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Are There Significant Gravitational Corrections to the Unification Scale?

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Abstract

Higher dimension operators may be induced in the Lagrangian of a unified theory by gravity. We find that $c\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}H)/M_{Planck}$ can lead to substantial corrections to the unification scale. For ordinary SU(5) a second solution unifying at 10^{17}Gev can exist which has no observable proton decay but an unacceptable value of $\sin^2\theta_w$. Analogous effects in supersymmetry, however, can lead to enormous corrections. We find that supersymmetric SU(5) can unify anywhere between $5\times10^{16}<M_X<10^{18}$Gev with an acceptable $\sin^2\theta_w$ for $|c|\leq1.0$.

Though we know precious little about quantum gravity it is clear, by virtue of the successes of conventional nonabelian gauge theories at energies low compared to M_{Planck}, that gravity must be highly decoupled at such energies. This implies that the effective Lagrangian of the world at energy, $E<M_{Planck}$, consists of a set of $d\leq 4$ operators of some theory containing SU(3)xSU(2)xU(1), and possibly supersymmetry, at presently accessible energies, which may have nontrivial new physics at higher energies (e.g. grand unification), but is essentially nongravitational up to M_{Planck}. In addition, there may be $d\geq 5$ operators that are induced by gravity and enter the Lagrangian scaled by factors of $(M_{Planck})^{-(d-4)}$ with order unity coefficients and which are subject only to the constraints of the symmetries (e.g. gauge invariance, supersymmetry, etc.) of the low energy theory. Such effects are known to occur, for example, in the presence of gravitational instantons.[1] Furthermore, if we have a unified theory associated with a scale M_X which pre-unifies at a scale $M_{X'}$, similar higher dimensional operators will occur in the lower energy theory scaled by $(M_{X'})^{-(d-4)}$ with calculable coefficients that depend upon the gauge, Higgs, and Higgs-Yukawa couplings (and are probably small).

This general observation has been emphasized by Ellis and Gaillard.[2] As a specific and clever application, Ellis and Gaillard have shown that the "unsuccessful" SU(5)[3,4] prediction for m_d/m_e may be understood as a consequence of assumed gravitationally induced d=5 operators in the Lagrangian. To paraphrase ref(1), since the Higgs-Yukawa couplings of d and e to the $\bar{\underset{\sim}{5}}$ of Higgs are only of order 10^{-5} to 10^{-6}, corrections of order M_X/M_{Planck} are important. Ellis and Gaillard show that the operator

$Tr(\underset{\sim}{10}_{fermion} \times \underset{\sim}{\bar{5}}_{fermion} \times \underset{\sim}{\bar{5}}_{Higgs} \times \underset{\sim}{24}_{Higgs})/M_{Planck}$ is precisely such as to destroy the offending condition $m_d = m_e$ at M_X, and give rise to the large corrections that are observed. The heavy quark to lepton mass ratios are immune to this correction and remain successful predictions of the model.

Ellis and Gaillard do not compute the coefficient of this operator, but its presence is certainly plausible, for example, per the kinds of effects discussed in ref.(1). To account for the observed mass ratio the coefficient of this new operator must be of order 0.3. Of course, the situation is not very satisfactory, because the presence and size of such $d \geq 5$ operators cannot be determined in quantum gravity, but is analogous to determining the structure of the weak Hamiltonian with sub-electron volt energies. We cannot a priori rule such terms out and should adopt the view that they are present with the most general structure compatible with the "known" low energy theory. It is thus important to identify all of the physical implications of such terms.

Presently we wish to point to a d=5 operator that may occur with potentially significant implications for grand unification, in particular, in estimating the scale of the unification of the theory. In supersymmetric theories when $M_X = 10^{17}$Gev these effects become pronounced and we find large correction factors of order $\exp(\mathcal{O}(1)\, \alpha_G^{-1} M_X/M_{Planck})$. Also, for a large range of coefficients of this operator we find that the unification condition of a general grand unified theory[5] admits two solutions; in ordinary SU(5) the conventional solution of order 10^{15}Gev remains, while a second solution of order 3×10^{17}Gev appears. Though observable proton decay disappears in this second solution, so too does the successful prediction for $\sin^2\theta_W$. The effects of this term on the usual unification scale of SU(5) are essentially negligible in $\sin^2\theta_W$, but can lead to about a 50% variation in M_X over a large range of gravitational perturbations. Alternatively, in supersymmetric SU(5) we find that M_X is extremely sensitive to the gravitational perturbations in a range comparable to the strength of the EG operator coefficient. For a coefficient less than -1.02, supersymmetric SU(5) cannot unify at all, and for $|c| \leq 1$, we find almost a two order of magnitude range of unification scales with acceptable $\sin^2\theta_W$ values.

Consider the unification of SU(3)xSU(2)xU(1) at a scale M_X into a group G broken by an adjoint representation Higgs field, H, of G. H we assume develops a vacuum expectation value, V (which is related in SU(5) to the unification scale by $M_X = g\sqrt{5/6}\,V$; our normalization of H is specified below). Then there may occur in the Lagrangian of the full unified theory for energies $M_X < E < M_{Planck}$ the term:

$$\mathcal{L} = \mathcal{L}_0 + c\,\frac{\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}H)}{M_{Planck}} \qquad (1)$$

where $G_{\mu\nu} = G^A_{\mu\nu}\lambda^A/2$ is the field strength of G. This term may arise with c of order unity in quantum gravity or in a preunified theory when G imbeds into some larger group G' at a new scale $M_{X'} \gg M_X$. In the latter case M_{Planck} must be replaced by $M_{X'}$ in eq.(1) and c becomes calculable once G and G' are specified. In either case, there will be renormalization group enhancements or suppressions (presumably small) of the form $(\alpha(M_X)/\alpha(M_{Planck}\text{ or }M_{X'}))^{\gamma/2b_0}$ where γ is the calculable log dimension of the d=5 operator in eq.(1). We have not evaluated this anomalous dimension, but mention that there can be nontrivial mixings in two loops to the previously proposed operator of Ellis and Gaillard; these mixings are potentially large if there are heavy fermions ($m \sim 200$Gev), but are otherwise expected to be negligible. It would be amusing to adopt the point of view that we "know" in SU(5) the EG operator coefficient from m_d/m_e and then ask what values c can have in eq.(1) for consistency with the renormalization group mixing effects. Presently we will ignore these renormalization effects.

We will now examine the effects of such a term for arbitrary c. We work only to one loop accuracy since the unknown value of c is the principal uncertainty in the analysis. Consider the breaking of the theory at a scale M_X including the term of eq.(1). In general we will then have the following modified kinetic terms for the SU(3)xSU(2)xU(1) gauge fields for $E \ll M_X$:

$$-\frac{1}{2}(1+\varepsilon_3)\,\mathrm{Tr}\,G_{\mu\nu}G^{\mu\nu}_{SU(3)} - \frac{1}{2}(1+\varepsilon_2)\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu}_{SU(2)} - \frac{1}{4}(1+\varepsilon_1)\,F_{\mu\nu}F^{\mu\nu}_{U(1)} \qquad (2)$$

where the ε_i are given by $C_i(cV)/M_{Planck}$, the C_i being Clebsch-Gordon coefficients associated with the breaking by the adjoint Higgs field H, and generally these differ for differing i. For example, in the case of SU(5) where H develops the vacuum expectation value $\langle H \rangle = V\,(2,2,2,-3,-3)/\sqrt{15}$, the ε_i are given by:

$$\varepsilon_3 = \frac{-4}{\sqrt{15}} \frac{cV}{m_{Planck}}, \quad \varepsilon_2 = \frac{6}{\sqrt{15}} \frac{cV}{m_{Planck}}, \quad \varepsilon_1 = \frac{1}{\sqrt{15}} \frac{cV}{m_{Planck}} \tag{3}$$

where,

$$V = \left(\frac{3}{10\pi\alpha_G} \right)^{1/2} M_X \tag{4}$$

Thus, the new term of eq.(1) induces effective dielectric constants which differ slightly for the three coupling constants below M_X, but which will vanish as a power of energy above M_X.

The effect of these dielectric constants below M_X is to redefine the physically observed coupling constants relative to the values that actually meet at the unification scale. By a finite field and coupling constant renormalization the physical coupling constants and fields are given by:

$$g_i = (1+\varepsilon_i)^{-1/2} g_{io}, \qquad A_{\mu i} = (1+\varepsilon_i)^{1/2} A_{\mu io} \tag{5}$$

where g_{io} and $A_{\mu o}$ are the "bare" coupling constants and fields. Having absorbed the dielectric constants in the Lagrangian of SU(3)xSU(2)xU(1) we can now perform the usual perturbative renormalizations to compute the evolution equations of the g_i. Thus it is the g_i that satisfy the usual renomalization group equations, while the g_{io} are the bare coupling constants that join to define the unification scale.

Hence, the unification scale M_X, the unification value of the running coupling constant, $\alpha_G = g^2/4\pi$, and $\sin^2\theta_W$ are determined implicitly by the system of coupled equations:[5]

$$\alpha_3^{-1}(M_W) = (1+\varepsilon_3)\alpha_G^{-1} - b_3 \log(M_X/M_W)$$

$$\frac{\sin^2\theta_W}{\alpha_{EM}} = (1+\varepsilon_2)\alpha_G^{-1} - b_2 \log(M_X/M_W) \qquad (6)$$

$$\mathcal{C}^{-1}\frac{\cos^2\theta_W}{\alpha_{EM}} = (1+\varepsilon_1)\alpha_G^{-1} - b_1 \log(M_X/M_W)$$

where $\mathcal{C}$ is the "imbedding coefficient" of U(1) into G. For SU(5) we have $\mathcal{C}=5/3$, $b_3=(11-4n_g/3)/2\pi$, $b_2=(22/3 - 4n_g/3 - n_h/6)/2\pi$ and $b_1=(-4n_g/3- n_g/10)/2\pi$. The ε_i are dependent upon α_G and M_X through eq.(3). It is convenient to "solve" eq.(6):

$$\log(M_X/M_W) = D^{-1}\left[(1+\varepsilon_3)\alpha_{EM}^{-1}(M_W) - (1+\varepsilon_2+\mathcal{C}(1+\varepsilon_1))\alpha_S^{-1}(M_W)\right]$$

$$\alpha_G^{-1} = D^{-1}\left[b_3\alpha_{EM}^{-1}(M_W) - (b_2+\mathcal{C}b_1)\alpha_S^{-1}(M_W)\right]$$

(7a, b)

$$\sin^2\theta_W = D^{-1}\left[b_3(1+\varepsilon_2) - b_2(1+\varepsilon_3) + c\,\alpha_{EM}\,\alpha_S^{-1}\left((1+\varepsilon_1)b_2 - (1+\varepsilon_2)b_1\right)\right]$$

$$D = b_3(1+\varepsilon_2 + c(1+\varepsilon_1)) - (b_2 + c\,b_1)(1+\varepsilon_3)$$

(7c,d)

In general the above equations may possess none, one or two solutions. For example, eq.(7a) is depicted graphically in Fig.(1) where the dashed line represents the lhs. of eq.(7a) and the curves are the rhs. for various choices of c. We have used SU(5) parameters with $\alpha_{EM}^{-1}(M_W)$=127, and $\alpha_S^{-1}(M_W)$=7. The intersections of the curves with the dashed lines represent solutions for the given c coefficient. We see that for $c<c_{critical}=-17.21$ there is no solution. For $c>c_{critical}$ there are in general two solutions, however, when $c>c'=-.05$ the upper second solution corresponds to $M_X>10^{19}$Gev and is unphysical since our gravitational perturbation is only meaningful below M_{Planck}. In Table I we present the values of M_X, $\sin^2\theta_W$ and α_G^{-1} for various coefficients, c, for the upper solution. We also give $\delta M_X/M_X$ for the standard unification point in SU((5). Here we find that $|\delta\sin^2\theta_W/\sin^2\theta_W| \leq .015$ over the entire range of c values. These are first order corrections to M_X which are expected to apply even when one computes the usual unification point to two-loop accuracy. We see that the largest effect upon the usual value of M_X occurs for the very large and unrealistic value, $c=-10$, and corresponds to only a 30% increase in the unification scale, thus a 3-fold increase in the proton lifetime. The second solution never admits an acceptable value of $\sin^2\theta_w$.[6] Thus, we conclude that the effects of such perturbations upon the conventional SU(5) model are not very pronounced except for the unrealistically large values of $|c|$. The second solution can be dismissed on phenomenological grounds.

The more interesting case of supersymmetric SU(5)[7,8] is displayed in Fig.(2). Here we have $b_3=(9-m/2)/2\pi$, $b_2=(6-h/2-m/2)/2\pi$, and $b_1=(-3h/10-m/2)/2\pi$. We will adopt the minimal choice of $m=12$(7). We find now that for $c<c_{critical}=-1.02$ the theory cannot unify. In Table II. the values of the unification parameters for both solutions are given. We see that now the upper solution has an acceptable value for $\sin^2\theta_w$ for $-1\lesssim c\lesssim -.5$. Furthermore, the lower solution is subject to large corrections over the full range of $|c|\leq 1.0$. We see that acceptable unification points exist for $5\times10^{16}\text{Gev}<M_X<10^{18}\text{Gev}$ in the small range of $|c|\leq 1$.

In supersymmetric theories the operator of eq.(1) must respect the low energy supersymmetry of the theory. We find that the analogue to the term in eq.(1) is now:

$$\frac{\mathrm{Tr}(W^\alpha W_\alpha \tilde{H})|_{\theta\theta}}{M_{\mathrm{Planck}}} + h.c.$$

$$= \frac{1}{M_{\mathrm{Planck}}} \mathrm{Tr}\Big(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} H + HD^2 + i\bar{\lambda}\not{D}\lambda H - \frac{1}{4}\lambda\lambda F$$

$$+ i\sqrt{2}\,\lambda\Psi D + \frac{1}{\sqrt{2}}\lambda(\sigma^\mu\bar{\sigma}^\nu)\Psi F_{\mu\nu} \Big) + h.c. \qquad (8)$$

where W^α and H are gauge and Higgs superfields and the trace is over the external SU(5) indices.[9] In addition to the gauge fields acquiring dielectric constant corrections, now the gauge-inos also are subject to such effects. These effects are extremely small since the F-term vacuum expectation value is associated with the scale of supersymmetry breaking, $O(M_W)$ in the present model, and corrections of order M_W/M_{Planck} are unlikely observables.

It is of interest to develop the full effective Lagrangian of d=5 and d=6 operators that may induce perturbations in supersymmetric theories where these effects are likely to be observable. Such effects may give ultimately an experimental handle on quantum gravitation, should low energy supermultiplets emerge in collider experiments. Methods of estimating the coefficents of such terms are also desired, particularly in supergravity where it may be possible to obtain reliable results.

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FIGURE CAPTIONS

Fig. 1: Graphic representation of eq. (7a) for SU(5). Solutions are the intersection points of the solid curves (rhs) with the dashed line (lhs) for the indicated values of c. $c=-17.21$ is the limiting case of no unification solution.

Fig. 2: Graphic representation of eq. (7a) for supersymmetric SU(5). (See Fig. 1 caption). No solution exists for $c<-1.02$ and large effects upon the usual unification scale are seen for $|c|\leq 1.0$.

Table I. SU(5) unification values for given gravitational perturbation, c. The "new solution" clearly does not exist for c≥0. $\delta M_X/M_X$ is given for the usual unification point where $M_X \sim 4\times10^{14}$ GeV in two loops.

c	Usual Solution: $\delta M_X/M_X$	New Solution: M_X	α_G^{-1}	$\sin^2\theta_W$
-10	.32	2.13×10^{16}	42.08	.176
-8	.23	3.05×10^{16}	42.2	.173
-6	.16	4.67×10^{16}	42.34	.168
-4	.10	8.19×10^{16}	42.53	.162
-2	.04	2.01×10^{17}	42.84	.153
0	0	X	X	X
2	-.04	X	X	X
4	-.08	X	X	X
6	-.11	X	X	X
8	-.14	X	X	X
10	-.16	X	X	X

Table II. Supersymmetric SU(5) unification values for gravitational perturbation, c, to one-loop precision.

c	Lower Solution: M_X	α_G^{-1}	$\sin^2\theta_W$	Upper Solution: M_X	α_G^{-1}	$\sin^2\theta_W$
-1.0	1.60×10^{17}	23.22	.218	2.42×10^{17}	23.13	.215
-.8	1.11×10^{17}	23.30	.222	4.68×10^{17}	22.99	.209
-.6	9.41×10^{16}	23.34	.223	8.11×10^{17}	22.87	.204
-.4	8.35×10^{16}	23.36	.224	1.55×10^{18}	22.73	.198
-.2	7.60×10^{16}	23.38	.225	4.16×10^{18}	22.51	.190
0.0	7.02×10^{16}	23.40	.226	$>M_{Planck}$	X	X
.2	6.56×10^{16}	23.41	.226	X	X	X
.4	6.17×10^{16}	23.43	.227	X	X	X
.6	5.85×10^{16}	23.44	.227	X	X	X
.8	5.67×10^{16}	23.45	.227	X	X	X
1.0	5.32×10^{16}	23.46	.228	X	X	X

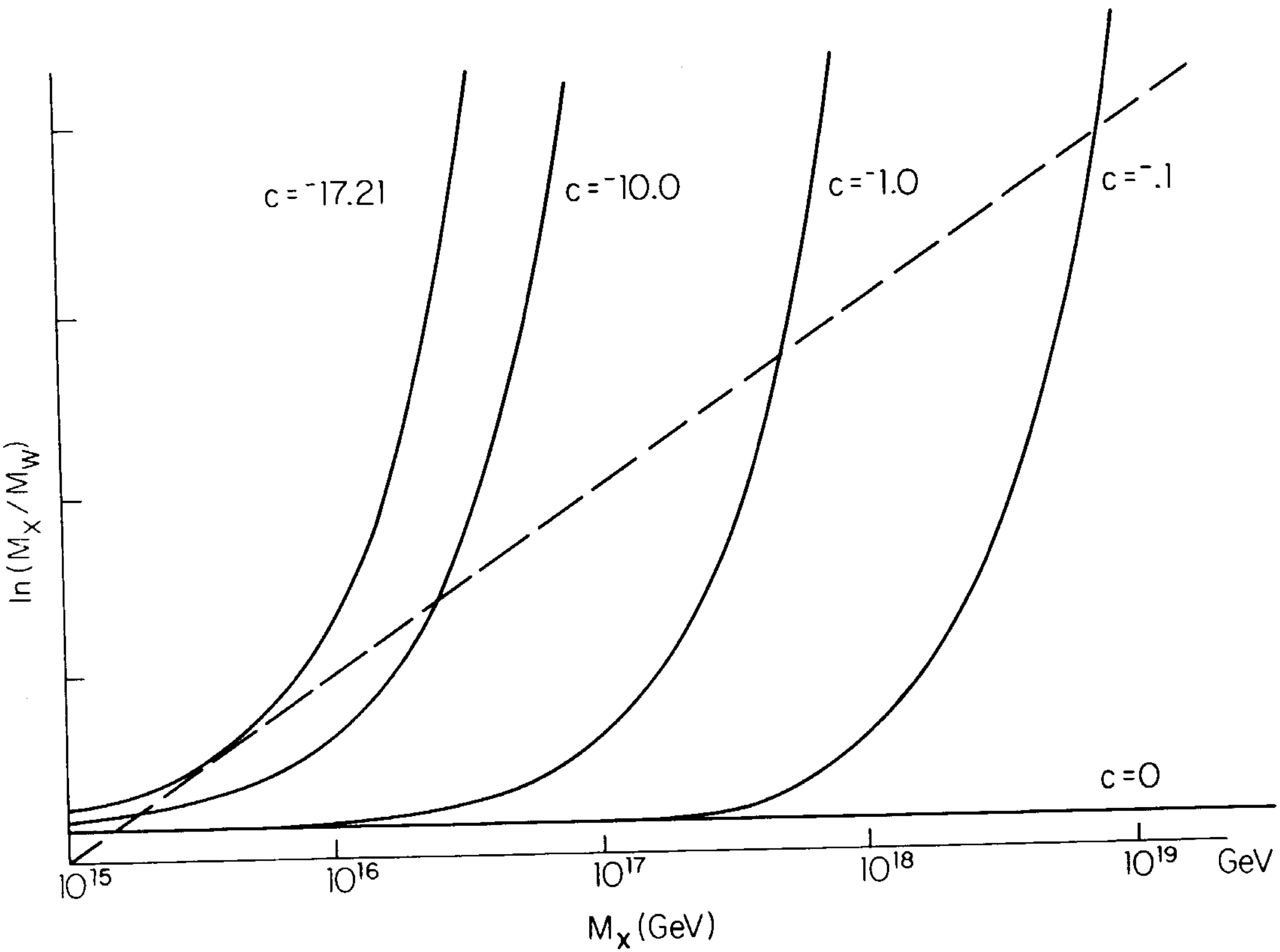
ln (M_X / M_W)
c = ⁻17.21
c = ⁻10.0
c = ⁻1.0
c = ⁻.1
c = 0
10^15
10^16
10^17
10^18
10^19
GeV
M_X (GeV)

Fig. 1

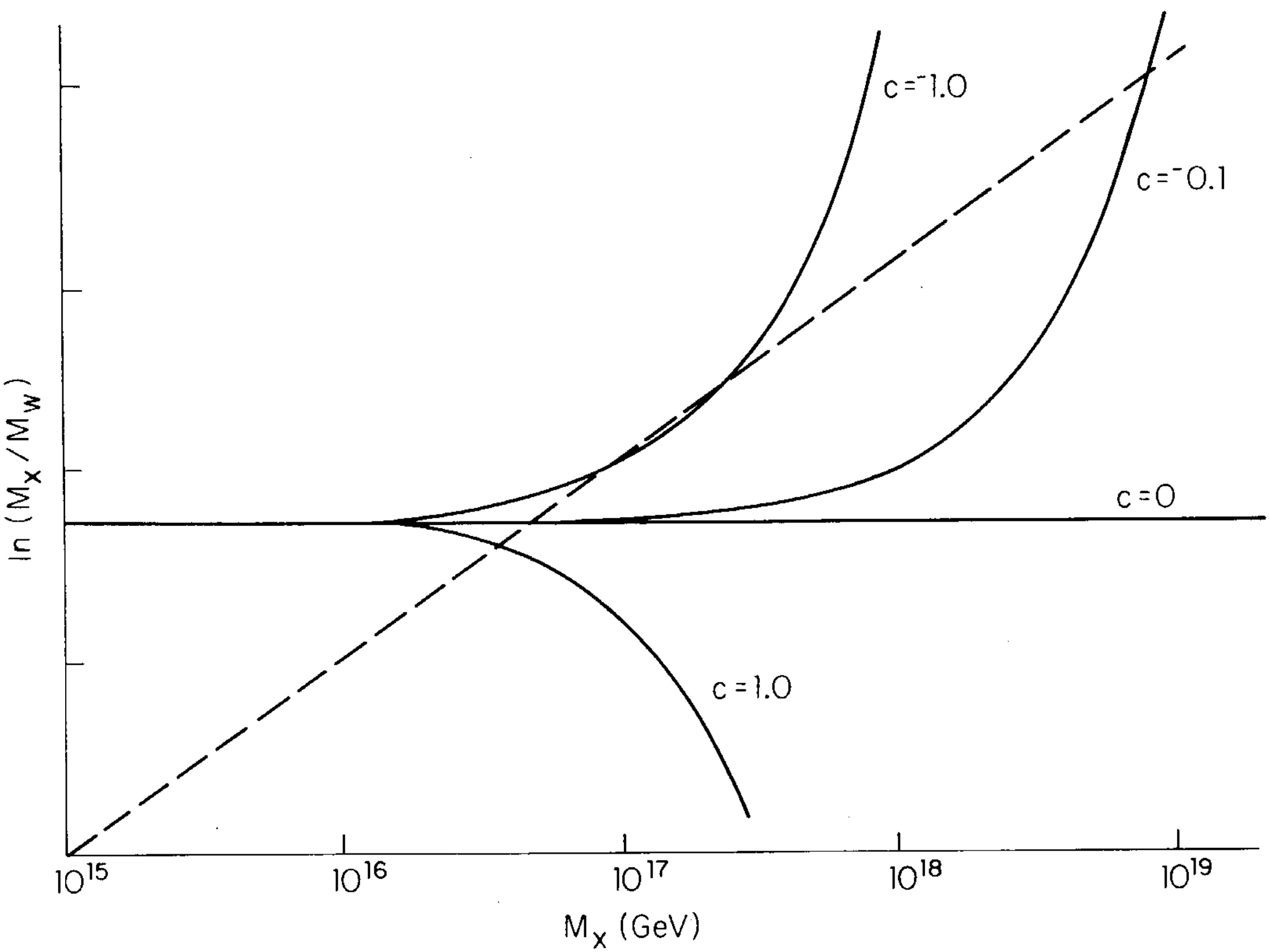

Fig. 2