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QUARK-HADRON AND CHIRAL TRANSITIONS AND THEIR RELATION TO THE EARLY UNIVERSE

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ABSTRACT

A review is made of the relation of the quark-hadron and chiral symmetry transitions and the big bang universe. Possible signatures at de-confinement are mentioned and related to cosmology. It is noted that to produce surviving perturbations at this epoch requires black hole production which in turn would require a first-order phase transition. The effect of a quark matter phase in neutron stars is also discussed and related to monopole flux limits through its role in cooling.

1. INTRODUCTION

Given the enormous complexity of trying to understand the dynamics of a heavy ion collision, we are continuously tempted to look at models which are similar but more easily understood. A good example of such a model is the Big Bang model of the early Universe. The physics of the early Universe is quite well known up to temperatures ~ 1 MeV, i.e. the era of primordial nucleosynthesis.¹⁾ From this point, one simply extrapolates known physics up to higher energies. In fact, to understand why there exists a very small but finite baryon number in the Universe, one must extrapolate up to temperatures of the order of 10^{15} GeV, i.e. the era of baryon generation.

At present, we will try to limit ourselves to a discussion of the early Universe at temperature around 1 GeV. It is at this time one expects that the Universe underwent a transition from quark matter to hadronic matter. Indeed this is also the relevant energy scale needed to understand the present status of heavy ion collisions. We will try therefore, to point out the many similarities between the early Universe and a heavy ion collision, as well as the differences. We will begin with a discussion of the stan-

standard Big Bang model with some emphasis on what we expect at $T \sim 1$ GeV. Regarding both the similarities and the differences, we will discuss what we might learn about heavy ion collisions from the early Universe and vice-versa from future experiments.

2. THE STANDARD BIG BANG MODEL

The standard Big Bang model is described by the Friedmann-Robertson-Walker metric for a homogeneous and isotropic Universe, which is of the form

$$ds^2 = dt^2 - R^2(t)[dr^2/1-kr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad (1)$$

where $R(t)$ is the scale factor and $k = 0, \pm 1$ is a measure of curvature. Using the Einstein field equations, one can solve for the behavior of $R(t)$

$$(\dot{R}/R)^2 = (8\pi G/3)\rho - k/R^2 + (1/3)\Lambda \quad (2)$$

where ρ is the total energy density and Λ is the cosmological constant. Because of the way ρ scales with R ($\rho \sim 1/R^4$, radiation dominated Universe or $\rho \sim 1/R^3$, matter dominated Universe) and we know that the ρ -term dominates over the k and Λ terms today we can neglect these terms at all earlier times when $R \rightarrow 0$. In addition, the equation for energy conservation tells us that

$$\dot{\rho}/\rho + 3(1 + p/\rho)\dot{R}/R = 0 \quad (3)$$

where p is the isotropic pressure. These two equations ((2) and (3)) are sufficient for describing the evolution of the early Universe given the equation of state.

At early times ($T > \text{few eV}$) the Universe is generally supposed to be radiation dominated, i.e. the equation of state is given by $p = (1/3)\rho$. In an adiabatically expanding Universe $\rho \sim T^4 \sim R^{-4}$. For this case $R(t) \sim t^{1/2}$ and one has a time-temperature relation which is given by

$$t = (3/32\pi G\rho)^{1/2} \quad (4)$$

The energy density ρ will in general depend on a) the particle spectrum and b) particle interactions. For a free gas of massless particles the energy density is given by its black body form

$$\rho = \pi^2/30(g_B + 7/8 g_F)T^4 \quad (5)$$

where $g_B(F)$ are the total number of boson (fermion) degrees of freedom. In general, the energy density of a free gas is given by

$$\rho = \sum \rho_i = \sum_i \int E_{q_i} dn_{q_i} \quad (6)$$

where $E_{q_i} = (m_i^2 + q_i^2)^{1/2}$ and

$$dn_{q_i} = g_i/2\pi^2 [\exp(E_{q_i}/T) \pm 1]^{-1} q^2 dq \quad (7)$$

and the sum runs over each particle state i . For a continuous spectrum, such as in the bootstrap model,²⁾ eq. (6) becomes

$$\rho = \sum \rho_i + \int dn_q \, dm \, \rho(m) E_q \quad (8a)$$

$$\rho(m) = \theta(m - m_0) c m^{-1} (T_0/m)^b \exp(m/T_0) \quad (8b)$$

where m_0 separates the discrete states i from the continuous spectrum $\rho(m)$, c is a constant and T_0 some temperature scale generally taken to be ~ 160 MeV.

The free gas approximation is a good one so long as one is far from a phase transition. Near a phase transition, one must certainly take into account the effects of interactions. A possible approach to the inclusion of interactions is the use of effective interactions³⁾ (mean fields). If the potential energy can be expressed as a function of number density only, one can express the energy density (for a single particle type) as

$$\rho_i = \int E_{q_i} \, dn_{q_i} + f(n) \quad (9)$$

where now $E_{q_i} = (q_i^2 + m_i^2)^{1/2} + U(n)$ and

$$\partial f / \partial n = - \sum_i \int (\partial E_{q_i} / \partial n) \, dn_{q_i} \quad (10)$$

where $n = \sum_i n_i$ is the total number density.

Independent of the form of the particle interactions, we know that

$$t \sim 1/T^2 \quad (11)$$

so that at early times we have high energies and temperatures. Because $\rho \sim T^4$, early times also imply high temperatures as well. It is because the study of the early Universe almost necessarily involves hot dense matter, one is led to analogies with heavy ion physics. The major phenomena which are now being considered are the quark-hadron and chiral symmetry transitions. There is still a great amount of uncertainty regarding these transitions. One still does not really know their order or at what temperatures they occur. Needless to say there is as yet no experimental evidence of either transition.

3. NATURE OF THE TRANSITION

Despite all of the uncertainty, there are a number of qualitative features about which we are confident. The most obvious is that the hadron phase exists. We live in it. If grand unification at $T \sim 10^{15}$ GeV provides us with a solution to the baryon asymmetry and we can then believe extrapolations to those energy

scales, we have a strong indication that the quark phase must exist as well. At $T \sim 10^{15}$ GeV the age of the Universe is only $t \sim 10^{-35}$ sec. and the length scale of a causally connected region is only $ct \sim 10^{-12}$ fm whereas a hadron has a size ~ 1 fm. Indeed, at the GUT epoch the number of hadrons per "hadronic volume" is about 10^{46} !

Given the existence of the quark-hadron transition it is fairly straightforward to see that any description of either phase (in particular the quark phase) will depend heavily on the inclusion of particle interactions. For example, if one considers only free gases of hadrons and quarks and gluons. One will find that at low temperatures $T \lesssim m_\pi$, the pressure in the hadron phase is less than the quark-gluon pressure, simply because there are more quark-gluon degrees of freedom at $T \lesssim m_\pi$. At high temperatures as hadronic states become excited the hadron pressure exceeds the quark-gluon pressure. Thus one has a low temperature quark phase with a high temperature hadron phase. Clearly the interactions need to be taken into account and the best hope for understanding the transition at present lies in the Monte Carlo calculations.^{4,5)}

Of particular importance to the early Universe (as well as heavy ion collisions) is the order, temperature and density of the transition. Monte Carlo calculations for SU(3) gluon matter indicate that there is a first order transition at⁵⁾

$$T_c \approx 80 \text{ MeV}, \quad \Lambda_c \approx 0.96 \Lambda_{\text{mom}} \tag{12}$$

There is of course still a large uncertainty⁶⁾ in Λ_{mom} , $\Lambda_{\text{mom}} \sim 100 - 500$ MeV. The density of the transition also reflects this uncertainty and $n_c \sim 2 - 20 n_0$ ($n_0 = 0.17 \text{ fm}^{-3}$). The inclusion of fermions may alter these results.^{4,9)} Indeed some recent calculations seem to indicate a higher order for the transition when (light) fermions are included.

4. SURVIVING PERTURBATIONS - BLACK HOLES

A first order phase transition for either the confinement or chiral transition could however, have interesting cosmological implications.⁷⁾ In particular if the transition is first order, fluctuations can develop at the transition. The horizon size at this epoch is $\lesssim 10^{30}$ g, thus less than or comparable to the mass of the largest planets. This is therefore the maximum scale that can

be directly affected by either transition. Unlike the GUT phase transition at $T \sim 10^{15}$ GeV which may undergo a period of exponential expansion or inflation,⁸⁾ the quark-hadron and chiral transitions cannot undergo significant inflation even if they are first order and thus their effects are bound by the horizon size. The reason they cannot inflate is that they conserve baryon number and thus they could not regenerate the observed baryon-photon ratio n_b/n_γ of $\sim 5 \times 10^{-10}$ if significant entropy generation or inflation dropped the ratio coming into this epoch. Since most models of baryosynthesis do not overproduce n_b/n_γ at the end of the GUT transition, we cannot add many photons (entropy) or dilute the baryons. Therefore any fluctuations generated at the quark matter transitions cannot directly produce galaxies or clusters.

However planetary scale fluctuations may still be very interesting. Crawford and Schramm¹⁰⁾ carried out a crude numerical N-body simulation with a quark-antiquark ensemble with only 2-body linear potential interactions. They found that clusters of N quarks grew with a probability proportionate to $N^{-0.6 \pm 0.1}$. Thus while small clumps were favored in this calculation it was possible to form arbitrarily large clumps, up to horizon size, but with decreasing probability. They also noted that the largest clumps had an increasing probability to be within their Schwarzschild radius since higher mass black holes have lower critical densities. These black holes were forming from strong interactions, however once formed gravity would of course, disconnect their matter from further interactions. Thus these planetary mass black holes would not enter into nuclear reactions during big bang nucleosynthesis and their density would not be constrained by the nucleosynthetic arguments. Black holes of this size would not blow up via the Hawking mechanism since their mass exceeds Hawking's limit of 10^{15} g.

It is conceivable that these could be the dark matter of the Universe and even achieve $\Omega = 1$ (where $\Omega = \rho/\rho_c$ is the ratio of the present density and the critical density ρ_c needed to close the Universe). The present observed cosmological density limits tell us that $\lesssim 10^{-8}$ of the quark-transition material could have gone into such black holes with the limit corresponding to the planetary black hole dominated Universe. Freese, Price and Schramm¹¹⁾ have gone on to show that such black holes could even be the seeds that cause the initial baryon condensates of the Universe which sub-

sequently explode and create large structures due to their generated shock waves¹²⁾.

To retain a signature from the transitions that affects the later Universe requires locking things up into relics that are not destroyed in subsequent events like nucleosynthesis. Black holes do this very well and it is not easy to conceive of any other way for the transition to have a significant effect on the later Universe. As the formation of such objects requires very large fluctuation growth which in turn requires a first order transition, cosmology will be very interested in the order of the transition. Note also that such fluctuation growth might manifest itself in heavy ion collisions in the form of jets when the phase transition occurred.

5. SIMILARITIES WITH HEAVY ION COLLISIONS

In addition to the order of the quark-hadron transition, the similarities between the Big Bang and heavy ion collisions which beg experimentation is the particle spectrum and equation of state at high temperatures. In the early Universe, the expansion is assumed to be adiabatic and when far from a phase transition, particle interactions are in equilibrium, i.e.

$$\Gamma > H \tag{13}$$

where Γ is a typical interaction rate and the Hubble parameter $H \sim t^{-1}$ is the expansion rate of the Universe. The equation of state then takes on its radiation dominated form $p = \rho/3$. Near a phase transition, one expects departures from this equation of state and one expects certain rates to go out of equilibrium. The particle abundances of course depend heavily on the equation of state and whether or not rates are in equilibrium.

Just near the quark-hadron phase transition, there remains the interesting possibility of a "limiting" temperature. Such an effect may occur in the Big Bang and/or a heavy ion collision. The effect, however, may be due to one or more of a number of causes. The earliest discussion of a limiting temperature is that in connection with the bootstrap theory.²⁾ In the bootstrap model, one generally encounters singularities in the thermodynamic quantities as $T \rightarrow T_0$. For example in eq. (8), for $b = 2$, while the number density of hadrons remains finite as $T \rightarrow T_0$, the pressure (and energy and entropy densities) diverge. On the basis of this divergence, it was argued¹³⁾ that the Universe had a limiting

temperature of T_0 . As was discussed earlier, when one examines the phase diagram (including quark-gluon matter) around T_0 , one finds that because of the divergence in the pressure, the Universe would have preferred the hadronic state at high temperatures and the quark-gluon state at low temperatures. A possible way out of this apparent contradiction is to consider effective potentials (describing repulsive interactions) as in eqs. (9 and 10). It was shown¹⁴⁾ that for certain types of potentials, the limiting temperature may be avoided.

The other possible sources for a limiting temperature are related solely to heavy ion collisions. These are the transparency of nuclei at high energies¹⁵⁾ and/or the quark-hadron transition itself.¹⁶⁾ At moderately low energies (few GeV/nucleon) one can consider as a simple model of heavy ion collisions two nuclei overlapping and stopping in the center of mass frame. That is, all of the kinetic energy converted to particle production and heating. If one extrapolates the data on p-p and p-A collisions to A-A collisions, one expects that at energies of a few tens of GeV/nucleon, the nuclei will no longer stop, but rather become transparent¹⁴⁾. If such a picture is correct, the amount of energy which actually gets deposited for particle production may be limited. Thus in the dense central regions of the collisions a limiting temperature may become apparent.

One troubling aspect of the transparency in heavy ion collisions is that the limiting temperature may be below the critical temperature and density for the quark-hadron transition. Indeed, a possible signature of a transition might be a limiting temperature independent of the transparency.¹⁶⁾ For example if one assumes complete stopping of the nuclei (no transparency) and neglects surface effects, one can calculate the relative abundance of particle-types for a given projectile energy. As one increases the projectile energy the temperature in the compound nucleus will continue to increase. Once $T > T_c$ and/or $n > n_c$, the particle spectrum will freeze out with a distribution characteristic of T_c . The reason being that at higher temperatures one will have produced a quark-gluon plasma which must cool down to T_c before hadrons will condense out and be observed. Thus T_c will act like a limiting temperature for the hadron spectrum because of the formation of a quark-gluon plasma.

It is evident, however, that the transparency of nuclei will mimic this signature of a quark-gluon plasma. In fact, if a limiting temperature is observed, it will become an interesting task to discern which of the three causes discussed is responsible. Alternative¹⁷⁾ signatures of the phase transition can however recover the importance of heavy ion collisions to cosmology.

6. DIFFERENCES BETWEEN THE BIG BANG AND HEAVY ION COLLISIONS

Up to this point, we have only concentrated on areas in which there are similarities between the Big Bang and heavy ion collisions. There are however, some important differences. For example, the net baryon density in collision may differ greatly from that in the early Universe. Today, we presume that there is little or no antimatter present, i.e.

$$n_B - n_{\bar{B}} \approx n_B \gg n_{\bar{B}} \quad (14)$$

where $n_{B(\bar{B})}$ is the number density of baryons (anti-baryons). In fact we also know that n_B itself is very small and when compared to the number density of photons¹⁾

$$n_B/n_\gamma \sim (3 - 5) \times 10^{-10} \quad (15a)$$

$$n_\gamma = (2f(3)/\pi^2) T_0^3 \quad (15b)$$

where $T_0 \approx 2.7^\circ \text{K}$ is the present temperature of the microwave background. In comparison we expect¹⁸⁾ as a relic left over from the very hot early stages of the Universe

$$n_{\bar{B}}/n_\gamma \sim 10^{-20} \quad (16)$$

because of the smallness of these two numbers (15) and (16). It is to a very good approximation safe to set the chemical potential corresponding to baryon number $\mu = 0$.

In a heavy ion collision it is not so obvious as to what condition must be imposed. At lower energies and in non-transparent collisions, one must certainly take into account a non-zero value for μ . For example, eq. (1) becomes

$$dn_{q_i} = (g_i/2\pi^2) [\exp(E_{q_i} - \mu_i)/T \pm 1]^{-1} q_i^2 dq_i \quad (17)$$

with corresponding changes in the definitions of the other thermodynamic quantities¹⁹⁾. The major effect of course is that the phase transition to quark-gluon matter can occur at low temperatures if the baryon density is sufficiently high. If, however, a collision at high energies is transparent, one must discuss two regions in which the baryon density is different. There will be a

fragmentation region with large n_B (μ) and a central region where most of the energy gets deposited but with $\mu = 0$.

The chemical potential represents only a calculational (but tangible) difference between the early Universe and heavy ion collisions. Other differences exist which may be much more complicated. These have to do with surface effects and equilibrium timescales within the compound nucleus. To be sure, the comparisons between a collision and the Big Bang are fruitful but a real understanding of the dynamics of a collision will have to rely on detailed hydrodynamic studies²⁰⁾.

7. NEUTRON STAR INTERIORS

The final point we would like to discuss is the astrophysical possibilities and consequences of a quark-gluon interior of a neutron star. In contrast to the Big Bang, the quark-hadron transition in a neutron star is expected to occur at a very low temperature (few MeV) and with large net baryon density. Whether or not a star with a quark-gluon interior is stable against gravitational collapse will depend on the critical density of the phase transition^{21,22)}. If one considers a star with uniform energy density and pressure the mass of the star will be just $M = (4\pi/3)\rho R^3$ where R is a complicated function of p/ρ . The conditions for gravitational stability are reduced to a condition on the adiabatic index²³⁾

$$\Gamma = (\partial \ln P / \partial \ln n_B)_S \quad (18a)$$

$$\Gamma > \Gamma_c(p/\rho) \text{ for stability.} \quad (18b)$$

Independent of the details of the equation of state for the quark-gluon gas, one finds that condition (18) can be met if the critical density of the phase transition $n_c \lesssim 9n_0$. That is, if the quark-hadron transition occurs at relatively low densities, there exists the possibility that there are stable neutron stars with a quark-gluon interior. For high density transitions, there will be no such possibility.

Let us for the moment assume that "quark stars" may exist and ask what are the consequences. The major effect of a quark-gluon interior is an enhanced cooling rate for the star²⁴⁾. In a normal neutron star the most efficient energy loss mechanism is the modified Urca process



The total energy loss rate in neutrinos is²⁵⁾

$$Q_\nu \approx 6.6 \times 10^{39} M^{-1/3} (r/12)^3 T_c^8 \text{ erg s}^{-1} \quad (20)$$

where M is the mass of the star in solar masses, n is the baryon density in nuclear densities, r is the radius of the star in km and T_c is the central temperature in units of 10^9 °K. If one includes the possibility of quark matter, the quark Urca process will dominate

$$d + u + e^- + \bar{\nu}_e; e^- + u \rightarrow d + \nu_e \quad (21)$$

The total energy loss rate in this case is²⁶⁾

$$Q_\nu \approx 1.6 \times 10^{45} (\alpha Y_e^{1/3}) T_c^6 \quad (22)$$

where $\alpha \sim 2$, is the SU(3) gauge coupling and $Y_e \sim .01$, is the electron to quark ratio.

The effects of increased cooling on the age-luminosity relation has been discussed in detail elsewhere²⁷⁾. Here we would just like to comment on the relation between increased cooling and the abundance of magnetic monopoles²⁸⁾. Without going into too much detail, magnetic monopoles are supposed to have been produced in the very early stages of the Universe during a grand unified phase transition such as $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. In addition, these monopoles are expected to have large baryon number violating cross-sections. There are needless to say, a wide variety of limits on the abundance of monopoles from experiments, cosmology and astrophysics. The strongest constraint however comes from considering the effect of a monopole inside a neutron star²⁹⁾.

Monopoles in general will be trapped by neutron stars. Because of the baryon number violating interactions, monopoles will tend to heat up neutron stars. The limit on the abundance of monopoles is then derived by demanding that the x-ray output of a hot neutron star not exceed the observed x-ray background. This places a limit on the flux of monopoles to be²⁹⁾

$$F \lesssim 10^{-23} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (23)$$

This is to be compared with other limits coming from the overall density of the Universe

$$F \lesssim 5 \times 10^{-15} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (24)$$

for a monopole mass of 10^{16} GeV or from galactic magnetic fields³⁰⁾

$$F \lesssim 10^{-16} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (25)$$

If quark matter exists in the interior of neutron stars, the limit

due to baryon catalysis is raised to

$$F \lesssim 10^{-19} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (26)$$

while this is still stronger than the other limits (24) and (25), it may not be so desparately small so as to create problems with models of breaking SU(5).

8. CONCLUSION

In conclusion, we have tried to point out where some relationships between heavy ion collisions and cosmology and astrophysics lie. To be sure the most important pieces of information are the order, temperature, and density of the quark-hadron transition. The most interesting case being a first order transition in which some density perturbations are produced. In addition, a great deal of input regarding the equation of state of dense matter is needed. Finally, it seems to us remarkable, that through neutron stars information yielded by an A-A collision may tell us something about the breaking of a symmetry whose relevance occurred during the first 10^{-35} s of the Universe.

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