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# DYNAMICAL SYMMETRY BREAKING AND THE SIGMA MESON MASS IN QCD

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Abstract

The spontaneous breakdown of chiral symmetry is analyzed dynamically via bound state Bethe-Salpeter equations. While in general spontaneous mass generation is linked to a massless $0^-$ pion and no specific constraint on a massive $0^+$ meson, for the particular theory of asymptotically free QCD we show that a $0^+$ $\sigma$ meson should exist with mass $m_\sigma \approx 600$ MeV to 700 MeV.



We are now accustomed to viewing the spontaneous breakdown of chiral symmetry from two radically different theoretical frameworks:

i) Lagrangian symmetry breaking.

In the Gell-Mann-Lévy $\sigma$ model,[1] the $0^-$ pion and $0^+$ $\sigma$ meson are assumed to be elementary particles which couple in a chiral invariant manner to elementary quarks (or nucleons) in the fundamental Lagrangian. Spontaneous breakdown of chiral symmetry then means that the Lagrangian remains chirally invariant, but the $\sigma$ field develops a nonzero vacuum expectation value, $\langle\sigma\rangle_0 = f_\pi \neq 0$ such that $f_\pi g_{\pi qq} = m_{qk} \neq 0$ with the pion remaining massless. There is, however, no constraint on the value of $m_\sigma$ in the $\sigma$ model. In fact in the nonlinear $\sigma$ model the $\sigma$ field is a function of the $\pi$ field and is obtained by taking the limit $m_\sigma \to \infty$ on the linear model.

ii) Dynamical symmetry breaking.

In the Nambu-Jona-Lasinio (NJL) four-fermion (Hartree-Fock) approach,[2] which can be extended to vector-gluon theories[3] including QCD, gluons and quarks are assumed as elementary. The quark mass is then dynamically generated along with a bound state massless pion such that the equations which dress the quark, giving it all of its mass (DE) are precisely the same equations which bind the quark and antiquark together in a pseudoscalar s wave at zero momentum transfer ($\mathrm{PBE}|_{q\to 0}$) to form the massless pion,

$$\mathrm{DE} = \mathrm{PBE}|_{q\to 0}. \tag{1}$$

With an $m_{qk}\,\bar{q}q$ term then occurring in the renormalized Lagrangian, appearing to break the chiral symmetry, the Nambu-Goldstone pion arises as a massless pole in the axial current. But the unanswered question is: Must the $0^+$ $\sigma$

meson also necessarily exist as a dynamical bound state in order to restore an apparent chiral symmetry to the renormalized Lagrangian? If so, then what is the dynamical value of $m_\sigma$ in the quark model?

In this paper we shall work within the above dynamical symmetry breaking picture specifically for the quark vector-gluon nonabelian theory of QCD and demonstrate that in the chiral limit, just as (1) is valid, then the asymptotic freedom property also requires

$$\mathrm{PBE}|_{q\to 0} = \mathrm{SBE}|_{q^2 = 4m_{qk}^2}, \tag{2}$$

where SBE is the $0^+$ scalar p wave binding equation evaluated at $q^2 = 4m_{qk}^2$. Thus, given (1) and (2), spontaneous breakdown of chiral symmetry for QCD means that when the quark acquires all of its mass $m_{qk} \neq 0$ via the gluon dressing equation, quark and antiquark automatically bind together to form in the chiral limit,

(1) a $0^-$ s wave pion with $m_\pi = 0$

(2) a $0^+$ p wave $\sigma$ meson with $m_\sigma = 2m_{qk}$.

Indeed, in the original four-fermion model of NJL, the $\sigma$ meson result (2) was discovered but not significantly exploited because no mass scale was then associated with $m_\sigma$ or $m_{qk}$. Furthermore the NJL $(\bar{\psi}\psi)^2$ theory is nonrenormalizable and it was not clear to what extent (2) followed for "realistic" model field theories. Here we wish to stress that QCD is one such renormalizable theory for which (2) is valid and for which $m_\sigma = 2m_{qk}$ makes good phenomenological sense.

To begin with, we remind the reader of the subtleties in proving the first spontaneous breakdown condition (1) in the context of chiral-invariant quark-gluon field theories. In particular, the bound state (homogeneous) Bethe-Salpeter equation depicted in Fig. 1 at $q \to 0$ only relates the $0^-$ spinless wave function $P(p^2)$ to the spinless component $C(p^2)$ of the dressing equation with inverse quark propagator $S^{-1}(p) = C(p^2) + \not{p}D(p^2)$. The spin one nature of the gluon complicates the spontaneous breakdown condition (1) via the form factor $D(p^2)$. But it is the (spin one) axial-vector Ward



identity and an inhomogeneous $\gamma_\mu\gamma_5$ addition to Fig. 1 which disentangles the relations between the form factors, thus preserving (1) but only providing[3] $P(p^2) = 2C(p^2)$.

In order to establish the $\sigma$ meson relation (2), it is necessary to demonstrate that the $0^+$ wave function $U(p^2)$ satisfies the same bound state Bethe-Salpeter equation corresponding to Fig. 2 as the Bethe-Salpeter equation for the $0^-$ wave function $P(p^2)$ of Fig. 1. However to verify this relation in a general way, we must consider wave functions P and U which depend upon both invariants $p^2$ and $q^2$: $P = P(p^2,q^2)$, $U = U(p^2,q^2)$. We shall discard the $p\cdot q$ dependence of P and U since we are dealing with spin 0 scalar states.

Then the most transparent way to decouple the $q^2$ from the $p^2$ dependence on the right-hand side of Figs. 1 and 2 is to employ dispersion relations at fixed $q^2$. Suppressing the nonabelian character of the gluon, ignoring the momentum variation of the gluon coupling, and working in the Feynman gauge for simplicity, we find

$$\mathrm{Im}\,P(p^2,q^2) = 2g^2 \int d\rho_2(k)\,(k-p)^{-2}\,(-q^2)\,P\,(k^2,q^2 = 4m^2 - 4k^2) \tag{3a}$$

$$\mathrm{Im}\,U(p^2,q^2) = 2g^2 \int d\rho_2(k)\,(k-p)^{-2}\,(4m^2 - q^2)\,U\,(k^2,q^2 = 4m^2 - 4k^2), \tag{3b}$$

where $d\rho_2$ repesents two body phase space and $(k\pm\frac{1}{2}q)^2 = m^2$ requires $k\cdot q = 0$ and $q^2 = 4m^2 - 4k^2$. Next we construct (unsubtracted) dispersion relations for P and U, respectively evaluated at $q^2 = 0$ and $q^2 = 4m^2$ so that the $-q^2$ and $4m^2 - q^2$ factors in (3a) and (3b) are eliminated:

$$P(p^2,q^2 = 0) = \frac{1}{\pi}\int \frac{\mathrm{Im}P(p^2,q'^2)dq'^2}{q'^2 - q^2}\bigg|_{q^2 = 0}$$
$$= -\frac{2g^2}{\pi}\int dq^2 \int \frac{d\rho_2(k)}{(k-p)^2}\,P(k^2,q^2 = 4m^2 - 4k^2) \tag{4a}$$

$$U(p^2,q^2 = 4m^2) = \frac{1}{\pi}\int \frac{\mathrm{Im}U(p^2,q'^2)dq'^2}{q'^2 - q^2}\bigg|_{q^2 = 4m^2}$$
$$= -\frac{2g^2}{\pi}\int dq^2 \int \frac{d\rho_2(k)}{(k-p)^2}\,U(k^2,q^2 = 4m^2 - 4k^2). \tag{4b}$$

At this point it is clear that <u>if</u> both P and U are in fact independent of the meson momentum-transfer invariant $q^2$, then (4a) and (4b) have identical structures, implying that

$$P(p^2) \propto U(p^2). \qquad (5)$$

This result is obviously valid for the NJL $(\bar{\psi}\psi)^2$ model where all such momenta invariants, $p^2$ or $q^2$ are suppressed. But we maintain that (5) is also true for the asymptotically free theory of QCD with running coupling constant $\alpha_s(p^2) = \pi d/\ell n p^2/\Lambda^2$ and (low) energy scale of[4]

$$\Lambda \approx 150 \text{ MeV}. \qquad (6)$$

Since the latter observation is our key point, we provide further elaboration. Asymptotic freedom requires not only a logarithm fall off of the quark-gluon coupling for <u>large</u> $p^2/\Lambda^2$, but also that such couplings $g(p^2,q^2,p\cdot q)$ depend upon only <u>one</u> momentum invariant, i.e., $p^2$ in this case. Thus the desired $q^2$ suppression in the pseudoscalar and scalar wave functions of (4) follows for QCD when $p^2 >> \Lambda^2$. However, the spontaneous generation of quark mass occurs for $m_{dyn}(p^2) = -C(p^2)/D(p^2) \neq 0$, where $m_{dyn}$ is the dynamically generated quark mass appearing in the quark propagator and in the induced (Nambu-Goldstone) pseudoscalar component of the axial current

$$J_{\mu 5} \propto \gamma_\mu \gamma_5 - \frac{2m_{dyn}(p^2)\gamma_5 q_\mu}{q^2} \qquad (7)$$

such that[5-7]

$$m_{dyn}(p^2 = m^2_{dyn}) \equiv m_{dyn} \approx 300\text{-}320 \text{ MeV}. \qquad (8)$$

The latter mass scale is consistent with $m_{dyn} \sim \frac{1}{3}m_N$ being the chiral-limiting nonstrange constituent quark mass. Then the asymptotic freedom condition $p^2 >> \Lambda^2$ follows from (6) and (8) even for <u>low $p^2$ in the spontaneous breakdown region.</u>

If instead $\Lambda$ turns out to be larger, say $\Lambda \sim$ 300-500 MeV $\sim m_{dyn}$ the absence of $q^2$ dependence in P and U and hence the validity of (5) still follows for QCD because of



the coupling constant freeze-out[8] for $p^2 \leq 1$ GeV$^2$. Thus the main feature of QCD which leads to (5) is the ultraviolet (or deep Euclidean) structure of the QCD coupling or equivalently the ultraviolet behavior of the dynamically generated quark mass,[9] which behaves for large-$p^2$ as[9]

$$m_{dyn}(p^2) = \frac{M^2}{p^2}\, m_{dyn}(M^2) \left[\frac{\ell n M^2/\Lambda^2}{\ell n p^2/\Lambda^2}\right]^{1-d} \qquad (9)$$

with d the anomalous dimension $d = 12(33-2n_f)^{-1}$. Indeed, when one combines (9) with the flavor-dependent current quark masses, the large $p^2$ behavior of all such masses properly extrapolates[7] down to the spontaneous breakdown region of $p^2 \sim (300 \text{ MeV})^2$.

To summarize in a slightly different manner, the asymptotic freedom property of QCD allows us to ignore the $q^2$ dependence of P and U so that (4) leads to (5): $P(p^2) \propto U(p^2)$. At the same time the axial Ward identity or equivalently (7) requires $P(p^2) = 2C(p^2)$ (i.e., $m_{dyn} = -C/D$) and all must be non-vanishing in order that massless pions exist. Note that there is no definite relation between P and U as there is between P and C. (This is because the $\sigma$ meson does not appear as a pole in the vector current as the pion occurs in the axial-vector current (8).) Thus (5) connects Figs. 1 and 2 via the binding equation relation (2) for m in (4) identified as $m_{qk}$. The latter quark mass in the quark loops of Figs. 1 and 2 must then correspond to $m_{dyn} \approx 300$ MeV in the chiral limit. Finally, therefore, we may make the identification for QCD

$$q^2 = 4m^2 \text{ in (4b)} \quad \leftrightarrow m_\sigma = 2m_{dyn} \approx 600\text{-}640 \text{ MeV}. \qquad (10)$$

Chiral-breaking corrections to (10) could increase (10) to at most 700 MeV.

With regard to this numerical estimate for $m_\sigma$, nuclear theorists have long discovered the need for a $0^+$ $2\pi$ exchange isobar of mass 500-600 MeV in order to explain the $^3S_1$ NN nuclear force.[10] Moreover, low energy (and subthreshold) $\pi N$ scattering data leads to a $\pi N$ $\sigma$-term of magnitude[11] $\sigma_{\pi N} \approx 65$ MeV. Then in the context of the unrenormalized $\sigma$ model, we deduce that [12]

$$\sigma_{\pi N} = (m^2_\pi/m^2_\sigma)m_N \approx 65 \text{ MeV}, \qquad m_\sigma \approx 530 \text{ MeV}. \qquad (11)$$

This result is slightly modified by factors of $g_A \approx 1.25$ when the σ model is renormalized so that (11) corresponds to a renormalized mass $m_\sigma \sim 600$-700 MeV. As a last piece of evidence, charged ππ phase shift analyses enjoy a possible I = 0 solution which resonates[13] in the 600-800 MeV region (the σ?), although it is wide and buried under the $\rho^0 \to \pi^+\pi^-$ background. It is experimentally viable,[14] however, to investigate $\sigma \to \pi^0\pi^0$ (which is uncontaminated by the ρ) by constructing four-photon detection devices. We hope that serious consideration is given to detecting the σ meson in this manner in the near future.

If in fact the σ meson is finally confirmed in the 600-700 MeV region, then our above analysis suggests the following theoretical conclusions:

a) Because the dynamical binding equation relation (2) is a model-dependent result, the presently accepted theory of asymptotically free QCD with a low energy Λ scale would then be indirectly verified in that it, and perhaps few other renormalizable field theories, requires that a $0^+$ σ meson should exist with $m_\sigma \sim 600$-700 MeV.

b) Such a massive scalar $\bar{q}q$ meson is the chiral symmetry-spontaneous breakdown analog of the Higgs meson (minus the Schwinger mechanism giving mass to the vector bosons) now sought after to verify the SU(2) x U(1) spontaneously broken gauge theory.[15] If indeed the latter Higgs meson does exist, then perhaps its mass can be dynamically determined[16] as the σ mass is by QCD. This may be done in conjunction with the gauge technique.[17]

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Figure Captions

Fig. 1. Spinless component of the $0^-$ pseudoscalar Bethe-Salpeter bound state equation for $P(p^2)$.

Fig. 2. The $0^+$ scalar Bethe-Salpeter bound state equation for $U(p^2)$.

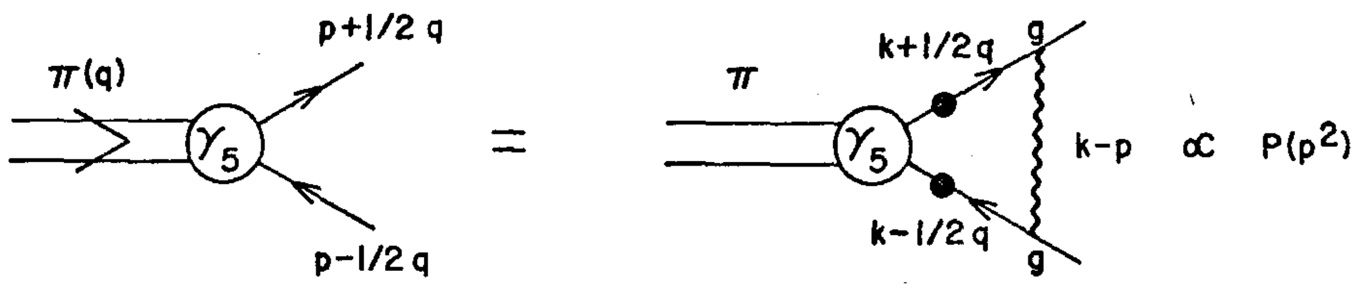


FIGURE 1

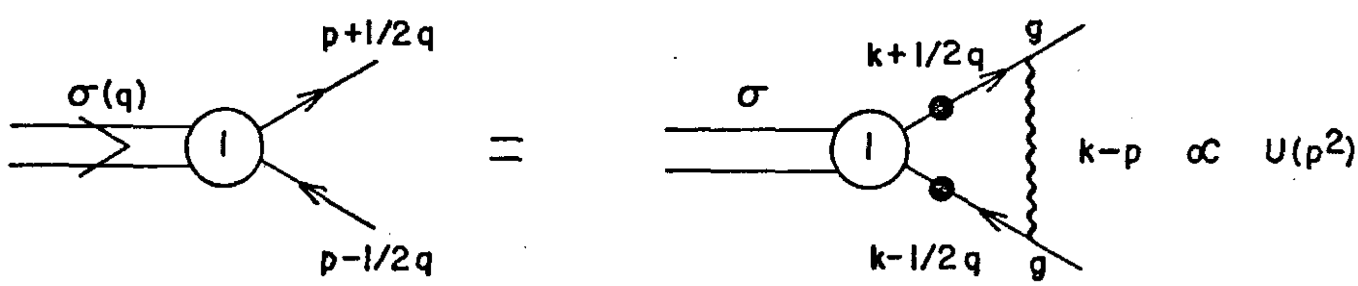


FIGURE 2