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Monopolonium

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ABSTRACT

We examine the general properties of a monopole anti-monopole boundstate. Lifetimes grow as the cube of the initial diameter and range, for an SU(5) GUT monopole with mass=2×10^{16} GEV, from about 43 days for d=1 fm. to 10^{11} years for d=.1 angstrom. We find about 10^{7} hadrons are produced by fragmentation of gluons that are radiated by classical Larmor radiation. In the final burst when the extended cores overlap about 25 fundamental degrees of freedom of the full unified gauge group are produced. We find that such objects would have been produced in the early Universe at about the time of Helium synthesis and their decay products and Larmor radiation may be observable.

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I. Introduction

If the Universe is widely populated by magnetic monopoles it becomes conceiveable that a monopole-antimonopole boundstate, i.e. monopolonium, can be formed in the laboratory or may have been formed naturally in the Universe at large. Such objects, though unstable, have an interesting physical evolution in time, dependent upon their masses, their initial classical radii and their core structure. For GUT monopoles with masses of the order of 10^{16} GEV, the lifetimes of monopolonium systems range from days, for an initial diameter of about a fermi, up to many times the Universe' lifetime with diameter of about a tenth of an angstrom or more. While behaving as a classical system as long as $r > r_{core}$, they will radiate characteristic dipole radiation up to high energies, $M_{monopole} \sim 10^{16}$. Thus a monopolonium system provides a window on the physics of elementary processes up to the extremely high energy scale characterized by it's mass and could, in principle, yield information about all of the physics between current accelerator energies and the grand unification scale! A single event would produce, for example, about 10^5 Z-bosons by classical dipole radiation alone.

In the final annihilation stages the extended cores of the monopole and anti-monopole overlap and one expects to produce the elementary gauge and Higg's bosons of the full unifying gauge group. This "last gasp" of the monopolonium system is expected to be cataclysmic, releasing 2×10^{16} Gev (about a kilowatt-hour) in less than 10^{-38} sec. We expect here a spectacular explosion of hadrons with a total hadron multiplicity from the entire process of order 10^7. In the present paper we will discuss the expected yields and spectra of hadrons, photons, Z-,X-,Y-,and Higgs-bosons. Thus, while not quite a "table-top" experiment, the study of monopolonium decay processes, should nature avail, would afford the best experimental view of grand unification that one can presently imagine.

Moreover, in the early Universe we will argue that a sizeable and potentially detectable abundance of ultra long-lived monopolonium may have been formed. Remarkably, we find that this process would have occured during the relatively late period of Helium synthesis and depends only upon the assumption of the existence of an acceptable abundance of ordinary heavy monopoles at that time. We do not address the question of how the Universe may have arrived at that epoch with a monopole abundance well below the closure density. Rather, we adopt the view that we know the Universe did indeed pass through such a phase, and if the monopole abundance is near the closure density today (if it is not then the detection of monopoles in experiments will be virtually impossible), then the existence of a substantial relic abundance of monopolonium follows as a logical consequence. For GUT monopoles we find that in a typical cosmologically averaged cubic light year containing on average 10^{32} monopoles, there will be today about 10^{15} monopolonia and roughly 400 decays per year. In galaxies and clusters these abundances and rate densities may be significantly larger. There may also exist mechanisms to significantly enhance the formation and we view the above results as conservative lower limits. The objects of larger diameter are spinning down producing radio frequency radiation from which we may place lower bounds on the masses of GUT monopoles. The cataclysmic decay events may produce visible cosmic ray and high energy gamma ray events in large scale earth-bound or orbiting detectors. Indeed, monopolonium may be easier to find than monopoles themselves. In this paper we will only discuss the formation of monopolonium in some detail and will defer a systematic survey of observational signatures and constraints to a forthcoming work(1).

Much of our discussion will be sufficiently general that it applies to any magnetic monopole, regardless of detailed structure, dependent only upon masses, magnetic charges and cosmological density of monopoles. Also, much of this discussion is presumed valid even for monopoles that are dressed by a few nucleons. The further corrections for monopoles dressed by heavy nuclei are no doubt estimable, but insofar as relic monopolonium is concerned, we do not expect any such phenomenon due to the rarity of heavy nuclei in the helium synthesis phase of the early Universe. The remaining discussion will specialize to the case of an SU(5) GUT monopole, but may be readily taken over to any other grand unified theory gauge group(2). We shall neglect such complications as the Rubakov-Callan effect(3), which could conceivebly enhance the monopolonium formation rates but which we otherwise do not expect will significantly change things, eg. as in hadroproduction, given the extremely short time scales that will be involved.

II. Profile of Monopolonium

Assume for the sake of discussion that we have an SU(5) monopole separated a distance r from an anti-monopole. For SU(5) we assume:

$$
\begin{aligned}
M_X &= 5 \times 10^{14}\ \text{GEV} \\
\alpha_{GUT} &= 1/40 \quad \text{at} \quad M_X \\
M_m &\approx \alpha^{-1} M_X = 2 \times 10^{16}\ \text{GeV}
\end{aligned}
\tag{1}
$$

The effective Rydberg for the monopolonium system at large separation ($r \gg 10^{-13}$ cm) is:

$$
\begin{aligned}
R &= \bar{M} g_m^4 / 2\hbar^2 = M_m\,(\text{GeV}) \times 293 \\
\bar{M} &= M_m/2 = \text{reduced mass}
\end{aligned}
\tag{2}
$$

where g_m is the magnetic charge and N the "monopole number":

$$
\vec{B} = \frac{g_m \hat{r}}{r^2}; \quad g_m e = \frac{N}{2}\hbar c; \quad g_{Dirac} = 3.28 \times 10^{-8}\ \text{esu} \tag{3}
$$

For SU(5) monopolonium the above Rydberg is valid only at distances larger than a few fermi. As r becomes comparable to (Λ_{QCD}) the SU(3) color chromomagnetic field turns on. The chromomagnetic field terminates at a distance scale of $.2\ \text{fm} < r < 1\text{fm}$ due to the confinement effects of QCD, believed generally to be a shielding by color-magnetic monopole like fluctuations in the ordinary QCD vacuum[4].

For $r = 1/M_W$, the U(1) group of electromagnetism decomposes into the U(1) and diagonal generator of SU(2) of the full Weinberg-Salam electroweak model. For all scales less than a fermi the various operant coupling constants are evolving with energy by the usual logarithmic renormalization effects. These renormalization effects lead to a net evolution of the effective magnetic charge, $g_m^{(5)}$.

Remarkably, however, the evolution of g_m is very small over the full range of the desert, even though these various heirarchical effects are setting in and the individual coupling constants are evolving considerably in this range. With λ a threshold parameter of O(1), the magnetic coupling constant is:

$$
\begin{array}{ll}
E \lesssim 1\,\mathrm{GeV} & g_m^2 = 1/4e^2 \\
1\,\mathrm{GeV} \lesssim E \lesssim \lambda M_{Z^0} & g_m^2 = 1/4e^2 + 1/3g_3^2 \\
\lambda M_{Z^0} \lesssim E \lesssim \lambda M_X & g_m^2 = 1/4g_1^2 + 1/4g_2^2 + 1/3g_3^2 \\
E \sim \lambda M_X & g_m^2 \approx 1/\alpha_{GUT}
\end{array}
\tag{4}
$$

where E is the characteristic energy scale = 1/r.

These follow by considering the SU(5) monopole vector potential at the scale $E = \lambda M_X$ where the three coupling constants, $\tilde{g}_1$, g_2, and g_3 of U(1)xSU(2)xSU(3) are all equal to g_{GUT} ($\tilde{g}_1$ is the SU(5) normalized U(1) coupling constant). We write:

$$\frac{\tau^{(3)}}{g_{GUT}} = -\frac{1}{2}\sqrt{\frac{5}{3}}\,\frac{\lambda^{(1)}}{\tilde{g}_1} - \frac{1}{2}\,\frac{\lambda^{(2)}}{g_2} - \frac{1}{\sqrt{3}}\,\frac{\lambda^{(3)}}{g_3} \tag{5}$$

where:

$$\begin{aligned}
\tau^{(3)} &= \mathrm{diag}(0,\ 0,\ 1,\ -1,\ 0) \\
\lambda^{(1)} &= \sqrt{\frac{12}{5}}\,\mathrm{diag}\left(-\tfrac{1}{3},\ -\tfrac{1}{3},\ -\tfrac{1}{3},\ \tfrac{1}{2},\ \tfrac{1}{2}\right) \\
\lambda^{(2)} &= \mathrm{diag}(0,\ 0,\ 0,\ 1,\ -1) \\
\lambda^{(3)} &= \sqrt{\frac{1}{3}}\,\mathrm{diag}(1,\ 1,\ -2,\ 0,\ 0)
\end{aligned} \tag{6}$$

Thus, the resulting force between a monopole anti-monopole pair is:

$$|\vec{F}| = g_m^2/r^2, \qquad g_m^2 = \frac{1}{2}\left\{\mathrm{Tr}\left(\frac{\tau^{(3)}}{g_{GUT}}\right)^2\right\} \tag{7}$$

Finally, we restore the Weinberg-Salam coupling constant normalization to obtain eq.(4):

$$g_1 \equiv g_{1\,\mathrm{W.S.}} = \sqrt{\frac{3}{5}}\,\tilde{g}_1 \tag{8}$$

We thus see that at very short distances, $g_m = 1/g_{GUT}$, which is the correct t'Hooft-Polyakov result for an adjoint of Higgs bosons and which corresponds to N=2, or an effective Schwinger charge in terms of the gauge group charge g_{GUT}. But at very large distances we have the net evolution of the coupling constants and the confinement effects of QCD which shield the $1/g_3$ terms and we have a pure electromagnetic monopole with $g_m=1/2e$. Hence, the SU(5) monopole has the Dirac value for the magnetic charge. Numerically we see that $g_m{}^2(r>>1\text{fm}) = 137/4 = 34.25$, while $g_m{}^2(r = M_X{}^{-1}) = 40$. The various heirarchy effects lead to only a net 15% change in $g_m{}^2$ over the full range of the desert, and we shall ignore these in our analysis of monopolonium energetics. However, we will have to include these effects in our discussion below of gamma, Z-boson, and hadron production via gluon jets.

Assume now that the monopole anti-monopole pair is in a circular orbit about the center of mass. We have:

$$g_m^2/r^2 = \bar{M}\omega^2 r \tag{9}$$

and the energy:

$$E = \frac{1}{2}\bar{M}\omega^2 r^2 - g_m^2/r = -\frac{1}{2}g_m^2/r \tag{10}$$

The system will lose energy by classical dipole radiation and the Larmor power formula is indeed valid for monopoles as well as electric charges (the polarization of the outgoing photons is of course flipped from electric to magnetic). Thus:

$$\frac{dE}{dt} = -2\cdot\left(\frac{2}{3}\right) g_m^2 \left(\omega^2 \frac{r}{2}\right)^2 / c^3 = -\frac{64}{3} E^4 / (g_m^2 M_m^2 c^3) \quad (11)$$

by use of eq.(9). Neglecting the slight renormalization evolution effects of g_m^2, we thus have:

$$-\int_{E_0}^{E_f} dE/E^4 = \frac{1}{3}\left(E_f^{-3} - E_0^{-3}\right)$$

$$= \frac{64}{3}\left(t_f - t_0\right)/\left(g_m^2 M_m^2 c^3\right) \quad (12)$$

or:

$$\tau \approx M_m^2 c^3 r_0^3 / 8 g_m^4 \quad (13)$$

where in the last expression we've made use of eq.(10).

Hence, the lifetime of the state is determined completely classically and grows as the cube of the system's initial diameter. In Table(I) we give numerical values of the lifetime vs. classical diameter, energy, principal quantum number and v/c. Remarkably, a system of a GUT monopole with $r = 10^{-13}$ cm lives about 43 days while with r= 1/10 Angstrom, about a tenth the size of a hydrogen atom, we obtain 10^{11} years! This latter result raises the spectre of relic monopolonium produced in the very early Universe surviving up to the present and decaying today. We return to this question in Section IV.

The classical decay of the system may be viewed quantum mechanically as a cascade of jumps through sequentially decreasing principal quantum numbers. The energy is given by the virial theorem and by the Bohr formula[6]:

$$E = -\frac{1}{2} g_m^2/r = -R/n^2 \qquad (14)$$

We see in Table(I) that the principal quantum number of the instantaneous orbit is O(40) as v/c-->1. Simultaneously the orbital diameter approaches the core size of a the GUT monopole, $r \to 1/M_X$.

The instantaneous transition energy is given by:

$$E' = R\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) \simeq 2R\frac{1}{n^3} \qquad (15)$$

The classical Larmor power formula for the rate of producing photons of energy, E', is expected to be quite accurate for n>1 (we note in passing that even the 2P-->1S hydrogen transition rate can be computed to 30% accuracy classically[6]). The instantaneous value of n gives, of course, the total number of quanta radiated with energy E>E'. For all of our subsequent discussion, excluding the core burst, n is safely >40.

From eq.(15) we have the differential number of photons radiated in a window of energy E' to E'+dE':

$$\frac{dn}{dE'} = \frac{2^{1/3}R^{1/3}}{3}(E')^{-4/3} \qquad (16)$$

The system decays by emitting photons until E' becomes greater than the pion threshold, when $r \to 10^{-18}$ cm and the lifetime remaining is 10^{-9} sec, and $n=4.2\times10^{6}$. Here the system is beginning to radiate both photons and gluons by classical dipole transitions with relative probabilities that may be read off from eq.(4):

$$P_\gamma : P_{gluon} \sim 1/4e^2 : (1/3g_3^2)f(E') \qquad (17)$$

Here, f(E') is an unknown hadronic threshold function which is zero for $E' < m_\pi$, and should approach unity rapidly as $E' \to 1$ GEV. A few low energy hadrons are expected in this phase of the decay, but rapidly as E' exceeds 10 GEV, the gluon production leads to the formation of jets of hadrons. Since this behavior sets in when the remaining lifetime is only 10^{-12} sec while the principal quantum number is of order 10^{6} we do not expect that the jets would be resolvable as "spikes" of hadrons until very high energies (see Section III.), but rather as a continuous distribution in space which may follow the $1-\cos^2(\theta)$ spatial distribution of the Larmor radiated gluons reasonably well. Below we consider both the multiplicity and energy distribution of the hadrons in the resulting jets.

Furthermore, as the system passes through the principal quantum number, $n = 4\times10^{5}$, the Z^0 threshold opens up as the U(1) of electromagnetism decomposes into the U(1) and diagonal generator of the

SU(2) of the electroweak theory. Now the relative probabilities of photons, Z-bosons, and gluons become:

$$P_\gamma : P_{Z^0} : P_{gluon} \sim \left(\frac{\cos^2\vartheta_w}{g_1^2} + \frac{\sin^2\vartheta_w}{g_2^2}\right) : \left(\frac{\sin^2\vartheta_w}{g_1^2} + \frac{\cos^2\vartheta_w}{g_2^2}\right) : 4/3 g_3^2 \tag{18}$$

where ϑ_w, g_1, g_2, g_3 are, of course,energy dependent. In Fig.(1) the normalized probabilities to produce the three different quanta are plotted vs. ln(E) from 1 GEV up to the GUT scale. Here we've built in threshold factors of the form:

$$\theta(E - E_{thr})\left(1 - \frac{E_{thr}^2}{E^2}\right)^p \tag{19}$$

and we've taken p=2 though the overall physics is insensitive to the choice of p>1. We further choose E_{thr}=1 for gluons and $M_Z{}^0$=95 GEV for the Z-boson. Above all thresholds we note that the three normalized probabilities are simply expressed in terms of the running coupling constants:

$$P_\gamma = \frac{1}{g_1^2 + g_2^2}\left(\frac{g_2^2}{g_1^2} + \frac{g_1^2}{g_2^2}\right)\frac{1}{D}$$

$$P_{Z^\circ} = \left(2/(g_1^2 + g_2^2)\right) \cdot \frac{1}{D}$$

$$P_{gluon} = \left(4/3 g_3^2\right) \frac{1}{D} \qquad (20)$$

$$D = 1/g_1^2 + 1/g_2^2 + 4/3 g_3^2$$

From eq.(16) we therefore have the number of quanta of species i produced in an energy window E to E+dE:

$$\frac{dn_i}{dE} = \frac{2^{1/3} R^{1/3}}{3} p_i(E)\, E^{-4/3} \qquad (21)$$

Quantitatively we note that the direct rate of production of gamma's exceeds that of Z-bosons which in turn greatly exceeds that of gluons until very high energies (provided we are above the Z°-threshold). We find by a numerical integration that from a scale of 1 GEV up to M_X that the total number of direct photons is 4×10^6 while there are 2.3×10^5 Z-bosons and 1.3×10^5 gluons produced. In Fig(2) we plot the three multiplicity distributions

The fragmentation of gluons into high multiplicity jets of hadrons and secondary decay products, including photons from π° decays, substantialy modifies the spectrum. Most of the relatively soft photons will be secondaries in this range. We first estimate the total yield of hadrons. In QCD the multiplicity of charged hadrons produced in a gluon jet of energy E is expected to be given in leading log QCD[7]:

$$N_h(E) = a \exp\left(b\sqrt{\ln(E/\Lambda)}\right) + n_0 \tag{22}$$

where a and n_0 are uncalculated and b is determined:

$$b = 4\left(C_A/b_0\right)^{1/2}; \quad b_0 = 11 - \tfrac{2}{3}n_f; \quad C_A = 3 \tag{23}$$

Phenomenologically, the Petra data including quark jets is well fit by $b=2.7\pm.28$, fully consistent with the above result, with $a=.027$ and $n_0=2$(8). However, the Petra data is equally well fit by a naive statistical model of jet fragmentation which predicts:

$$N_h(E) = a' E^{1/2}$$

and phenomenologically $a'=2.2$. Though we expect the leading log QCD result to be correct up to the enormous energies of the gluons we are considering, it is nonetheless useful to see how sensitive our results are to the choice of fragmentation multiplicity. The total hadron yield is thus determined by convoluting the gluon distribution with the fragmentation multiplicity:

$$N_h = \int_{E_0\sim 10}^{E_f\sim M_X} N_h(E)\, \frac{2^{1/3} R^{1/3}}{3}\, \rho_{\text{gluon}}(E)\, E^{-4/3}\, dE \tag{25}$$

We find a total yield of $\sim 10^7$ hadrons for the leading log QCD fragmentation and 7×10^8 for the $E^{(1/2)}$ fragmentation distribution (the naive parton model predicts a ln(E) multiplicity in a jet which is already inconsistent with the low energy data and we thus exclude it.)

We may further estimate the spectrum of hadrons and secondary photons, though here we are on somewhat thinner ice. The exact x-distribution for fragmentation of a gluon jet is not known, and only a few properties, such as the total multiplicity and more recent observations of a peak at very low x have been determined.(7) Indeed, it is not clear how much can be determined theoretically. For our purposes the important features are to realize the correct multiplicity, assure that the first moment of the distribution be normalized properly to unity, and try to guess the correct large-x behavior, which we take to be $(1-x)^2$. We will build the multiplicity into the low-x behavior of the distribution. For the leading log QCD multiplicity formula we find that the following distribution works reasonably well:

$$\frac{dN_h}{dx} = N(b) \exp\left(b\sqrt{\ln\frac{1}{x}}\right) \frac{(1-x)^2}{x\sqrt{\ln\frac{1}{x}}} \tag{26}$$

where N(b) is determined by the condition that the first moment of the distribution is normalized to unity (energy conservation). We obtain:

$$N(b) = \frac{1}{2}\left[e^{b^2/4} I(b) - \sqrt{2}\, e^{b^2/8} I(b/\sqrt{2}) + \frac{1}{\sqrt{3}} e^{b^2/12} I(b/\sqrt{3})\right]^{-1}$$

$$I(b) = \frac{\sqrt{\pi}}{2}\left(\mathrm{erf}\left(\frac{b}{2}\right) + 1\right) \tag{27}$$

Since N(b) is determined uniquely by b, the overall normalization of the multiplicity, i.e. the parameter a in eq.(22), is now fixed to be:

$$a = \left(\frac{2}{3}\right) N(b)/b \tag{28}$$

where the factor 2/3 comes from counting in eq.(22) only the charged hadrons. We find: for b=2.6 (n_f=6), a=.041; b=2.8, a=.026; b=3.0, a=.019. Since a_{expt}=.027±.01 and b_{expt}=2.7±.28 (note that our definition of b in eq.(22-26) differs by a factor of $\sqrt{2}$ relative to that in ref.(8)) we see that the simple one-parameter distribution of eq.(26) predicts correctly the overall normalization of the multiplicity. This suggests that energy conservation is a sufficient constraint with the low-x behavior of the distribution to determine the overall multiplicity. For the $E^{(1/2)}$ multiplicity we may use:

$$\frac{dN_h}{dx} = \frac{15}{16}\, x^{-3/2}(1-x)^2 \tag{29}$$

The total hadron multiplicity is taken to be:

$$N_h = \int_{\varepsilon}^{1} dx \left(dN_h / dx \right) \tag{30}$$

which may be seen to yield the correct multiplicity growth with energy when the infra-red cutoff, ε , is taken to be:

$$\varepsilon = \mu / E_{jet} \qquad \mu \sim 1 \text{ GeV} \tag{31}$$

The fragmentation distributions are converted into hadron energy distributions and are convoluted with the gluon distribution from eq.(21) to obtain the hadron energy spectrum, e.g. for the leading log QCD distribution we obtain:

$$\frac{dN_h}{d\ln(E)} = \frac{(2R)^{1/3}}{3} N(b) \int_{E}^{m_x} \left(1 - \frac{E}{E'}\right)^2 (E')^{-4/3} \frac{\exp\left(b\sqrt{\ln E'/E}\right)}{\sqrt{\ln(E'/E)}} \rho_{gluon}(E')\, dE' \tag{32}$$

We choose as the upper limit of this convolution the energy scale corresponding to the point at which the cores of the monopoles are overlapping. To this we will add below the contributions from the final burst, but this will be found to be a small correction to the total hadron spectrum. The hadrons we count do not include the neutrals that end up as photons. Here we may again appeal to the Petra data in which the naive expectation that about 30% of the total distribution converts quickly to photons is born out. The results are plotted in Fig.(3) along with the gluon distribution, for both multiplicity growth assumptions. The photon distribution can be taken to be 30% of the hadron spectrum. We see that the hadron spectrum is somewhat softer than the gluon distribution, as expected, though it tails up to $E=5\times10^{13}$. The hadrons ultimately end up as gammas, electrons, nucleons and neutrinos, as well as π's and muons, at a distance range applicable to astrophysical detection. We note that at accelerator energies the baryon yield in jets is anomalously higher than one would have expected from naive hadronization ideas and constitutes about 10% of the spectrum. This may continue up to the enrgies under consideration here and, if so, we may have a novel mechanism for producing cosmic rays by the decays of relic monopolonium (see Section IV.).

III. The Burst

Reference to Table(I) shows that at a principal quantum number of $n=40$ the classical diameter of the monopolonium system has shrunk to $r=10^{-28}$ cm = $1/M_X$ and the cores of the monopoles themselves are now overlapping. Simultaneously $v/c \to 1$ and our classical approximations are invalid. At this stage we still have about 75% of the system's total energy to liberate. Here we expect to produce a burst of particles of all types contained in the bare unified gauge theory.

A simple approximation to the physics of the burst is to assume that the system's total energy is uniformly distributed throughout a local region of diameter $2/M_X$. Particle multiplicities are then determined by a universal amplitude and by phase space alone. Similar statistical models are successful in hadroproduction at high energy(9). Such a model neglects coherent or thermalization phenomena. Should the system go into a "fireball" phase at this point we expect a much higher yield of lower energy particles and a lower yield of the unified group gauge bosons, X- and Y-, etc.

The universal amplitude, A, will be seen to have dimensions of area and is expected to be related to the volume of the system:

$$A \sim \left(\frac{4}{3}\pi R^3\right)^{2/3} \sim 2.6 R^2 \sim \frac{10.4}{M_x^2} \tag{33}$$

In what follows we closely parallel the analysis of ref.(10).

The partial width to produce n identical bosons is:

$$\Gamma^n = \frac{A^n}{n!}\left\{\prod_{i=1}^{n}\int\frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4(Q-\Sigma p_i)\right\} \tag{34}$$

We neglect here an overall common normalization which we will not need to know. It is expedient to consider the quantity:

$$e^{-aE_0}\Gamma^n = \frac{A^n}{n!}\left\{\prod_{i=1}^{n}\int\frac{d^3p_i}{(2\pi)^3 2E_i}e^{-aE_i}\int d^4x\, e^{-i(Q-\Sigma p_i)\cdot x}\right\}$$

$$= \frac{A^n}{n!}\int d^4x\, e^{-iQ\cdot x}\left[\int\frac{d^3p}{(2\pi)^3 2E}e^{ip\cdot x-aE}\right]^n \tag{35}$$

where $Q = (E_0,0,0,0)$ and use has been made of energy conservation, $E_0=\sum E_i$ and the energy-momentum delta function has been replaced by it's integral representation. In the end the parameter, a, will cancel. We may perform the integral in the bracket for massless particles, $p_0=E_0$:

$$e^{-aE_0}\Gamma^n = \frac{A^n}{n!}\int dt\, e^{-iE_0 t}\int d^3x \left\{\frac{1}{4\pi^2((a-it)^2+x^2)}\right\}$$

$$= \frac{A^n}{n!}\int dt\, \frac{2\pi^2(2n-4)!\, e^{-iE_0 t}}{(4\pi^2)^n (a-it)^{2n-3}\, 2^{2n-3}(n-1)!\,(n-2)!}$$

$$= \frac{A^n}{n!}\,\frac{1}{8}\,\frac{E_0^{2n-4}\, e^{-aE_0}}{(16\pi^2)^{n-2}(n-1)!\,(n-2)!} \tag{36}$$

or:

$$\Gamma^n = \frac{(16\pi^2)^2}{8E_0^4}\left(\frac{AE_0^2}{4\pi}\right)^n \left[n!\,(n-1)!\,(n-2)!\right]^{-1} \tag{37}$$

We can apply the above result to each of the degrees of freedom of the grand unified gauge group, including the superheavy bosons, provided the mean energy of each particle is large compared to it's mass. If the mean energy is comparable to the heavy masses, we expect the heavies to be suppressed by an additional factor of $\exp(-M_h/E)$ in the above expression. For the massless spin-one gauge bosons there is of course an extra statistical factor of 2 relative to spin-zero Higgs bosons. In general we have K "flavor-color" degrees of freedom each of which may produce n particles. Thus, the overall multiplicity is readily computed as follows:

$$\bar{n} = \left(\sum_{n=1}^{\infty} \Gamma^n n \Big/ \sum_{n=1}^{\infty} \Gamma^n \right) \tag{38}$$

and $\bar{n}$ is roughly the value of n maximizing Γ^n. From:

$$\Gamma^n \propto \exp\left(n \ln\left(\frac{AE_0^2}{4\pi}\right) - n \ln\left(\frac{n}{e}\right) - (n-1) \ln\left(\frac{n-1}{e}\right) - (n-2) \ln\left(\frac{n-2}{e}\right) \right) \tag{39}$$

we obtain:

$$\bar{n} \sim \left(\frac{AE_0^2}{4\pi}\right)^{1/3} \cdot e \sim 25 \tag{40}$$

Eq.(39) is, we emphasize, the total number of fundamental degrees of freedom of equal statistical weight that are produced. The reader may note that we should in principle have calculated the width into n particles of flavor-color 1, m particles of flavor-color 2, etc. From this we would then calculate the total multiplicity over all flavor-colors. However, for flavor-colors having the same statistical weight the result would be identical to that of eq.(40). We assume that each helicity state of a vector has the same weight as a single Higgs scalar. We note that this result reasonably close to the multiplicity we would have obtained by naively extrapolating the classical Larmor result down from n=40. Here, however, the energy is uniformly shared by the outgoing particles. The average energy per particle is E = $1.2\text{x}10^{15}$ GEV which makes our neglect of superheavy masses reasonable.

In SU(5) we have 24 gauge and 24 Higgs bosons. In counting the number of X- and Y- bosons we must take a total of 12x2 degrees of freedom from the gauge bosons and 12x1 (longitudinal) degrees of freedom from the Higgs. We further have a total of 24x3 degrees of freedom altogether. Thus the fraction of X- and Y- bosons produced is 12x(2+1)/(24x3)=1/2. Thus 25/2 = 12 X- and Y-bosons are expected. Table(II) presents the approximate yields and fractions in the burst phase of the various SU(5) gauge and Higgs bosons.

The decays of the superheavy gauge and Higgs bosons as well as the fragmentation of the gluons will produce very high energy hadron jets as well has leptons. With an average energy of $O(10^{15})$ GEV the expected multiplicity per jet is $\simeq 10^4$ from the leading log QCD and, though it somewhat increases the multiplicity at very high energies, it is a negligible correction to the hadron spectrum of Fig.(3).

The particles produced in the burst will decay into leptons, quarks, and the lighter gauge and Higgs bosons. Of the 25 degrees of freedom initially excited, roughly 25x2x8/(2x24+24), or $\simeq$5 are gluons. The remaining 20 objects will typically decay into two body final states. Ignoring gauge bosons, we expect typically 25% of these will be leptons and 75% quarks. Thus we get roughly 10 leptons and 30 quark jets in addition to the 5 original gluon jets. These jets should be distributed more or less isotropically in space and might be cleanest along the z-axis of the system where the $1-\cos^2(\theta)$ Larmor distribution is zero (of course p_T effects and multiple scattering will give a nonzero background here). The typical jet opening angle at these energies is[11]:

$$\delta\theta \sim a E^{-1/4} \sim .38 E^{-1/4} (\text{rad}) \sim 22^\circ E^{-1/4}$$

$$\sim 4 \times 10^{-3} \text{ degrees} \tag{41}$$

thus we have very highly collimated jets. We note that even for the Larmor spin-down the azimuthal distribution of hadrons will not be entirely uniform. The last revolution corresponds to all jets of energy $E > E'$ where:

$$\int_{E'}^{M_X} \frac{d\theta}{dn} \frac{dn}{dE} dE = \int_{E'}^{M_X} \frac{d\theta}{dn} \frac{(2R)^{1/3}}{3} E^{-4/3} dE \langle \rho_{gluon} \rangle = 2\pi \tag{42}$$

and

$$\frac{d\theta}{dn} \sim n^2/g^6 \sim \left(\frac{2R}{E}\right)^{2/3} \frac{1}{(40)^3} \tag{43}$$

thus:

$$2\pi \sim \frac{1.2 \times 10^{13}}{E'} \qquad E' \sim 1.9 \times 10^{12} \text{ GeV} \tag{44}$$

and the fraction of the full 2π azimuthal angle occupied by the hadrons in jets is:

$$f = \frac{1}{2\pi}\int_{E'}^{m_X} (.37\,E^{-1/4})\,\frac{dn_3}{dE}\,dE \sim .068 \ll 1 \qquad (45)$$

The enormous yield of hadrons would require that to detect the leading decay fragments of the X-,Y- and superheavy Higgs bosons in a monopolonium decay event one must be able to cut on all hadrons of energy less than about 5×10^{13} to 10^{14} GEV and focus upon the very low multiplicity particles in the 10^{14} to 10^{16} decades. These objects will carry the information about the grand unified gauge group. For example, it may be possible to detect the CP-violation in the decays of superheavy Higgs (or gauge) bosons by counting a net baryon excess in the leading particles (expected optimistically at a 10^{-3} level). Thus one could test in principle the mechanisms by which the Universe acquired a net baryon number with = 1000 monopolonium decay events. This would be extremely difficult at best.

IV. Relic Monopolonium

The crucial observation of the present section is that, as stated above, objects of an initial size of order a tenth of an angstrom or more, have lifetimes equal to or exceeding that of the Universe. This suggests the possibility that such monopolonia were formed in the early Universe and may have survived up to the present. Some fraction of these will be decaying presently and the high multiplicity of final fragments may be observable. Alternatively, the larger objects are presently spinning down and should be producing a diffuse radio background. From this we can place joint limits on the masses and closure fractions of arbitrary monopoles as this part of the annihilation is completely insensitive to the GUT assumption. We will concentrate presently on the formation of relic monopolonium and leave many questions of related interest to future discussions[1].

Our approach will be to give first a very general estimate of the expected abundance of relic monopolonia from statistical mechanics by computing the equilibrium fraction of monopolonia at a fixed temperature in the solution of monopoles and antimonopoles via a classical-differential version of the Saha equation. This will immediately translate into the decay rate in a typical comoving volume. We will not see in this estimate the specific physical formation mechanisms, and we shall follow with an estimate based upon a definite process. Here we will consider the same mechanism by which various authors have attempted to rid cosmology of an over-abundance of primordial GUT monopoles. We will look at collisions between "cool" monopoles and antimonopoles that are hard enough to capture into loosely bound states by the emission of classical Larmor radiation. Though the impact of such events upon the background density of free monopoles is insignificant, nonetheless a substantial abundance of monopolonium is generated. This specific process will yield a comparable supply to that suggested by our more general statistical argument.

We see that the binding energy of monopolonia with sizes between 1/10 to 10 Angstrom ranges from ≈ 340 KEV to 3.4 KEV. Thus, this is the relevant temperature scale for the formation and corresponds to the Universe age of from 10 to 10^4 seconds. We believe we have a reliable understanding of Cosmology in this epoch since the primordial Helium abundance can be reliably calculated and agrees with observation. It is known that for our GUT monopoles, the ratio of the monopole density to photon density must be less than 10^{-19} during this phase(12), and is probably at least as small as 10^{-24}, since that is the present allowed fraction given the closure density today and there is no known mechanism for reliably reducing monopole abundances since that time. Our point of view is to assume that they must have been in an acceptable abundance during this epoch and to proceed to obtain essentially a lower limit on the resulting monopolonium abundance.

First we shall assume a uniform distribution of monopoles and antimonopoles with a common density r_M. At a temperature $T < M$ we will have a Maxwell-Boltzmann distribution:

$$r_M \left(\frac{M}{2\pi T}\right)^{3/2} \exp\left(\frac{-Mv^2}{2T}\right) d^3v\, d^3r \tag{46}$$

assuming thermal equilibrium. This does not require necessarily a thermal equilibrium with radiation, since it can arise by the general red-shifting down from the primodial monopole production epoch. Indeed, a collision between a monopole and a photon of energy T exchanges only the infinitesimal energy T^2/M and it requires many collisions, $\simeq M/T$, for a thermalization to take place for small T. However, collisions between monopoles themselves lead to efficient energy exchanges and the thermal distribution should be maintained by these, treating the interaction as an instantaneous Coulomb potential.

Consider a monopole and antimonopole pair, each described by the distribution of eq.(45). The equilibrium distribution of pairs that are bound with binding energy in the range E_b to E_b+dE_b is given by:

$$df = \left(\frac{M}{2\pi T}\right)^3 \int d^3v_1 d^3v_2 d^3r_1 d^3r_2 \exp\left\{\frac{-\frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_2^2 + g_m^2/|r_1-r_2|}{T}\right\}$$

$$\cdot\, \delta\left(E_b - \frac{1}{2}\bar{M}(v_1-v_2)^2 + g_m^2/|r_1-r_2|\right) dE_b\, r_m r_{\bar{m}} \qquad (47)$$

It is convenient to go over to center of mass positions and velocities, $V=(v_1+v_2)/2$, $u=(v_1-v_2)$, $R=(r_1+r_2)/2$, $r=(r_1-r_2)$:

$$df = \left(\frac{M}{2\pi T}\right)^3 \int d^3V d^3R\, d^3u\, d^3r \exp\left(-\frac{(2M)V^2}{2T}\right)\exp\left(-\frac{1}{2}\frac{\bar{M}u^2}{T} + \frac{g_m^2}{rT}\right)$$

$$\cdot\, \delta\left(-|E_b| - \frac{1}{2}\bar{M}u^2 + \frac{g_m^2}{r}\right) dE_b \quad r_m r_{\bar{m}}$$

$$= \left(\frac{2M}{2\pi T}\right)^{3/2} \int d^3V d^3R \exp\left(-\frac{1}{2}(2M)V^2 \cdot \frac{1}{T}\right)\; dr_{m\bar{m}} \qquad (48)$$

where the result has been factorized into a Maxwell-Boltzmann distribution in the boundstate with mass 2M times the density fraction of the boundstates in the background monopole anti-monopole gas:

$$dr_{m\bar{m}} = \left(\frac{\bar{M}}{2\pi T}\right)^{3/2} \int d^3u\, d^3r\; \delta\left(-|E_b| - \frac{\bar{M}u^2}{2} + \frac{g_m^2}{r}\right) e^{-\left(\frac{\bar{M}u^2}{2T} - \frac{g_m^2}{rT}\right)} r_m r_{\bar{m}}\, dE_b \quad (49)$$

Performing the d^3u integration gives:

$$dr_{m\bar{m}} = \int \left(\frac{1}{2\pi T}\right)^{3/2} \cdot 2\pi\, \theta\left(\frac{g_m^2}{|E_b|} - r\right)\left(\frac{g_m^2}{r} - |E_b|\right)^{1/2} e^{|E_b|/T} r_m r_{\bar{m}}\, d^3r\, dE_b \quad (50)$$

$$= \frac{dE_b}{E_b}\left(\frac{1}{2\pi T E_b}\right)^{3/2} e^{|E_b|/T}\, 8\pi^2 g_m^6 \int_0^1 y^2\left(\frac{1}{y}-1\right)^{3/2} dy \qquad (51)$$

Thus, performing the y integral gives:

$$dr_{m\bar{m}} = \frac{dE_b}{E_b}\left(\frac{\pi^3}{2}\right)\left(\frac{1}{2\pi T E_b}\right)^{3/2} r_m r_{\bar{m}}\, e^{|E_b|/T} g_m^6 \qquad (52)$$

This is simply a classical version of the Saha equation, which may be viewed as formation by multibody monopole collisions in thermal equilibrium. In a comoving volume we have N_M monopoles. We may write:

$$r_m = aT^3\left(r_m/r_\gamma\right) \qquad a = \pi^2/30 \qquad (53)$$

Thus the number of boundstates in the comoving volume with binding energy E_b to E_b+dE_b becomes:

$$dN_{m\bar{m}} = \frac{dE_b}{E_b}\left(\frac{\pi^5}{30}\right)\left(\frac{T}{2\pi E_b}\right)^{3/2} e^{|E_b|/T} g_m^6 N_m\left(\frac{r_m}{r_\gamma}\right) \qquad (54)$$

The ratio r_M/r_γ is cosmologically invariant for the low formation rates discussed here.

We see that the binding energy enters an exponential and thus the most probable states by eq.(54) have infinite binding energy. This is the well known disease of the Coulomb gas and it is readily interpreted. In general, as the members of a pair get very close together the Boltzmann factor in eq.(47) is diverging. But such a pair is also no longer in thermal equilibrium and is dominated by the local mutual Coulomb force. Only when the energy is within an order of magnitude of the temperature is the process expected to be reliably described as an equilibrium one. Thus we may only apply eq.(54) for $E = \eta T$, where η is a parameter of O(1). Thus we have for the differential number of objects instantaneously bound with energy E to E+dE:

$$dN_{M\bar{M}} = \frac{dE_b}{E_b}\left(\frac{\pi^5}{60}\right)\left(\frac{1}{2\pi\eta}\right)^{3/2} e^{\eta} g_m^6 N_m \left(\frac{r_m}{r_\gamma}\right) \tag{55}$$

where N_m is the number of monopoles in a comoving volume.

Strictly speaking, these objects are not bona-fide boundstates, but only thermal fluctuations of an arbitrary pair into a state of locally negative total energy. They are the equilibrium fraction of the monopolonia dissolved in the monopole anti-monopole gas. However, as the system cools we expect that for an appropriate choice of $\eta \sim O(1)$ this abundance of objects will be left behind as boundstates since their binding energy will exceed the thermal energy available to dissociate them. Thus eq.(55) is indeed expected to describe the relic abundance of monopolonia formed as the Universe cooled through a temperature of order the binding energy.

The formula of eq.(55) possesses a pleasant scaling behavior. Reference to the lifetime formula of eq.(12) shows that we may write:

$$\frac{3\,dE_b}{E_b} = \frac{d\tau}{\tau} \tag{56}$$

Thus, the decay rate of monopolonia in a comoving volume follows immediately:

$$\frac{dN_{m\bar{m}}}{d\tau} = \frac{1}{\tau}\left(\frac{\pi^5}{180}\right)\left(\frac{1}{2\pi\eta}\right)^{3/2} e^{\eta} g_m^6 N_m \left(\frac{r_m}{r_\gamma}\right) \tag{57}$$

We see that the decay rate increases as the age of the comoving volume decreases. Today, a typical cubic light-year would contain roughly $3\mathrm{x}10\mathrm{e}^{32}$ GUT monopoles if we saturate the closure density. From eq.(57) we obtain ~ 350 decays of monopolonia per year per cubic light-year, assuming the conservative $r_M/r_\gamma = 10^{-24}$, and 3.5×10^{12} saturating the Helium abundance limit of $r_M/r_\gamma = 10^{-19}$. We may further estimate the total fraction of monopolonia by converting from lifetime to diameter through the convenient scaling law:

$$\frac{3\,dr}{r} = \frac{d\tau}{\tau} \tag{58}$$

Thus, we may integrate from a radius of $r = 1/10$ Angstrom up to an upper limit, R, for which we expect the formation to terminate:

$$\frac{N_{m\bar{m}}}{N_m} = g_m^6 \left(\frac{\pi^5}{60}\right)\left(\frac{1}{2\pi\eta}\right)^{3/2} e^{\eta}\left(\frac{r_m}{r_\gamma}\right) \ln\left(\frac{R}{r_0}\right) \tag{59}$$

We thus see that the specific choice of R is irrelevant as we are only logarithmically sensitive to it. In practice we would expect R to correspond to a value for which a monopolonium is readily ionized by a magnetic field or other traumatizing event (we note that at 1/10 A it requires a B-field of order 10^{10} Gauss to ionize). Putting in numbers yields about 10^{14} (or 10^{19} assuming the larger monopole to photon ratio) GUT monopolonia per cubic light year, or a fraction of 10^{-18} monopolonia to monopoles.

In fact, we expect that eq.(59) with $\eta=1$ probably underestimates the abundance of monopolonium since it makes no reference to specific reactions such as capture by the emission of radiation, scalars or other particles, and considers only monopole collisions in thermal equilibrium implicitly via their instantaneous Coulomb potential. We may directly estimate the formation of monopolonium by radiative capture essentially making use of the cross-section:

$$\langle \sigma v \rangle = \pi\left(\frac{\sqrt{2}}{3} g_m^2\right)^{2/5} \left\{ \frac{g_m^4}{\bar{M}^2}\left(\frac{\bar{M}}{E_0}\right)^{9/10} \right\} \tag{60}$$

where the quantity in braces is familiar[12], but we've obtained the extra overall numerical factor =9.6. E_0 is the kinetic energy of an incident monopole at infinity which is of order the temperature (it would be a straightforward analysis to fold in the complete thermal distribution here). This follows by demanding a classical collision between monopole and anti-monopole during which there is sufficient acceleration to radiate away the energy by the Larmor power formula. In fact, we may use more general differential cross-section:

$$\frac{d\langle\sigma v\rangle}{dE_b} = 9.6\left(\frac{2}{5}\right)\bar{M}^{-7/5}\frac{g_m^4}{E_0}\left(\frac{1}{E_0+|E_b|}\right)^{7/5}\left(\frac{E_0}{\bar{M}}\right)^{1/2} \tag{61}$$

which is the cross-section of capture into boundstates of binding energy E_b. The rate of formation of monopolonia into these states at temperature T in a given comoving volume is:

$$\frac{dN_{m\bar{m}}}{dt\,dE_b} = r_m\frac{d\langle\sigma v\rangle}{dE_b}$$

$$= (3.8)\left(\frac{g_m^4}{\bar{M}^2}\right)\left(\frac{1}{T}\right)^{1/2}\left(\frac{1}{T+|E_b|}\right)^{7/5}\left(\frac{r_m}{r_\gamma}\right)\left(\frac{\pi^2}{30}\right)T^3\bar{M}^{-9/10} \tag{62}$$

We may convert the differential formation rate in time into a rate in temperature in a comoving volume (thus the Hubble expansion terms do not appear). This may be integrated over all temperatures and we note that the integral receives most of it's contribution for T = E_b as expected. The result is:

$$\frac{dN_{m\bar{m}}}{dE_b} = (3.8)\left(\frac{\pi^2}{15}\right)\left(\frac{8\pi^3}{90}\right)^{-\frac{1}{2}} g_f^{-\frac{1}{2}} \frac{g_m^4 \, M_{pl} \, E_b^{-9/10}}{(\bar{m})^{11/10}} \left(\frac{r_m}{r_\gamma}\right) N_m \, I \tag{63}$$

$$g_f = \frac{1}{2}\left(\#\text{ boson helicities} + \frac{7}{8}\#\text{ fermion helicities}\right)$$

where the integral is:

$$I = \int_0^\infty \frac{dx}{x^{1/2}(1+x)^{7/5}} \approx 2.2 \tag{64}$$

The final monopolonium abundance is numerically:

$$\frac{N_{m\bar{m}}}{N_m} \sim \frac{20.8 \, g_m^4 \, m_{pl}}{(\bar{m})^{11/10}} (E_b)^{1/10} \left(\frac{r_m}{r_\gamma}\right) \sim 10^{-18} \tag{65}$$

and we have the decay rate:

$$\frac{dN_{m\bar{m}}}{d\tau} = \left(\frac{2.08}{3\tau}\right) \frac{g_m^4 \, m_{pl} (E_b)^{1/10}}{(\bar{m})^{11/10}} \left(\frac{r_m}{r_\gamma}\right) N_m \sim 2.9 \times 10^{2} / \text{cu. ly. yr.} \tag{66}$$

This is remarkably close to our preceding result even though it involves somewhat different physics. The exponent 9/10 is sufficiently close to unity that our scaling is approximately valid as well. The rough agreement between these results encourages us that they are probably correct though a more detailed analysis of specific mechanisms is desired. Moreover, perhaps our choice of $\eta = 1$ in eq.(57) is overly restrictive. If $\eta=10$ the rates and fractions are increased by a large factor of 256, and as $\eta \to \infty$ all monopoles become bound into monopolonia. We can envision a number of additional mechanisms that might significantly enhance the formation rates and we regard the quoted results as probable lower limits on the formation.

We will not give here a detailed description of the observability of relic monopolonium. However we will remark that we can place nontrivial limits on the monopole masses and closure fractions from the above considerations and that we believe that the decay products of these systems might be detectable in a variety of experimental configurations. A systematic discussion of the observational implications, constraints, signatures and other general considerations is in preparation(1).

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FIGURE CAPTIONS

Fig. 1 Normalized probabilities, p_γ, p_{Z^0}, p_{gluon}.

Fig. 2 γ, Z^0, gluon $d(\ln(n))/d(\ln E)$ vs. $(\ln E.)$

Fig. 3 Charged Hadron spectrum (a) leading log QCD, (b) $E^{1/2}$ multiplicity, and (c) gluon spectrum. γ-distribution $\sim$ 1/3 hadron distribution.

Table 1. Monopolonium Properties

Classical Diameter (cm)	Lifetime (sec)	Binding Energy (GeV)	Transition Energy (eV)	Principal Quantum Number	V/C
10^{-8}	3.71×10^{22}	3.35×10^{-5}	1.61×10^{-7}	4.17×10^{11}	4.10×10^{-11}
10^{-9}	3.71×10^{18}	3.35×10^{-4}	5.09×10^{-6}	1.32×10^{11}	1.30×10^{-10}
10^{-10}	3.71×10^{15}	3.35×10^{-3}	1.61×10^{-4}	4.17×10^{10}	4.10×10^{-10}
10^{-11}	3.71×10^{12}	3.35×10^{-2}	5.09×10^{-3}	1.32×10^{10}	1.30×10^{-9}
10^{-12}	3.71×10^{9}	3.35×10^{-1}	1.61×10^{-1}	4.17×10^{9}	4.10×10^{-8}
10^{-13}	3.71×10^{6}	3.35	5.09	1.32×10^{9}	1.30×10^{-8}
10^{-14}	3.71×10^{3}	3.35×10^{1}	1.61×10^{2}	4.17×10^{8}	4.10×10^{-8}
10^{-15}	3.71	3.35×10^{2}	5.09×10^{3}	1.32×10^{8}	1.30×10^{-7}
10^{-16}	3.71×10^{-3}	3.35×10^{3}	1.61×10^{5}	4.17×10^{7}	4.10×10^{-7}
10^{-18}	3.71×10^{-9}	3.35×10^{5}	1.61×10^{8}	4.17×10^{6}	4.10×10^{-6}
10^{-20}	3.71×10^{-15}	3.35×10^{7}	1.61×10^{11}	4.17×10^{5}	4.10×10^{-15}
10^{-22}	3.71×10^{-21}	3.35×10^{9}	1.61×10^{14}	4.17×10^{4}	4.10×10^{-4}
10^{-24}	3.71×10^{-27}	3.35×10^{11}	1.61×10^{17}	4.17×10^{3}	4.10×10^{-3}
10^{-26}	3.71×10^{-33}	3.35×10^{13}	1.61×10^{20}	4.17×10^{2}	4.10×10^{-2}
10^{-28}	3.71×10^{-39}	3.35×10^{15}	1.61×10^{23}	4.17×10^{1}	4.10×10^{-1}

Table 2. Fractions and Approximate Yields in Burst

Species	Fraction	Approx. Yield
$X, \bar{X}$	1/4	6
$Y, \bar{Y}$	1/4	6
W^+, W^-, Z^0	1/12	2
gluons	2/9	6
γ	1/36	0
color 8 Higgs	1/9	3
weak Higgs	1/24	1

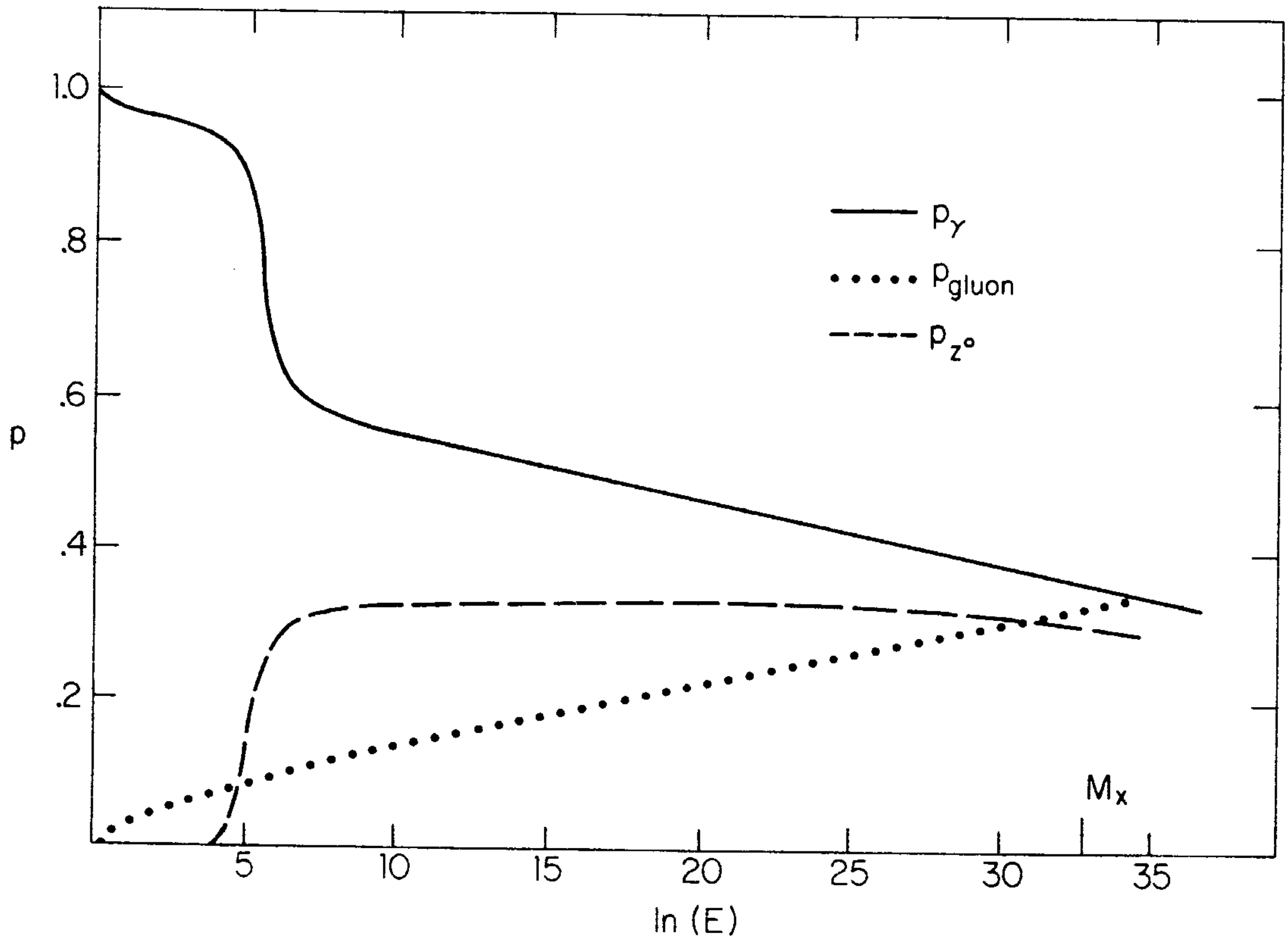

Fig. 1

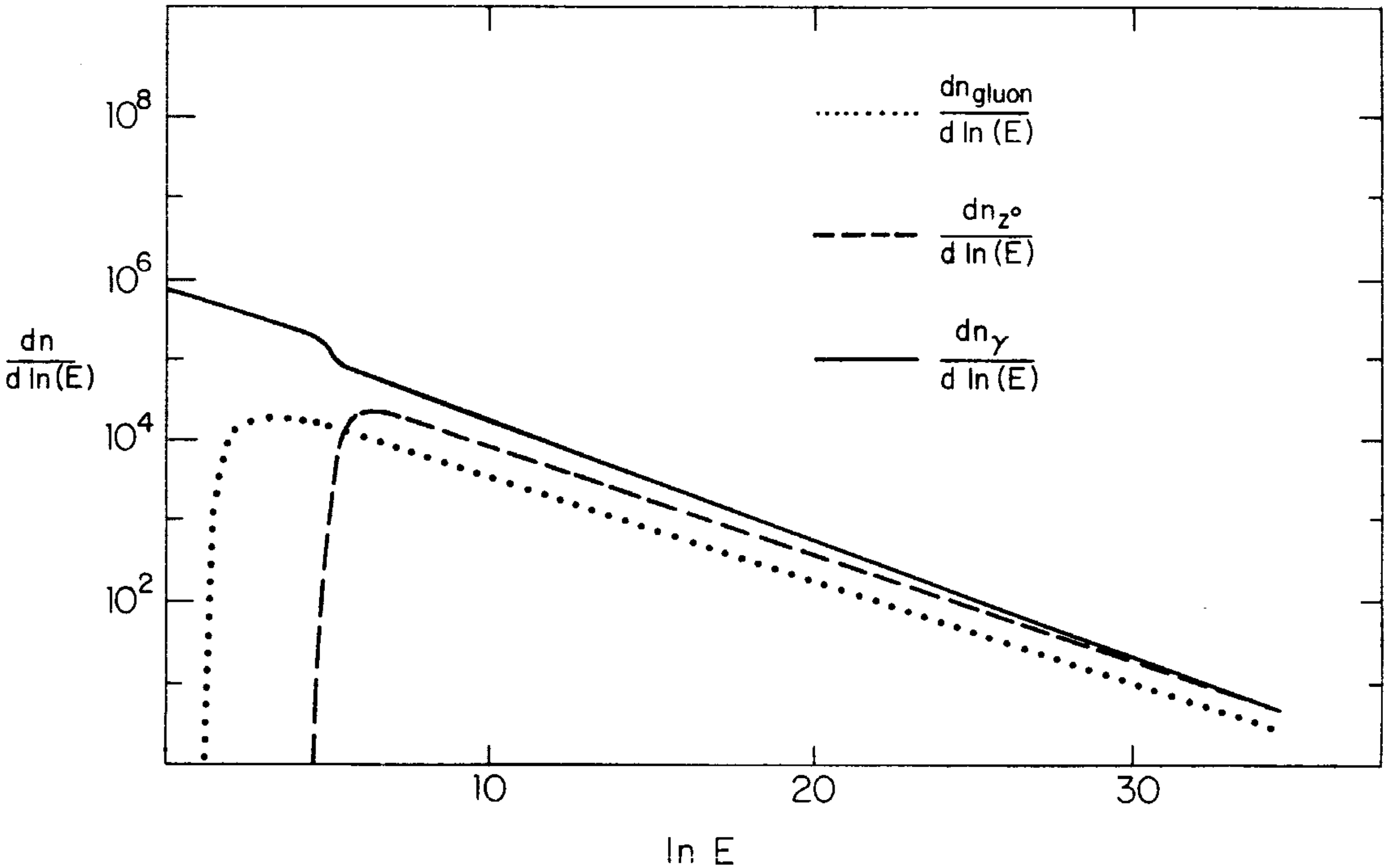
$\frac{dn}{d\ln(E)}$
10^8
10^6
10^4
10^2
10
20
30
$\frac{dn_{gluon}}{d\ln(E)}$
$\frac{dn_{Z^\circ}}{d\ln(E)}$
$\frac{dn_\gamma}{d\ln(E)}$
ln E

Fig. 2

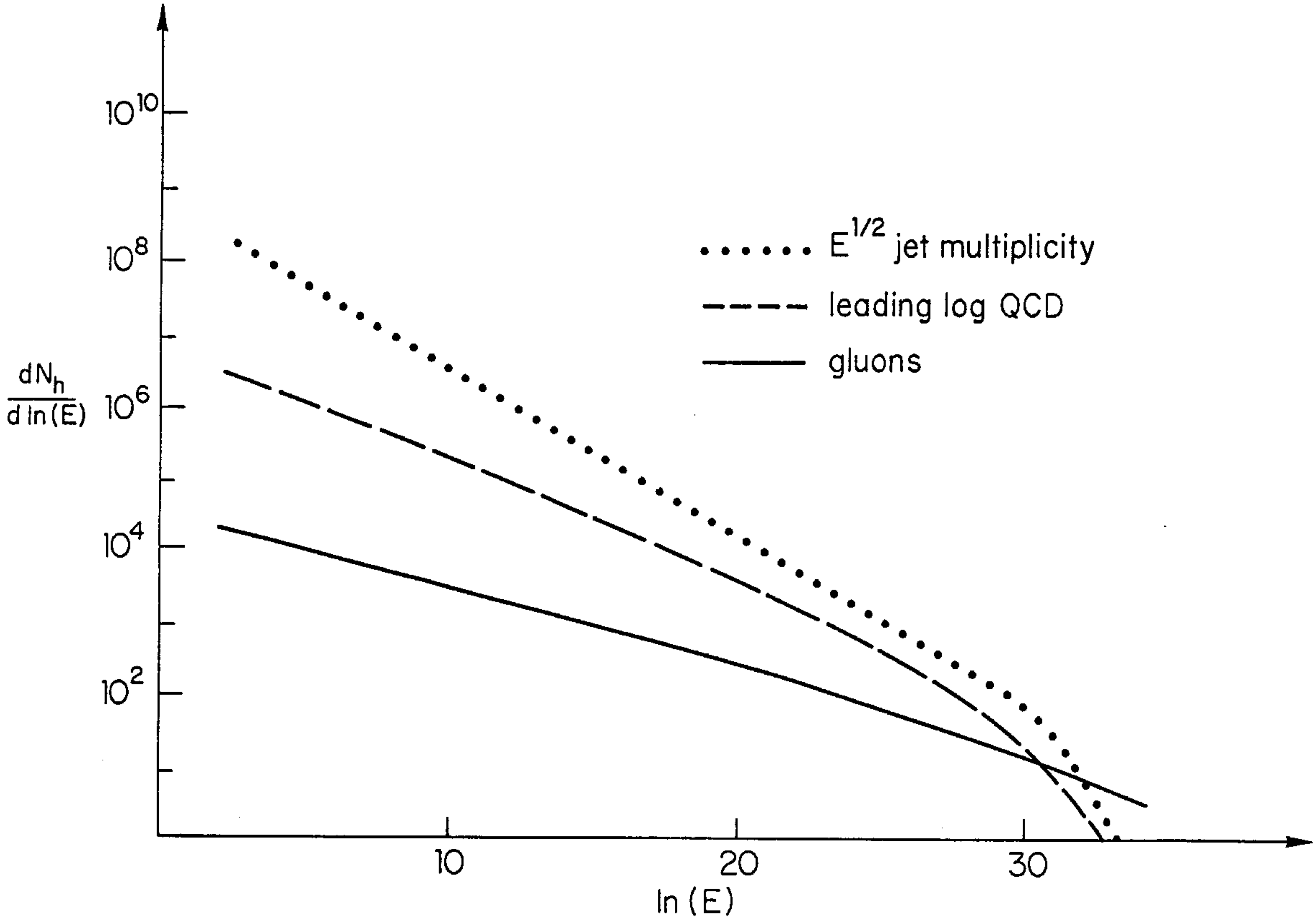
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
$\frac{dN_h}{d\ln(E)}$
$E^{1/2}$ jet multiplicity
leading log QCD
gluons
10
20
30
ln (E)

Fig. 3