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How to Measure the Polarization of Top Quarks

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ABSTRACT

The production of polarized top quarks will be reflected in the polarization of T*-mesons and can be observed experimentally through their dominant weak decays. It is shown that energy and transverse momentum distributions of top meson decay products are effected and eventually enhanced or depleted by a factor between 0.5 and 1.5. Parity violating effects in the lepton distribution from top decay could be observed.

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I. INTRODUCTION

The advent of a new generation of experiments at hadron colliders in the TeV region may well lead to the experimental observation of top mesons. This possibility has stimulated several theoretical papers, [1] which give strong and electroweak production cross sections and discuss potential signatures for heavy quark production based on hard leptons, missing transverse energy through neutrinos and on event topologies.

Within these model calculations, production and decay of T and T* mesons has been simulated by the production and decay of t-quarks, assuming however isotropic decay of the t-quark in its rest frame. This assumption would be correct, if only scalar mesons (T) would decay weakly, whereas vector mesons (T*) decayed dominantly through strong or electromagnetic interactions into T analogous to the situation in charm and bottom decays. However, the hyperfine splitting between T and T* becomes extremely small, the M1 transition rate $\Gamma(T^*\to T+\gamma)$ decreases proportional m^{-3} and the weak decay rate $\Gamma(T\to B+X)$ increases with m^5 where m denotes the heavy quark mass.

The ratio between weak and electromagnetic decays is roughly one for $m\approx13$ GeV and exceeds unity by a large factor for $m>20$ GeV. [2]

Hence any quark polarization will result in a polarization of T* and - through the parity violating decay of T* - will be visible in the angular distribution of its decay products. Some consequences of this effect on top production in e^+e^- collisions have been discussed in ref. [3]. Here we want to concentrate on the effects of quark polarization on various distributions in hadronic and specifically in proton antiproton collisions. The motivation for this more detailed discussion is threefold:

1. It would be quite useful to verify experimentally the dominance of weak T* decays and the hadronization model for polarized t quarks as discussed below.
2. The measurement of top-quark polarization could give us interesting clues on the production mechanism.
3. Some of the experimentally observable quantity, in particular, the transverse momentum distributions of leptons from the decay chain $W^+ \rightarrow t(\rightarrow \ell^+ + X) + \bar{b}$ are substantially modified and hence quark polarization should be taken into account in Monte Carlo programs.

II. HADRONIZATION OF TOP QUARKS AND THEIR DECAY

We shall assume, that a polarized t quark with $S_Z=1/2$ will convert into $T^*(S_Z=1)$, $T^*(S_Z=0)$, $T^*(S_Z=-1)$ and $T(S=0)$ with relative weight 2:1:0:1. Angular distributions of decay products from $T^*(S_Z=1)$ will just look like those from $t(S_Z=1/2)$, whereas those from $T^*(S_Z=0)$ and $T(S=0)$ will be isotropic around the Z-axis. Hence in 50% of the decays, the quark spin information is preserved and transfered to the decay products. In the following analysis, we shall therefore always give the distributions for the decay of quarks, including polarization effects, of unpolarized mesons and of the statistical mixture, as described above.

The decay of a polarized quark (or lepton) is described by

$$d\Gamma = (\omega + R^{\mu} S_{\mu}) dPS \tag{1}$$

where the spin vector S_{μ} is constrained by

$$SP_t = 0 \quad ; \qquad S^2 = -1 \quad . \tag{2}$$

The scalar amplitude ω and the vector R_{μ} depend on the momenta of t and the decay products. R_{μ} can be chosen to be purely spacelike in the t-rest frame (RP_t=0). Positivity of $d\Gamma$ for all S then implies

$$-1 \leq R^2/\omega^2 \leq 0 \quad . \tag{3}$$

Specifically for leptonic top decay $d\Gamma$ is given by

$$d\Gamma = \frac{16G_F^2}{m} P_b P_{\nu} (P_t P_{\ell} - mSP_{\ell}) dPS(b, \ell^+, \nu) \quad . \tag{4}$$

If one is only interested in lepton distributions, one may integrate the other variables and finds for m_b=0

$$\frac{d\Gamma}{\Gamma} = dN = 12\xi^2(1-\xi)(1+\cos\theta)/2 \; d\xi \; d\cos\theta$$

$$\xi = 2E_{\ell}/m \quad ; \qquad \cos\theta = \vec{S}\cdot\hat{n}_{\ell} \quad . \tag{5}$$

In the following, effects of quark polarization on hadronic and electroweak production will be discussed in a systematic manner. We are mainly interested in finding situations where large effects due to quark polarization can be expected, and discuss their qualitative behavior. In order to avoid extensive Monte Carlo calculations, we will either give the fully differential rate, derive model independent bounds or calculate partially integrated distributions without the various cuts usually imposed in experimental analysis.

III. HADRONIC $t\bar{t}$ PRODUCTION

Both quark annihilation ($q\bar{q}\to t\bar{t}$) and gluon fusion ($gg\to t\bar{t}$) have been found to contribute substantially to open top production. As long as one considers tree diagrams, there will be no polarization of t quarks due to the absence of final state interactions and time reversal invariance. Hence in this approximation, all distributions which involve decay products of one T or T* meson only are unaffected by polarization effects. However, even tree diagrams may (and in general will), lead to correlations between the spins of t and $\bar{t}$, and hence to correlations between their decay products.[f1] These correlations affects all distributions, which involve decay products of t and $\bar{t}$, e.g., the invariant mass of the lepton pair from the leptonic decay of both t and $\bar{t}$. Could these effects grossly enhance or deplete distributions compared to those from uncorrelated decays? At the quark level, hadronic production cross sections and distributions will be of the form

$$dN \propto (A+S_+^{\mu}S_-^{\nu}B_{\mu\nu})dPS \quad , \tag{6}$$

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f1) Similar effects have been studied in detail for the reaction $e^+e^-\to\tau^+\tau^-$ with subsequent decay of τ^+ and τ^-. [4]

where S_+ and S_- denote the spin vectors of t and $\bar{t}$ and are subject to the constraints

$$S_\pm^2 = -1 \quad ; \quad S_+P_t = 0 \quad ; \quad S_-P_{\bar{t}} = 0 \quad . \tag{7}$$

The scalar A and the tensor $B_{\mu\nu}$ may depend on incoming and outgoing momenta. Positivity of dN leads to the following constraints on B:

$$-A \leq S_+^\mu S_-^\nu B_{\mu\nu} \leq A \tag{8}$$

for all S_+ and S_- which fulfill eq. (7). The differential distribution for t and $\bar{t}$ decay products is given by

$$dN_q \propto (A\omega_+\omega_- + R_+^\mu R_-^\nu B_{\mu\nu})dPS\ dPS_+dPS_- \quad . \tag{9}$$

Comparing dN_q to the distribution which ignores spin correlations

$$dN_0 \propto A\omega_+\omega_+ dPS\ dPS_+dPS_- \quad , \tag{10}$$

one derives the following bound from eqs. 3, 8 and 9

$$0 \leq dN_q \leq 2dN_0 \quad . \tag{11}$$

This bound also holds for all partially integrated distributions. As far as decays of real mesons are concerned, the correlated distribution is only relevant if both t and $\bar{t}$ convert into vector mesons with aligned spins, and thus only in 25% of the cases. Experimentally observable distributions are therefore given by

$$dN_E = 3/4\ dN_0 + 1/4\ dN_q \tag{12}$$

and thus bounded by the uncorrelated distribution

$$3/4\ dN_0 \leq dN_E \leq 5/4\ dN_0 \quad . \tag{13}$$

Note that this bound depends only on our assumption about the hadronization of t quarks into mesons, as described before, but is <u>independent of the hadronic production mechanism and of possible experimental cuts</u>. Furthermore, the bounds from eq. (13) represent extreme cases already, and mostly dN_E will deviate from dN_0 by less than 25%. As an illustrate example, consider the reaction

$$q\bar{q} \rightarrow t(\rightarrow e^+ + X) + \bar{t}(\rightarrow e^- + X) \quad , \tag{14}$$

close to $t\bar{t}$ threshold. The combined lepton distribution is of the form

$$\frac{dN_q}{d\xi_+ d\xi_- d\cos\theta_{+-}} = \frac{1}{2}(1 - \frac{1}{3}\cos\theta_{+-})\frac{dN}{d\xi_+}\frac{dN}{d\xi_-} \quad , \tag{15}$$

where $\xi = 2E_{lepton}/m$, θ_{+-} denotes the angle between the momenta of ℓ^+ and ℓ^- in the $t\bar{t}$ rest frame and $dN/d\xi$ is given through eq. (5). The leptons are preferentially emitted in opposite directions and thus large invariant lepton pair masses are enhanced, compared to the calculation which ignores this effect. Figure 1 shows the distributions of $\mu^2 = (P_{\ell^+} + P_{\ell^-})^2/m^2$ for uncorrelated (f=0) and correlated (f=1) decay

$$\frac{dN}{d\mu^2} = 4[1 + 27\mu^4 - 28\mu^6 + 6\mu^4(3+2\mu^2)\ln\mu^2]$$

$$+ f^2 4[-\frac{1}{3} + 6\mu^2 + 15\mu^4 - \frac{62\mu^6}{3} + 2\mu^4(9 + 4\mu^2)\ln\mu^2] \quad , \qquad (16)$$

together with the one resulting from the statistical mixture of T and T* decays (f=1/2). As expected, dN_E and dN_O are quite similar, and differ by less than 10%.

If we allow for final state interactions or relative phases, due to higher order corrections, also hadronic interactions can lead to quark polarization and show up in (parity conserving) terms of the form[f2)]

$$dN \propto \ldots + C(\vec{n}_t \times \vec{n}_{beam}) \cdot \vec{S} \; dPS \qquad (17)$$

Combined with the parity violating decay distribution from eq. (4), this leads to a parity violating term in the lepton distribution

$$dN_{lept} \propto \ldots + C(\vec{n}_t \times \vec{n}_{beam}) \cdot \vec{n}_{lept} dPS \; dPS(b,\ell^+,\nu) \quad . \qquad (18)$$

The distribution of lepton momenta perpendicular to the production plane will be one of the first and clear signals for top production. It will be worthwhile to look for linear terms or asymmetries in this distribution.

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f2) Instead of $\vec{n}_{beam}$ also other momenta, e.g., of additionally produced jets or particles could appear.

No calculation of the coefficient C in eq. (18) has been performed to date. Obviously only quark annihilation contributes. From QCD, one might expect C to be of order α_S/π and thus quite small. However, a sizeable Λ polarization has been observed in hadronic collisions out to rather large values of $P_\perp$. Eventually, also top quark polarization could turn out to be larger than expected. The resulting parity violating asymmetry as given in eq. (19) cannot be simulated by any other background, and would constitute an unambiguous proof of T and T* production.

IV. ELECTROWEAK PRODUCTION OF TOP MESONS THROUGH W-DECAY

In contrast to top quarks produced in hadronic reactions, those from the decay of W

$$u + \bar{d} \to W^+ \to t + \bar{b} \tag{19}$$

will be strongly polarized, and this will affect the distributions of top meson decay products. It is again possible to give bounds for the maximal modification, due to this effect. With arguments quite similar to those leading to eq. (11), and under the same assumptions, one can demonstrate that distributions from quark decay-including polarization are bounded by the distributions neglecting these effects:

$$0 \leq dN_q \leq 2dN_0 \quad . \tag{20}$$

In this case, decay products of only one top meson are involved, and thus all experimentally observable distributions dN_E are bounded by

$$\frac{1}{2}\, dN_0 \leq dN_E \leq \frac{3}{2}\, dN_0 \quad . \tag{21}$$

We will now evaluate some of these effects more quantitatively. The distribution of polarized top quarks is given in lowest order by

$$dN \propto \{M^4 - m^4 + (\Delta p)^2 - 2M^2 \Delta p$$

$$-2m(SP_W - Sp)(M^2 - m^2 - p\Delta)\} dPS(t,\bar{b})$$

$$\Delta = P_b - P_b \quad ; \quad p = P_u - P_d \quad . \tag{22}$$

For the distribution of final states in the reaction

$$u + \bar{d} \rightarrow W^+ \rightarrow t(\rightarrow b + \ell^+ + \nu) + \bar{b} \quad , \tag{23}$$

one obtains from eqs. (4) and (22)

$$dN \propto \{(M^4 - m^4 + (\Delta p)^2 - 2M^2 \Delta p) P_t P_\ell$$

$$-f2m^2(P_\ell P_W - P_\ell p)(M^2 - m^2 - p\Delta)\} P_b P_\nu dPS(t,\bar{b}) dPS(b,\ell^+,\nu) \quad , \tag{24}$$

where f=1,0,1/2 for quark decay, spinless meson decay, and for the statistical mixture of T and T*. This fully differential distribution should be most useful for Monte Carlo calculations. However, a number of consequences of eq. (24) are quite obvious: In principle, the W restframe can be reconstructed in most of the cases, if energy and direction of the b-jet are measured.[f3] If we then restrict ourselves to lepton distributions, the following effects are apparent.

a) The t quark is dominantly produced with negative helicity

$$r = (n_- - n_+)/n_{tot} = [1-m^2/2M^2]/[1+m^2/2M^2] \tag{25}$$

Since the ℓ^+ is preferentially emitted in the direction of $\vec{S}_t$ (cf. eq. 5), the lepton spectrum is softened considerably

$$\frac{dN}{dZ} = \begin{cases} 0 & \text{for } Z > \frac{1+\beta}{2} \\ \frac{2}{\beta}[1-3(\frac{2Z}{1+\beta})^2 + 2(\frac{2Z}{1+\beta})^3] & \text{for } \frac{1+\beta}{2} > Z > \frac{1-\beta}{2} \\ \quad -f\ r\ \frac{2}{\beta^2}[-1+6Z-3(1+2\beta)(\frac{2Z}{1+\beta})^2 + (1+3\beta)(\frac{2Z}{1+\beta})^3] & \\ 32Z^2\gamma^4[3-2Z\gamma^2(3+\beta^2)]-f\ r\ 32Z^2\gamma^4[-3+8Z\gamma^2] & \text{for } \frac{1-\beta}{2} > Z > 0 \end{cases}$$

$$\xrightarrow[m^2/M^2 \to 0]{} 2(1-Z)^2[(2Z+1)-f(4Z-1)]$$

where $Z = 2E_\ell/M$. (26)

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f3) This is quite analoguous to the reconstruction of the W kinematics from the electron momentum described in ref. [5].

and dN/dZ is shown for f=1,1/2 an 0 in fig. 2 in the limit $m^2/M^2 \to 0$.

b) Furthermore, the spin of t points preferentially against the direction of the incoming u-quark. This effect is most apparent in eq. 22 for $\vec{p}_t \perp \vec{p}_u$. Let $n_{\uparrow\downarrow}$ ($n_{\uparrow\uparrow}$) denote the number of t quarks with spin parallel (opposite) to $\vec{p}_u$. The net alignment (for $\vec{p}_t \perp \vec{p}_u$) is given by

$$(n_{\uparrow\uparrow} - n_{\uparrow\downarrow})/n_{tot} = -\frac{2m/M}{1+m^2/M^2} \quad . \tag{27}$$

Since decay leptons tend to follow the spin direction of t, this asymmetry will be reflected e.g., in a forward backward asymmetry of ℓ^+, which has to be distinguished from the one resulting from the production asymmetry of t quarks. In fact it even has the opposite sign, and will dominate for large top masses.

c) Since all spin terms in eqn. 22 vanish for $\vec{s}$ perpendicular to the production plane, all distributions of momenta out of the production plane remain unaffected. Just as in the case of hadronic production, this is a consequence of T invariance and the absence of final state interactions.

d) The quark polarization will affect the transverse momentum distribution of leptons, a quantity which can be measured without W-reconstruction. like the energy distribution in the W restframe (eq. 26), also the $P_\perp$ distribution will be softended and - for large $P_\perp$ - reduced by nearly a factor 1/2. In the limiting case $m_t \ll M_W$ and vanishing transverse momentum of W^+ it can be calculated analytically.

$$\frac{dN}{dZ_\perp} = Z_\perp \int_{Z_\perp}^{1} \frac{dZ}{Z}\frac{dN}{dZ}\frac{1}{\sqrt{Z^2-Z_\perp^2}}\frac{3}{4}\left(1+\frac{Z^2-Z_\perp^2}{Z^2}\right)$$

$$= \frac{9}{4} Z_\perp\{[(1/Z_\perp+2Z_\perp)\arccos Z_\perp - 3\sqrt{1-Z_\perp^2}]$$

$$-f[(-1/Z_\perp+6Z_\perp)\arccos Z_\perp + 8\ln\left(\frac{1+\sqrt{1-Z_\perp^2}}{Z_\perp}\right) - 13\sqrt{1-Z_\perp^2}]\}$$

$$Z_\perp = 2p_\perp/M \quad . \tag{28}$$

and is shown in Fig. 3 for f=1,1/2 and 0.

e) Although no $\vec{S}\cdot\vec{P}_t\times\vec{P}_b$ - term is present in eq. 22, higher order corrections at the $u\bar{d}$ or $t\bar{b}$ vertex could in principle induce such effects. Apart from the obvious factor α_s/π, such terms will be further suppressed by a factor m_b/M_W. The coupling of the practically massless u- and d-quarks to the intermediate boson remains of the form V-A after higher order corrections have been taken into account. For $m_b/M_W\to 0$, also the $t\bar{b}W^+$ vertex remains of the form V-A. Hence - up to terms of order $\alpha_s/\pi m_b/M_W$ - the production amplitude is only modified by a global factor and the distribution (22) remains unchanged.

To summarize:

The polarization of t-quarks is retained in 50% of the cases through the production and subsequent weak decay of T*.

For <u>hadronic associated</u> top production, this affects correlated decay distributions, the maximal change, however is only 25%. Polarization of t quarks perpendicular to the production plane is expected to be of order α_s/π and leads to parity-odd terms in the lepton distribution, e.g., of the form $n_{beam} \times n_t \cdot n_\ell$.

Lepton distributions from the W decay chain $W \to T$ (or T^*) $+X \to e^+ + X$ may increase or decrease by as much as 50% through polarization effects. Specifically, the rate for large $p_\perp$ leptons will be depleted. An additional forward backward asymmetry of leptons is induced, which is opposite in sign to the t-quark production asymmetry. The quark polarization perpendicular to the production plane and the resulting parity odd terms in the lepton distribution are expected to be rather small in this case.

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FIGURE CAPTIONS

Fig. 1a: Lepton pair mass distribution for the reaction $q+\bar{q} \rightarrow t(\rightarrow \ell^+ + X) + \bar{t}(\rightarrow \ell^- + X)$ close to threshold, as given in eq. (16). The solid, dashed and dotted lines correspond to $f^2=1$, 0.25 and 0.

Fig. 1b: Lepton energy spectrum in the W-restframe from the decay $W \rightarrow t(\rightarrow \ell + X)+b$ as given in eq. (26). The solid, dashed and dotted lines correspond to f=1, 0.5 and 0.

Fig. 1c: Lepton transverse momentum spectrum from the decay $W \rightarrow t(\rightarrow \ell + X)+b$ as given in eq. (28). The solid, dashed and dotted lines correspond to f=1, 0.5 and 0.

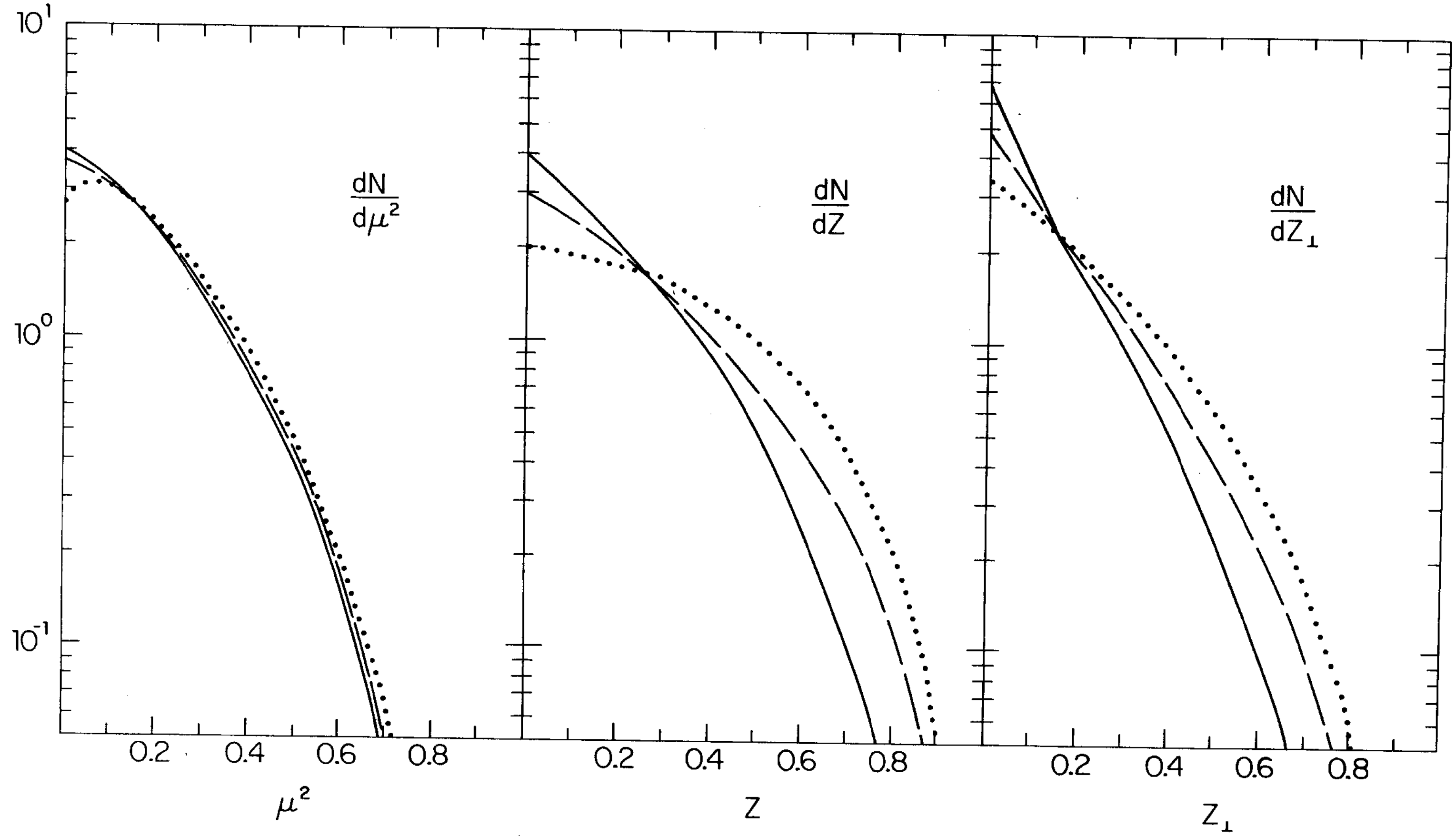

Fig. 1a

Fig. 1b

Fig. 1c