

Fermi Motion Effects in Deep Inelastic Lepton
Scattering from Nuclear Targets*

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Abstract

The ratio of deep inelastic structure functions of nuclear targets to the sum of free neutron and proton structure functions has been calculated, using a modified form of the Atwood and West technique for deuterium. Fermi gas momentum distributions were used with modifications to include high momentum tails resulting from nucleon-nucleon correlations. Tables of smearing ratios for W_1 , W_2 and W_3 are given as a function of x and Q^2 for deuterium and several heavy nuclei. We find that for $x > 0.5$ the scaling violations for heavy nuclei are smaller than those for free nucleons. The shapes of the antiquark distributions are also changed.

*Work supported in part by the U. S. Department of Energy

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I. INTRODUCTION

The effects of nuclear binding in the case of deep inelastic lepton scattering from the deuteron has been discussed in detail by Atwood and West¹. In this communication we generalize the method to the case of heavy nuclei and present numerical results that can be used by recent^{2,3} experiments which make use of heavy nuclear targets.

Deep inelastic lepton scattering is an important tool in the study of nucleon structure. Structure function measurements using electron, muon and neutrino beams have been used to test the quark parton model and to investigate scaling violation effects predicted by quantum chromodynamics (QCD). The comparison of neutrino measurements with measurements with electrons and muons provides information on the charges of nucleon constituents. The comparison of high energy muon experiments with the lower energy experiments at SLAC provide tests of the scaling violations predicted in QCD.

An experimental difficulty in such comparisons is that different experiments use different nuclear targets. The experiments at the Stanford Linear Accelerator Center (SLAC) use electron beams in conjunction with liquid hydrogen and liquid deuterium targets⁴. The high statistics neutrino and muon experiments typically use steel targets³. In addition to complicating the comparison of results from different experiments, Fermi motion effects make QCD tests more difficult since they change the shape of the structure function and thus can alter the magnitude of the scaling violations.

Fermi motion effects are also of interest in another field which is the study of nuclear shadowing in inelastic electron and muon scattering experiments⁵ in the low x , low Q^2 region. Here the separation of shadowing effects from Fermi motion effects is important. This work was initially undertaken for the analysis of a nuclear shadowing experiment at SLAC (Ditzler et al⁵).

In this communication, we describe the Fermi motion effects which modify the measured structure functions for nuclear targets from the results expected for free neutrons and protons. We calculate the effects in the incoherent impulse approximation. We extend the Atwood-West technique for the deuteron¹ to the case of a heavy nucleus. For the nuclear wave functions we use a Fermi gas momentum distribution⁶, modified to include a high momentum component⁷ that arises from nucleon-nucleon correlations inside the nucleus⁸. We will use off-shell kinematics to ensure that energy and momentum are conserved in the scattering process in the impulse approximation, but we will assume on-shell dynamics, i.e., we will make a correspondence between off-shell structure functions and structure functions that are measured for free nucleons^{9,10,11}. The results are given in tables which can readily be used by various experiments.

II. Kinematics

A. Kinematics-free protons

Before we discuss the kinematics of the scattering from an off-shell nucleon in a deuterium or heavy target nucleus we consider the case of scattering from a free proton. We will take the case of electron scattering to represent the general lepton-nucleon scattering at high energies. The kinematics of the scattering from a free proton of mass M_p is shown in figure 1a. The incident electron energy is E_0 and the final scattering energy in the laboratory system is E' . The scattering angle in the laboratory is defined as θ . The four-momentum transfer to the target proton is $q = (\vec{q}_3, q_0)$. We define the following variables in terms of laboratory energies and angles. The square of the invariant four-momentum transfer q is

$$q^2 = -4 E_0 E' \sin^2(\theta/2) = -Q^2 \quad (1)$$

The square of the initial target proton four-momentum P_i is

$$P_i^2 = M_p^2.$$

The square of the final state proton momentum P_f (which is equal to the final state invariant mass) is

$$P_f^2 = W^2 = (P_i + q)^2 = P_i^2 + 2P_i \cdot q + q^2 = M_p^2 + 2Mv - Q^2 \quad (2)$$

where $v = E_0 - E' = q_0$ (in the lab system), and $X = Q^2/(2q \cdot P_i) = Q^2/(2M_p v)$.

B. Kinematics - scattering from an off-shell nucleon in the deuteron

In the impulse approximation, the spectator nucleon in the deuteron is free and is on the mass shell. It is totally unaffected by the interaction. The interacting nucleon with momentum P_i must be off the mass shell in order to conserve energy and momentum in the scattering process. The kinematics is shown in figure 1b. The Fermi motion does not change Q^2 but it does change the final state invariant mass W and the quantity $P_i \cdot q$. Because the interacting nucleon is off the mass shell, its effective mass is less than the mass of the proton and is a function of its momentum (see figure 2). The on-shell spectator has momentum \vec{p}_s and on-shell energy $E_s = (\vec{p}_s^2 + M_p^2)^{1/2}$. The off-shell interacting proton has momentum $-\vec{p}_s$ and off-shell energy in the lab $E_i = M_d - E_s$, where M_d is the mass of the deuteron, i.e.

$$\vec{p}_i = -\vec{p}_s \text{ and } E_i = M_d - (\vec{p}_s^2 + M_p^2)^{1/2} \quad (3)$$

After the scattering the invariant mass of the final state (neglecting the free spectator) is

$$P_f^2 = W'^2 = (P_i + q)^2 = P_i^2 + 2 P_i \cdot q - Q^2,$$

$$W'^2 = (E_i^2 - \vec{P}_i^2) + 2 E_i v - 2 P_3 |\vec{q}_3| - Q^2, \quad (4)$$

where P_3 is the momentum along the direction of the \vec{q}_3 vector.

C. Kinematics - scattering from an off shell nucleon in the nucleus ($P < K_F$)

For momenta less than the Fermi momenta K_F , the nucleon interacts with the average potential of all the nucleons in the nucleus of atomic weight A . Therefore, in the impulse approximation, the interacting nucleon has momentum P_i which is balanced by a recoiling nucleus of atomic weight $A-1$ and momentum $P_{A-1} = -P_i$. The interacting nucleon is off the mass shell, and the recoiling $A-1$ nucleus is on the mass shell. After the collision the recoiling nucleus is not in a highly excited state, and all the particles are on the mass shell (see figure 1c). In the lab system we have

$$\vec{P}_i = -\vec{P}_s, \quad E_i = M_A - \sqrt{\vec{P}_s^2 + M_{A-1}^2},$$

and (5)

$$W'^2 = (E_i^2 - \vec{P}_s^2) + 2 E_i v - 2 P_3 |\vec{q}_3| - Q^2.$$

D. Kinematics - scattering from an off shell nucleon in the nucleus ($P \gg K_F$)

In the simple Fermi gas model the nucleons cannot have momenta greater than the Fermi momentum K_F . Such high momenta can only come from the interaction between individual nucleons through their hard core potential. In the case

where the nucleon has acquired its high momentum by interacting with another single nucleon we can assume that a single nucleon is recoiling against it. This case can be treated as having a quasi-deuteron in the nucleus with a spectator nucleus of atomic weight $A-2$ which is at rest in the laboratory system. The kinematics (shown in figure 1d) are the same as the scattering from a nucleon bound in the deuteron.

III. NUCLEAR MOMENTUM DISTRIBUTIONS

We have used Fermi gas momentum distributions that were obtained from fits⁶ to quasielastic electron scattering data from heavy nuclei. In the Fermi gas model the momentum distribution is constant up to the maximum Fermi momentum K_F and is zero above K_F . We have added a high momentum tail to the momentum distribution according to Moniz⁷ which is based on calculations of nucleon-nucleon correlations in nuclear matter⁸. The normalized momentum distributions used are:

$$\begin{aligned} |\phi(\vec{P})|^2 &= \frac{1}{c} \left[1 - 6 \left(\frac{K_F a}{\pi} \right)^2 \right] && \text{for } 0 < |\vec{P}| < K_F \\ &= \frac{1}{c} \left[2R \left(\frac{K_F a}{\pi} \right)^2 \left(\frac{K_F}{P} \right)^4 \right] && \text{for } K_F < |\vec{P}| < 4 \text{ GeV}/c \\ &= 0 && \text{for } |\vec{P}| > 4 \text{ GeV} \end{aligned} \quad (6)$$

with $a = 2 \text{ GeV}^{-1}$, $c = \frac{4}{3} \pi K_F^3$ and $R = 1/[1 - K_F/4]$. These momentum distributions satisfy the normalization

$$\int_0^{4 \text{ GeV}/c} |\phi(\vec{P})|^2 4\pi P^2 dP = 1.0 \quad (7)$$

The Fermi momenta K_F given in reference 6 are plotted in figure 3 versus the atomic weight A. As in reference 6, the difference in the momentum distributions for protons and neutrons was taken into account as follows:

$$K_F^P = K_F \left(\frac{2Z}{A} \right)^{1/3}$$

$$K_F^N = K_F \left(\frac{2N}{A} \right)^{1/3} \quad (8)$$

where $A = Z + N$ is the atomic weight, Z is the number of protons and N is the number of neutrons. For an isoscalar target $Z = N = A/2$ and $K_F^P = K_F^N = K_F$. The momentum distributions for carbon (C^{12}), silicon (Si^{28}), iron (Fe^{56}) and lead (Pb^{208}) are shown on linear and logarithmic scales in figure 4a and 4b. The Fermi momenta that are shown are: 0.221 GeV for C^{12} , 0.239 GeV for Si^{28} , 0.257 GeV for Fe^{56} and 0.265 GeV for Pb^{208} .

IV. IDENTIFICATION OF OFF-SHELL STRUCTURE FUNCTIONS

The on-shell structure functions w_1 and w_2 are functions of two variables Q^2 and $q \cdot P$ (or Q^2 and W). The off-shell structure functions can depend on three variables Q^2 , $q \cdot P_1$ and P_1^2 (or Q^2 , $q \cdot P_1$ and w'). The Fermi motion does not affect Q^2 but it does affect $q \cdot P_1$ and w' .

A reasonable procedure^{9,10,11} is to assume that the off-shell structure functions are the same as on-shell structure functions with the same Q^2 and final state mass $W' = W$. This identification ensures that when the final state mass is that of a free proton (i.e., no pion production), the process is clearly identified as quasielastic scattering and the nucleon elastic form factors are used for the structure functions. The same procedure

ensures proper treatment⁹ of electroproduction of resonances and single pion production.

In the case of scattering from bound nucleons inside the deuteron, the kinematics (and therefore W') are uniquely defined since the recoil spectator is a free nucleon (equation 4).

In the case of the scattering from an off-shell nucleon in the nucleus, the invariant mass of the final state depends on the mass of the recoil spectator (see figure 2). We expect that for $|\vec{P}_1| \ll K_F$ the recoil is a nucleus with $A-1$ nucleons since the nucleon interacts with the average potential of all the other nucleons. For the case of $|\vec{P}_1| \gg K_F$ it is most likely a single nucleon since such high momentum typically come from a single scattering. However, in the intermediate region we can have recoil spectator masses which are between these two extremes. In addition to quasi-deuterons from nucleon-nucleon correlations, multi-nucleon correlations such as quasi-alpha particles etc., can also lead to high momentum components^{12,24}. In general we need a nuclear wave function which is a function not only of the three momentum $|\vec{P}|$ but also a function of the off-shell energy. As an approximation, we apply the case of the heavy $A-1$ nucleus spectator (figure 1c) to all momenta less than the Fermi momentum K_F , and apply the case of a single nucleon spectator (figure 1d) to all momenta greater than K_F .

V. THE SMEARING EXPRESSIONS

The general expression for the cross section for scattering of high energy electrons from a nuclear target of momentum P^A can be written in terms of two structure functions w_1^A and w_2^A .

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{MOTT} [w_2(Q^2, q \cdot P^A) + 2 \tan^2 \theta w_1(Q^2, q \cdot P^A)] \quad (9)$$

where

$$\sigma_{\text{MOTT}} = \frac{\sigma^2}{4E_0} \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \quad (10)$$

The above expression is derived¹³ from the general electromagnetic tensor

$$W_{\mu\nu}^A(p, q) = -W_1^A(Q^2, q \cdot p^A) \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + \quad (11)$$

$$\frac{W_2^A(Q^2, q \cdot p^A)}{M_A^2} \left(p_\mu^A - \frac{q \cdot p^A}{q^2} q_\mu \right) \left(p_\nu^A - \frac{q \cdot p^A}{q^2} q_\nu \right)$$

A generalization of the West¹ approach for the deuteron to the case of a heavy nucleus is to express the heavy target tensor $W_{\mu\nu}^A$ in terms of off-shell nucleon tensors.

$$W_{\mu\nu}^A = Z \int |\phi(\vec{p})|^2 d^3\vec{p} [W_{\mu\nu}^p(p_1, q)] + \text{similar terms for the neutrons} \quad (12)$$

Equating individual tensor components we obtain equations for W_1 and W_2 which are identical to those derived for the deuteron in Ref. 1 except that nuclear momentum distributions are used for $|\phi(\vec{p})|^2$. In addition, the identification of the off-shell kinematics is as described in the previous section.

$$W_1^A = Z \int |\phi(\vec{p})|^2 d^3\vec{p} \left[W_1^p + \frac{W_2^p}{2M_p^2} (\vec{p}^2 - p_3^2) \right] + \text{similar term for the neutrons} \quad (13)$$

$$W_2^A = Z \int |\phi(\vec{p})|^2 d^3\vec{p} \left[\left(1 - \frac{p_3 Q^2}{M_p v^1 q_3} \right) \left(\frac{v^1}{v} \right)^2 + \frac{p^2 - p_3^2}{2M_p^2} \left(\frac{Q^2}{q_3^2} \right) \right] W_2^p \quad (14)$$

+ similar term for the neutrons

Here p_3 is the momentum component along the \vec{q}_3 direction and $v^1 = p_1 \cdot q / M_p$.

The identification of the off-shell structure functions ($W_1^p, W_2^p, W_1^n, W_2^n$) was described in the previous section

We express the Fermi motion effects in terms of a ratio of the sum of structure functions for free neutrons and protons to the structure functions calculated for a heavy target. We define the smearing ratios for W_1 and W_2

$$S_1 = \frac{ZW_1^p + NW_1^N}{W_1^A} \quad (15)$$

$$S_2 = \frac{ZW_2^p + NW_2^N}{W_2^A}$$

We also define the normalized structure function for a nucleus as follows

$$\bar{W}_2^A = W_2^A \cdot (2/A) \quad (16)$$

This average structure function should be about the same for all nuclei except for the Fermi motion effects and the difference between neutron and proton structure functions.

We have calculated the smearing ratios S_1 and S_2 for the deuteron using Hamada-Johnston¹⁴ wave functions and compare the results to the smearing ratios calculated for heavy nuclei. Detailed studies of the smearing ratios for the deuteron with various different wave functions have been performed by Atwood and West^{1,15} and Bodek^{9,10,11}.

VI. STRUCTURE FUNCTION FITS

We have used fits to deep inelastic structure functions of Bodek et al¹¹ which used a parametrization due to Atwood and Stein^{16,17}. The parametrization fits all the SLAC data published in Reference 11. It includes scaling violations in terms of a modified scaling variable¹⁸ ω_w and describes the data

over the entire SLAC Q^2 range ($0.1 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$). The structure function W_2 is described by

$$\sqrt{W_2}(\nu, Q^2) = B(W, Q^2) g(\omega_w) \omega_w / w$$

$$g(\omega_w) = \sum_{n=3} C_n (1 - 1/\omega_w^n)$$

$$\omega_w = \frac{2M_p \nu + a^2}{Q^2 + b^2} \quad (17)$$

The modulating function $B(W, Q^2)$ contains 12 parameters representing the masses, widths and amplitudes of the cross section for electroproduction of the four most prominent nucleon resonances, and eight parameters representing the W dependence of the low W non-resonant contribution and single pion production threshold. The modulating function $B(W, Q^2)$ is close to unity in the deep inelastic region ($W > 2 \text{ GeV}$). The parameters from Bodek et al¹¹ are: $a^2 = 1.642$ and $b^2 = 0.376$ for both the neutron and the proton. The other parameters are: $C_3 = 0.256$, $C_4 = 2.178$, $C_5 = 0.898$, $C_6 = -6.716$, and $C_7 = 3.756$ for the proton; and $C_3 = 0.064$, $C_4 = 0.225$, $C_5 = 4.106$, $C_6 = 7.079$, and $C_7 = 3.055$ for the neutron. (See figure 7)

VII. RESULTS for W_1 and W_2

We have used the structure function fits for the neutron and proton to calculate the smeared structure function for the deuteron and heavier elements. The structure function W_1 was calculated assuming $R = \frac{\sigma_L}{\sigma_T} = 0.18$, which is the assumption that was made when the fits to the SLAC data were performed.

Tables 1 and 2 give the values of the proton and neutron structure functions that were used in the calculations for representative values of X and Q^2 .

The structure functions are presented in the form $\sqrt{W_2}$ and $2M_p \cdot X \cdot W_1$. The

structure functions in the resonance region ($W < 2 \text{ GeV}$) vary rapidly across the resonance peaks. The resonance contribution was included in detail in the calculations of the structure functions for the deuteron and heavier nuclei. The structure function including the resonance region are shown in figure 7.

Tables 3 and 4 give the calculated normalized structure functions (\bar{W}_1 and \bar{W}_2) for deuteron and steel nuclei respectively. Tables 5, 6, 7, 8 and 9 give the smearing ratios for W_1 and W_2 in the deep inelastic region ($W > 2 \text{ GeV}$) for deuterium, carbon, silicon, steel and lead respectively. As can be observed from the tables the corrections for W_1 and W_2 are the same. Consequently, the Fermi motion effects change $R = \sigma_L/\sigma_T$ by less than 0.01. The Fermi motion effects are similar to within 5% for all nuclei except at large values of X ($X > 0.5$) where the effects are larger for the heavier nuclei. The smearing ratios are fairly independent of Q^2 for fixed X at small X but have a larger Q^2 dependence at large X . The variations are due to the fact that the shape of the X distributions vary with Q^2 due to the observed scaling deviations of the nucleon structure functions. The Fermi motion effects tend to reduce the deviations from scaling for heavy nuclei at large values of X ($X > 0.5$). The effects are larger at large Q^2 because the X distributions are steeper. This illustrates the importance of calculating the corrections for finite X and Q^2 . The smearing ratios S_2 for deuterium and steel are shown in figure 5 for the case of $Q^2 = 100 \text{ GeV}^2$.

The sensitivity of the smearing ratios to various effects are shown in table 10. The ratios are calculated for steel at a Q^2 of 100. The first column gives the nominal values, the second column shows the effect of calculating the kinematics with a heavy nucleus ($A-1$) as a spectator for all momenta (including $|\vec{E}| > \kappa_p$). The third column gives the ratios calculated

with no nucleon-nucleon correlations (i.e., no momenta greater than the Fermi momentum K_F). As can be seen from the table, the addition of the high momentum components changes the corrections considerably. The effect of the high momentum components is reduced when a single nucleon rather than the whole nucleus is balancing the momentum.

VIII. DISCUSSION W_1 and W_2

We have applied the smearing technique of Atwood and West¹ to the case of a general heavy nucleus. We find that the Fermi momentum effects in heavy nuclei are similar to those in the deuteron, but are larger. These are most important for large X where the difference between the effects in the various nuclei are also large. However, that is the region where most experiments have little data. At large values of X ($X > 0.5$) the effects are such as to reduce the scaling deviation for heavy nuclei (see tables). This means that Fermi motion corrections must be applied to the structure functions before QCD tests are done.

We suggest that experimental results be published without the application of Fermi motion correction as well as with the application of such corrections in order to facilitate more direct comparisons between experiments which use the same and different nuclear targets. The smearing ratios are provided in tables in a form which can be readily used by various experiments.

Our Fermi motion corrections for nuclei are very similar to those for the deuteron for $X < 0.5$. The corrections are equal to unity at around x of 0.5 for high A nuclei (0.6 for Deuterium). At large X ($X > 0.5$) the deviations from unity change sign. This is in contrast to a recent calculation for carbon by Savin and Zacek¹⁹, who have applied the Atwood and West technique

to the case of a carbon nucleus. They obtain larger corrections (for $X < 0.5$) which never equal unity. Although they used a different form for the C^{12} wave functions²⁰, we think that the major difference is that they used on-shell kinematics in their calculation and did not conserve energy in the scattering process. We believe that care must be taken in conserving energy in the scattering process by taking the nucleon off-the mass shell.

We recognize that, although we have used off-shell relativistic kinematics, our approach is inherently non-relativistic in that we make use of non-relativistic nuclear wave functions. There have been studies of deuteron binding effects using relativistic vertex functions for the deuteron^{21,22}. However, numerical results for the smearing ratio for deuteron in the deep inelastic scattering case using relativistic vertex functions have not been published yet, but are expected soon²². Frankfurt and Strickman²³ have studied deuteron binding effects using a light cone approach. Their calculations yield smearing ratios which are about 2% smaller than those calculated using the Atwood and West technique used here. The case of the nucleus is more complicated. Frankfurt and Strickman²⁴, using a light cone approach, also find that the high momentum components from nucleon-nucleon and other correlations are important. They only present results for a heavy nucleus in the $Q^2 \rightarrow \infty$ limit. These results compare well with our $Q^2=100$ calculation for Pb^{208} (see figure 10). However, detailed calculations for finite X and Q^2 have not been done. We hope that detailed numerical results from other approaches to the problem of nuclear binding corrections will be presented in the future at finite X and Q^2 in a form which can be directly compared to our tables.

We note that experiments²⁵ which have data using hydrogen as well as heavy targets can directly rule out Fermi motion calculations which yield

extremely large corrections. This can be done by trying to extract the neutron structure function¹¹ from the data according to

$$W_{2n}^A = \frac{W_2^A(\text{measured}) - Z \cdot W_2^P(\text{measured})/S_2^{PA}}{N \cdot S_2^{NA}} \quad (18)$$

where S_2^{PA} and S_2^{NA} are the smearing ratios for the proton and neutron structure functions in a nucleus A. In the large X region, one might extract unphysical negative neutron structure functions if the Fermi motion corrections are not properly calculated. The resulting ratio of neutron and proton structure functions can be compared to the SLAC results^{11,15} which indicate that W_{2n}/W_{2p} approaches 1/4 as $X \rightarrow 1$. The calculated smearing ratios for the proton and neutron S_2^N and S_2^P are presented in tables 11, 12 and 13 for Deuterium, Carbon (C¹²) and Steel (Fe⁵⁶) respectively. Note that we have only smeared the inelastic structure functions, including contributions from the resonance region. We have not included the small contribution of the elastic scattering (which leads to a quasielastic peak). This is because the contribution of the quasielastic tail including the radiative effects is typically subtracted from all deep inelastic electron scattering^{5,16} and muon scattering data. The subtraction of the quasielastic radiative tail, including Fermi motion effects for the case of the deuteron is described in detail in Ref. 11.

IX. DEEP INELASTIC NEUTRINO SCATTERING

The general $W_{\mu\nu}$ tensor for the scattering of neutrinos from a target A can be written in terms of three structure functions²⁶

$$W_{\mu\nu}^A(q, P^A) = -(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) W_1 + (P_\mu^A - \frac{P^A q_\mu}{q^2}) (P_\nu^A - \frac{P^A q_\nu}{q^2}) \frac{W_2}{M_A^2} - i \frac{\epsilon_{\mu\nu\alpha\beta} P_\alpha^A q_\beta}{2M_A^2} W_3 \quad (19)$$

For an incident neutrino energy E_0 and a final state muon energy E' and a laboratory scattering angle θ we obtain

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dQ^2 d\nu} = \frac{G^2 E'^2}{2\pi^2} \cos^2 \frac{\theta}{2} \left[W_2^A(Q^2, q \cdot P^A) + 2 \tan \frac{\theta}{2} W_{1\nu}^A(Q^2, q \cdot P^A) + \frac{E_0 + E'}{M_A} \tan \frac{\theta}{2} W_{3\nu}^A(Q^2, q \cdot P^A) \right] \quad (20)$$

We can rewrite the last term for W_3 with the proton mass in the denominator by defining

$$W_{3\nu}^A = W_{3\nu}^A \left(\frac{M_P}{M_A} \right) \quad (21)$$

We write a tensor for $W_{\mu\nu}^A$ equation analogous to equation (12). Equating the XX and 0,0 components of the tensor leads to equations (13) and (14) for W_1 and W_2 . Equating the off-diagonal elements leads to an equation for W_3

$$W_{3\nu}^A = Z \int |\phi(\vec{p})|^2 d^3p \left[\frac{E_i}{M_P} - \frac{P_{3\nu}}{M_P q_3} \right] W_{3\nu}^P \quad (22)$$

We define the smearing ratio for W_3 to be

$$S_3 = \frac{ZW_{3\nu}^P + NW_{3\nu}^N}{W_{3\nu}^A} \quad (23)$$

The structure functions W_1 , W_2 and W_3 satisfy the inequality

$$\nu W_2 \geq \frac{2M_p XW_1}{(1+Q^2/\nu^2)} \geq \frac{M_p XW_3}{(1+Q^2/\nu^2)^{1/2}} \quad (24)$$

In terms of the variables x and y the cross section is

$$\frac{d^2\sigma_{\nu\bar{\nu}}}{dx dy} = \frac{G^2 M_p E}{\pi} \left[(1-y)\nu W_2 + \frac{y^2}{2} (2M_p XW_1) + (y^2 - y^2/2) XW_3 \right] \quad (25)$$

where $y = \nu/E_0$.

For light spin 1/2 quarks we have $R = \sigma_L/\sigma_T = Q^2/\nu^2$ and therefore

$$\nu W_2 = 2M_p XW_1 \quad (26)$$

Within the quark-parton model, the structure functions can be written in terms of quark and antiquark distributions.

$$\begin{aligned} \nu W_2 &= Q + \bar{Q} \\ XW_3 &= Q - \bar{Q} \end{aligned} \quad (27)$$

In our calculations we use

$$\bar{Q}_\nu(x, Q^2) = \left(\frac{18}{5}\right) \frac{1}{2} (1-x_w)^7 B(W, Q^2) g(0) \frac{\omega_w}{\omega} \quad (28)$$

where B and g are defined in equation 17, and

$$XW_{3\nu} = \nu W_{2\nu} - 2\bar{Q} \quad (29)$$

We take $\nu W_{2\nu} = \frac{18}{5} \nu W_2^{em}$. These structure functions are plotted in figure 7. These structure function fits yield (at large Q^2)

$$\int_0^1 (\nu W_2^{ep} + \nu W_2^{en}) dx = 0.275 \quad (30)$$

$$\nu W_2^{ep}(x_w \approx 0) + \nu W_2^{en}(x_w \approx 0) = 0.743$$

and our choice of antiquark distribution yields (at large Q^2)

$$\frac{\int \bar{Q}(x) dx}{\int [Q(x) + \bar{Q}(x)] dx} = 0.17$$

We define the smearing ratio for the antiquark distribution S_Q

$$S_Q^- = \frac{Z \cdot \bar{Q}^P + N \bar{Q}^N}{\bar{Q}^A}$$

Tables 14, 15, 16, 17 and 18 give the smearing ratios S_3 and S_Q^- for the deuteron, C^{12} , Si^{28} , Fe^{56} and Pb^{208} respectively. The smearing ratios S_3 for the deuteron and steel are shown in figure 8 for $Q^2 = 100$. The antiquark smearing ratios S_Q^- are shown in figure 9.

We find that the smearing ratios for W_3 are similar to those for W_1 and W_2 . However, the Fermi motion effects on the antiquark distributions are significant for values of $x > 0.3$. This is because the antiquark distributions have a very steep x dependence.

Frankfurt and Strickman²⁴, using a light cone approach, calculate the smearing correction to the antiquark distribution for a heavy nucleus in the $Q^2 \rightarrow \infty$ scaling limit. They also find large effects at $x > 0.3$ which are similar to what we obtain for Pb^{208} at $Q^2 = 100$ (see figure 11).

Conclusions

We find that the Fermi motion corrections for W_1 , W_2 and W_3 affect the pattern of the scaling violation in heavy nuclei for $X > 0.5$. They are such as to reduce the magnitude of the scaling violations. Therefore, these corrections must be applied before QCD tests are performed.

The corrections for W_1 , W_2 and W_3 are similar. For W_1 and W_2 the corrections are almost the same and therefore the Fermi motion effects change $R = \sigma_L/\sigma_T$ by less than 0.01.

The corrections for the antiquark distribution are important for $X > 0.3$. They are such as to make the antiquark distributions in heavy nuclei fall less steeply with X than the corresponding distributions for free nucleons.

We expect that the correction can be used in the region where smearing ratios are close to 1.0 (e.g. $S_1, S_2 > 0.75$). At very large values of X , the corrections are very large and are therefore subject to theoretical uncertainties. However, presently there is little experimental data at such large value of X .

This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-3065. One of us (AB) thanks the Alfred P. Sloan Foundation for their support. We appreciate the help of Dr. E. J. Moniz in providing us with the modified Fermi-gas nuclear wave functions. We thank Ms. Deena Dubin and Mr. D. Williams for programming support and Dr. W. B. Atwood for numerous helpful discussions on Fermi motion effects in the deuteron.

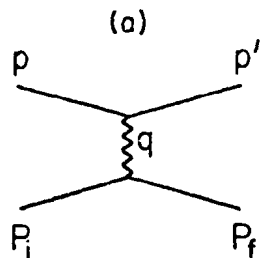
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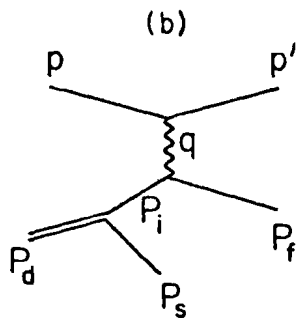
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Figure Captions

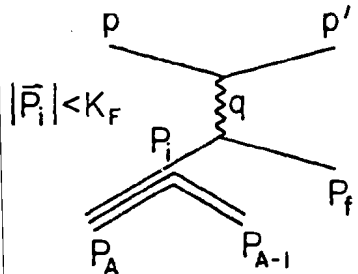
1. Kinematics for on-shell and off-shell scattering.
 - (a) Free nucleons (b) A nucleon bound in the deuteron (c) A nucleon with momentum $|\vec{p}_i| < K_F$ in a heavy nucleus of atomic weight A.
 - (d) A nucleon bound in a heavy nucleus having momentum $|\vec{p}| > K_F$ due to an interaction with another nucleon.
2. A comparison of on-shell and off-shell kinematics. (a) The invariant mass square (b) The laboratory energy. Shown are the case of a heavy steel nucleus as a spectator (Fe) and a single nucleon as a spectator.
3. The Fermi momenta K_F for various nuclei of atomic weight A from Moniz et al. (Ref. 6).
4. Momentum distributions used in the calculation for C^{12} , S_i^{28} , Fe^{56} and Pb^{208} . (a) $|\phi(\vec{p})|^2$, (b) $4\pi|\vec{p}|^2|\phi(\vec{p})|^2$.
5. Smearing ratios for W_2 for deuterium and steel calculated for $Q^2 = 100 \text{ GeV}^2$. $[S_2^{Fe} = \frac{Z W_2^P + N W_2^N}{W_2^{Fe}}, S_2^D = \frac{W_2^P + W_2^N}{W_2^D}]$.
6. The smearing ratio S_2 for carbon (C^{12}) calculated by us (solid line) and as calculated by Savin and Zacek (Ref. 19), for the $Q^2 = 100 \text{ GeV}^2$ case.
7. Representative values of the structure functions $v(W_2^{ep} + W_2^{en})$, $\frac{5}{18} \times (W_3^{vp} + W_3^{vn})$ and $\frac{5}{18} (\bar{Q}^{vp} + \bar{Q}^{vn})$. The values of W_2^{ep} and W_2^{en} were obtained from fits to the SLAC e-p and e-d data (Bodek et al.). The functional forms for the other structure functions are discussed in the text.
8. Smearing ratios for W_3 for the deuteron and Fe^{56} for $Q^2 = 100 \text{ GeV}^2$.
9. Smearing ratios for the antiquark distributions for the deuteron and Fe^{56} for $Q^2 = 100 \text{ GeV}^2$.
10. The smearing ratio S_2 calculated by us for lead (Pb^{208}) at $Q^2 = 100 \text{ GeV}^2$ (solid line) and as calculated by Frankfurt and Strickman (Ref. 24) for a heavy nucleus in the $Q^2 \rightarrow \infty$ scaling limit using a light cone approach.
11. The smearing ratio calculated by us for lead (Pb^{208}) at $Q^2 = 100 \text{ GeV}^2$ (solid line) and as calculated by Frankfurt and Strickman (Ref. 24) for a heavy nucleus in the $Q^2 \rightarrow \infty$ limit using a light cone approach.



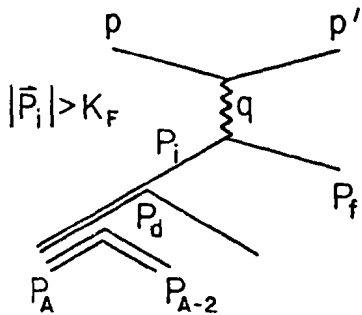
$P_i = (0, M_p)$
 $P_i^2 = M_p^2$



$P_i = (-\bar{P}_s, M_d - \sqrt{\bar{P}_s^2 + M_p^2})$
 off shell: $P_i^2 < M_p^2$
 on shell: $P_s = (\bar{P}_s, \sqrt{\bar{P}_s^2 + M_p^2})$



$|\bar{P}_i| < K_F$
 $P_i = (-\bar{P}_s, M_{A-1} - \sqrt{\bar{P}_s^2 + M_{A-1}^2})$
 off shell: $P_i^2 < M_p^2$
 on shell: $P_{A-1} = (\bar{P}_s, \sqrt{\bar{P}_s^2 + M_{A-1}^2})$



$|\bar{P}_i| > K_F$
 $P_i = (-\bar{P}_s, M_d - \sqrt{\bar{P}_s^2 + M_p^2})$
 off shell: $P_i^2 < M_p^2$
 on shell: $P_{A-2} = (0, M_{A-2})$
 on shell: $P_i = (\bar{P}_s, \sqrt{\bar{P}_s^2 + M_p^2})$

Figure 1

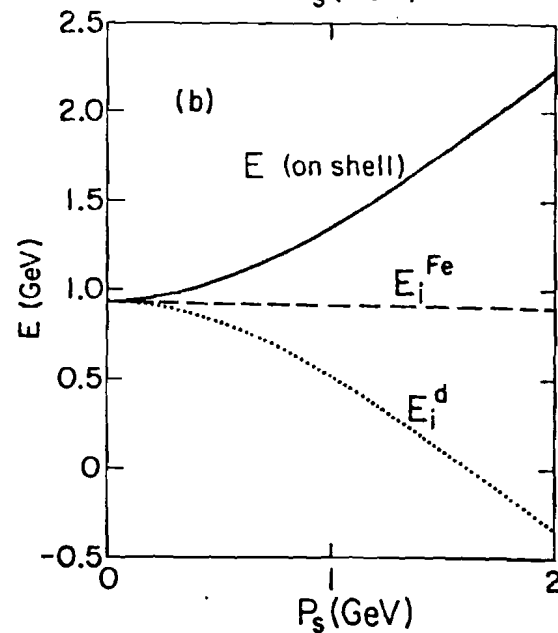
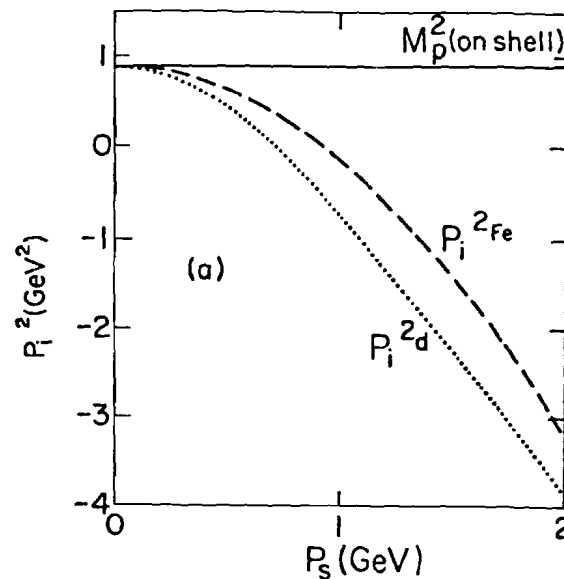


Figure 2

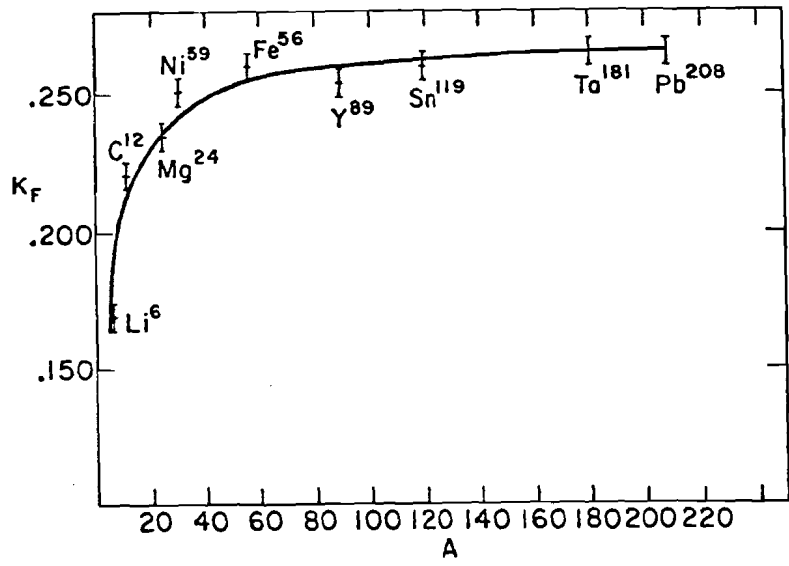


Figure 3

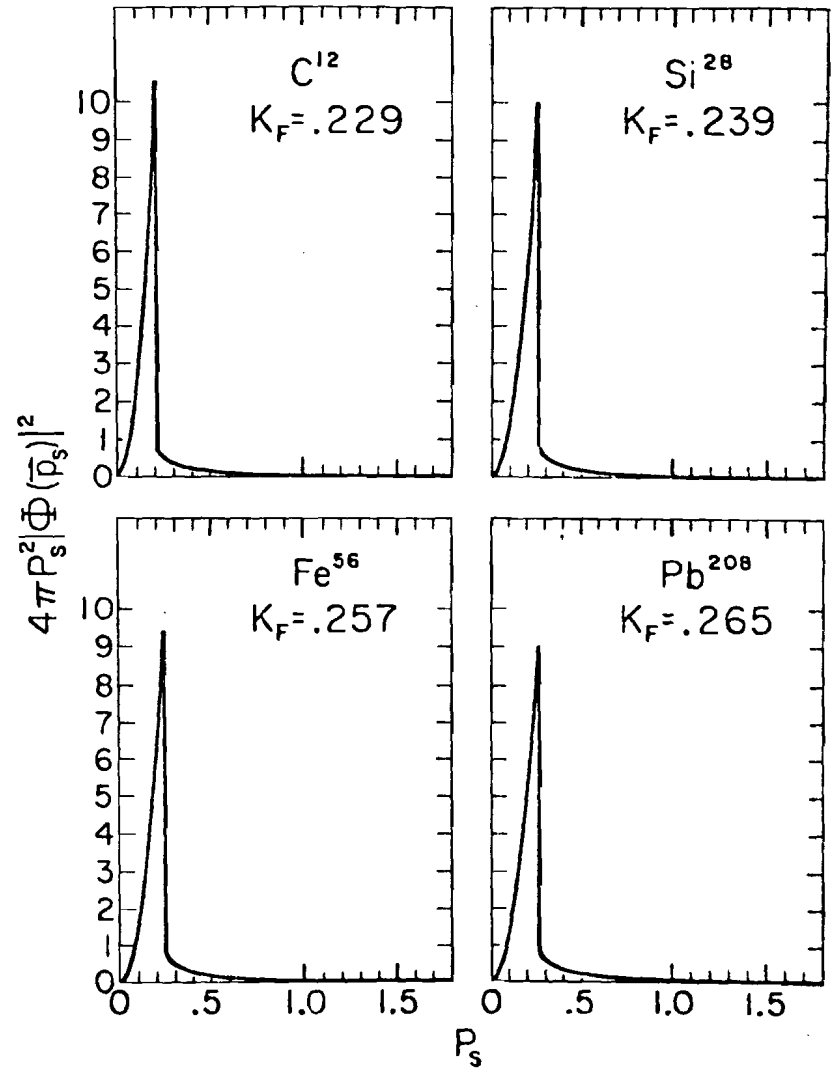


Figure 4a

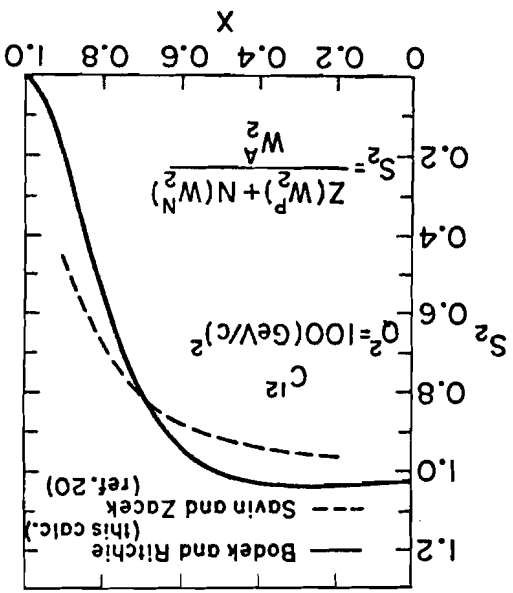


Figure 6

Figure 5

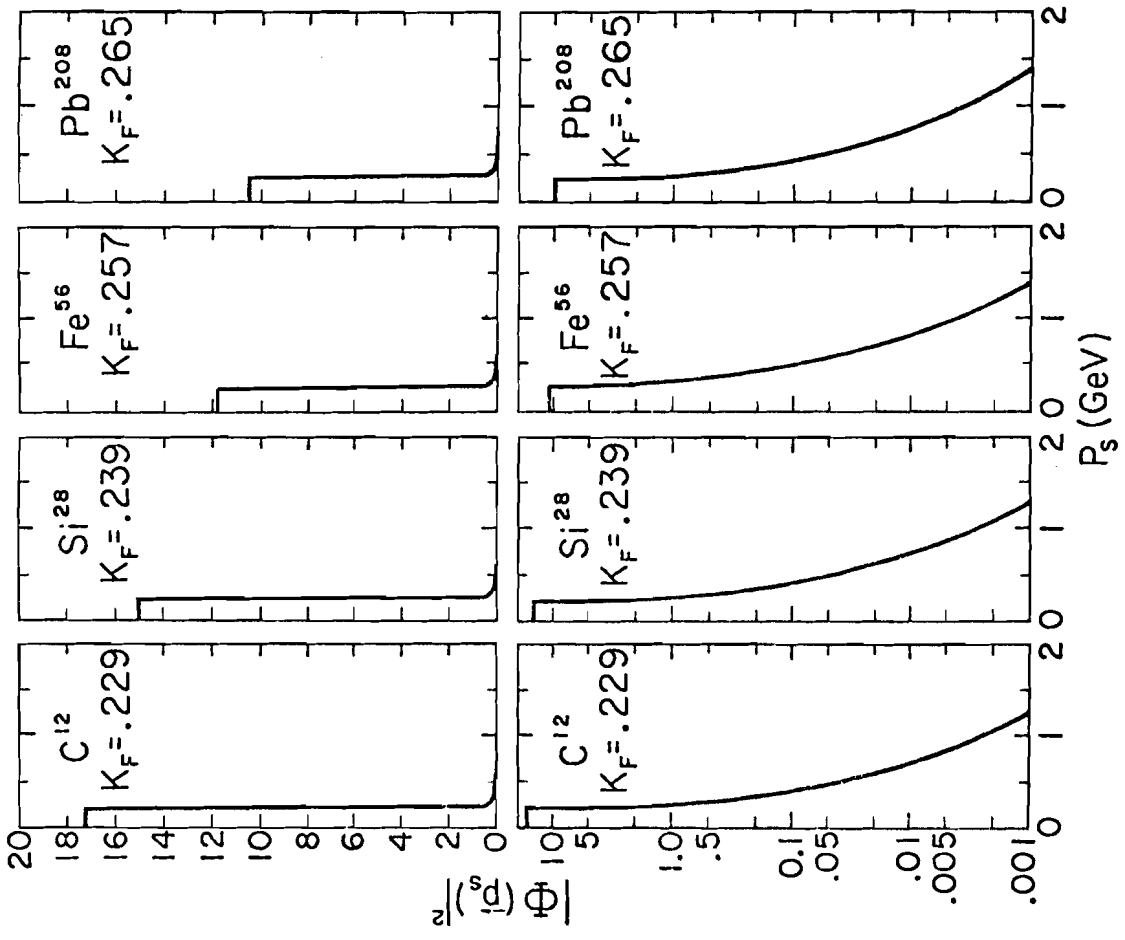
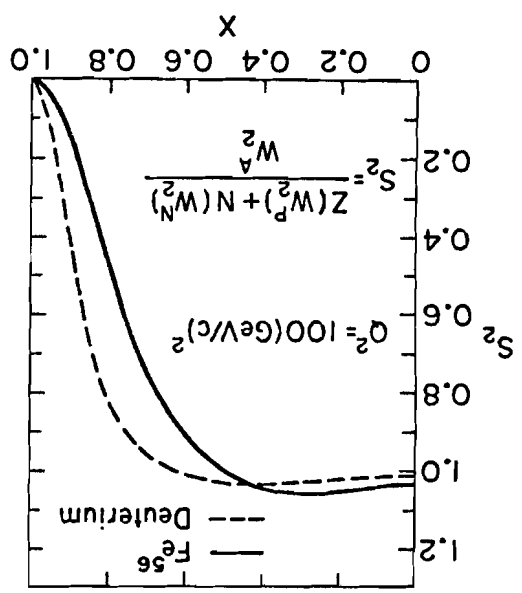


Figure 4b

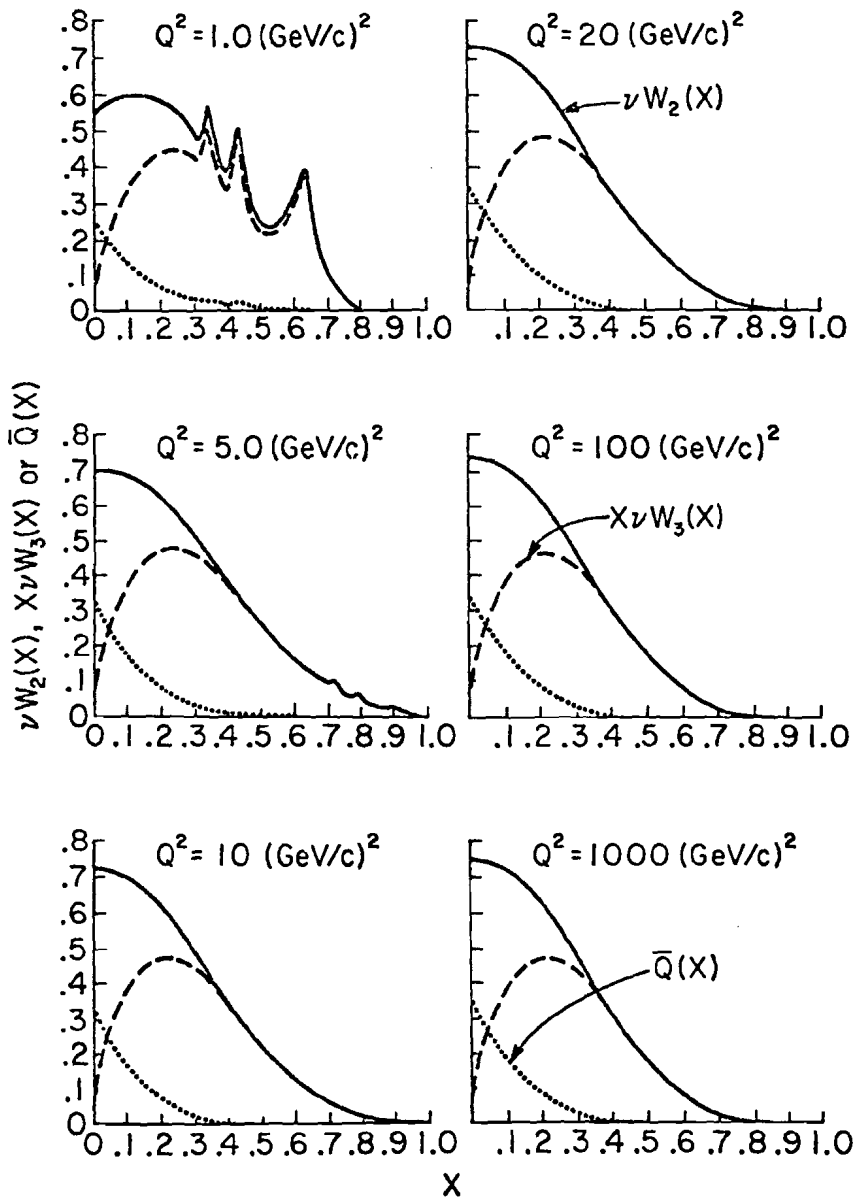


Figure 7

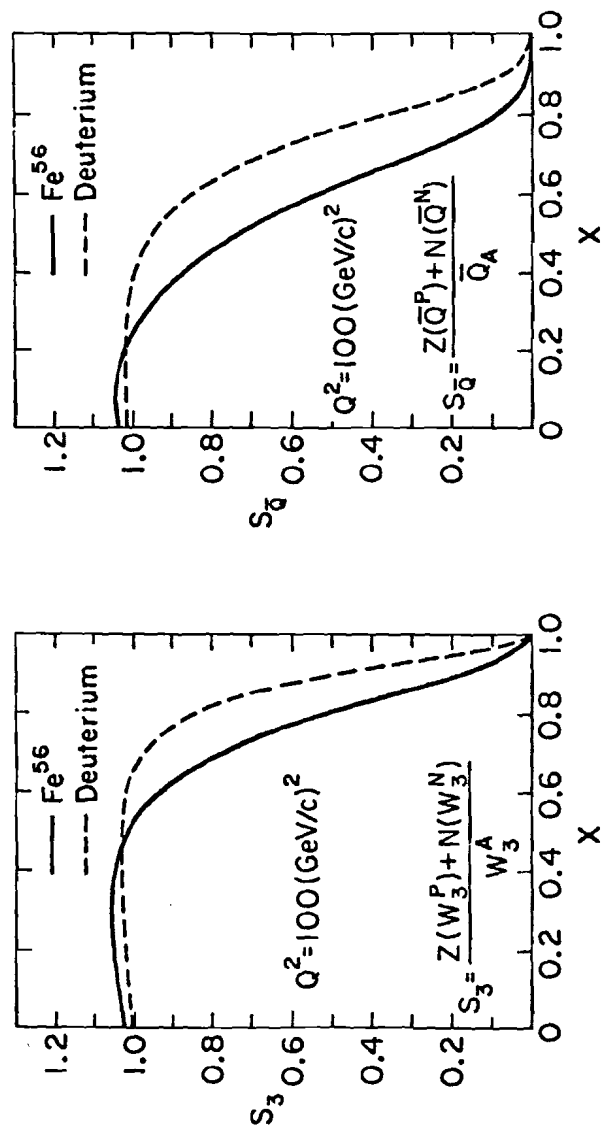


Figure 8

Figure 9

Table 1

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂
0.01	.2333	.2752	.2945	.3475	.3047	.3595	.3100	.3658	.3145	.3711	.3155	.3723
0.05	.2519	.2946	.3006	.3541	.3087	.3639	.3129	.3691	.3164	.3734	.3172	.3743
0.10	.2750	.3435	.3056	.3581	.3108	.3654	.3135	.3693	.3158	.3725	.3163	.3732
0.15	.2953	.3228	.3047	.3540	.3064	.3587	.3074	.3612	.3081	.3633	.3083	.3638
0.20	.3108	.3214	.2969	.3407	.2945	.3422	.2933	.3438	.2923	.3445	.2921	.3447
0.25	.3208	.3102	.2826	.3194	.2754	.3180	.2718	.3172	.2689	.3166	.2682	.3164
0.30	.3089	.2767	.2630	.2919	.2506	.2867	.2444	.2838	.2394	.2815	.2382	.2810
0.35	.3384	.2789	.2395	.2602	.2220	.2511	.2131	.2461	.2060	.2420	.2044	.2411
0.40	.3477	.2625	.2136	.2265	.1913	.2137	.1800	.2066	.1711	.2007	.1691	.1994
0.45	.2751	.1895	.1863	.1924	.1603	.1765	.1471	.1676	.1368	.1603	.1345	.1586
0.50	.2296	.1441	.1589	.1594	.1304	.1414	.1160	.1312	.1049	.1228	.1025	.1208
0.55	.2667	.1524	.1322	.1286	.1027	.1092	.0880	.0986	.0769	.0897	.0744	.0877
0.60	.4297	.2236	.1068	.1005	.0782	.0819	.0639	.0710	.0534	.0622	.0511	.0602
0.65	.3582	.1699	.0839	.0763	.0571	.0587	.0481	.0485	.0348	.0404	.0328	.0387
0.70	.1327	.0575	.0743	.0669	.0398	.0401	.0287	.0312	.0209	.0243	.0194	.0228
0.75	.0468	.0185	.0450	.0380	.0262	.0258	.0173	.0186	.0114	.0132	.0103	.0121
0.80	.0000	.0000	.0331	.0270	.0162	.0154	.0095	.0100	.0054	.0062	.0047	.0055
0.85	.0000	.0000	.0180	.0141	.0092	.0086	.0045	.0047	.0021	.0024	.0017	.0020
0.90	.0000	.0000	.0125	.0094	.0038	.0035	.0021	.0022	.0006	.0007	.0004	.0005
0.95	.0000	.0000	.0000	.0000	.0011	.0010	.0005	.0005	.0001	.0001	.0000	.0000

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂	2m _π w ₁	v _w ₂
0.01	.2284	.2694	.2896	.3417	.2998	.3537	.3051	.3600	.3096	.3653	.3106	.3665
0.05	.2275	.2662	.2762	.3254	.2943	.3352	.2986	.3404	.2921	.3446	.2929	.3456
0.10	.2283	.2603	.2592	.3037	.2644	.3109	.2671	.3146	.2694	.3177	.2699	.3185
0.15	.2292	.2506	.2404	.2793	.2424	.2838	.2434	.2861	.2443	.2881	.2445	.2885
0.20	.2284	.2363	.2197	.2522	.2182	.2539	.2174	.2548	.2168	.2555	.2167	.2557
0.25	.2254	.2180	.1975	.2232	.1923	.2220	.1897	.2214	.1876	.2209	.1871	.2207
0.30	.2088	.1871	.1744	.1936	.1657	.1895	.1612	.1873	.1577	.1855	.1569	.1851
0.35	.2213	.1825	.1513	.1644	.1393	.1576	.1333	.1539	.1285	.1509	.1274	.1502
0.40	.2209	.1667	.1208	.1366	.1142	.1276	.1069	.1227	.1011	.1186	.0998	.1177
0.45	.1703	.1173	.1074	.1109	.0911	.1003	.0829	.0945	.0766	.0897	.0752	.0886
0.50	.1388	.0871	.0876	.0879	.0705	.0764	.0620	.0701	.0555	.0650	.0541	.0638
0.55	.1578	.0902	.0697	.0678	.0528	.0563	.0445	.0499	.0383	.0448	.0370	.0436
0.60	.2493	.1297	.0538	.0506	.0380	.0398	.0305	.0338	.0250	.0291	.0238	.0280
0.65	.2040	.0968	.0403	.0367	.0262	.0270	.0197	.0217	.0151	.0176	.0142	.0167
0.70	.0743	.0322	.0349	.0306	.0172	.0173	.0119	.0130	.0084	.0098	.0077	.0091
0.75	.0258	.0102	.0195	.0165	.0106	.0104	.0067	.0071	.0042	.0049	.0037	.0044
0.80	.0000	.0000	.0136	.0111	.0061	.0058	.0033	.0035	.0018	.0021	.0015	.0018
0.85	.0000	.0000	.0070	.0055	.0032	.0030	.0014	.0015	.0006	.0007	.0005	.0006
0.90	.0000	.0000	.0046	.0034	.0012	.0011	.0006	.0006	.0002	.0002	.0001	.0001
0.95	.0000	.0000	.0000	.0000	.0003	.0003	.0001	.0001	.0000	.0000	.0000	.0000

Table 2

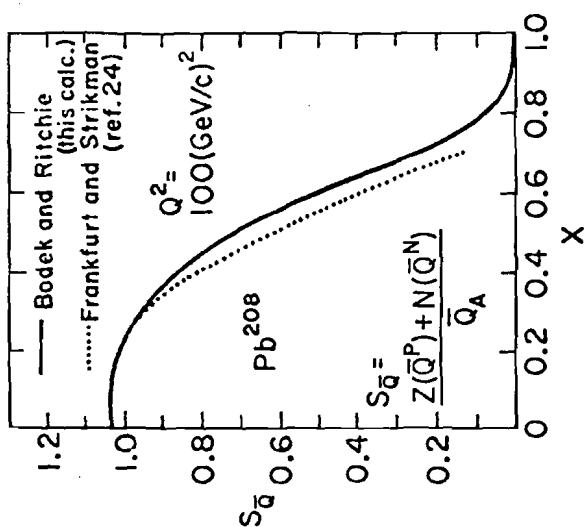


Figure 11

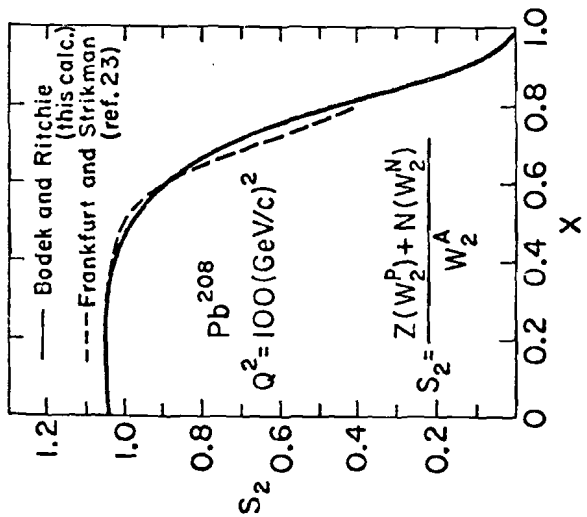


Figure 10

Table 3

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂
0.01	.4516	.5445	.5841	.6892	.6045	.7132	.6152	.7259	.6240	.7364	.6261	.7388
0.05	.4794	.5608	.5768	.6794	.5929	.6971	.6015	.7094	.6085	.7180	.6101	.7200
0.10	.5034	.5738	.5648	.6618	.5751	.6763	.5806	.6839	.5851	.6902	.5862	.6916
0.15	.5244	.5734	.5452	.6333	.5488	.6425	.5508	.6474	.5525	.6514	.5529	.6523
0.20	.5392	.5577	.5166	.5929	.5126	.5965	.5107	.5984	.5092	.6000	.5088	.6003
0.25	.5461	.5282	.4801	.5426	.4677	.5400	.4614	.5386	.4564	.5374	.4553	.5371
0.30	.5177	.4639	.4375	.4854	.4163	.4761	.4056	.4711	.3970	.4670	.3951	.4661
0.35	.5597	.4614	.3909	.4246	.3613	.4087	.3464	.4001	.3344	.3930	.3318	.3913
0.40	.5686	.4292	.3424	.3631	.3055	.3412	.2869	.3293	.2722	.3194	.2689	.3171
0.45	.4454	.3068	.2937	.3033	.2513	.2768	.2300	.2621	.2134	.2500	.2097	.2472
0.50	.3684	.2312	.2465	.2473	.2008	.2178	.1781	.2012	.1605	.1877	.1566	.1846
0.55	.4246	.2426	.2019	.1964	.1555	.1658	.1326	.1485	.1152	.1345	.1114	.1313
0.60	.6790	.3533	.1606	.1512	.1162	.1217	.0944	.1048	.0783	.0912	.0749	.0883
0.65	.5622	.2667	.1243	.1130	.0834	.0857	.0639	.0702	.0499	.0580	.0470	.0554
0.70	.2070	.0896	.1112	.0975	.0570	.0574	.0406	.0441	.0294	.0341	.0271	.0319
0.75	.0726	.0287	.0645	.0545	.0367	.0362	.0240	.0257	.0156	.0180	.0140	.0165
0.80	.0000	.0000	.0468	.0381	.0222	.0214	.0128	.0136	.0072	.0083	.0062	.0073
0.85	.0000	.0000	.0250	.0195	.0124	.0116	.0060	.0062	.0027	.0031	.0022	.0026
0.90	.0000	.0000	.0170	.0128	.0075	.0046	.0027	.0028	.0007	.0008	.0005	.0006
0.95	DEUTERON		.0000	.0000	.0015	.0013	.0004	.0006	.0001	.0001	.0001	.0001

Table 5

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
0.01	1.015	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014
0.05	1.016	1.012	1.015	1.014	1.015	1.015	1.015	1.015	1.015	1.015	1.015	1.015
0.10	1.018	1.010	1.016	1.015	1.016	1.016	1.016	1.016	1.016	1.016	1.016	1.016
0.15	1.020	1.011	1.018	1.016	1.018	1.017	1.018	1.018	1.018	1.018	1.018	1.018
0.20	1.023	1.013	1.021	1.018	1.020	1.019	1.020	1.020	1.020	1.020	1.020	1.020
0.25	-	-	1.023	1.021	1.023	1.022	1.023	1.022	1.023	1.022	1.023	1.023
0.30	-	-	1.026	1.023	1.025	1.024	1.025	1.024	1.025	1.024	1.025	1.025
0.35	-	-	1.028	1.026	1.028	1.026	1.027	1.026	1.027	1.026	1.026	1.026
0.40	-	-	1.031	1.028	1.029	1.028	1.028	1.028	1.027	1.027	1.027	1.027
0.45	-	-	1.033	1.031	1.030	1.029	1.029	1.028	1.027	1.027	1.027	1.026
0.50	-	-	1.035	1.032	1.030	1.029	1.027	1.026	1.024	1.024	1.023	1.023
0.55	-	-	1.035	1.033	1.028	1.027	1.023	1.023	1.018	1.018	1.017	1.017
0.60	-	-	1.031	1.029	1.024	1.023	1.016	1.015	1.007	1.007	1.005	1.005
0.65	-	-	-	-	1.015	1.014	1.002	1.002	0.988	0.988	0.984	0.984
0.70	-	-	-	-	1.000	0.999	0.980	0.979	0.955	0.955	0.949	0.949
0.75	-	-	-	-	0.970	0.969	0.943	0.943	0.901	0.901	0.890	0.890
0.80	-	-	-	-	-	-	0.882	0.882	0.812	0.812	0.791	0.791
0.85	-	-	-	-	-	-	0.778	0.778	0.663	0.663	0.627	0.627
0.90	-	-	-	-	-	-	-	-	0.430	0.430	0.374	0.374
0.95	DEUTERON		-	-	-	-	-	-	0.146	0.146	0.096	0.096

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂	2m ₁ v ₁	v ₂
0.01	.4653	.5489	.5879	.6937	.6083	.7177	.6190	.7304	.6279	.7409	.6299	.7433
0.05	.4926	.5762	.5910	.6961	.6073	.7160	.6159	.7264	.6230	.7351	.6246	.7371
0.10	.5195	.5921	.5830	.6831	.5936	.6980	.5992	.7059	.6039	.7123	.6050	.7138
0.15	.5385	.5888	.5608	.6514	.5647	.6611	.5668	.6662	.5686	.6704	.5690	.6713
0.20	.5497	.5686	.5272	.6051	.5232	.6088	.5213	.6108	.5198	.6125	.5194	.6128
0.25	.5529	.5347	.4856	.5489	.4730	.5461	.4666	.5447	.4616	.5434	.4604	.5432
0.30	.5210	.4668	.4390	.4871	.4175	.4775	.4067	.4724	.3980	.4682	.3960	.4672
0.35	.5605	.4621	.3896	.4232	.3598	.4070	.3448	.3982	.3328	.3910	.3301	.3894
0.40	.5672	.4281	.3394	.3599	.3025	.3379	.2839	.3259	.2692	.3159	.2660	.3137
0.45	.4428	.3050	.2899	.2994	.2477	.2729	.2266	.2582	.2101	.2461	.2064	.2434
0.50	.3653	.2292	.2424	.2432	.1972	.2139	.1747	.1975	.1574	.1841	.1536	.1811
0.55	.4200	.2400	.1979	.1925	.1522	.1623	.1296	.1452	.1126	.1314	.1089	.1283
0.60	.6705	.3489	.1571	.1478	.1134	.1188	.0921	.1021	.0763	.0889	.0729	.0859
0.65	.5543	.2629	.1212	.1102	.0811	.0833	.0621	.0682	.0484	.0563	.0456	.0537
0.70	.2038	.0882	.1082	.0949	.0553	.0557	.0393	.0427	.0284	.0329	.0262	.0308
0.75	.0714	.0282	.0626	.0529	.0355	.0350	.0231	.0248	.0150	.0173	.0134	.0158
0.80	.0000	.0000	.0453	.0368	.0214	.0206	.0122	.0130	.0069	.0079	.0059	.0070
0.85	.0000	.0000	.0241	.0188	.0118	.0111	.0057	.0059	.0026	.0030	.0021	.0024
0.90	.0000	.0000	.0163	.0123	.0048	.0044	.0026	.0027	.0007	.0008	.0005	.0006
0.95	.0000	.0000	.0000	.0000	.0014	.0013	.0006	.0006	.0001	.0001	.0000	.0001

Table 4

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
0.01	1.026	1.024	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025
0.05	1.029	1.019	1.026	1.024	1.026	1.026	1.025	1.026	1.025	1.026	1.025	1.026
0.10	1.033	1.017	1.029	1.025	1.028	1.026	1.028	1.027	1.027	1.027	1.027	1.027
0.15	1.038	1.018	1.032	1.027	1.031	1.028	1.030	1.029	1.030	1.030	1.030	1.030
0.20	1.044	1.023	1.034	1.029	1.033	1.030	1.033	1.031	1.032	1.032	1.032	1.032
0.25	-	-	1.037	1.031	1.035	1.032	1.034	1.033	1.033	1.033	1.033	1.033
0.30	-	-	1.040	1.033	1.037	1.034	1.035	1.034	1.034	1.033	1.033	1.033
0.35	-	-	1.041	1.035	1.037	1.034	1.034	1.033	1.032	1.032	1.032	1.032
0.40	-	-	1.042	1.036	1.035	1.032	1.031	1.029	1.027	1.027	1.026	1.026
0.45	-	-	1.042	1.036	1.031	1.027	1.024	1.022	1.018	1.017	1.016	1.016
0.50	-	-	1.039	1.033	1.023	1.019	1.012	1.010	1.002	1.001	0.999	0.999
0.55	-	-	1.031	1.025	1.009	1.006	0.992	0.990	0.976	0.975	0.972	0.972
0.60	-	-	1.012	1.007	0.988	0.984	0.962	0.960	0.937	0.936	0.930	0.930
0.65	-	-	-	-	0.954	0.951	0.918	0.916	0.879	0.879	0.869	0.869
0.70	-	-	-	-	0.905	0.902	0.854	0.853	0.797	0.797	0.782	0.782
0.75	-	-	-	-	0.833	0.831	0.766	0.765	0.685	0.685	0.664	0.664
0.80	-	-	-	-	-	-	0.649	0.648	0.540	0.540	0.512	0.512
0.85	-	-	-	-	-	-	0.501	0.501	0.368	0.368	0.335	0.335
0.90	-	-	-	-	-	-	-	-	0.191	0.191	0.159	0.159
0.95	CARBON		-	-	-	-	-	-	0.053	0.053	0.033	0.033

Table 6

Table 7

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
0.01	1.031	1.029	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.05	1.035	1.023	1.032	1.030	1.032	1.030	1.031	1.031	1.031	1.031	1.031	1.031
0.10	1.041	1.020	1.035	1.031	1.034	1.034	1.033	1.033	1.033	1.033	1.033	1.033
0.15	1.047	1.023	1.038	1.033	1.037	1.034	1.037	1.035	1.036	1.036	1.036	1.036
0.20	1.053	1.028	1.042	1.035	1.040	1.037	1.039	1.038	1.039	1.038	1.038	1.038
0.25	-	-	1.045	1.038	1.042	1.039	1.041	1.039	1.040	1.040	1.040	1.040
0.30	-	-	1.047	1.040	1.044	1.040	1.042	1.040	1.040	1.040	1.040	1.040
0.35	-	-	1.049	1.041	1.044	1.040	1.041	1.038	1.038	1.037	1.037	1.037
0.40	-	-	1.050	1.042	1.041	1.037	1.036	1.034	1.031	1.031	1.030	1.030
0.45	-	-	1.049	1.042	1.036	1.031	1.027	1.025	1.020	1.019	1.018	1.018
0.50	-	-	1.046	1.038	1.026	1.021	1.012	1.010	1.000	0.999	0.997	0.997
0.55	-	-	1.035	1.028	1.009	1.005	0.989	0.986	0.969	0.969	0.964	0.964
0.60	-	-	1.014	1.007	0.983	0.979	0.953	0.951	0.924	0.923	0.916	0.916
0.65	-	-	-	-	0.944	0.941	0.902	0.900	0.858	0.858	0.847	0.847
0.70	-	-	-	-	0.889	0.885	0.831	0.829	0.768	0.768	0.752	0.752
0.75	-	-	-	-	0.810	0.807	0.735	0.734	0.649	0.648	0.626	0.626
0.80	-	-	-	-	-	-	0.612	0.611	0.500	0.500	0.472	0.472
0.85	-	-	-	-	-	-	0.462	0.461	0.331	0.331	0.299	0.299
0.90	-	-	-	-	-	-	-	-	0.166	0.166	0.136	0.136
0.95 SILICON	-	-	-	-	-	-	-	-	0.044	0.044	0.027	0.027

Table 8

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
0.01	1.039	1.035	1.037	1.023	1.037	1.037	1.037	1.037	1.037	1.037	1.037	1.037
0.05	1.044	1.029	1.037	1.034	1.039	1.038	1.039	1.038	1.038	1.038	1.038	1.038
0.10	1.050	1.026	1.038	1.045	1.042	1.040	1.042	1.040	1.041	1.041	1.041	1.041
0.15	1.057	1.028	1.040	1.053	1.046	1.042	1.045	1.043	1.044	1.044	1.044	1.044
0.20	1.065	1.035	1.043	1.060	1.049	1.045	1.048	1.046	1.047	1.046	1.046	1.046
0.25	-	-	1.045	1.065	1.051	1.047	1.050	1.047	1.048	1.048	1.048	1.048
0.30	-	-	1.048	1.069	1.052	1.048	1.050	1.047	1.048	1.047	1.047	1.047
0.35	-	-	1.049	1.072	1.052	1.047	1.048	1.045	1.044	1.044	1.043	1.043
0.40	-	-	1.050	1.072	1.049	1.043	1.042	1.039	1.036	1.036	1.035	1.035
0.45	-	-	1.049	1.070	1.041	1.036	1.031	1.029	1.022	1.021	1.020	1.020
0.50	-	-	1.044	1.064	1.029	1.024	1.013	1.011	0.999	0.998	0.995	0.995
0.55	-	-	1.032	1.050	1.009	1.004	0.986	0.983	0.963	0.963	0.958	0.958
0.60	-	-	1.008	1.023	0.979	0.975	0.945	0.943	0.912	0.911	0.903	0.903
0.65	-	-	-	-	0.935	0.931	0.888	0.886	0.839	0.839	0.827	0.827
0.70	-	-	-	-	0.874	0.870	0.810	0.808	0.741	0.741	0.724	0.724
0.75	-	-	-	-	0.788	0.786	0.707	0.706	0.616	0.616	0.593	0.593
0.80	-	-	-	-	-	-	0.579	0.578	0.465	0.465	0.437	0.437
0.85	-	-	-	-	-	-	0.429	0.429	0.301	0.301	0.270	0.270
0.90	-	-	-	-	-	-	-	-	0.147	0.147	0.120	0.120
0.95 IKON	-	-	-	-	-	-	-	-	0.038	0.038	0.023	0.023

Table 9

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2	S1	S2
0.01	1.044	1.040	1.042	1.042	1.042	1.042	1.042	1.042	1.042	1.042	1.042	1.042
0.05	1.049	1.033	1.045	1.041	1.044	1.042	1.044	1.043	1.043	1.043	1.043	1.043
0.10	1.056	1.029	1.049	1.043	1.047	1.045	1.047	1.045	1.046	1.046	1.046	1.046
0.15	1.065	1.033	1.053	1.045	1.051	1.047	1.050	1.048	1.049	1.049	1.049	1.049
0.20	1.073	1.040	1.056	1.048	1.054	1.049	1.053	1.050	1.051	1.051	1.051	1.051
0.25	-	-	1.059	1.050	1.056	1.051	1.054	1.051	1.052	1.052	1.052	1.052
0.30	-	-	1.062	1.052	1.056	1.051	1.053	1.051	1.051	1.050	1.050	1.050
0.35	-	-	1.063	1.053	1.055	1.049	1.050	1.047	1.046	1.046	1.045	1.045
0.40	-	-	1.063	1.053	1.051	1.045	1.043	1.040	1.037	1.036	1.035	1.035
0.45	-	-	1.060	1.050	1.042	1.036	1.031	1.028	1.021	1.020	1.018	1.018
0.50	-	-	1.054	1.044	1.028	1.022	1.011	1.008	0.995	0.994	0.991	0.991
0.55	-	-	1.040	1.031	1.006	1.001	0.981	0.978	0.957	0.956	0.951	0.950
0.60	-	-	1.014	1.006	0.974	0.969	0.937	0.934	0.901	0.901	0.893	0.893
0.65	-	-	-	-	0.927	0.922	0.877	0.874	0.825	0.825	0.813	0.813
0.70	-	-	-	-	0.862	0.858	0.795	0.793	0.725	0.724	0.707	0.707
0.75	-	-	-	-	0.774	0.771	0.690	0.688	0.597	0.597	0.574	0.574
0.80	-	-	-	-	-	-	0.561	0.560	0.447	0.447	0.419	0.419
0.85	-	-	-	-	-	-	0.412	0.411	0.286	0.286	0.256	0.256
0.90	-	-	-	-	-	-	-	-	0.138	0.138	0.112	0.112
0.95 LEAD	-	-	-	-	-	-	-	-	0.035	0.035	0.022	0.022

Table 10

x	A	B	C
0.01	1.037	0.997	1.000
0.05	1.038	0.997	1.001
0.10	1.040	0.998	1.001
0.15	1.044	0.998	1.002
0.20	1.046	0.996	1.003
0.25	1.048	0.992	1.003
0.30	1.047	0.985	1.001
0.35	1.044	0.974	0.998
0.40	1.036	0.956	0.992
0.45	1.021	0.929	0.982
0.50	0.998	0.891	0.964
0.55	0.963	0.837	0.936
0.60	0.911	0.764	0.895
0.65	0.839	0.667	0.836
0.70	0.741	0.550	0.754
0.75	0.616	0.412	0.644
0.80	0.465	0.269	0.505
0.85	0.301	0.141	0.345
0.90	0.147	0.052	0.183
0.95	0.038	0.009	0.053

Table 11

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N
0.01	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.015	1.014	1.015
0.05	1.011	1.012	1.014	1.015	1.014	1.015	1.014	1.015	1.014	1.015	1.014	1.015
0.10	1.009	1.011	1.014	1.016	1.015	1.016	1.015	1.017	1.015	1.017	1.015	1.017
0.15	1.009	1.012	1.016	1.017	1.016	1.018	1.017	1.018	1.017	1.019	1.017	1.019
0.20	1.012	1.014	1.018	1.019	1.019	1.020	1.019	1.020	1.020	1.021	1.020	1.021
0.25	1.017	1.020	1.020	1.021	1.021	1.022	1.022	1.023	1.022	1.023	1.022	1.023
0.30	0.969	0.972	1.023	1.024	1.024	1.024	1.024	1.025	1.025	1.025	1.025	1.025
0.35	-	-	1.025	1.026	1.026	1.026	1.026	1.026	1.027	1.026	1.027	1.026
0.40	-	-	1.028	1.028	1.028	1.027	1.028	1.027	1.028	1.026	1.028	1.026
0.45	-	-	1.031	1.030	1.030	1.028	1.029	1.026	1.028	1.025	1.028	1.024
0.50	-	-	1.033	1.031	1.030	1.027	1.028	1.024	1.026	1.020	1.025	1.020
0.55	-	-	1.034	1.031	1.029	1.024	1.025	1.018	1.021	1.012	1.020	1.011
0.60	-	-	1.031	1.026	1.025	1.017	1.018	1.008	1.011	0.998	1.009	0.995
0.65	-	-	1.019	1.012	1.018	1.006	1.007	0.990	0.994	0.973	0.990	0.969
0.70	-	-	-	-	1.005	0.986	0.987	0.962	0.964	0.933	0.958	0.925
0.75	-	-	-	-	0.977	0.951	0.953	0.917	0.915	0.868	0.904	0.854
0.80	-	-	-	-	0.922	0.886	0.897	0.844	0.830	0.761	0.810	0.738
0.85	-	-	-	-	-	-	0.797	0.725	0.686	0.655	0.650	0.557
0.90	-	-	-	-	-	-	-	0.453	0.359	0.396	0.307	-
0.95	DEUTERON	-	-	-	-	-	-	0.157	0.112	0.104	0.072	-

Table 12

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N
0.01	1.023	1.024	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025	1.025
0.05	1.018	1.020	1.024	1.025	1.025	1.025	1.025	1.026	1.025	1.026	1.025	1.026
0.10	1.015	1.018	1.024	1.026	1.026	1.027	1.026	1.028	1.027	1.028	1.027	1.028
0.15	1.016	1.020	1.026	1.028	1.028	1.029	1.029	1.030	1.029	1.030	1.029	1.030
0.20	1.021	1.025	1.029	1.029	1.031	1.030	1.031	1.031	1.032	1.031	1.032	1.031
0.25	-	-	1.032	1.031	1.033	1.031	1.034	1.031	1.035	1.031	1.035	1.031
0.30	-	-	1.034	1.032	1.035	1.031	1.036	1.030	1.036	1.030	1.036	1.030
0.35	-	-	1.037	1.032	1.036	1.029	1.036	1.028	1.035	1.026	1.035	1.026
0.40	-	-	1.039	1.032	1.036	1.026	1.034	1.022	1.032	1.019	1.031	1.018
0.45	-	-	1.039	1.030	1.032	1.019	1.028	1.012	1.024	1.006	1.023	1.005
0.50	-	-	1.038	1.025	1.026	1.008	1.017	0.997	1.010	0.986	1.008	0.983
0.55	-	-	1.031	1.014	1.014	0.990	1.000	0.972	0.986	0.954	0.983	0.950
0.60	-	-	1.014	0.992	0.995	0.964	0.973	0.935	0.950	0.907	0.945	0.900
0.65	-	-	-	-	0.965	0.924	0.932	0.883	0.897	0.840	0.888	0.829
0.70	-	-	-	-	0.919	0.865	0.873	0.808	0.819	0.745	0.805	0.729
0.75	-	-	-	-	0.852	0.784	0.789	0.708	0.712	0.620	0.691	0.598
0.80	-	-	-	-	-	-	0.675	0.579	0.569	0.467	0.541	0.438
0.85	-	-	-	-	-	-	0.529	0.427	0.394	0.299	0.359	0.268
0.90	-	-	-	-	-	-	-	0.207	0.145	0.172	0.119	-
0.95	CARBON	-	-	-	-	-	-	0.057	0.040	0.036	0.026	-

Table 13

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N	S2P	S2N
0.01	1.033	1.038	1.034	1.039	1.034	1.040	1.034	1.040	1.034	1.040	1.034	1.040
0.05	1.026	1.032	1.033	1.040	1.034	1.041	1.035	1.041	1.035	1.041	1.035	1.041
0.10	1.021	1.030	1.034	1.042	1.036	1.043	1.036	1.044	1.037	1.045	1.037	1.045
0.15	1.023	1.033	1.037	1.044	1.039	1.046	1.040	1.047	1.040	1.047	1.040	1.048
0.20	1.030	1.040	1.040	1.046	1.042	1.048	1.043	1.048	1.044	1.049	1.044	1.049
0.25	1.036	1.046	1.043	1.048	1.045	1.048	1.046	1.048	1.047	1.048	1.047	1.048
0.30	-	-	1.047	1.049	1.048	1.047	1.048	1.046	1.048	1.046	1.048	1.045
0.35	-	-	1.050	1.049	1.049	1.044	1.048	1.042	1.047	1.039	1.047	1.038
0.40	-	-	1.051	1.048	1.047	1.038	1.044	1.033	1.041	1.028	1.041	1.026
0.45	-	-	1.052	1.044	1.042	1.028	1.036	1.018	1.030	1.008	1.029	1.006
0.50	-	-	1.049	1.036	1.032	1.011	1.021	0.994	1.010	0.979	1.008	0.975
0.55	-	-	1.039	1.020	1.016	0.985	0.997	0.959	0.980	0.935	0.975	0.929
0.60	-	-	1.019	0.991	0.990	0.948	0.962	0.909	0.934	0.872	0.926	0.863
0.65	-	-	-	-	0.952	0.894	0.911	0.841	0.868	0.786	0.857	0.773
0.70	-	-	-	-	0.896	0.821	0.839	0.749	0.776	0.674	0.760	0.656
0.75	-	-	-	-	0.817	0.724	0.743	0.634	0.656	0.537	0.634	0.513
0.80	-	-	-	-	-	-	0.618	0.496	0.506	0.382	0.477	0.355
0.85	-	-	-	-	-	-	0.467	0.347	0.335	0.229	0.302	0.202
0.90	-	-	-	-	-	-	-	-	0.166	0.103	0.136	0.083
0.95	IRON	-	-	-	-	-	-	-	0.043	0.026	0.026	0.016

Table 14

x	Q ² 1.0		5.0		10.0		20.0		100.0		1000.0	
	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ
0.01	1.003	1.015	1.003	1.015	1.003	1.015	1.003	1.015	1.003	1.015	1.003	1.015
0.05	1.008	1.013	1.007	1.017	1.007	1.018	1.007	1.018	1.007	1.018	1.007	1.018
0.10	1.014	1.006	1.012	1.018	1.012	1.019	1.012	1.020	1.011	1.020	1.011	1.020
0.15	1.020	0.992	1.016	1.017	1.016	1.019	1.016	1.021	1.016	1.022	1.016	1.022
0.20	1.026	0.975	1.020	1.012	1.020	1.017	1.020	1.019	1.020	1.021	1.020	1.022
0.25	-	-	1.024	1.005	1.024	1.012	1.023	1.016	1.023	1.019	1.023	1.020
0.30	-	-	1.028	0.993	1.027	1.004	1.027	1.010	1.026	1.015	1.026	1.016
0.35	-	-	1.031	0.977	1.030	0.992	1.029	1.000	1.028	1.007	1.028	1.009
0.40	-	-	1.035	0.957	1.032	0.975	1.031	0.966	1.030	0.975	1.029	0.977
0.45	-	-	1.037	0.931	1.033	0.952	1.031	0.945	1.029	0.977	1.029	0.960
0.50	-	-	1.039	0.901	1.035	0.923	1.030	0.917	1.027	0.950	1.026	0.954
0.55	-	-	1.040	0.866	1.032	0.886	1.026	0.899	1.021	0.912	1.019	0.915
0.60	-	-	1.035	0.828	1.027	0.843	1.018	0.859	1.009	0.857	1.007	0.850
0.65	-	-	-	-	1.018	0.796	1.005	0.789	0.990	0.779	0.986	0.775
0.70	-	-	-	-	1.002	0.752	0.980	0.719	0.957	0.670	0.951	0.655
0.75	-	-	-	-	0.971	0.726	0.944	0.647	0.903	0.524	0.892	0.490
0.80	-	-	-	-	-	-	0.803	0.604	0.813	0.345	0.793	0.290
0.85	-	-	-	-	-	-	0.778	0.604	0.664	0.164	0.628	0.110
0.90	-	-	-	-	-	-	-	-	0.431	0.041	0.375	0.017
0.95	DEUTERON	-	-	-	-	-	-	-	0.146	0.002	0.096	0.000

Table 15

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ
0.01	1.015	1.024	1.014	1.025	1.014	1.026	1.014	1.026	1.014	1.026	1.014	1.026
0.05	1.024	1.016	1.022	1.026	1.021	1.027	1.021	1.027	1.021	1.027	1.021	1.027
0.10	1.034	0.993	1.029	1.021	1.029	1.024	1.029	1.025	1.028	1.026	1.028	1.026
0.15	1.043	0.962	1.035	1.010	1.035	1.016	1.034	1.018	1.034	1.021	1.034	1.021
0.20	1.053	0.924	1.040	0.991	1.039	1.001	1.039	1.006	1.038	1.010	1.038	1.011
0.25	-	-	1.045	0.966	1.043	0.981	1.042	0.989	1.041	0.995	1.040	0.996
0.30	-	-	1.048	0.932	1.045	0.952	1.043	0.963	1.041	0.973	1.041	0.975
0.35	-	-	1.050	0.890	1.045	0.915	1.042	0.930	1.040	0.943	1.039	0.946
0.40	-	-	1.052	0.841	1.044	0.868	1.039	0.886	1.035	0.903	1.034	0.907
0.45	-	-	1.051	0.787	1.039	0.812	1.032	0.831	1.025	0.850	1.023	0.855
0.50	-	-	1.048	0.732	1.030	0.749	1.019	0.764	1.008	0.782	1.006	0.788
0.55	-	-	1.038	0.679	1.015	0.682	0.998	0.687	0.982	0.699	0.978	0.702
0.60	-	-	1.017	0.631	0.993	0.615	0.967	0.603	0.942	0.598	0.935	0.598
0.65	-	-	-	-	0.958	0.557	0.922	0.516	0.883	0.482	0.874	0.477
0.70	-	-	-	-	0.906	0.524	0.857	0.432	0.800	0.358	0.786	0.343
0.75	-	-	-	-	0.833	0.524	0.767	0.364	0.688	0.233	0.667	0.212
0.80	-	-	-	-	-	-	0.649	0.341	0.542	0.124	0.514	0.102
0.85	-	-	-	-	-	-	0.501	0.341	0.369	0.045	0.336	0.032
0.90	-	-	-	-	-	-	-	-	0.192	0.008	0.159	0.004
0.95	CARBON	-	-	-	-	-	-	-	0.053	0.000	0.033	0.000

Table 16

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ
0.01	1.019	1.030	1.018	1.031	1.018	1.031	1.018	1.031	1.018	1.031	1.018	1.031
0.05	1.030	1.019	1.027	1.031	1.027	1.032	1.027	1.033	1.027	1.033	1.027	1.033
0.10	1.042	0.992	1.036	1.025	1.036	1.028	1.035	1.030	1.035	1.031	1.035	1.031
0.15	1.053	0.955	1.043	1.011	1.042	1.018	1.042	1.021	1.042	1.024	1.041	1.025
0.20	1.065	0.912	1.049	0.989	1.048	1.001	1.047	1.007	1.046	1.012	1.046	1.013
0.25	-	-	1.054	0.958	1.051	0.976	1.050	0.985	1.049	0.993	1.049	0.995
0.30	-	-	1.058	0.919	1.054	0.943	1.051	0.956	1.049	0.967	1.049	0.969
0.35	-	-	1.060	0.871	1.054	0.900	1.050	0.916	1.047	0.931	1.046	0.935
0.40	-	-	1.061	0.817	1.051	0.847	1.046	0.866	1.040	0.885	1.039	0.890
0.45	-	-	1.060	0.758	1.045	0.784	1.036	0.804	1.028	0.825	1.026	0.831
0.50	-	-	1.056	0.698	1.034	0.715	1.021	0.731	1.008	0.751	1.005	0.757
0.55	-	-	1.044	0.641	1.016	0.642	0.996	0.648	0.976	0.660	0.972	0.665
0.60	-	-	1.020	0.592	0.989	0.571	0.959	0.558	0.930	0.554	0.922	0.556
0.65	-	-	-	-	0.948	0.507	0.907	0.466	0.863	0.437	0.852	0.432
0.70	-	-	-	-	0.890	0.466	0.834	0.379	0.772	0.315	0.756	0.303
0.75	-	-	-	-	0.810	0.466	0.737	0.304	0.651	0.198	0.629	0.181
0.80	-	-	-	-	-	0.612	0.256	0.502	0.100	0.474	0.084	0.084
0.85	-	-	-	-	-	0.462	0.256	0.332	0.035	0.300	0.025	0.025
0.90	-	-	-	-	-	-	-	0.166	0.006	0.137	0.003	0.003
0.95	SILICON	-	-	-	-	-	-	0.044	0.000	0.028	0.000	0.000

Table 17

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ
0.01	1.024	1.037	1.023	1.038	1.023	1.038	1.023	1.038	1.023	1.038	1.023	1.038
0.05	1.038	1.024	1.034	1.038	1.034	1.039	1.033	1.040	1.033	1.040	1.033	1.041
0.10	1.052	0.991	1.045	1.030	1.044	1.034	1.044	1.037	1.043	1.038	1.043	1.038
0.15	1.065	0.948	1.053	1.014	1.052	1.022	1.051	1.026	1.051	1.029	1.051	1.030
0.20	1.079	0.900	1.060	0.988	1.058	1.002	1.057	1.009	1.056	1.014	1.056	1.015
0.25	1.090	0.848	1.065	0.952	1.062	0.972	1.060	0.983	1.059	0.992	1.058	0.994
0.30	-	-	1.069	0.907	1.064	0.934	1.061	0.949	1.059	0.961	1.058	0.964
0.35	-	-	1.072	0.854	1.064	0.885	1.059	0.904	1.055	0.921	1.054	0.925
0.40	-	-	1.072	0.794	1.060	0.826	1.053	0.847	1.047	0.869	1.045	0.874
0.45	-	-	1.070	0.730	1.052	0.758	1.042	0.779	1.032	0.803	1.030	0.809
0.50	-	-	1.064	0.666	1.039	0.683	1.023	0.700	1.008	0.722	1.004	0.728
0.55	-	-	1.050	0.606	1.018	0.605	0.994	0.611	0.971	0.626	0.966	0.631
0.60	-	-	1.023	0.555	0.986	0.530	0.952	0.518	0.918	0.516	0.910	0.519
0.65	-	-	-	-	0.940	0.462	0.893	0.423	0.845	0.398	0.833	0.396
0.70	-	-	-	-	0.876	0.414	0.813	0.334	0.746	0.280	0.729	0.271
0.75	-	-	-	-	0.788	0.409	0.709	0.256	0.619	0.170	0.596	0.158
0.80	-	-	-	-	-	-	0.580	0.197	0.467	0.083	0.439	0.071
0.85	-	-	-	-	-	-	0.429	0.186	0.302	0.028	0.271	0.021
0.90	-	-	-	-	-	-	-	0.147	0.005	0.120	0.003	0.003
0.95	IRON	-	-	-	-	-	-	0.038	0.000	0.023	0.000	0.000

Table 18

x	1.0		5.0		10.0		20.0		100.0		1000.0	
	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ	S3	SQ
0.01	1.028	1.041	1.026	1.043	1.026	1.043	1.026	1.043	1.026	1.043	1.026	1.043
0.05	1.043	1.027	1.039	1.043	1.038	1.044	1.038	1.045	1.038	1.046	1.038	1.046
0.10	1.059	0.991	1.051	1.034	1.050	1.038	1.050	1.041	1.049	1.042	1.049	1.043
0.15	1.074	0.944	1.060	1.015	1.059	1.024	1.058	1.029	1.057	1.032	1.057	1.033
0.20	1.088	0.894	1.067	0.986	1.064	1.001	1.063	1.009	1.062	1.015	1.062	1.016
0.25	-	-	1.072	0.948	1.068	0.969	1.066	0.981	1.064	0.990	1.064	0.992
0.30	-	-	1.075	0.900	1.069	0.928	1.066	0.943	1.063	0.956	1.062	0.960
0.35	-	-	1.077	0.844	1.068	0.876	1.063	0.895	1.058	0.913	1.057	0.917
0.40	-	-	1.077	0.782	1.063	0.814	1.055	0.836	1.048	0.858	1.047	0.863
0.45	-	-	1.074	0.717	1.054	0.743	1.042	0.765	1.031	0.789	1.029	0.795
0.50	-	-	1.066	0.652	1.039	0.667	1.021	0.683	1.005	0.705	1.001	0.712
0.55	-	-	1.050	0.591	1.015	0.588	0.989	0.593	0.965	0.607	0.959	0.613
0.60	-	-	1.022	0.539	0.981	0.512	0.944	0.498	0.909	0.497	0.900	0.500
0.65	-	-	-	-	0.932	0.443	0.882	0.403	0.811	0.380	0.819	0.377
0.70	-	-	-	-	0.864	0.391	0.799	0.314	0.729	0.263	0.712	0.255
0.75	-	-	-	-	0.775	0.380	0.692	0.235	0.601	0.158	0.578	0.147
0.80	-	-	-	-	-	-	0.562	0.173	0.449	0.076	0.421	0.065
0.85	-	-	-	-	-	-	0.412	0.144	0.287	0.025	0.257	0.019
0.90	-	-	-	-	-	-	-	-	0.138	0.004	0.113	0.002
0.95	LEAD	-	-	-	-	-	-	-	0.035	0.000	0.022	0.000