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Ultra High Energy Cosmic Ray Neutrinos

Christopher T. Hill
Fermi National Accelerator Laboratory, P.O. Box 500
Batavia Illinois, 60510

David N. Schramm
The University of Chicago, Chicago, Illinois
and
Fermi National Accelerator Laboratory

Abstract

Neutrinos, produced by the collisions of ultra high energy cosmic rays and the $3^{O}K$ background radiation, require careful treatment of the evolving cosmic ray spectrum. The resulting neutrino differential energy spectrum is flat up to energies of order 10^{19}ev, thus most events are expected at this energy. The total ν_e-flux should be significantly larger than the proton flux at $\sim 5\times10^{19}$ev when the recoil proton is correctly treated in photomeson production and when the Bethe-Heitler process (e^+e^- pair production) is incorporated. Based upon the observed primary proton spectrum we obtain a lower limit on the neutrino flux of $\sim 5.2/\mathrm{km}^2\mathrm{yr\ sr}$ implying roughly .4 detected upward moving events per year in the present Fly's eye range of sensitivity.

Ultra high energy cosmic rays, presumably protons, can undergo meson photoproduction reactions during collisions with the $3^{o}K$ cosmological background photons. These processes become important above energies of order $2x10^{19}$ev for cosmic rays traversing a few interaction lengths (an interaction length corresponding to the peak value of the total photoproduction cross-section, $\sim 500\ \mu b$, and a photon density of $400/cm^3$, is L=1.6 Mpc compared to the Virgo cluster range of 20 Mpc; asymptotically the total cross-section drops to $100\ \mu b$, corresponding to an 8 Mpc interaction length. Most of the effect occurs near the resonance). First suggested by Greisen and Kuzmin and Zatsepin(1) and later analyzed by Stecker and Strong et. al., Hillas, and others(2) the general conclusion was that at energies exceeding $3x10^{19}$ev the extragalactic component of the CR spectrum (believed to be 100% of the spectrum at these energies) must be cut-off. This seems to be at odds with the reported observations of events extending up to 10^{20}ev and a general flattening of the CR spectrum above $2x10^{19}$ev(3).

However, recently we have reanalyzed the spectrum evolution at these energies, introducing a transport equation which properly includes the recoil proton(4) (which is simply discarded in the mean energy loss, or attenuation length analyses of ref.(2). Indeed, there is considerable confusion in the literature over the simple kinematics of $\gamma p \to \pi p$ in this reference frame; the proton energy loss is severely limited and the Universe is not opaque to protons below $5x10^{19}$ev via this process alone). We find that the quantity $E^3 dN/dE$, which is essentially flat below 10^{19}ev, develops a bump between $2x10^{19}ev < E < (6\pm 1)x10^{19}$ev, corresponding to the pile-up of recoil protons which have dropped below threshold to undergo further photoproduction reactions (see Fig.(1)). Above $(6\pm1)\times10^{19}$ev we expect the rapid cut-off as anticipated in ref.(1). The shape of the pile-up is relatively insensitive to the shape of the input primary spectrum after a few interaction lengths, apart from overall normalization. Events above 10^{20}ev would imply an evolution through fewer than 10 interaction lengths (which marginally accomodates the Virgo cluster as source, however the spectrum will be considerably evolved even at this range).

An important corollary phenomenon is the production of neutrino, electron and photon secondaries by the processes:

$$
\begin{aligned}
&p+\gamma \to n+\pi^+ \to p+2e^+ + 2\nu_e + \nu_\mu + \bar{\nu}_\mu \\
&\bar{p}+\gamma \to \bar{n}+\pi^- \to \bar{p}+2e^- + 2\bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu \\
&p+\gamma \to p+\pi^0 \to p+2\gamma \\
&\bar{p}+\gamma \to \bar{p}+\pi^0 \to \bar{p}+2\gamma
\end{aligned}
\tag{1}
$$

These have been discussed elsewhere[(5)] though not in the present context of the transport evolution of the proton spectrum, which is essential. Also, several new observations will be presented concerning the Bethe-Heitler (production of an e^+e^- pair in the Coulomb field of the proton) process here.

The analysis of the electron spectrum is complicated by the evolution due to Compton scattering off of the backgound photons and synchrotron radiation energy loss in galactic and intergalactic B-fields. Similarly, the 2-photon process, $\gamma+\gamma\to e^+ + e^-$ has a large cross-section since the scale is set by m_e. Thus, the evolution of both electrons and photons involves mixing terms in the appropriate transport equation. While clearly of interest to gamma ray observers, we will defer consideration of these to a later paper. Presently, we will focus attention on the neutrino spectrum.

Neutrinos are unaffected by magnetic fields and have sufficiently small cross-sections that interactions with cosmic matter may be neglected. Even at galactic densities, ~ 1 baryon/cm^3 ($\rho_{gal} \sim 10^6 \times \rho_{critical}$), the total weak cross-section $\sim 10^{-33}$cm^2 implies an interaction length of 10^{15}lyr.$\gg R_U$, the radius of the Universe. Thus, the neutrino spectrum will survive unevolved from the most distant point of production, whether a primary neutrino production spectrum or the induced spectrum from photoproduction reactions. The detection and observation of such neutrinos is clearly of enormous interest.

A cosmic ray proton can undergo meson photoproduction if the energy, E_p, exceeds the threshold energy, $E_t = m_p m_\pi / 2\bar{E}_\gamma$, where $\bar{E}_\gamma$ is the average of the photon energy plus longitudinal momentum, $\bar{E}_\gamma = E_\gamma(1+\cos\theta)/2$, which for 3°K is 7×10^{-4}ev; hence $E_t \sim 10^{20}$ev. In practice the high energy Boltzmann tail allows collisions to be likely down to $E_p \sim (6\pm 1)\times 10^{19}$ev. In a typical collision with incident proton energy E_p the recoil proton will have energy E_p' in the range:

$$E_- \leq E_p' \leq E_+ \qquad (2)$$

where:

$$
\begin{aligned}
E_\pm &= E_p\left\{\frac{1}{2}\left(1+\frac{\Delta_-}{s}\right) \pm \frac{\sqrt{F}}{s}\right\} \\
\Delta_\pm &= m_p^2 \pm m_\pi^2 \\
F &= \frac{1}{4}\left\{(s-\Delta_+)^2 - \Delta_+^2 + \Delta_-^2\right\} \\
s &= 4E_\gamma E_p + m_p^2
\end{aligned}
\tag{3}
$$

Similarly, the produced π's will have energy in the range:

$$
E_{\pi-} \le E'_\pi \le E_{\pi+} \tag{4}
$$

where:

$$
E_{\pi\pm} = E_p\left\{\frac{1}{2}\left(1-\frac{\Delta_-}{s}\right) \pm \frac{\sqrt{F}}{s}\right\} \tag{5}
$$

The only approximation employed above is the self consistent one, $E_p' \gg m_p$; $E_\pi' \gg m_\pi$.

The predominant processes are those in the resonance region of the photoproduction cross-section because of the large size of the cross-section here and the fact that the primary spectrum is falling with energy rapidly and the high energy behavior of the cross-section is thus ineffective. We shall assume that the process, $p+\gamma \rightarrow \Delta \rightarrow \pi + N$ on or near the Δ resonance is the only important contribution. Here the total cross-section is between 300 and 500 μb and we may take $s = m^2 \sim 1.5$ Gev^2 and the average recoil energy we approximate by:

$$\bar{E}'_p \sim \frac{1}{2}\left(E_+ + E_-\right) = \frac{1}{2}\left(1 + \frac{m_p^2 - m_\pi^2}{m_\Delta^2}\right) E_p \qquad (6)$$

(this is not unreasonable here and corresponds to the approximation of an even spherical harmonic angular distribution in the ρ rest frame; in fact the angular distribution evolves rapidly through the resonance with large even components). Hence, defining the inelasticity:

$$(1-\kappa) \equiv \frac{1}{f} \sim \bar{E}'_p / E_p \sim .78; \quad \kappa \sim .22 \qquad (7)$$

A proton of incident energy, E_p will thus suffer a number, n, collisions before it decouples by dropping below threshold. n is given by:

$$(1-\kappa)^n \sim E_t / E_p; \quad n = n(E_p) = \ln(E_p/E_t)/\ln f \qquad (8)$$

Finally, if the primary spectrum has a differential distribution, dN/dE = c/E^p, then the total number of produced π's per proton during the evolution of the spectrum is given by:

$$\bar{n} = \frac{1}{\ln f}\left\{\int_{E_t}^{\infty} \ln(E/E_t)\,\frac{1}{E^p}\,dE \Big/ \int_{E_t}^{\infty}\frac{1}{E^p}\,dE\right\}$$

$$= \frac{1}{(p-1)\ln f} \qquad (9)$$

Thus, for a primary spectrum falling like $1/E^3$ we obtain $\bar{n} = 2$. A flatter spectrum such as $1/E^2$ to $1/E^{1.5}$, which can account with the proper normalization for the ankle structure in the spectrum gives $\bar{n} = 4$ to $\bar{n} = 8$.

Furthermore, we expect the average π energy to be given, for an incident proton energy E_p, by:

$$E_\pi' \sim \frac{1}{2}\left(E_{\pi^-} + E_{\pi^+}\right) = \frac{1}{2}\left(1 - \frac{m_p^2 - m_\pi^2}{m_\Delta^2}\right)E_p$$

$$\sim .22\ E_p \qquad (10)$$

Convoluting with a differential distribution of index p gives:

$$\overline{E}_\pi' \sim .22 \int_{E_t}^{\infty} E \cdot \frac{1}{E^p} dE \Big/ \int_{E_t}^{\infty} \frac{1}{E^p} dE$$

$$\sim .22 \; E_t \left(\frac{p-1}{p-2}\right) \qquad (11)$$

or $\overline{E}_\pi' \sim .44\ E_t \sim 2.6\times10^{19}$ev with p=3. With p=2 we must impose an upper limit, E_c, on the integration spectrum to obtain $\overline{E}_\pi' \sim .22(p-1)\ln(E_c/E_t)E_t$ and for $p < 2$ this approximation clearly breaks down and we must consider the transport of the recoil proton which effectively reduces the cut-off energy.

The above discussion should be regarded as a sketch of the more complete results expected from a detailed numerical integration of the full spectral transport equations including the produced pion spectrum and incorporating the full laboratory data from low energy photoproduction experiments. This work is in progress (and preliminary results support the above remarks).

We thus see that the pion distribution will be centered in a range $2.6\times10^{19}\text{ev} < E < 6.5\times10^{19}\text{ev}$. A measure of the width of the distribution is seen by noting that the minimum energy of produced pions is given by $E_{\pi-}$ at threshold, or:

$$E_{\pi^-} \rightarrow \frac{1}{2} E_t \left(1 - \frac{m_p^2 - m_\pi^2}{m_\Delta^2}\right) \sim 7.8 \times 10^{18} \text{ev} \qquad (12)$$

In the π rest frame a given neutrino energy is defined by E_{ν_0} and the angular distribution is isotropic, $dN/d\cos(\theta)$ = constant. In the boosted frame the given neutrino has energy:

$$E_\nu = \gamma E_{\nu_0}(1 - \cos\theta); \qquad \gamma = \frac{E_\pi}{m_\pi} \qquad (13)$$

Thus, for fixed E_{ν_0} we have $dE_\nu = -\gamma E_{\nu_0} d\cos(\theta)$ and the energy distribution in this frame is constant, dN/dE= constant/γE .

In the rest frame of the decaying π^+ there will be initially a μ^+ and ν_μ ; the ν_μ energy is, $E_{\nu_0} = (m_\pi^2 - m_\mu^2)/2m_\pi$ and the μ energy is $E_\mu = (m_\pi^2 + m_\mu^2)/2m_\pi$. The μ^+ of course decays into $e^+ + \nu_e + \bar{\nu}_\mu$ and we may neglect the μ^+ recoil and assume that the energies of the decay fragments are all equal to $(m_\pi^2 + m_\mu^2)/6m_\pi$. In the boosted frame we have $m_\pi = \bar{E}_\pi =$ 2.6 to 6.5×10^{19}ev. Thus, the boosted neutrino spectrum may be described by:

$$dn_i/dE = \mathcal{N}\,\theta(E_{\nu i} - E)/E \qquad (14)$$

where $\mathcal{N}$ is an overall normalization and we take a step function approximation to the actual smooth distribution. The cut-off energies are:

$$E_{\nu_\mu} = \frac{\bar{E}_\pi}{m_\pi}\left(\frac{m_\pi^2 - m_\mu^2}{2m_\pi}\right) \sim (.43 \text{ to } 1.5)\times 10^{19}\,\text{ev}$$

$$E_{\nu e} = \frac{\bar{E}_\pi}{m_\pi}\left(\frac{m_\pi^2 + m_\mu^2}{6m_\pi}\right) \sim (.52 \text{ to } 1.8)\times 10^{19}\,\text{ev}$$

$$E_{\bar{\nu}_\mu} = E_{\nu e} \tag{15}$$

The recoil neutron decays to a proton, positron and electron neutrino. In the neutron rest frame the neutrino energy satisfies:

$$E_{\nu e o} = \frac{(m_n - m_p)^2 - m_e^2}{2(m_n - m_p)} \sim .55 \text{ Mev} \tag{16}$$

The maximum ν_e energy is therefore:

$$E_{\nu e} = \left(\frac{E_p}{m_p}\right)\times 5.5\times 10^{-4}\,\text{Gev} \sim 3.5\times 10^{15}\,\text{ev} \tag{17}$$

Thus, we have several distinct species with flat spectra; the ν_e, ν_μ, and $\bar{\nu}_\mu$ spectra extend up to .5 to $1.8\text{x}10^{19}$ev and a second ν_e spectrum extends up to $3.5\text{x}10^{15}$ev. Each differential spectrum is flat due to the isotropy in the parent rest frame. A flat differential distribution implies that most of the observed events will be in the highest energy bins, i.e. the number of events between E_1 and E_2 is $(E_2-E_2)/E_{max}$ and is only significant when $E_2 \sim E_{max}$.

In our previous analysis[4] we have neglected the Bethe-Heitler process, also first suggested by Greisen in his classic letter[1] and extensively analyzed by Blumenthal[6]. Though a large cross-section, $\sim 10^{-27}cm^2$, this process leads to very small energy loss per collision. Nonetheless, the accumulated effect of many collisions will displace the pile-up of Fig.(1) toward the left for cosmic rays whose total path length exceeds $2x10^9$lyr. and their contribution to the pile-up will be obscured by the primary $1/E^3$ spectrum of nearer sources. We may normalize to the number of protons in the spectrum above $5x10^{19}$ev to count the total number of produced neutrinos. This we do using Blumenthal's results below.

Furthermore, as analyzed by Giler, Wdowczyk and Wolfendale[7], charged cosmic rays are subject to diffusion in the intergalactic magnetic field. Though we do not agree with the analysis of the spectrum of ref.(7), as it does not consider the transport of the recoil protons as in ref.(4) and the assumed intergalactic magnetic field strength of 10^{-8}gauss may be somewhat large, nonetheless the central ideas of magnetic diffusion we believe are correct. A diffusion constant appropriate to an average intergalactic field of order 10^{-9} gauss is of order $\kappa \sim 10^{34}cm^2s^{-1}$ implying an upper limit on the range of charged protons of order 10^9 lyr. This does not imply a reduction of the flux relative to a line-of-sight flux in the limit of infinite diffusion constant because we are sampling within a given diffusion volume element locally produced cosmic rays over the entire lifetime of the Universe. It should be noted that we can formulate direct tests from the observational data of charged cosmic rays that measure an intergalactic field strength (For example, the non-imaging of the Virgo cluster at 5×10^{19} ev suggests the existence of a 10^{-9} gauss intergalactic field of large correlation length. The formulation of such tests is work currently in progress[4]).

Let τ be the effective "lifetime" of a particle emitted at energy E to remain within $E_o > E > E_o - \Delta E$. The observed flux of such particles is given in terms of the diffusion Green's function by:

$$\dot{j}_p = \frac{c}{4\pi}\int_0^T G(0,T;\vec{x},t)\,\rho_{sources}(\vec{x})\,\eta(t)\,e^{-\left(\frac{T-t}{\tau}\right)}\,dt\,d^3x \tag{18}$$

where $\rho_{sources}$ is the spatial density of sources and $\eta(t)$ is the activity in produced particles per source per second. T is the present Hubble age. Provided ρ is constant on the large scale of $\sqrt{\kappa T}$ and $\eta(t)$ is relatively constant for $(T-t) < \tau$, we will obtain:

$$\dot{j}_p = \frac{c}{4\pi}\int_0^T \langle\rho_s\rangle\,\eta(t)\,e^{-\left(\frac{T-t}{\tau}\right)}dt \sim \frac{c}{4\pi}\langle\rho_s\rangle\,\eta(T)\,\tau \tag{19}$$

Clearly, j_p is controlled by the recent activity, $T-t < \tau$. For neutrinos, $\tau = \infty$ and we are sensitive to i) larger ranges of source distances and therefore an expected "Olber's" enhancement of the flux due to summing over many fainter sources at large distance and ii) possibly a greater activity in earlier epochs, $\eta(0) \gg \eta(T)$ as in "bright phase models"[8]. For ν's we expect the flux:

$$\dot{j}_\nu = \left(\frac{c}{4\pi}\right)\left(\frac{\bar{n}}{2}\right)\langle \rho_s \rangle \int_0^T \eta(t)\,dt$$

$$\gtrsim \left(\frac{T}{\tau}\right)\left(\frac{\bar{n}}{2}\right)\dot{j}_p \tag{20}$$

The factor (1/2) is the fraction of collisions producing a charged π. The principal enhancement factors are T/τ and $\bar{n}$. Thus, from the observed flux of protons at $\sim 5\times10^{19}$ev we should be able to place a rough lower limit on the total flux normalization of neutrinos, $\mathcal{N}$, which should significantly exceed the corresponding flux of protons.

Blumenthal[6] has shown that the effects of the Bethe-Heitler process lead to a finite energy loss scale length, L, less than the Hubble horizon for a primary proton of energy E greater than 10^{19}ev. Though this process should be similarly treated with a transport evolution equation as we have previously done for photomeson production, we expect generally that it implies that the pile-up of Fig.(1) will receive contributions from sources within a finite range $\lesssim$ 300Mpc, while more distant sources will have their contributions energy degraded and fall below the $1/E^3$ background. The pile-up peak will still be located at $\sim 5\times10^{19}$ev but the enhancement should onset at lower energies, $\sim 2\times10^{19}$ev (we remark that it is easy to construct a simple model of the behavior of the spectrum by treating the pile-up as a delta function and the $1/E^3$ background with a theta-function cutoff at the same energy. The delta function then becomes smoothed into a continuous distribution, still peaking however at the original energy).

The scale length of the distortion of the observed spectrum is actually at L/3, since as a proton of incident energy E traverses to photon background and is reduced to energy λE, it's contribution to the spectrum is reduced relative to the $1/E^3$ background by an amount λ^3. L/3 is read off of Blumenthal's Fig.(4); for $10^{19}ev \leq E \leq 10^{20}ev$ we find $1.3\times10^{27}cm \leq L/3 \leq 1.8\times10^{27}cm$, or assuming $T_H = 10^{10}$lyr. we have $5.5 \leq T/\tau \leq 7.7$.

The integrated flux of protons above 5×10^{19}ev based upon the observed ankle structure is $1.7\times10^{-1}/km^2$yr sr implying an induced neutrino flux of .94 to $5.2/km^2$yr sr using $\bar{n}=2$ to $\bar{n}=8$. However, the ankle structure[3] is subject to large experimental uncertainties and we should further note that a primary spectrum falling like $1/E^3$ with a normalization given by the better data below the ankle (uncertain by a factor of ~ 2) implies a pile-up between 2×10^{19}ev and 6×10^{19}ev and a cut-off above the pile-up and a total flux of recoil protons of $(5.5$ to $11.0)\times10^{-2}/km^2$yr sr. This implies a lower limit on the neutrino flux of (.3 to .6) to $1.6/km^2$yr sr.

Recently Sokolsky[(9)] has discussed the detectability of upward moving events initiated by electron neutrinos in the ground below the Fly's Eye detector resulting in an atmospheric EAS. Assuming a detection radius of 30km, an ontime efficiency of 10% and an interaction depth of 300m (which is enlarged by coherent multiple scattering effects, the Landau-Migdal-Pomeranchuk effect; this may be a source of considerable uncertainty) we find that the two neutrino flux estimates above yield between 2.4×10^{-2} and .42 detected upward moving EAS events per year. We emphasize that these are lower limits because the activity of sources is presumably greater during the earlier epochs[(8)], and the protons from these periods are not visible while the neutrinos are.

The electron neutrinos from the process described above have a flat differential spectrum and should produce events cleanly in a range of order .5 to 2.0x 10^{19}ev. (A precise spectral shape at high energies is in preparation). We clearly predict two muon neutrinos per electron neutrino at these energies, which are not subject to the LPM effect but which may be detectable in DUMAND.

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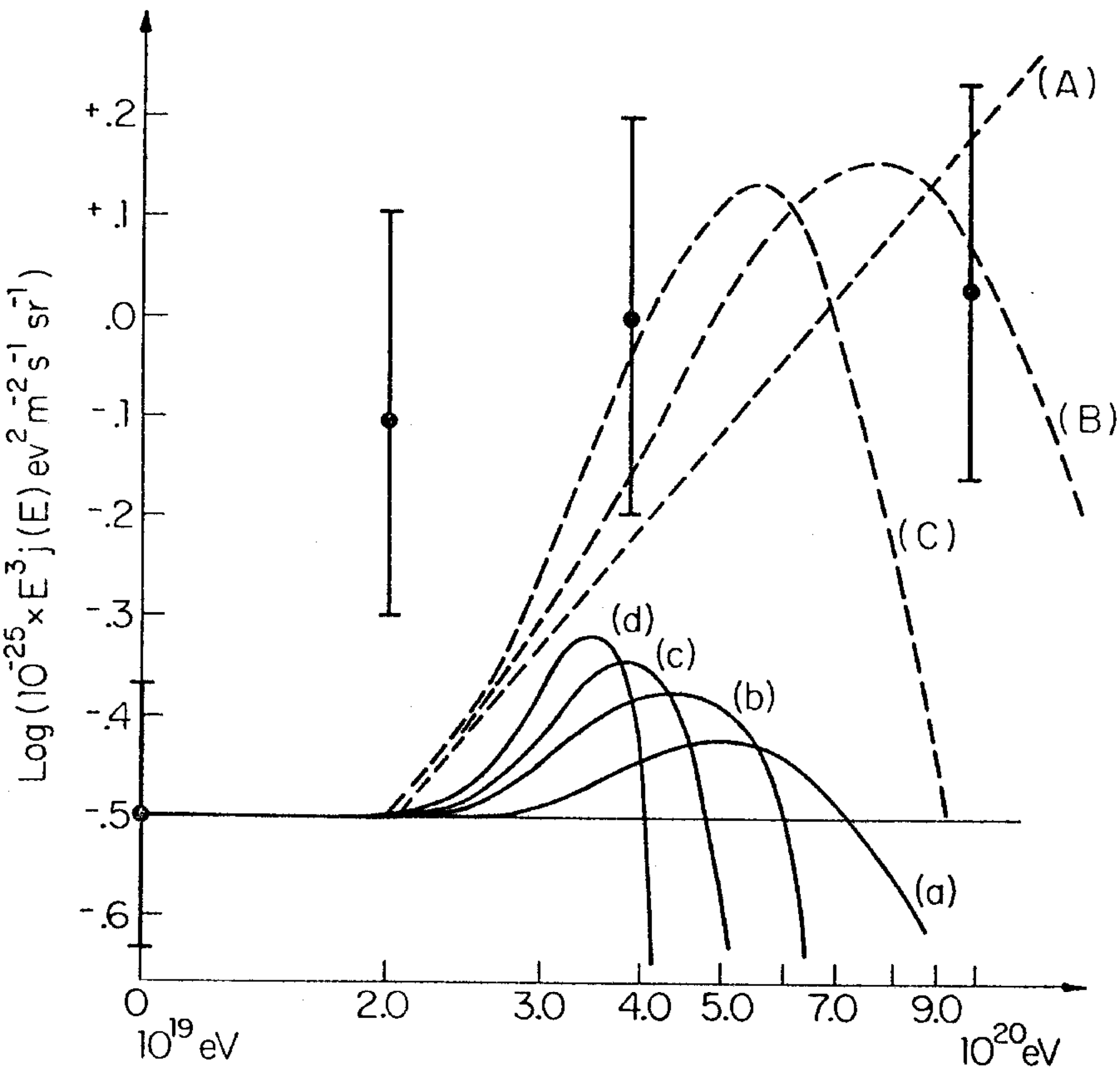
Figure Caption

Photomeson production evolved proton spectra showing the recoil proton pile-up. Solid lines denote $1/E^3$ spectrum (overall normalization is uncertain upward by a factor of 2.) for (a) 24 Mpc. (b) 80 Mpc. (c) 160 Mpc. (d) 400 Mpc. Dashed lines denote (A) primary spectrum with $1/E^2$ component (B) evolved through 8 Mpc. (C) evolved through 64 Mpc.



Log $(10^{-25} \times E^3 j(E)\ \mathrm{ev}^2 \mathrm{m}^{-2} \mathrm{s}^{-1} \mathrm{sr}^{-1})$
+.2
+.1
.0
-.1
-.2
-.3
-.4
-.5
-.6
(A)
(B)
(C)
(a)
(b)
(c)
(d)
0
2.0
3.0
4.0
5.0
7.0
9.0
10^{19} eV
10^{20} eV

Figure 1