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PREON MODELS WITH DYNAMICAL SYMMETRY BREAKING

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ABSTRACT

We have investigated all composite models based on complex, anomaly-free and asymptotically free representations of the gauge groups SU(3) to SU(8), SO(4N+2) and E_6, with not more than two different preons. We discuss in detail the role of Fermi-statistics in the determination of the (meta) flavor representations of the composites. Under certain assumptions about the possible ground states, the solutions to 't Hooft's anomaly equations are presented for the complete flavor group and all non-Abelian subgroups, to which the flavor group can break. We find several models which satisfy anomaly-matching when the flavor group is broken by the simplest condensate.

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I. Introduction.

Although there is no direct experimental indication that quarks and leptons are composites, this idea has attracted much attention recently.[1-4] The motivation is that, although relatively simple models exist to describe the interactions of quarks and leptons, these models leave at least three questions unanswered; why does spontaneous symmetry breaking occur at a scale of $\sim$ 100 GeV, why are there three generations of quarks and leptons instead of one, and why do they have a rather remarkable mass-spectrum. The answer to these questions may be that a hidden substructure exists, and that quarks and leptons are not fundamental.

Clearly a rather unusual kind of dynamics is necessary to understand compositeness of quarks and leptons, since their masses are known to be much smaller than their inverse size. The only known way to explain that is to assume that they are kept massless by a chiral symmetry which is not spontaneously broken.[3] They can then be given a mass by introducing a small explicit breaking of that symmetry. A very restrictive constraint on models with unbroken chiral symmetries was formulated by 't Hooft.[3] He pointed out that the anomalies of the fundamental fermions -- hereafter referred to as "preons" -- have to be reproduced by bound states.

Many mechanisms have been suggested which might provide answers to the questions mentioned above. The most attractive feature of composite models is that they might have a calculable and realistic mass spectrum. The fermion masses will be proportional to some power of the parameter which breaks the chiral symmetry, and that power may be different for different generations. Examples of this principle have been given in several papers[5,6] (not necessarily in the context of composite models). A prerequisite of such a mechanism is an understanding of the existence of generations. An interesting aspect of the anomaly equations is the fact that they allow bound state representations to appear only with definite multiplicites. Therefore, the anomaly equations might determine the number of generations. However, that alone can not be considered to be a solution to the generation problem. One should also be able to show that the bound states can actually be constructed with the required multiplicity. Since radial or orbital excitations are generally considered to be unacceptable as massless bound states this is a non-trivial problem. In this paper we will look for solutions to the anomaly equations with an acceptable set of composites. In models of this type, the appearance of generations does not seem to be the most natural thing to happen, but nevertheless several ideas have been proposed which do not require unacceptable bound states.[5,7]

In many cases the anomaly equations can not be satisfied by a reasonable set of bound states. In such cases the confining symmetry -- hereafter referred to as "metacolor" -- is inconsistent with the chiral symmetry, and at least one of the two symmetries has to break. When one assumes that the confining symmetry breaks one obtains a tumbling gauge theory.[8] We will assume, supported by arguments given in Ref. 9, that this is not what will happen, but that the chiral symmetry will break. In models with a (pseudo) real[10] metacolor representation it is then most likely that the chiral symmetry breaks down to a real symmetry.[11,12] This is what happens in QCD. In such models there are no anomalies left after symmetry breaking, and therefore the existence of massless bound states is not required. If the metacolor representation is complex, the bilinear condensate which breaks the symmetry for real models does not exist, but it is possible that multi-fermion condensates are formed. These condensates may break the chiral symmetry group to a complex subgroup, the anomalies of which can be matched by a set of massless composites. In a model of this kind the original anomaly conditions are satisfied by a combination of composite fermions and composite Goldstone bosons.[3,13] (In some models, solutions with unbroken symmetry group exist;[4,14,15] a solution with broken symmetry has been studied in Ref. 16.) The anomaly equations can be used to determine which complex subgroups are allowed as the result of symmetry breaking. The advantage of this kind of models is, that the symmetry breaking pattern is not arbitrary, but is restricted to a finite and usually small set of possibilities. Of course, even if the anomaly-matching restrictions can force certain symmetries to break, they do certainly not exclude the possibility that more symmetries are broken. On the other hand, a model in which only those symmetries break that are forced to break would be very attractive.

In this paper we will examine the restrictions, which the anomaly equations impose on the unbroken subgroups for the simplest models with complex metacolor representations, and a few models with real representations. The anomaly equations -- without the additional requirements of Ref. 3 -- are only restrictive when they are used in combination with some assumptions regarding the allowed bound states. We will discuss these assumptions in Section II. In Section III we describe the procedure to construct the (meta) flavor representations of the composites, allowed by Fermi-statistics, and present the results of this construction. We then consider anomaly-matching for all non-Abelian subgroups of the flavor group (the prefix "meta" will be omitted in the following), using a method described in Section IV. The results of this search are presented in Section V. Clearly the description of a model of this type is only complete when the condensates, which break the unwanted symmetries, are given. We discuss this problem in Section VI, and give a few interesting examples of models for which the simplest condensate one can form provides sufficient symmetry breaking.

II. Dynamical Assumptions.

In addition to the anomaly equations 't Hooft[3] uses two other conditions on the massless bound state spectrum. One of these, the decoupling condition, can not be formulated for models with a complex metacolor representation since the metacolor symmetry does not allow any mass terms for preons. For real representations the condition has been criticized in Ref. 17. We will also relax the condition which requires the bound state dynamics to be independent of the flavor group ("N-independence"). There are at least two sources of N-dependence of the dynamics: Fermi-statistics, which forbids certain composites -- either fermions or Goldstone bosons -- for small N, and asymptotic freedom, a property which one would expect to be important for the spectrum of a model and which is lost if N becomes too large. (N is the multiplicity of a metacolor representation.) Whether N-independence is a reasonable condition for some intermediate range of N-values is an open question, but we do not impose this requirement, nor do we find any solutions satisfying it. We will however require a mild form of flavor independence for the solutions with broken symmetry. This will be discussed in Section IV.

To look for solutions to the anomaly equations one has to make some assumptions about the bound state spectrum. We will assume that the massless bound state spectrum does not contain states which are radial or orbital excitations, states which contain valence metagluons,[18] and exotic states. (A valence gluon is defined as a gluon without which it is impossible to construct a metacolor singlet state with certain flavor-spin quantum numbers and a ground state spatial wave function; an exotic state is a composite which contains subsets of preons which can form metacolor singlets.) Furthermore we will limit the number of valence preons in a composite to n_c, if the metacolor group is $SU(n_c)$. States with more preons will always contain subsets with vanishing n_c-ality and are therefore in most cases -- but not always -- exotic. These assumptions are inspired by the groundstates of QCD, but since we consider different confining groups, complex fermion representations and unbroken chiral symmetries it is not completely obvious that they are valid. A better understanding of the dynamics is necessary to justify them. Similar assumptions have been made in several papers.[5,15,16,19]

Since by assumption the spatial wave functions we consider are completely symmetric in all particles, the flavor representation of the bound state is determined by imposing Fermi-statistics for identical preons. The maximum multiplicity of each bound state representation is in general one, unless more than one metacolor-spin wave function exists for the same flavor representation. The indices, as defined by 't Hooft[3] can not be negative and have to be smaller than the maximum multiplicity of the representation.

III. Construction of the Bound States.

As explained in the previous section, the massless bound states which we allow have a metacolor-flavor-spin wave function which is totally antisymmetric in the identical preons. Moreover the composites have to be metacolor singlets and left-handed spin-1/2 particles (of course for every left-handed state, a right-handed one with complex conjugate representation can be constructed). Before we discuss the determination of the flavor representation we will first introduce some notations.

In a model with p preons (i.e., p different irreducible representations of the metacolor group G_M) the complete symmetry group of the preons in the presence of metacolor forces but in the absence of any other interactions is

$$G_M \otimes SU(n_1) \otimes \cdots \otimes SU(n_p) \otimes [U(1)]^{p-1} \otimes Z \tag{3.1}$$

where n_i is the multiplicity of the i^{th} irreducible metacolor representation. The p - 1 U(1)-factors are chosen in such a way that they are anomaly-free with respect to G_M. The discrete group Z is the subgroup of the U(1) corresponding to axial preon number, which is left unbroken by G_M-instantons.[21]

We will denote SU(n) representations by Young diagrams, which we specify by a sequence of integers indicating the length of their rows. For example, 0 is the trivial representation, 1 the fundamental one, 2 the symmetric tensor of rank 2, 11 (sometimes written as 1^2) the antisymmetric second rank tensor. The complex conjugate of a representation r is denoted as $\bar{r}$; r^T is the representation obtained by interchanging rows and columns of the Young-diagram of r.

The i^{th} preon transforms according to the representation

$$(r_i, 0, \cdots, 0, 1, 0, \cdots, 0, Q_i^1 \cdots, Q_i^{p-1}, 1) \tag{3.2}$$

of the symmetry group (3.1), in an obvious notation. In the following we will use the quantities d_i, the dimension of the representation r_i of G_M, and $N_i = 2 \times d_i \times n_i$, the number of components of the i^{th} preon field.

To construct totally antisymmetric metacolor-flavor-spin wave functions we use the antisymmetric representations of the group $SU(N_i)$ for each preon in the bound state. This group is not a symmetry group, but all we use are the permutation properties of the representations. To obtain the metacolor, flavor and spin representations seperately we have to calculate the branching of $SU(N_i)$-representations to representations of the subgroup $G_M \otimes SU(n_i) \otimes SU(2)$ (the SU(2) is the left-handed or right-handed factor of the Lorentz group, for particles and antiparticles respectively). This branching consists of three steps:

$$\begin{array}{ccccc} SU(N_i) & \overset{1}{\rightarrow} & SU(d_i) & \otimes & SU(2n_i) \\ & & \downarrow 3 & & \downarrow 2 \\ & & G_M & & SU(n_i) \otimes SU(2) \end{array} \qquad (3.3)$$

Branchings 1 and 2 are special cases of $SU(NM) \rightarrow SU(N) \otimes SU(M)$, for which sufficiently extensive tables are available.[22] Because we are only considering completely antisymmetric representations of $SU(N_i)$, a general rule can be given for branching 1:

$$1^m = \sum_k (Y_{m,k}, Y^T_{m,k}) \qquad (3.4)$$

where m is the number of preon i in the bound state, and $Y_{m,k}$ is any m-box Young-diagram. The sum is over all possible Young-diagrams.

Branching 3 is of a different kind. From the point of view of Lie groups it corresponds to the embedding of a representation r with dimension d of a group G in the fundamental representation of SU(d); from the point of view of the permutation group it corresponds to tensor products of r with definite symmetry properties. The branching rules we need can be obtained by calculating a suitable tensor product and use the sum rules for the dimension, index and anomaly (a derivation of the index and anomaly sum rule is given in the appendix) to identify the terms which belong to a certain SU(d) Young-diagram. The interpretation of this branching as a symmetrized tensor product allows us to generalize the results to arbitrary rank. For Lie groups of type A_n for example, all branching rules can be written in terms of Young-diagrams, without reference to a particular value of n. This implies that the dimension, index and anomaly sum rules can be used for arbitrary n, which makes them much more powerful. We explain the details of this procedure in a separate publication,[23] which will contain tables of tensor products with definite symmetry for asymptotic free and complex representations of SU(n), SO(4n + 2) and E_6.

The final step in the calculation of the flavor representation of a composite is the multiplication of all metacolor, flavor and spin representations obtained for each set of identical preons, and selecting the left-handed spin-1/2 metacolor singlets.

We have considered all anomaly-free, asymptotically free and complex metacolor representations[24] which are direct sums of not more than two different irreducible representations. We consider all simple Lie groups, but restrict ourselves to rank smaller than 8 for the unitary groups. Beside these, only the groups E_6 and SO(4n+2) have complex representations. Each anomaly-free representation can be repeated N times. The maximum value of N is determined by asymptotic freedom. When this maximum is less than 3 we do not consider the model.

A useful concept for the determination of candidate bound states is the congruence class,[25,26] the generalization of the more familiar "n-ality" for SU(n) to any Lie algebra. Most of the possible SU(4), SU(6) and SU(8) models can be shown to have only bosons as bound states. The same is true for most of the SO(4n+2)-models. The congruence class of a SO(4n+2) irreducible representation is given by two integers, defined modulo two and four respectively. The congruence class of a tensor product of two representations is given by the sum of their congruence numbers. Since the spinor representations belong to the congruence classes $(1, \pm 1)$, one cannot construct a singlet (which has congruence numbers (0,0)) out of an odd number of them. It is then clear that to construct a spin-1/2 metacolor singlet an additional representation with first congruence number 0 is needed. This is not allowed by asymptotic freedom for all n except n = 2 (notice that n = 1 gives a metacolor group SO(6), which is isomorphic to SU(4)).

The bound state representations of the complex models we have considered are listed in Table I. For each model the table gives the complete symmetry group (the first factor is always the metacolor group; discrete symmetries are omitted) and the asymptotic freedom limit. Then the representations of the preons for this group are given. For SU(N)-groups we use Young-diagrams, in the notation introduced in the beginning of this section, and for E_6 and SO(4n+2) we use the Dynkin-label.[26,27] Finally the table gives the preon-content of the composites, their representation and maximum multiplicity, and a label. In a few cases we have not been able to determine the multiplicity because our methods were not powerful enough to obtain the branching rule we needed. In the table these multiplicities are indicated by a variable n or m.

The real representations we consider are the $1 + \bar{1}$ and $2 + \bar{2}$ representations for SU(3), SU(5) and SU(7). For these two cases the preons are

$$\text{I:}\quad \alpha = (1,1,0,1) \qquad \text{II:}\quad \alpha = (2,1,0,1)$$
$$\beta = (\bar{1},0,1,-1) \qquad\qquad \beta = (\bar{2},0,1,-1) \tag{3.5}$$

where the notation is the same as in Table I. The left-handed composites are

$$A_n = \alpha^n(\bar{\beta})^{k-n} \qquad B_n = \beta^n(\bar{\alpha})^{k-n} \tag{3.6}$$

where the metacolor group is SU(k) and n is any odd number not larger than k.

The SU(k)-singlet states are the completely antisymmetric combination (1^k) of the fundamental representation, and the completely symmetric representation (k) of the symmetric second rank tensor. Fermi-statistics forces the flavor-spin representation to be completely symmetric for models of type I and completely antisymmetric for type II. This means that the flavor representation has a Young-diagram which is the same and the transposed of the one for the spin representation for type I and II respectively. Since the spin representation is completely fixed, so is the flavor representation:

$$\text{type I} \qquad A_n = \left(0, \left[\frac{n+1}{2}, \frac{n-1}{2}\right], \overline{\left[\frac{k-n}{2}, \frac{k-n}{2}\right]}, k\right),$$

$$B_n = \left(0, \overline{\left[\frac{k-n}{2}, \frac{k-n}{2}\right]}, \left[\frac{n+1}{2}, \frac{n-1}{2}\right], -k\right) \tag{3.7}$$

$$\text{type II} \qquad A_n = \left(0, \left[\frac{n+1}{2}, \frac{n-1}{2}\right]^T, \overline{\left[\frac{k-n}{2}, \frac{k-n}{2}\right]}^T, k\right)$$

$$B_n = \left(0, \overline{\left[\frac{k-n}{2}, \frac{k-n}{2}\right]}^T, \left[\frac{n+1}{2}, \frac{n-1}{2}\right]^T, -k\right) \tag{3.8}$$

where the notation is as in Table I, but with Young-diagrams denoted $[r_i, \cdots, r_n]$, where r_i is the length of the i^{th} row. As before a bar means complex conjugation, a 'T' indicates the transposed diagram. We will refer to these real models with types I and II preon representation of SU(k) metacolor group as k,I and k,II respectively in Table II.

This construction does not work for antisymmetric tensor representations since singlets can be formed in many ways. Consequently the number of candidate massless bound state representations becomes very large (16 for k = 5, 98 for k = 7), which makes a systematic search for solutions extremely cumbersome. We have only looked for solutions which do not require symmetry breaking, for k = 5. No such solutions were found. For k = 6, the representation $11 + \overline{11}$ allows three-preon composites. The flavor representation of the composites is identical to model 3, II. This SU(6)-model is referred to as 6,III in Table II.

IV. Anomaly Matching for Subgroups.

When the unrestricted anomaly equations can not be satisfied one may expect that the chiral symmetry is forced to break. A model with such a forced dynamical symmetry breaking has to meet at least three requirements.

(1) The reduced anomaly equations must have solutions.

(2) A set of composite Goldstone bosons must exist which can break all the symmetries which violate the anomaly equations.

(3) These composites must condense and, when the unbroken subgroup is not unique, choose the required breaking.

The last criterion requires a detailed understanding of the dynamics. At present we can only assume that it is satisfied for cases of interest.

All complex, non-Abelian subgroups which satisfy condition 1 can be found in the following way. Consider the anomaly equations for the non-Abelian part of the chiral flavor group.

$$\sum_{k=1}^{b} \ell_k \, A_i(r_i^{\,k}) = \sum_{\ell=1}^{p} A_i(r_i^{\,\ell}) \quad , \qquad i = 1,\cdots,p \tag{4.1}$$

where b is the number of bound states. The k^{th} bound state is a representation $(r_i^{\,k},\cdots,r_p^{\,k})$ of the flavor group; ℓ_k is the index for that bound state;

A_i is the anomaly of the i^{th} flavor group factor. The multiplicity of r_i is implicitly included in A_i. Now consider an arbitrary embedding of a group G in the entire flavor group. As is shown in the Appendix, the anomaly A_s of that subgroup is related in a simple way to those of the original flavor group:

$$A_s = \sum_{i=1}^{p} C_i \; A_i \tag{4.2}$$

where the coefficients C_i are integers, which are equal to the anomalies of the embedding of G in the fundamental representations of the flavor group. Because these coefficients are independent of the representation, the anomaly matching condition for the subgroup -- assuming all other symmetries are broken down to anomaly-free ones -- is:

$$\sum_{i=1}^{p} C_i \left[\sum_{k=1}^{b} \ell_k \; A_i(r_i{}^k) \; - \; \sum_{\ell=1}^{p} \; A_i(r_i{}^\ell) \right] \; = 0 \tag{4.3}$$

When the deviations from anomaly-matching for each of the p SU(n)-factors of the flavor group -- given a set of indices ℓ_n -- are combined to form a p-dimensional vector, then the C_i's form a vector in its orthogonal subspace. If $p = 2$ this determines the ratio of C_1 and C_2, and hence the solutions are

$$C_1 = \ell n_1 \; , \quad C_2 = \ell n_2 \; , \text{ for any integer } \ell, \tag{4.4}$$

where n_1 and n_2 are the smallest integers satisfying (4.3). Now we know the anomalies of the subgroup in each of the groups, and also the maximum dimension of its representation in each group. Since the representation in both groups has to be non-trivial -- otherwise one of the two SU(n)-factors would itself be a solution -- we also know that the rank of the subgroup can not be larger than the rank of the smallest of both group. Only a very limited search is then required to find all solutions.

When the flavor symmetry breaks, both the preon representations and the bound state representations will in general break into several components. We have implicitly assumed that these components are either all massless or all massive, i.e., we will not find solutions for which the anomaly equations are satisfied by only part of a broken multiplet (real components are of course an exception). Notice that all these states have identical metacolor-spin wave functions and that all preons remain massless if the metacolor representations is complex. Therefore all these components are dynamically equivalent. This seems to justify this restriction, but on the other hand an extension of these arguments would lead to "N-independence",[3] a condition which we have not required in general.

A possibility which is not included in this search is a complex U(1)-symmetry protecting the composites from getting massive. To get an interesting model of this type one would like the unbroken group to contain a real or at least anomaly-free non-Abelian subgroup G in addition to this U(1)-factor. The generator of the complex U(1) group can be a linear combination of generators of all (Abelian and non-Abelian) factors of the full flavor group. A relation similar to (4.3) can be derived, expressing the two anomaly equations of $G \otimes U(1)$ in terms of all anomaly equations of the unbroken flavor group, but this does not lead to any useful restrictions in the general case. For our two-preon models we only investigate the special case that the U(1)-factor of the flavor group is not broken, whereas the non-Abelian part breaks down to an anomaly-free subgroup.

Finally the possibility exists, that the flavor group breaks down to a subgroup for which the preons are in an anomaly-free representation. The anomaly equations do then not require any massless bound states, but one may add any anomaly-free bound state representation, if there is one. (The fact that the preons are in an anomaly-free representation of the subgroup does not necessarily imply the same for the composites, in non-left-right-symmetric models.) To keep these additional, unnecessary composites massless their representation must be complex or there must be an unbroken discrete complex subgroup. In this way one can add massless composites to any minimal solution of the anomaly equations.

V. Results.

The solutions to the (restricted) anomaly-matching equations for the models presented in Section III are shown in Table II. The first two columns indicate the model and the integer N which specifies the flavor group. For complex models the first column refers to Table I; for real models the dimension of the metacolor group and the representation are given in the notation of Section IV. Column 3 and 4 contain the subgroup for which a solution was obtained and the way it is embedded in the flavor group. The embedding is given by two, in general reducible, representations of the subgroup to which the fundamental representations of each of the non-Abelian factors of the flavor group branch. The order of the two factors is as in Table I. Reducible representations are given by direct sums of Young-diagrams; $n * r$ means that representation r occurs with multiplicity n. Column 5 shows the bound states which match the anomalies of the preons. The labels refer to Table I and definition (3.5) for complex and real models respectively. The last column contains references to papers in which the solution was first presented or discussed.

A few remarks concerning the completeness of the table are necessary. As explained in the previous section, the only U(1)-subgroup we consider is the one which is present in the unbroken flavor group. When U(1)-factors are being ignored, the most general continuous subgroup is a product of simple groups. Each of these simple groups has to satisfy the anomaly equations, with the same set of bound states. When a subgroup H of the full flavor group yields a solution, then the same set of massless composites is a solution for any subgroup of H. In Table II we have only listed the largest subgroup in each subgroup chain. Furthermore we have omitted all subgroups with respect to which the preons are in an anomaly-free representation. Therefore, out of each solution in the table, other groups can be constructed by considering subgroups, and by adding non-Abelian factors which satisfy the anomaly equations in a trivial (or non-trivial) way. For example, when a group SU(k) appears in the table, which is embedded in one of the chiral symmetry groups as $n * 1$, one can add the rotations of these n fundamental representations to the unbroken symmetry group, obtaining a group $SU(k) \otimes SO(n)$.

For a few less interesting cases we have omitted the complete list of embeddings and the list of bound states; for model 13 we have only considered the embeddings 1;0, 0;1, 1;1 and $1;\bar{1}$ because of the huge number of possibilities. Furthermore we have not listed sets of bound states differing from those in the table only by an additional anomaly-free combination of composites.

For real (left-right-symmetric) models we have allowed solutions which violate the left-right symmetry. This leads of course to parity doubling of solutions, and we give only one solution out of each pair. When the embedding of the subgroup is left-right symmetric (like the axial-vector SU(N)-subgroup of $SU(N) \otimes SU(N)$), the multiplicities of A_i- and B_i-composites can be interchanged for each i separately. The table contains only one solution out of this class.

Finally we have to point out that, although the composites in column 5 are the only solutions for the subgroup defined in columns 3 and 4, there may be additional solutions when smaller subgroups are considered. This can happen when this subgroup requires a smaller number of anomaly equations to be satisfied.

VI. Dynamical Symmetry Breaking.

To construct realistic models out of the solutions in Table II one has to find a way to break the symmetrics which violate anomaly matching. By selecting a sufficiently large set of composite scalars -- if necessary exotics, in the sense of Section II -- one can usually satisfy the second of the conditions formulated in Section IV. This condition is not extremely restrictive, since it does not require symmetries to remain unbroken. For example, in cases where only the U(1)-factor has to be broken, any charged scalar will usually be sufficient[28] (of course one has to make sure that there is no other U(1) left unbroken which has a component in the original U(1)-factor). When this scalar condenses, it may break part of the non-Abelian group as well, but the unbroken group will in any case satisfy anomaly matching,

A large part of the solutions have an unbroken group SU(3) or SU(4) in rather complicated reducible representations. Although we have listed these solutions for completeness, it does not seem to be very likely that the symmetry will actually break that way. Moreover the group theory of symmetry breaking is not sufficiently developed to allow us to handle such complicated breaking patterns in general. In the remainder of this section we will adopt the philosophy that the simplest scenarios are most likely to be correct, and give some illustrative examples. In all these examples the symmetries can be broken by a scalar in an irreducible representation. When possible, we use the results of Ref. 29 to determine the possible breaking patterns. In more complicated situations we show that the unbroken subgroup we are interested in is a maximal little group of the scalar boson representation.[30]

Model 15 provides two very interesting examples. The simplest composite scalar one can construct in any model which is not left-right symmetric consists of at least four preons. In this model there is only one non-exotic four-preon scalar boson, α^4. By requiring Fermi-statistics one finds that this beson forms a representation (22,0,-8) of the flavor group $SU(N) \times SU(4N) \times U(1)$. If one ignores the U(1)-factor, then this scalar is a singlet for N = 2 and a second rank symmetric tensor with respect to the first group for N = 3. Therefore, for N = 2 the group $SU(2) \times SU(8)$ is left unbroken whereas, according to Ref. 29, for N = 3 the group $SU(3) \times SU(12)$ breaks down either to $SU(2) \times SU(12)$ or to $SO(3) \times SU(12)$. Now we have to consider the U(1)-factor. For N = 2 the U(1)-factor is broken, because the scalar has a U(1)-charge. The same is true for N = 3, but when the symmetry breaks down to $SU(2) \times SU(12)$ there is a new unbroken U(1), generated by a linear combination of an SU(3)-generator and the generator of the original U(1)-factor. The anomaly matching equations involving this U(1) are linear combinations of all five original equations, which are not all satisfied. When the symmetry breaks down to $SO(3) \times SU(12)$ however, it is impossible to embed an additional U(1) in the non-Abelian part of the flavor group, and therefore the U(1)-factor breaks completely. According to Table II, both for N = 2 and N = 3 the unbroken groups -- $SU(2) \times SU(8)$ and $SO(3) \times SU(12)$ respectively -- allow the anomaly equations to be satisfied.

Other examples of simple composite scalars, which can break the symmetries as required can be found in model 10. The simplest non-exotic scalar is $\alpha^2\bar{\beta}^2$, which is a representation $(2,\overline{11}, -6)$ of the flavor group $SU(N) \times SU(2N) \times U(1)$. A maximal little group of the representation is $SU(N) \times SU(2)$, embedded in the following way

$$\begin{aligned} SU(N) \times SU(2N) &\rightarrow SU(N) \times SU(2) \\ (1,0) &\rightarrow (1,0) \\ (0,1) &\rightarrow (1,1) \end{aligned} \tag{6.1}$$

It is a simple matter to check that the scalar breaks down in the following way

$$(2,\overline{11}) = (2 \otimes \bar{2},0) + (2 \otimes \overline{11},2) \tag{6.2}$$

The first term contains a singlet, which means that the subgroup $SU(N) \times SU(2)$ is a little group for the representation $(2,\overline{11})$. To prove that it is a maximal little group one has to show that any larger subgroup does not yield singlets in the branching of the representation $(2,\overline{11})$. When one would enlarge the unbroken group by a U(1)-factor, it can only be the one of the original flavor group. But the composite scalar has a charge for that U(1), and therefore breaks it. Another obvious enlargement of the subgroup is $SU(N) \times SU(N) \times SU(2)$, but this is not a little group either. There are no other enlargements, which implies that $SU(N) \times SU(2)$ is a maximal little group. According to the conjectures, formulated in Ref. 26, it is then a possible unbroken subgroup. This subgroup appears in the table for $N = 6$ (the additional SU(2) is anomaly-free and is therefore allowed by the anomaly equations). Of course the same scalar condensate provides the necessary symmetry breaking for $N = 3$, leaving a subgroup $SU(3) \times SU(2)$ unbroken.

Another example is model 18. The massless composites which satisfy the anomaly matching equations transform both like the 10-dimensional representation of SU(4) and differ only by U(1)-quantum numbers. One of the two U(1)'s is broken to a discrete group by instantons, the other has to be broken spontaneously. This can be done by a condensate α^4 (or β^4). Notice that α^2 nor β^2 cannot form singlets; this is due to the fact that the 16 and 126 of SO(10) are both complex representations. This is also the reason why the $\alpha^3\beta^2$ -composite is not an exotic state. This model illustrates the kind of solution one would like to have to explain generations.

A search for other models in which the desired symmetry breaking can be done by the non-exotic scalar boson with the smallest number of preons yields the following result. In model 4 this boson is $\alpha^3\beta$, which has no U(1)-charge, and consequently can not break the U(1)-factor. In model 11, the boson is $\alpha^2\beta^2$, which transforms like a (2,-6) representation of the flavor group SU(5) $\times$ U(1). The only allowed unbroken subgroup with completely broken U(1)-factor is SO(5), which is a real group. In model 13 all scalar bosons are exotics. All other complex models either have solutions for completely unbroken flavor groups, or have no solutions at all.

To get anomaly matching in other cases than the examples discussed above one obviously has to relax some of the rules governing the construction of bosonic and fermionic composites. For several solutions in Table II one can construct irreducible scalar composites which can break the symmetry, but which are exotic states, according to our definition, formulated in Section.II. It is not clear whether the binding forces for such states are sufficient to assume that they condense; arguments against such condensates are given in Ref. 12.

It is conceivable that this is determined by much more subtle properties than just exoticness; for example, one may only want to discard exotic condensates, consisting of two or more clusters of preons which can form massless spin-1/2 composites, or scalars, which are condensates themselves.

An example of a model with symmetries which are broken by an exotic scalar is given in Ref. 16. The model is based on the SU(6) × SU(6) solution of six flavor QCD (model 3.I in Table II). Since the preon representation is real, the simplest condensate would cause complete chiral symmetry breaking, and one has to assume that this does not happen. The U(1)-symmetry corresponding to preon-number, can then be broken by the scalar α^6 (or β^6). These states are exotic, because they consist of two α^3-clusters, but α^3 does not correspond to a massless bound state in this model. The representation of α^6 for the flavor group SU(6) × SU(6) × U(1) is (6,0,6). The sixth rank symmetric tensor can break SU(6) to SU(5) or SO(6), but only in the latter case the U(1)-symmetry is completely broken, for reasons explained in the discussion of the first example. In Ref. 16 a detailed investigation of the phenomenology of this model is presented.

VII. Conclusions.

We have performed a search for preon models similar in spirit to Ref. 15, but we have extended the scope of this search in two important ways; we have considered bound states of more than three preons and we have included the possibility of dynamical symmetry breaking. The first extension has yielded only two new solutions (models 4 and 13, both with N = 2), but the second allows many new ones, although their number is still surprisingly small compared to the number of possibilities we have investigated.

The second step in our program, to find a potentially realistic dynamical symmetry breaking pattern for our solutions, has also been succesful in part. We have indeed found some non-trivial models which satisfy 't Hooft's equations if a simple four-preon condensate breaks part of the chiral symmetry.

The obvious next step is to look for embeddings of the standard model. Although we have not done that systematically, we do not consider any of our solutions as very satisfactory. In particular, generations do not appear in a natural way.

Further attempts to construct composite models along the lines of Ref. 3 will have to concentrate on the possibilities which we have not considered. There is a large number of complex, anomaly-free and asymptotically free representations[24] which we have not investigated because they consist of more than two irreducible representations. A systematic search which includes all these models is practically impossible.

The rules for the construction of composites which we have chosen are not on a very firm theoretical basis because of insufficient understanding of the dynamics, and they may be modified. An interesting possibility, which may shed some light on the generation problem is to relax the rule which forbids exotics. We have indeed found models which can have a generation-like spectrum in that case, with a mechanism similar to the one of Ref. 5, but those models are a bit too exotic to be discussed here.

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Appendix

In this appendix we formulate and derive a theorem[31] which relates the second and third index of the representation of a group to those of a subgroup. For a simple Lie algebra, these indices are defined as follows:

$$\mathrm{Tr}\ \lambda^a \lambda^b = C(R)\ \delta^{ab} \tag{A1}$$

$$\mathrm{Tr}\ \{\lambda^a \lambda^b\}\ \lambda^c = A(R)\ \ d^{abc} \tag{A2}$$

where λ^a are the generators of the Lie algebra in a representation R. The normalization of the generators and the symmetric tensor d^{abc} can be chosen in such a way that the second index C and the third index, or anomaly, A are integers, which are equal to 1 for the smallest representation.[26,32] In the following we will assume that this normalization is chosen in any group we consider. We denote the representation with indices equal to one as r_o.

Consider a simple subalgebra H of the semisimple Lie algebra $G = G_1 \otimes G_2 \otimes \cdots \otimes G_m$. We denote the generators of H as Λ^α, and those of G_i as λ_i^a, $i = 1 \cdots n$. The embedding of H is defined by the projection of the generators of G on those of H

$$\Lambda^\alpha = \sum_{i=1}^{m} \sum_a M_i^{\ \alpha a}\ \lambda_i^a \tag{A3}$$

Using the fact that the groups G_i have no anomalies with respect to each other, and that $\mathrm{Tr}\ \lambda_i^a\ \lambda_j^b = 0$ for $i \neq j$ one can derive the following relations:

$$C_H(R) = \sum_{i=1}^{m} \gamma_i\ C_i(r_i) \tag{A4}$$

$$A_H(R) = \sum_{i=1}^{m} \alpha_i\ A_i(r_i) \tag{A5}$$

where C_H and A_H are the indices of the group H and C_i and A_i those of G_i. The representation $(r_1 \cdots r_m)$ of G is assumed to reduce to a representation R of H. The coefficients γ_i and α_i are explicitly

$$\alpha_i = \sum_{\alpha\beta\gamma} \frac{d^{\alpha\beta\gamma} d_i^{\ abc}}{d^2} M_i^{\ \alpha a} M_i^{\ \beta b} M_i^{\ \gamma c} \qquad (d^2 = \sum_{\alpha\beta\gamma} (d^{\gamma\beta\gamma})^2) \tag{A6}$$

$$\gamma_i = \sum_{\alpha\beta} \frac{\delta^{\alpha\beta}\ \delta^{ab}}{\delta^2} M_i^{\ \alpha a} M_i^{\ \beta b} \qquad (\delta^2 = \sum_{\alpha\beta} (\delta^{\alpha\beta})^2\ ) \tag{A7}$$

where $d^{\alpha\beta\gamma}$ and $d_i^{\ abc}$ are the symmetric tensor of the groups H and G_i respectively.

Obviously the coefficients α_i and γ_i are independent of the representation. For the third index this is a direct consequence of the fact that the symmetric tensor is representation-independent. To calculate the coefficients we can use any representation we prefer, and we choose the fundamental representations of G, defined as follows:

$$f_j = (r_1^{\ j} \cdots, r_m^{\ j}) \qquad \text{with } r_i^{\ j} = \delta_{ij}\ r_o \tag{A8}$$

The representation of H contained in f_j is $R(f_j)$. Substituting this in (A4) and (A5) we find:

$$\gamma_i = C_H(R(f_i)) \tag{A9}$$

$$\alpha_i = A_H(R(f_i)) \tag{A10}$$

This implies that for our normalization of the generators the coefficients are integers, which can be interpreted as the indices of the representations of H, embedded in the fundamental representations of G. Relations (A4),(A5) and (A9),(A10) are the basic result of this Appendix. The extension of this result to indices of higher order is non-trivial, because the equivalent of (A1) and (A2) does not exist. Indeed, a straightforward generalization of (A4) and (A5) to the fourth index does not hold.[33] The results of this appendix are valid in exactly the same way if H is a U(1)-factor.

Table Captions

Table I List of the models with complex metacolor representations we have considered. Listed are the full symmetry group, the preon-representations and the candidates for massless composites. A more detailed explanation is given in Section III.

Table II List of solutions. A detailed explanation of this table is given in Section V. The references are:

(a) G. 't Hooft, Ref. 3;

(b) Dimopoulos, Raby and Susskind, Ref. 4;

(c) Banks, Yankielowicz and Schwimmer, Ref. 14;

(d) C. Albright, Ref. 15;

(e) C. Albright, B. Schrempp and F. Schrempp, Ref. 16;

(f) Dan-Di Wu, Ref. 19.

Table I

Model 1

Group:	SU(3) x SU(N) x SU(7N) x U(1)		$N \leq 2$
Preons:	α	(2,1,0,7)	
	β	($\bar{1}$,0,1,-5)	
Composites:	α^3	(0,21,0,21)	A
	β^3	(0,0,21,-15)	B
	$\alpha\beta^2$	(0,1,2,-3)	C
	$\alpha\beta^2$	(0,1,11,-3)	D

Model 2

Group:	SU(4) x SU(N) x SU(8N) x U(1)		$N \leq 3$
Preons:	α	(2,1,0,4)	
	β	($\bar{1}$,0,1,-3)	
Composites:	$\alpha\beta^2$	(0,1,2,-2)	A
	$\alpha\beta^2$	(0,1,11,-2)	B

Model 3

Group:	SU(5) x SU(N) x SU(9N) x U(1)		$N \leq 3$
Preons:	α	(2,1,0,-9)	
	β	($\bar{1}$,0,1,7)	
Composites:	$\alpha\beta^2$	(0,1,2,5)	A
	$\alpha\beta^2$	(0,1,11,5)	B
	β^5	(0,0,32,35)	C
	α^5	(0,221,0,-45)	D

Model 4

Group:	SU(5) x SU(N) x SU(N) x U(1)		$N \leq 13$
Preons:	α	(1,1,0,3)	
	β	($\overline{11}$,0,1,-1)	
Composites:	α^5	(0,32,0,15)	A
	$\alpha\bar{\beta}^2$	(0,1,$\bar{2}$,5)	B
	$\alpha^2\beta$	(0,11,1,5)	C
	$\alpha^2\beta$	(0,2,1,5)	D
	β^5	(0,0,32,-5)	E
	β^5	(0,0,221,-5)	F
	β^5	2x(0,0,311,-5)	G
	β^5	(0,0,41,-5)	H
	β^5	(0,0,211,-5)	J

Table I - cont'd

Model 5

Group:	SU(5) x SU(N) x SU(9N) x U(1)		$N \lesssim 1$	
Preons:	α	(2,1,0,-27)		
	β	$(\overline{11},0,1,7)$		
Composites:	$\alpha\bar{\beta}^4$	$2x(0,1,\overline{22},-55)$	A	
	$\alpha\bar{\beta}^4$	$(0,1,\bar{4},-55)$	B	
	$\alpha\bar{\beta}^4$	$(0,1,\overline{1111},-55)$	C	
	$\bar{\alpha}^2\beta^3$	$(0,\bar{2},21,75)$	D	
1	β^5	(0,0,R,35)	E - J	
		(R as for Model 4, E - J)		

Model 6

Group:	SU(5) x SU(N) x SU(16N) x U(1)		$N \lesssim 1$	
Preons	α	(21,1,0,8)		
	β	$(\bar{1},0,1,-11)$		
Composites:	α^5	mx(0,5,0,40)	A	
	β^5	(0,0,32,-55)	B	

Model 7

Group:	SU(5) x SU(N) x SU(15N) x U(1)		$N \leq 1$	
Preons:	α	(22,1,0,3)		
	β	$(\bar{1},0,1,-7)$		
Composites:	β^5	(0,0,32,-35)	A	
	$\alpha\beta^4$	(0,1,4,-25)	B	
	$\alpha\beta^4$	(0,1,1111,-25)	C	
	$\alpha\beta^4$	(0,1,211,-25)	D	
	$\alpha\beta^4$	(0,1,31,-25)	E	
	$\alpha\beta^4$	(0,1,22,-25)	F	
	$\alpha^3\beta^2$	2x(0,3,2,-5)	G	
	$\alpha^3\beta^2$	(0,3,11,-5)	H	
	$\alpha^4\beta$	mx (0,4,1,5)	J	$m \leq 2$
	α^5	nx (0,5,0,15)	K	

Table I - cont'd.

Model 8

Group:	$SU(5) \times SU(N) \times SU(6N) \times U(1)$		$N \leq 1$	
Preons:	α	(211, 1,0,1)		
	β	($\bar{1}$,0,1,-4)		
Composites:	β^5	(0,0,32,-20)	A	
	$\alpha\beta^4$	(0,1,4,-15)	B	
	$\alpha\beta^4$	2x(0,1,31,-15)	C	
	$\alpha\beta^4$	(0,1,22,-15)	D	
	$\alpha\beta^4$	2x(0,1,211,-15)	E	
	$\alpha^2\beta^3$	3x(0,2,3,-10)	F	
	$\alpha^2\beta^3$	5x(0,2,21,-10)	G	
	$\alpha^2\beta^3$	2x(0,2,111,-10)	H	
	$\alpha^3\beta^2$	5x(0,3,2,-5)	J	
	$\alpha^3\beta^2$	6x(0,3,11,-5)	K	
	$\alpha^4\beta$	mx(0,4,1,0)	L	$m \leq 5$
	α^5	nx(0,5,0,5)	M	

Model 9

Group:	$SU(5) \times SU(N) \times SU(6N) \times U(1)$		
Preons	α	(211,1,0,3)	
	β	($\overline{11}$,0,1,-4)	
Composites:	$\alpha\beta^2$	(0,1,11,-5)	A
	$\alpha\beta^2$	(0,1,2,-5)	B
	α^5	nx(0,5,0,15)	C
	β^5	(0,0,R,-20)	D-H
		(R as for Model 4, E-J)	

Model 10

Group:	$SU(6) \times SU(N) \times SU(2N) \times U(1)$		$N \leq 11$
Preons:	α	(11,1,0,-1)	
	β	($\bar{1}$,0,1,2)	
Composites:	$\alpha\beta^2$	(0,1,11,3)	A
	$\alpha\beta^2$	(0,1,2,3)	B
	α^3	(0,21,0,-3)	C
	$\alpha\bar{\beta}^4$	(0,1,$\overline{22}$,-9)	D

Table 1 - cont'd.

Model 11

Group:	SU(6) x SU(5N) x SU(N) x U(1)		$N \le 2$
Preons:	α	(11,1,0,2)	
	β	$(\bar{2},0,1,-5)$	
Composites:	α^3	(0,21,0,6)	A
	$\alpha^4\beta$	2x(0,211,1,3)	B
	$\alpha^4\beta$	2x(0,31,1,3)	C
	$\alpha^4\beta$	(0,22,1,3)	D
	$\alpha^4\beta$	(0,1111,1,3)	E

Model 12

Group:	SU(6) x SU(N) x SU(10N) x U(1)		$N \le 2$
Preons:	α	(2,1,0,5)	
	β	$(\bar{1}, 0,1,-4)$	
Composites:	$\alpha\beta^2$	(0,1,11,-3)	A
	$\alpha\beta^2$	(0,1,2,-3)	B

Model 13

Group:	SU(7) x SU(N) x SU(3N) x U(1)		$N \le 9$
Preons:	α	(11,1,0,3)	
	β	$(\bar{1},0,1,-5)$	
Composites:	$\alpha\beta^2$	(0,1,2,-7)	A
	$\alpha\beta^2$	(0,1,11,-7)	B
	$\bar{\alpha}^2\beta^3$	$(0,\bar{2},21,-21)$	C
	β^7	(0,0,43,-35)	D
	$\alpha^4\beta$	$(0,R_1,1,7)$	E_1-E_4
	α^7	$(0,R_2,0,21)$	F_1-F_{12}

where

R_1 = 22 + 2x 211 + 1111 + 2x 31

R_2 = 52 + 511 + 43 + 3x 421 + 2x 4111 + 2x 331 + 2 x 322 + 4x 3211 + 2x 31111 + 2x 2221 + 2x 2211 + 211111

Table I - cont'd.

Model 14

Group:	SU(7) x SU(N) x SU(11N)		$N \leq 3$
Preons:	α	(2,1,0,-11)	
	β	($\bar{1}$,0,1,9)	
Composites:	$\alpha\beta^2$	(0,1,11,7)	A
	$\alpha\beta^2$	(0,1,2,7)	B
	β^7	(0,0,43,63)	C
	α^7	(0,2221,0,-77)	D

Model 15

Group:	SU(8) x SU(N) x SU(4N) x U(1)		$N \leq 8$
Preons:	α	(11,1,0, 2)	
	β	($\bar{1}$,0,1,-3)	
Composites:	$\alpha\beta^2$	(0,1,2,-4)	A
	$\alpha\beta^2$	(0,1,11,-4)	B
	$\alpha^3\bar{\beta}^2$	(0,21,$\overline{11}$,12)	C
	$\alpha\bar{\beta}^6$	(0,1,$\overline{33}$,20)	D

Model 16

Group:	SU(8) x SU(N) x SU(12N) x U(1)		$N \leq 4$
Preons:	α	(2,1,0,6)	
	β	($\bar{1}$,0,1,-5)	
Composites:	$\alpha\beta^2$	(0,1,2,-4)	A
	$\alpha\beta^2$	(0,1,11,-4)	B

Model 17

Group:	SO(10) x SU(N_1) x SU(N_2) x U(1)		$N_1 = 1$; $N_2 \leq 4$
Preons:	α	(00002,1,0 ,-2N_2)	
	β	(00001,0,1,35N_1)	
Composites:	$\alpha\bar{\beta}^2$	(00000,1,$\bar{2}$,-70N_1-2N_2)	A
	$\alpha^3\bar{\beta}^2$	nx(00000,1,$\bar{2}$,-70N_1-6N_2)	B
	$\alpha^3\bar{\beta}^2$	mx(00000,1,$\overline{11}$,-70N_1-6N_2)	C

Table I - cont'd.

Model 18

Group:	$SO(10) \times SU(N_1) \times SU(N_2) \times U(1)$		$N = 1,\ N_2 \leq 4$
Preons:	α	$(00020,1,0,-2N_2)$	
	β	$(00001,0,1,35N_1)$	
Composites:	$\alpha\beta^2$	$(00000,1,2,70N_1-2N_2)$	A
	$\alpha\beta^2$	$(00000,1,11,70N_1-2N_2)$	B
	$\alpha^3\beta^2$	$n\times(00000,1,2,70N_1-6N_2)$	C
	$\alpha^3\beta^2$	$n\times(00000,1,11,70N_1-6N_2)$	D

Model 19

Group:	$E_6 \times SU(N)$		$N \leq 22$
Preon:	α	$(000010,1)$	
Composite:	α^3	$(000000,21)$	A

Table II

Model	N	(Sub) group	Embedding	Composites	Ref.
1	1	SU(7) ⊗ U(1)	trivial	D	c
2	1	SU(8) ⊗ U(1)	trivial	B	c
3	1	SU(9) ⊗ U(1)	trivial	B	c
4	1	U(1)	trivial	B; D	b,d
	2	SU(2) ⊗ SU(2) ⊗ U(1)	trivial	B + H	
	3	SU(3) ⊗ U(1)	0;1	C + G	
		SU(3)	$1;\bar{1}$	C+F; C+D+2G; A+D+F+G; A+C+E+F; A+C+D	
		G ⊗ U(1)	real	C + G	
	4	SU(4)	1;1	F+G+J	
		SU(4)	0;1	D; B+2G; B+E+F+G+J; B+C+F+2G+J	
		SU(4)	$1;\bar{1}$	D+E; C+D+G; B+C+E+F+G	
	5	SU(5) ⊗ SU(5)	trivial	C	
	6	SU(6)	$1;\bar{1}$	C+D+F+J; A+B+D+F+2G+J	
		SU(3)	1;2*1	B+C+F+J	
	7	SU(3)	$2;\bar{1}$	B+D+E+F	
	8	SU(8)	1;1	B + J	
		SU(8)	$1;\bar{1}$	A+F+G	
		SU(3)	$2*1;\bar{2}$	D	
	9	SU(9)	$1;\bar{1}$	B+C+J	
		SU(3)	$2;\bar{1}$	D+F+J	
	10	SU(5)	1;2*1 or 11;2*1	B + D	
		SU(3)	several embeddings		
	11	SU(3)	$3*1;\ 2*\bar{1}$	A+B+C+E+F+J	
	13	SU(4)	$3*1;\ \bar{2}$	D	
8	1	SU(6) ⊗ U(1)	trivial	K + 3L	
		SU(6)	trivial	2K + L; 5L	
		G ⊗ U(1)	real	K	
9	1	SU(6) ⊗ U(1)	trivial	B	d
10	1	SU(2) ⊗ U(1)	trivial	B	b
	3	SU(3) ⊗ SU(6)	trivial	A	
	4	SU(4)	$1;2*\bar{1}$	A + C	
	6	SU(6)	1;2*1	C	

Table II - cont'd.

Model	N	(Sub) group	Embedding	Composites	Ref.
		SU(6)	$1;\bar{1}$	B+C	
		SU(5)	$1;\overline{11}$	B+C	
	8	SU(3)	$2;\bar{1}+\bar{2}$	B+C	
	9	SU(3)	$1+2;\bar{1}+4$	C	
		SU(3)	$1+2;\overline{31}$	A+C	
		SU(3)	$1+2;2*\bar{2}$	A+C	
	10	SU(3)	$\bar{1}+2;\bar{1}+4$	C	
		SU(3)	$\bar{1}+2;2*1+\bar{2}$	B	
		SU(3)	$\bar{1}+2;5*\bar{1}$	B	
		SU(3)	$3;1+\bar{2}+\bar{3}$	B+C	
	11	SU(4)	$2;2*\bar{1}+\bar{2}$	B+C	
		SU(4)	$2;\bar{1}+\bar{2}$	A	
11	1	SU(5)	trivial	D;A + E	
12	1	SU(10) ⊗ U(1)	trivial	A	c
13	1	SU(3) ⊗ U(1)	trivial	A	b
	2	SU(2) ⊗ SU(6) ⊗ U(1)	trivial	$B+E_4$	
	3	SU(3)	$1;0$ or $1;1$ or $1;\bar{1}$	many solutions	
	4	SU(4)	$1;0$ or $1;1$ or $1;\bar{1}$	many solutions	
	5	SU(5)	$1;0$ or $1;1$ or $1;\bar{1}$	many solutions	
	6	SU(6)	$1;\bar{1}$	$B + E_3 + F_{11}$	
	7	SU(7)	$1,\bar{1}$	$B + E_3$	
14	1	SU(11) ⊗ U(1)	trivial	A	c
15	1	SU(4) ⊗ U(1)	trivial	A	b
	2	SU(2) ⊗ SU(8)	trivial	B	
	3	SU(12)	0;1	A + B + C	
16	1	SU(12) ⊗ U(1)	trivial	B	c
18	4	SU(4)	trivial	A+C	
19	6	SU(6)	trivial	A	

Table II - cont'd.

Model	N	(Sub) group	Embedding	Composites	Ref.
Left-right symmetric Models					
3,I	2	SU(2) ⊗ SU(2) ⊗ U(1)	trivial	$A_i + B_j$; i = 1,3; j = 1,3	a
	3	SU(3) ⊗ SU(3)	trivial	B_1	
	4	SU(4)	1;1	B_1	
	6	SU(6) ⊗ SU(6)	trivial	$A_1 + B_1$	e
	10	SU(3)	2*1;3*1	B_1	
		SU(3)	$2*1;3*\bar{1}$	$A_1 + B_3$	
	12	SU(6)	2*1;1	$B_1 + B_3$	
		SU(5)	2*1;11	$B_1 + B_3$	
		SU(3)	4 embeddings	$B_1 + B_3$ or $A_1 + B_3$	
	13	SU(3)	$2+2*\bar{1}$; 1+2	B_1	
	14	SU(7)	$1;2*\bar{1}$	$A_1 + B_3$	
		SU(5)	$11;2*\bar{1}$	$A_1 + B_3$	
		SU(3)	$3*1;1+\bar{2}$	$A_1 + B_3$	
		SU(3)	$4*1;\bar{1}+\bar{2}$	$A_1 + B_3$	
	16	SU(4)	$4*1;\bar{1}+\bar{2}$	$A_1 + B_3$	
		SU(4)	$\bar{1}+2;\bar{1}$	$A_3 + B_3 + B_1$	
		SU(3)	several embeddings	9 solutions	
3,II	3	SU(3)	0;1	B_1	
5,I	2	SU(2) ⊗ SU(2) ⊗ U(1)	trivial	$A_i + B_j$; i,j = 1,3,5	f
	3	G ⊗ U(1)	real	$A_1 + B_1 + A_3 + B_3$	
	10	SU(3)	1+2;3	$B_1 + A_3 + A_5$	
6,III	5	SU(5)	1;0	A_1	
	6	SU(3)	$1;2*\bar{1}$	$A_1 + B_1 + B_3$	
7,I	2	SU(2) ⊗ SU(2) ⊗ U(1)	trivial	$A_i + B_j$; i,j = 1,3,5,7	
	3	SU(3)	1;0	$B_3 + B_5 + A_7$	
7,II	3	SU(3)	1;1	A_3	

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