transformation properties. The C\(_T\)- and C\(_T\)-transformations leave the Majorana fermions, which do not have distinctive antiparticles, essentially unchanged. As a consequence, the cross-sections, e.g., for
inclusive production of Majorana particles in $e^+ - e^-$ and $p - \bar{p}$ collisions possess a certain symmetry, which is exact when the corresponding amplitudes are CP-symmetric and approximate if the production mechanism is perturbative and the $S$-matrix is CP-invariant\textsuperscript{*}. In the center of mass system of the initial particles this symmetry corresponds to a symmetry between the particle spatial distributions in the forward and in the backward hemispheres (i.e., implies zero forward-backward (F-B) asymmetry). As an illustration of our general results several examples corresponding to the production of supersymmetric Majorana fermions are considered.

It is convenient to begin our discussion by considering the specific process of $e^+ - e^-$ annihilation into a pair of arbitrary Majorana fermions $j$ and $j'$:

$$ e^+ - e^- \rightarrow j + j' \quad (1) $$

We shall assume that at least one of the two particles in the final state (say, $j'$) is massive, unstable and has visible decay products, so that its production rate in the two hemispheres in the $e^+ - e^-$ c.m.s. can be measured, in principle. Processes of this type are predicted to exist at relatively high energies ($\sqrt{s} > 30$ GeV), e.g., by a fairly wide class of supersymmetric theories/9,10,11/. In these theories $j$ and $j'$ can be two of the eigenstates of the $\gamma$, $\tilde{z}$ and $\tilde{z}_1$ Majorana mass matrix which has diagonal as well as nondiagonal elements. In particular/11/,

$j$ might have a large photino component being the lightest mass eigenstate, while $j'$ might be the second to the lightest mass eigenstate, having large Higgsino components. Then $j'$ would be unobservable directly as like the ordinary neutrinos, while $j$ would undergo a relatively fast decay into $j$ and a lepton pair:

$$ e^+ - e^- \rightarrow j + j' \rightarrow \left\{ \begin{array}{l} j + e^+e^- \text{, or} \\ j' + e^+e^- \text{, or} \\ j + \tilde{e}^+\tilde{e}^- \text{, or} \\ j' + \tilde{e}^+\tilde{e}^- \text{.} \end{array} \right. \quad (2) $$

Incidentally, this process might take place even at PEP and PESTRA with a cross-section in the picobarn region/11/.

If we separate the nontrivial part $R$ of the $S$-matrix, expressing the latter as

$$ S = I + R, \quad (3) $$

where $I$ is the identity operator, then the amplitude of the process

\textsuperscript{*} The proof of the second statement presented here for the case of $p - \bar{p}$ collisions is made under some rather general assumptions.
(1) can be written in the form:

\[(2\pi)^4 \delta(p_1+p_2-k-k') A(e^+e^-\rightarrow j'j) = \int \langle r,k \rangle \langle r',k' \rangle \left| \int e^-(s_2,s_2) e^-(s_1,p_1) \right|^2\]

Here \(p_1, p_2, k\) and \(k'\) are the 4-momenta of \(e^-, e^+, j\) and \(j'\), respectively, \(s_1^\downarrow\) is the projection of the spin of the electron on its momentum \(p_1\) \((s_1^\downarrow = \frac{1}{2} \text{ or } -\frac{1}{2})\), etc. The well-known set of invariant kinematical variables which can be used to describe the reaction (1) includes:

\[s_1 = -(p_1+p_2)^2, \quad t = (p_1-k)^2 \quad \text{and} \quad u = (p_2-k)^2,\]

only two of them being independent \((s-t-u = 2m_e^2 + m_1^2 + m_2^2\), where \(m_1^2\) is the mass of \(j'\)). In the \(e^-e^-\) c.m.s. we have:

\[t = \frac{1}{2} (s-m_e^2-m_1^2) + k \frac{1}{2} (s-4m_e^2) \cos \Theta - m_1^2,\]

\[u = \frac{1}{2} (s-m_e^2-m_1^2) - k \frac{1}{2} (s-4m_e^2) \cos \Theta - m_1^2,\]

where \(k\) is the absolute value of the \(j\) and \(j'\) momentum and \(\Theta\) is the angle formed by the \(e^-\) and \(j\) momenta. The differential cross-section, summed over the spin states of the final particles and averaged over the spin states of the initial \(e^+\) and \(e^-\), can be written as:

\[d\sigma = (2\pi)^4 \delta(p_1+p_2-k-k') f(s,m_e^2) F(s,t) \frac{dk}{k_0} \frac{dk'}{k_0} \cdot \]

Here \(f(s,m_e^2)\) is a well-known kinematical function determined by the relative flux of the initial particles, \(k_0\) is the \(j'\) energy and

\[F(s,t) = \frac{1}{4} \sum_{r,s} \left| \int e^+(e^-\rightarrow j'j) \right|^2.\]

We shall use also the CP and CPT transformation properties of one-particle free Majorana fermion states with definite momentum and spin projection on the momentum. Since a Majorana fermion has no distinctive antiparticle they read:

\[I \int (r,k) > \frac{U^{CP}}{\cp} \eta_{\cp}^{CP} \int (-r,k_p) > \quad \eta_p = (-\gamma, i\kappa_0), \]

\[I \int (r,k) > \eta_{\cp}^{CP} \int (-r,k) > , \]

where \(\eta_{\cp}^{CP} (j)\) and \(\eta_{\cp}^{CP} (j')\) are phase factors.

After these preparatory remarks we proceed to the discussion of the specific symmetry properties of the cross-section of the process (1) which are a direct consequence of the Majorana nature of the particles in the final state. Let us assume first that the couplings of \(j\) and \(j'\) are invariant under the CP transformations. Using eq. (9) we get then:
\[ \langle j(r,k) j'(r',k') I R I e^+(s_2,p_2) e^-(s_I,p_I) \rangle = \]
\[ = -\partial_{cp} \langle j(-r,k) j'(-r',k') I R I e^+(-s_I,p_I) e^-(-s_2,p_2) \rangle \]

where \( \partial_{cp} \) is a phase factor and the minus sign in the right-hand side appeared since the positions of the e\(^+\) and e\(^-\) creation operators in the initial state wave function have been interchanged. It follows from eqs. (II) and (8) that the function \( F(s,t) \) is symmetric with respect to the change of variables \( (s,t) \) to \( (s,u) \):

\[ F(s,t) = F((p_2 + p_I)^2, (p_2 - k_p)^2) = F(s,u) \]  

(12)

Since in the e\(^+\)-e\(^-\) c.m.s. the variable \( u \) can be obtained from \( t \) formally by replacing \( \cos \theta \) with \( (- \cos \theta) \), this implies that \( F(s,t) \) is an even function of \( \cos \theta \). Therefore the (F-B) asymmetry in the production of \( j' \) (or \( j \)) will be identically equal to zero independently of the structure of the interactions leading to (1).

This general conclusion is not valid if the \( j \) and \( j' \) couplings are not CP-symmetric. We are going to show, however, that it is still true to leading order in the perturbation theory if the process (1) is perturbative (i.e., \( j \) and \( j' \) do not possess couplings which cannot be treated perturbatively at the energies at which the process in question occurs*) and if the S-matrix is CPT-invariant. Indeed, the unitarity of the S-matrix is equivalent to the relation:

\[ R + R^* + RR^* = 0 \quad (RR^* = R^*R). \]  

(13)

Eq. (13) implies that to leading order in the perturbation theory the matrix elements of the operators \( K \) and \( R^* \) between the e\(^+\), e\(^-\) and \( j, j' \) states coincide. Together with the CPT-invariance of the S-matrix this leads to the approximate equality:

\[ \langle j(r,k) j'(r',k') I R I e^+(s_2,p_2) e^-(s_I,p_I) \rangle \approx \]
\[ = \partial_{cp} \langle j(-r,k) j'(-r',k') I R I e^+(-s_I,p_I) e^-(-s_2,p_2) \rangle, \]  

(14)

\[ \partial_{cp} \] being a common phase factor. It is trivial to show using eq. (14) that in this case

\[ F(s,t) \approx F(s,u) \]  

(15)

* For \( \sqrt{s} \geq \) several GeV this is supposed to be valid also for the particles whose strong interaction is governed or fixed (e.g., via the supersymmetry) by QCD (like gluinos).
and therefore the $(F-B)$ asymmetry in the production of $\gamma$ (or $\chi$) vanishes to leading order in the perturbation theory.

It was noticed in ref. /II/ that the leading contributions to the cross-sections of $e^+ - e^-$ annihilation into two different weak gauge and/or Higgs majorana fermions, calculated in the minimal SUSY extension of the standard model, exhibit the same symmetry property, although the couplings generating the reactions are not C-invariant and might not preserve CP-parity. Obviously, this is just one example of processes for which our general results are valid.

With minor modifications the preceding considerations can also be applied to the process of production of one majorana particle in association with a Dirac particle $N$ or its antiparticle $\bar{N}$

$$e^+ + e^- \rightarrow \gamma + \left\{\begin{array}{c}N \\ \bar{N}\end{array}\right.,$$

(16)

as well as* to the process of inclusive majorana particle production

$$e^+ + e^- \rightarrow \gamma + \text{anything}$$

(17)

They lead to exactly the same results for the $(F-B)$ asymmetry in the distribution of $\gamma$. For example, the analog of eq. (II) in the case of reaction (16) has the form:

$$\langle \gamma(r,k)N(r',k') | R \langle 1 e^+(s_2,p_2)e^-(s_1,p_1) > = \\
-\ell_p\langle \gamma(-r,k_p)\bar{N}(-r',k'_p) | R \langle 1 e^+(-s_1,p_1)e^-(-s_2,p_2) > \,$$

(18)

The differential cross-section describing the distribution of $\gamma$ represents a sum of two terms corresponding to the $e^+ - e^-$ annihilation cross-sections into the two different final states ($\gamma N$) and ($\gamma \bar{N}$). As a consequence of eq. (18) these terms transform into each other under the change of variables $u \rightarrow t$, $t \rightarrow u$ and therefore the differential cross-section is symmetric with respect to such a change.

The proof in the case of inclusive majorana particle production is based on similar analysis. It should be mentioned that the variables $t$ and $u$ are independent in this case. Therefore the relevant symmetry properties of the function $\tau(s,t,u)$, which determines the $t$- and $u$-dependence of the cross-section of the reaction (17), can be expressed

* The CPT-invariance of the S-matrix and the unitarity condition (13) imply that if one of the reactions (16) takes place (e.g., $e^+e^-\rightarrow \gamma$) then the other ($e^+e^-\rightarrow \gamma \bar{N}$) should also exist.
as follows: \[ f(s, t, u) \leq f(s, u, t). \] (15)

Obviously, the absence of (F-B) asymmetry in the production of a given neutral fermion in \( e^+ e^- \) annihilation can be a consequence of a specific dynamics governing the process. Moreover, it might even be simulated by specific circumstances. An example illustrating this is provided by the hypothetical reactions:

\[
\begin{align*}
\text{(20)} & \quad e^+ + e^- \rightarrow \bar{\nu} + \nu \quad \text{or} \quad \mu^+ \mu^- \bar{\nu} \\
\text{(21)} & \quad e^+ + e^- \rightarrow \bar{\nu} + \nu \quad \text{or} \quad \mu^+ \mu^- \bar{\nu}
\end{align*}
\]

where \( \bar{\nu} \) is a light (or massless) neutrino and \( N \) is a heavy neutral lepton of Dirac type (e.g., associated with a fourth sequential generation of leptons). Let us assume for definiteness that \( N \) has \((V, A)\) couplings only. Since the neutrinos are unobservable in the type of experiments under discussion, the signatures of these reactions will be the same as the signatures of the reaction (2) in which Majorana fermions are produced. It is not difficult to show that if the couplings leading to (20) and (21) are, e.g., CP-invariant, for each \( N (\bar{N}) \) appearing in the forward hemisphere there will be an \( \bar{N} (N) \) produced in the backward hemisphere. Thus the lepton pairs originating both from \( N \) and \( \bar{N} \) decays will be produced with equal cross-sections in the two hemispheres, exactly like in the process (2). In the particular example we are considering it is nevertheless possible to distinguish between the production of the Majorana and Dirac fermions by measuring the energy distributions of \( e^- (\mu^-) \) or \( e^+ (\mu^+) \) in each of the two hemispheres. These distributions will be identical if the decaying particle is a Majorana fermion (\( \nu \)); they will be different if the charged lepton originates from a weak (C- and P- nonconserving) decay of a Dirac neutral lepton (\( N \)).

Let us turn now to the process of inclusive production of Majorana particles in \( p\bar{p} \) collisions:

\[ p + \bar{p} \rightarrow \nu + \text{anything} \] (22)

From the point of view of the symmetry properties of the cross-sections

\* This interesting example was brought to my attention by Prof. Wolfenstein.
we are interested in, there exists a complete analogy between the processes (17) and (22), when the couplings of the Majorana fermion are CP-invariant. Independently of their specific form, \( J \) will be produced then in the \( p - \bar{p} \) collisions with equal cross-sections in the forward and backward hemispheres (in the \( p-\bar{p} \) c.m.s.).

The presence of strong interactions between the particles in the initial state makes the case of CP-noninvariant \( J \)-proton couplings more complicated than the corresponding case of \( e^+ - e^- \) annihilation. As is well-known, no exact methods of accounting for the initial state strong interaction effects have been developed to date. However, if the reaction (22) occurs at relatively high energies one can use the quark-gluon picture of the collision, i.e. QCD as a theory of the strong interactions. In this approach/12/ all nonperturbative initial state interaction effects are assumed to be accounted for by the distribution functions of the interacting quarks and/or gluons, which are extracted (directly or indirectly) from the data. The subprocesses corresponding to production of \( J \) in the interaction between quarks and/or gluons are supposed to be perturbative as (by assumption) they take place at energies at which the QCD effects can be treated perturbatively. Under these conditions it is possible to show that if the relevant subprocesses involve only two particles in the initial state, the \( (F - B) \) asymmetry in the distribution of \( J \) vanishes to leading order in the perturbation theory, as like in the corresponding case of \( e^+ - e^- \) annihilation. We shall give below a proof of this statement for the case when \( J \) is produced via quark - (anti)quark annihilation:

\[
q' + q'' \rightarrow J + \text{anything} \quad (q', q'' = u, d, \ldots, \bar{u}, \bar{d}, \ldots) (23)
\]

In the majority of cases of practical interest "anything" corresponds in the leading approximation to one particle and the \( J \) production cross-section represents a sum over all relevant pairs of initial quarks and two-particle final states. In particular, if \( J \) is expected to be produced predominantly in association with a given charged Dirac fermion \( X \), the reactions

\[
q + q' \rightarrow J + X \quad (24)
\]

\[
\bar{q} + \bar{q}' \rightarrow J + \bar{X} \quad (25)
\]

where \( \bar{X} (\bar{q}^{(m)}) \) is the antiparticle of \( X (\bar{q}^{(m)}) \), should actually be considered as subprocesses for the process (22). Let us focus first our
attention on this case. CPT-invariance and the unitarity of the S-matrix lead to the relation
\[ \langle \hat{f}(r',k)X(r',k') \rangle \equiv \langle \hat{f}(r',k)X(r',k') \rangle \]
\[ \equiv \langle \hat{f}(-r',k)\bar{X}(-r',k') \rangle \]
\[ \equiv \langle \hat{f}(-r',\bar{k})\bar{X}(-r',\bar{k}) \rangle \]
(25)
(25)

(the notations are obvious) valid in the leading order of the perturbation theory. Eq. (26) implies that in the leading approximation the cross-sections of the reactions (24) and (25), averaged over the initial quark spin states and summed over the spin states of the final particles, are described by the same function \( F_q((p_1'^2, p_2'^2, k')^2) = F_q(s', t') \):
\[ d\sigma (q' + q\to X) = 4\pi \delta(p_1'^2 + p_2'^2 - k')f_q(s')F_q(s', t') \frac{dk'}{k_0} \]
(27)
\[ (27) \]
\[ f_q(s') \text{ being a standard kinematical factor, identical for both reactions.} \]
In the approach we are following the cross-section of the process (22) with subprocesses (24) and (25) has the standard form of a sum of convolutions of the products of the initial state quark distribution functions in the proton and antiproton \( q^p(x) \) and \( q^\bar{p}(x) \), respectively, \( x \) being the fraction of the p or \( \bar{p} \) momentum \( ^\text{carried by} \) \( q^p(x) \) and the cross-sections (27). Assuming that \( p \) and \( \bar{p} \) have 4-momenta \( p_1 \) and \( p_2 \) and that \( f \) is produced with a definite momentum \( k \), it can be written as:
\[ d\sigma (p\bar{p} \to f + \text{anything}) \equiv \int_0^1 dx_1 \int_0^1 dx_2 \frac{dk}{k_0} (2\pi)^4 \delta(x_1 p_1 + x_2 p_2 - k')f_q(s')x\left[ \left[ q^p(x_1)q^\bar{p}(x_2) + q^\bar{p}(x_1)q^p(x_2) \right] F_q(s', (x_1 p_1 - k)^2) + \left[ q^p(x_2)q^\bar{p}(x_1) + q^\bar{p}(x_2)q^p(x_1) \right] F_q(s', (x_2 p_2 - k)^2) \right] \frac{dk}{k_0} = \]
\[ = F_p(s, (p_1 - k)^2, (p_2 - k)^2) \frac{dk}{k_0} \]
(26)
(26)

Here \( x_1 \) and \( x_2 \) are the fractions of the momenta of \( p \) and \( \bar{p} \) carried by the annihilating quarks, \( s' \) \( = x_1 x_2 s \) (the effects of the initial particle masses are neglected, which is a standard approximation in this approach).

\[ ^\text{If} \, \chi \, \text{does not carry an electric charge we should have, e.g.,} \]
\[ q'' = q^\gamma \text{ and the initial states in eqs. (24) and (25) should coincide.} \]
\[ ^\text{The dependence of the quark distribution functions on variables irrelevant to our discussion (like the q' - q'' invariant mass s')} \]
\[ \text{is not shown explicitly.} \]
and we have made use of the relations: \( q_{1,1}^{\dagger}(x) = q_{2,1}^{\dagger}(x) \) and \( q_{1,1}^{\dagger}(x) = q_{2,1}^{\dagger}(x) \). It is trivial to convince oneself that the function \( f_p(s, (p_1 - k)^2, (p_2 - k)^2) \) defined in eq. (28) is symmetric with respect to the interchange of \( p_1 \) and \( p_2 \), which implies that in the p-p c.m.s. it is an even function of \( \cos \theta = \frac{p_1 \cdot k}{(|p_1| |k|)} \). Consequently, in the leading approximation there will be no (F-B) asymmetry in the production of \( j \). It is easy to generalize this result to the case of gluon-quark as well as gluon-gluon annihilation and arbitrary possible particle \( X \), and to the case of arbitrary number of particles in the final state of the relevant subprocesses.

Our conclusions concerning the reaction (22) apply to the majority of cases of practical interest discussed in the literature. In particular, they are valid for the set of subprocesses considered in ref./13/, wherein the production of SUSY particles in p - \( \bar{p} \) collisions is studied in detail:

\[
\begin{align*}
q + q' &\rightarrow \tilde{\ell} + \tilde{\ell}' \quad \text{and} \quad \tilde{q} + \tilde{q}' \rightarrow \tilde{Q} + \tilde{Q}' \quad \text{and} \quad g + q \rightarrow \tilde{q} + \tilde{g},
\end{align*}
\]

where \( \tilde{w}^+ \), \( \tilde{q} \) and \( \tilde{g} \) are the supersymmetric partners of the \( w^+ \), \( q \) and gluons (g), respectively.

To conclude, the absence of (F-B) asymmetry is a possible signature for production of Majorana fermions (other than light neutrinos) in e\(^+\) - e\(^-\) and p - \( \bar{p} \) collisions.

Acknowledgements.

The author wishes to thank T.P. Cheng, S.K. Bilenky, C. Quigg, H. Suzuki and L. Wolfenstein for valuable and stimulating discussions. The hospitality of the Aspen Center for Physics and the Theory Groups at LBL and Fermilab, where parts of the present work were done, as well as the support of the Bartol Research Foundation of The Franklin Institute and especially of Prof. T. Gaisser, are gratefully acknowledged.
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