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DIMENSIONAL REDUCTION IN THE EARLY UNIVERSE: WHERE HAVE THE MASSIVE PARTICLES GONE?

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ABSTRACT

In many models based on reduction from greater than four dimensions, there are absolutely stable particles with masses of order R^{-1}. (R is the compactification scale.) If the temperature of the universe were ever close to R^{-1}, these massive states would have been present and some would have survived annihilation. We calculate the present mass density due to these particles and find both the 5-dimensional model and some versions of N=8 supergravity to be unacceptable. We discuss some possible solutions to this problem.

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One of the most attractive approaches for unifying gauge theories with gravitation is based on enlarging the dimensionality of space-time. This program is traced back to the work of Kaluza [1] and Klein [2], who proposed that gravitation and electrodynamics in 4 dimensions might be unified as a pure gravitational theory in 5 dimensions. The vacuum geometry determines the effective low-energy theory; 5-dimensional gravity reduces to 4-dimensional gravity and a peculiar version of electrodynamics if the vacuum is the space $M^4 \times S^1$. (M^4 is 4-dimensional Minkowski space and S^1 is the 1-sphere or circle.)

Recent work has centered on constructing higher-dimensional theories that include non-abelian gauge interactions [3]. For example, it is possible to formulate N=8 supergravity as a Kaluza-Klein theory in 11 dimensions [4]. Another example is the quantum superstring, which must be formulated in 10 dimensions [5]. Although there has not yet emerged a low-energy theory with the observed gauge and fermionic structure of the standard electroweak-QCD model, the attractiveness of these approaches has generated much interest.

Typically, the vacuum geometry of interest is a direct product of M^4 with a compact space that has a high degree of symmetry. Then each field has a harmonic expansion about the vacuum into 4-dimensional fields times the mass eigenfunctions of the extra dimensions. The zero modes correspond to the low-mass particle spectrum; the infinite sequences of higher modes have masses of order R^{-1}. It is usually assumed that R is of order ${G_N}^{1/2}$ = ${m_{Pl}}^{-1}$ = 1.62×10^{-33} cm times some power of the gauge coupling

(G_N is Newton's constant), although there are models where R is determined by the electroweak breaking scale $G_F^{1/2} = 6.74 \times 10^{-17}$ cm (G_F is Fermi's constant) [6].

We discuss a possible cosmological consequence resulting from the "4-dimensional particles corresponding to the non-zero modes of the harmonic expansions in mass eigenstates of the higher-dimensional fields," generically called "pyrgons" [7]. If the universe were ever at a temperature comparable to R^{-1}, the pyrgons would have been present. In the 5-dimensional theory with an unbroken local U_1 symmetry, the pyrgons cannot decay solely into zero modes, although they can annihilate with antipyrgons. The stable pyrgons that survive annihilation contribute to the present energy density of the universe. We calculate the present number density of the pyrgons, and then require their contribution be less than the observed bounds.

If annihilation were negligible, then today the number density of pyrgons would be comparable to the photon number density, which is acceptable only if the pyrgon mass satisfies $m_\psi \lesssim 100$ eV, just as for neutrino masses [8]. Thus, the pyrgons must be annihilated; we require that the annihilation rate ($\Gamma_A = n_\psi \sigma_A |v|$, where n_ψ is the pyrgon number density and σ_A is the annihilation cross section) must be comparable to the expansion rate of the universe ($\Gamma_E = T^2/m_{Pl}$, where T is the temperature). If we assume $\sigma_A|v| = \alpha^2/m_\psi^2$, then at $T = m_\psi$ and $n_\psi = T^3$, we estimate $\Gamma_A/\Gamma_E = \alpha^2\, m_{Pl}/m_\psi$. In the 5-dimensional Kaluza-Klein model, $\alpha = R_{Pl}^{\ 2}/R^2 = m_\psi^2/m_{Pl}^{\ 2}$, and $\Gamma_A/\Gamma_E = m_\psi^3/m_{Pl}^{\ 3}$. For $m_\psi < m_{Pl}$, the ratio is less than unity, and annihilation is ineffective at ridding the universe of stable pyrgons. However, if we modify the theory so the pyrgons carry a similar but larger charge such that α appearing in σ_A is order unity for any m_ψ, then $\Gamma_A/\Gamma_E \sim m_{Pl}/m_\psi$. If $m_\psi << m_{Pl}$, annihilation can be effective enough to reduce the pyrgon density below present observational bounds. For this to occur we will show $m_\psi \lesssim 10^6$ GeV.

The presence of absolutely stable pyrgons may appear to be a special feature of the 5-dimensional model, resulting from the fact that the zero modes do not carry the U_1 charge. However, there are somewhat more realistic theories with stable pyrgons in which the zero modes do carry the gauge quantum numbers: for example, 11-dimensional supergravity with vacuum $M^4 \times S^7$ has stable pyrgons even though most of the 256 zero modes do carry the SO_8 quantum numbers of the symmetry of S^7, as we show later.

We now turn to the details of the 5-dimensional model, where the vacuum is $M^4 \times S^1$, and the harmonic expansion is just a Fourier series in the extra coordinate ($y = 2\pi R\theta$) with coefficients that are fields on M^4. (Generalizations will be indicated as we go along.) Thus, the fields are expanded as

$$\Phi_I(x,y) = \sum_{k=-\infty}^{\infty} e^{ik\theta}\phi_I^k(x) \quad , \qquad (1)$$

where I is a space-time index and $|k|$ labels the mass eigenstate. (For a compact manifold with symmetry group G, the $e^{ik\theta}$ are replaced by representation matrices of G, $D^k(L_y^{-1})$, where L_y is the element of G that parameterizes the point y of the manifold. The sum on k is replaced by a sum or restricted sum over the representations of G.)

The equation of motion for small disturbances about the ground state geometry is given by [9]

$$\Box^{(5)} \Psi_1(x,y) = 0 \quad , \qquad (2)$$

where $\Box^{(5)}$ is the 5-dimensional d'Alembertian. In the free field limit, each field $\phi_1{}^k(x)$ satisfies a wave equation,

$$(\Box^{(4)} + M_k{}^2)\, \phi_1{}^k(x) = 0 \qquad (3)$$

where, in this case, the mass squared operator is just the d'Alembertian in the extra dimension. Therefore, each term in the harmonic expansion (1) corresponds to a particle with mass $M_k{}^2 = (k/2\pi R)^2$. The generator of the charge is just $i\partial_y$, so the charge of $\phi_1{}^k(x)$ is proportional to k. The mass spectrum in four dimensions [9] contains a massless, neutral spin-2 particle; a massless, neutral spin-1 particle; a massless neutral scalar; and an infinite tower of charged spin-2 pyrgons with masses $M_k{}^2 = (k/2\pi R)^2$, k = 1, 2, ...

Consider the decay of the pyrgons. We label each 4-dimensional field in the harmonic expansion by the quantum number k. The amplitude for the process,

$$\phi^k \rightarrow \phi^{k_1} + \phi^{k_2} + \ldots + \phi^{k_n} \quad , \qquad (4)$$

is contained in a term of the 5-dimensional effective action of the form,

$$I \sim 2\pi R \int d^4x \int d\theta \; e^{i(-k+k_1+\ldots+k_n)\theta} \; \phi^k(x)^* \cdot \phi^{k_1}(x) \ldots \phi^{k_n}(x) \quad , \qquad (5)$$

where the contraction on space-time indices is implicit in the notation. Upon integration over the extra dimension, the decay rate for (4) is proportional to the Kronecker δ, $\delta(k_1+k_2+\ldots+k_n-k)$. The appearance of derivative couplings in the action still leads to the same δ-function, since the derivative does not mix modes. The existence of the δ-function means that no pyrgon ($|k|>1$) can decay to zero modes (k=0) only. [For spaces with higher symmetry, the generalization of the Kronecker δ is a (3n-3)-symbol, which is nonzero only if it satisfies certain "triangle inequalities."] Of course, the annihilation of a pyrgon with its antiparticle can yield all zero modes.

We now calculate the number density n_ψ of remnant pyrgons from the big bang. There is no need to follow the evolution of the universe up to the time of compactification t_c if the initial conditions for n_ψ can be set at a time near t_c. At t_c the universe has become approximately 4-dimensional and the excitations of the vacuum geometry may be reinterpreted as 4-dimensional zero modes and pyrgons. If the 4-dimensional temperature T at t_c is near enough to $T_c \equiv R^{-1}$, then the calculation of n_ψ is insensitive to the initial conditions, as will be seen. Otherwise, the calculation places a <u>lower</u> limit on n_ψ.

Unless very special initial conditions control the evolution of the universe for $t < t_c$, there will be pyrgons at t_c, and the initial condition for the ratio, $r = n_\psi/n_\gamma$, will be of order unity, since, typically, the excitations of the 5-dimensional fields will be distributed over the modes in (1). Moreover, if the excited pyrgons ($|k| > 1$) decay rapidly enough into stable $|k| = 1$ pyrgons and zero modes, they will be present in a thermal distribution. The stable pyrgons decrease in number only by annihilation. For simplicity we ignore the $|k| > 1$ populations and compute n_ψ for the stable ones, assuming n_ψ for $k = +1$ and $k = -1$ are equal. Then n_ψ satisfies the equation,

$$\dot{n}_\psi = [(n_\psi^{eq})^2 - n_\psi^2]\ \sigma_A |v| - \Gamma_E\, n_\psi \quad , \qquad (6)$$

where the equilibrium number density n_ψ^{eq} is determined by m_ψ and T. As is typical in gauge theories, for $T < m_\psi$, the scale of the pyrgon-antipyrgon annihilation crossection is set by m_ψ^{-2}:

$$\sigma_A |v| \simeq (\alpha^2/m_\psi^2)\,|v| \simeq \alpha^2 R^2/\sqrt{RT} \quad , \qquad (7)$$

where $m_\psi \simeq R^{-1}$. The 4-dimensional expansion rate of the universe Γ_E is $(\rho_\psi + \rho_\gamma)^{1/2}/m_{Pl}$, where ρ_ψ is the ψ energy density ($\rho_\psi = m_\psi n_\psi$ for $T < m_\psi$) and ρ_γ is the radiation energy density ($\rho_\gamma \sim T^4$). We calculate the ratio r from n_ψ in (6) and the photon number density $n_\gamma = T^3$ [10].

It is typical of all Kaluza-Klein theories, including the 5-dimensional case, that the charge carried by the pyrgons is $\alpha = R_{Pl}^{\ 2}/R^2$, so $\sigma_A |v| = R_{Pl}^{\ 4}/R^2\sqrt{RT}$. Because of the small annihilation cross section, the final value of $r = n_\psi/n_\gamma$ will be near unity, independent of R. Thus, the model predicts primordial pyrgons to be nearly as abundant as primordial photons.

Although it is not in the spirit of Kaluza-Klein theories, one may make the *ad hoc* assumption that the pyrgons carry an additional charge that is similar to α, but not strongly dependent on R_{Pl}/R. If we assume for this charge that α is constant, then $\sigma_A|v| = \alpha^2 R^2/\sqrt{RT}$, and the annihilation rate *increases* with R.

The ratio $r = n_\psi/n_\gamma$ is shown as a function of T/T_c in Fig. 1 [11]; α is set to unity and the calculation is done for various values of R with the initial condition, r = 1. In the very special case that r = 0 at t_c, r rises quickly to the envelope of the curves; if r > 1 initially, this calculation gives a lower limit on the remnant pyrgons. After decoupling, r is constant in an isentropic expansion.

The calculation could be improved technically by considering pyrgon-antipyrgon capture into Coulomb bound states with large principle quantum numbers and subsequent annihilation [12], or by considering 3-body initial states [13]. However, on the basis of the effect on monopole-antimonopole annihilation, we do not expect the improvements to change substantially the results in Fig. 1.

Today ($T_\gamma \sim 10^{-13}$ GeV, $n_\gamma \sim 400$ cm^{-3}) the energy density contributed by the photons is $\rho_\gamma \sim n_\gamma T_\gamma \sim 10^{-4}\,\rho_c$, where ρ_c is the closure density, 10^{-29} g cm^{-3}. Since the total energy density of the universe is less than $2\rho_c$ [14], the present energy density of the ψ, $\rho_\psi = m_\psi n_\psi = R^{-1} n_\psi$, must satisfy

$\rho_\psi/\rho_\gamma \lesssim 10^4$, or

$$r = n_\psi/n_\gamma \lesssim 10^4\ T_\gamma R \quad . \tag{8}$$

Using the results from Fig. 1, this limit is satisfied only if $R > (10^6\ \mathrm{GeV})^{-1}$ at $r \lesssim 10^{-14}$. If the annihilation cross section is changed, for instance, by using a different value of α or by including more annihilation channels, the bound scales as $R > (\sigma_A m_\psi^2\ 10^6\ \mathrm{GeV})^{-1}$.

In summary, the above radius is much larger than the Planck radius. If the radius were R_{Pl} and the expansion of the universe were isentropic after ψ decoupling, the photon density would equal the density of charged spin-2 pyrgons with masses of order m_{Pl}. Moreover, this catastrophic prediction follows from other models with stable pyrgons.

Two possibilities for circumventing this bound on R come to mind. The first way is to relax the assumption that n_γ is constant. If a large amount of entropy were created after compactification, it would be possible to dilute the value of r to an acceptable level. However, the baryon asymmetry would be diluted by the same amount, so it is reasonable to require the entropy generation at an epoch prior to baryon number generation. This would seem to require compactification at energy scales > 10^{14} GeV. Another possibility is to relax the assumption that R is constant during the entire evolution of the universe. Indeed, Chodos and Detweiler [15] have discussed an interesting cosmological solution of the 5-dimensional theory in which the extra dimension is "large" at early times, and subsequently shrinks as the other three spatial dimensions grow. Perhaps, at the time of compactification, $R \gtrsim (10^6\ \mathrm{GeV})^{-1}$, but now R has diminished to m_{Pl}^{-1}.

Since the 5-dimensional model is very schematic, we now ask if these results can be generalized to more realistic theories with extra dimensions. The answer to the crucial question "Are there stable pyrgons?" is model dependent. There are many types of higher dimensional theories: various versions of supergravity in 11 dimensions; superstrings in 10 dimensions [16]; pure Einstein gravity in any number of dimensions, where one might use a non-standard action [17] or a non-standard <u>ansatz</u> for the vacuum [18]. In some models there are "external" matter fields present to force the compactification, while in others, the compactification is due to the presence of a cosmological constant.

In 11-dimensional supergravity with vacuum $M^4 \times S^7$, the zero modes and pyrgons are classified by helicity and SO_8. We show that if the SO_8 is unbroken or broken in a specific way, there are stable pyrgons. This can be seen as follows: the representations of SO_8 fall into 4 nonoverlapping classes that are congruent to the $\underline{1}$, $\underline{8}_v$, $\underline{8}_s$, or $\underline{8}_c$ [19]. <u>All</u> zero modes are in the $\underline{1}$ and $\underline{8}_s$ congruency classes [20,21]. The zero modes of helicity 2, 3/2, 1, 1/2, and 0, respectively, are in the $\underline{1}$, $\underline{8}_s$, $\underline{28}$, $\underline{56}_s$, and $\underline{35}_v$ + $\underline{35}_c$, where $\underline{1}$, $\underline{28}$, and all three $\underline{35}$'s are in the $\underline{1}$ class, and $\underline{8}_s$ and $\underline{56}_s$ are in the $\underline{8}_s$ class. Any tensor product of any number of representations in the $\underline{1}$ and $\underline{8}_s$ classes remains in the $\underline{1}$ and $\underline{8}_s$ classes; it is impossible to reach representations in the $\underline{8}_c$ or $\underline{8}_v$ classes in this way. Thus, by the generalization of (5), if there are any pyrgons in the $\underline{8}_c$ or $\underline{8}_v$ classes, then some must be stable.

The pyrgon spectrum for S^7 is easily computed. The $\underline{1}$ graviton induces a harmonic expansion of mass eigenstates (1), with the sum on k in (1) over all SO_8 representation containing an SO_7 singlet: $\underline{1}$, $\underline{8}_v$, $\underline{35}_v$, $\underline{112}_v$,..., or in terms of Dynkin labels, (0000), (1000), (2000), (3000),.... The (2k+1,0,0,0) representations are all in the $\underline{8}_v$ class, but the (2k,0,0,0) representations are in the $\underline{1}$ congruency class. (See [19] for a review.) The supermultiplet accompanying the (k000) term in the harmonic expansion for the graviton is a typical representation, which means that it is obtained by multiplying the zero mode representation by (k000). Now, the product of an $\underline{8}_v$-type representation with a representation in the $\underline{1}$, $\underline{8}_v$, $\underline{8}_s$, or $\underline{8}_c$ class, respectively, is in the $\underline{8}_v$, $\underline{1}$, $\underline{8}_c$, or $\underline{8}_s$ class. Thus, the pyrgons in the supermultiplets where the spin-2 member transforms as (2k+1,0,0,0) are in the $\underline{8}_v$ and $\underline{8}_c$ classes. For example, the "first" excited modes with helicity 2, 3/2, 1, 1/2, 0, respectively, are $\underline{8}_v$, $\underline{8}_c + \underline{56}_c$, $\underline{8}_v + \underline{56}_v + \underline{160}_v$, $\underline{8}_c + \underline{56}_c + \underline{160}_c + \underline{224}_{vc}$, $\underline{8}_v + \underline{56}_v + \underline{160}_v + \underline{112}_v + \underline{224}_{cv}$.

If the SO_8 is broken, but broken without collapsing the congruency-class distinctions, then there still remain stable pyrgons. The most likely breaking patterns correspond to the little groups of the spinless zero modes, $\underline{35}_v + \underline{35}_c$ [19]. There are 4 little groups of the $\underline{35}$'s: $SU_4 \times U_1$ and $SU_2 \times SU_2 \times SU_2 \times SU_2$, which preserve all the congruency class distinctions; and SO_7 and $SO_3 \times SO_5$, which collapse the distinctions. (Explicitly, the $\underline{8}_v$ and $\underline{1}$ classes are collapsed to a $\underline{1}$ class, and the $\underline{8}_s$ and $\underline{8}_c$ classes are collapsed to a spinor class.) Thus, in the former case, some pyrgons remain stable, but in the latter case, which includes the "squashed 7-sphere" with symmetry $SO_3 \times SO_5$, we conclude that all pyrgons are probably unstable [22].

In conclusion, interesting higher-dimensional theories may have stable pyrgons with masses of order R^{-1}; their cosmological implications can provide an important constraint on model building. The crucial ingredients for computing their contribution to the energy density of the universe are the structure of the harmonic expansion, the identification of the zero models and conservation laws, and the R dependence of σ_A. If there are stable pyrgons, then they become (yet further) candidates to dominate the dark matter of the universe.

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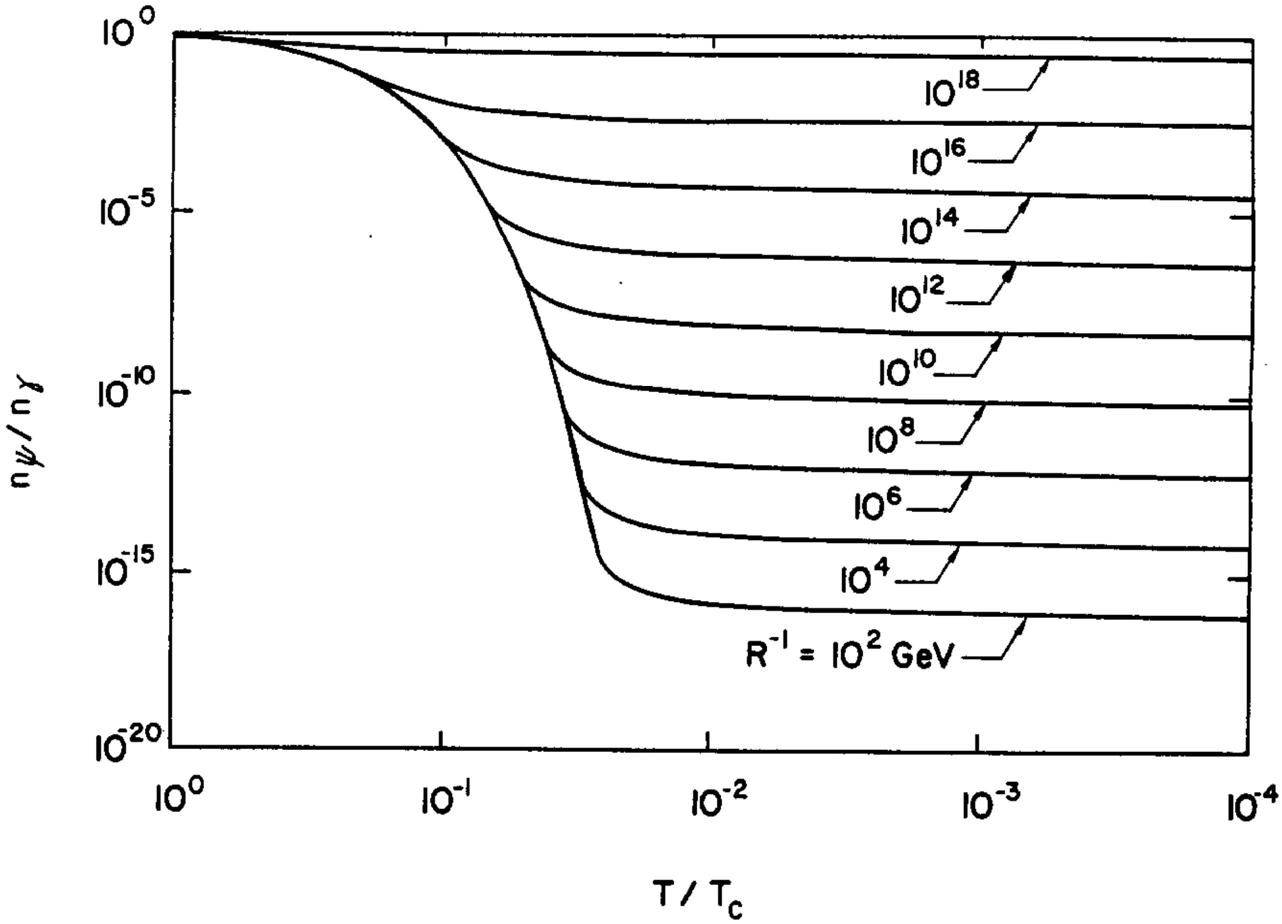

FIG. 1: The ratio r of the number density of the pyrgons n_ψ to the photon number density n_γ as a function of T/T_c for various values of the compactification scale R; $\alpha = 1$.