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WHY MASSES AND MAGNETIC MOMENTS SATISFY NAIVE QUARK MODEL PREDICTIONS*

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ABSTRACT

Relativistic and zero point energy corrections are shown to be absorbed in renormalizing phenomenological quark mass parameters appearing in quark model descriptions of baryon and meson masses and baryon and magnetic moments and do not affect successful relations. Analysis of small differences between effective quark masses in mesons and baryons gives two new successful relations between meson and baryon masses.

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A naive quark model[1,2] has given a relation between meson and baryon masses

$$M_\Lambda - M_N = 177 \text{ MeV} = m_s - m_u = (3/4)(M_{K^*} - M_\rho) + (1/4)(M_K - M_\pi) = 180 \text{ MeV} \quad (1a)$$

and two relations[3,4] for the magnetic moment of the Λ,

$$\mu_\Lambda = -0.61 \text{ n.m.} = (-1/3)\left[(1/\mu_p) + (M_\Lambda - M_p)/M_p\right]^{-1} = -0.61 \text{ n.m.} \quad (1b)$$

$$\mu_\Lambda = -0.61 \text{ n.m.} = -(\mu_p/3)\left(\frac{m_u}{m_s}\right) = -(\mu_p/3)(M_{\Sigma^{*+}} - M_{\Sigma^+})/(M_{\Delta^+} - M_p) = -0.61 \text{ n.m.} \quad (1c)$$

Equation (1a) is obtained from a universal mass formula for the mass M_h of any hadron in terms of the masses of the constituent quarks m_i and a hyperfine interaction depending on their spins $\vec{\sigma}_i$

$$M_h = \sum_i m_i + \sum_{i>j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \langle v_{ij} \rangle \quad (2a)$$

where $\langle v_{ij} \rangle$ is the value of the matrix element of the hyperfine interaction. Eqs.(1b) and (1c) are obtained by assuming that the magnetic moment μ_i of a quark with electric charge e_i is given by

$$\mu_i = e_i (M_p/m_i) \quad \text{nuclear magnetons} \quad (2b)$$

This remarkable agreement is very surprising in view of known neglected effects considerably larger than the difference between the theoretical and experimental values. It can be understood only if these neglected effects conspire to give contributions absorbed in the definition of the quark mass parameters m_i which are not determined by first principles but by fitting data. These quark mass parameters appear in both terms in Eq.(2a) and in Eq.(2b); i.e. as direct contributions to hadron masses, as coefficients in the strong hyperfine interaction responsible for spin splittings, and in the magnetic moments. The success of Eqs.(1) imply that the corrections to m_i in all three places in Eq.(2) for baryons and in the first term of Eq.(2a) for mesons are nearly the same. Note that Eqs.(1) do not involve the second term in Eq.(2a) nor Eq.(2b) for mesons.

We consider zero point kinetic and potential energies and relativistic effects neglected in Eqs.(2) and show that although these are large, their main contribution can be absorbed by changing the values of the mass parameter m_i in nearly the same way for mesons and baryons in the first term of Eq.(2a) and in the magnetic moment (2b). We find that the small difference in m_i between mesons and baryons does not affect the relation (1a) because m_s and m_u are shifted by about the same amount, and does not affect Eqs.(1b) and (1c) which involve only baryons. But this difference is observable in other experimental quantities calculated explicitly below to give two new relations which agree with experiment.

The first relation compares the calculated difference between the first term of Eq.(2a) for mesons and baryons with experiment by using spin averaged meson and baryon masses.

$$\left[m_i(\text{bar}) - m_i(\text{mes})\right]_{\text{theo}} = \frac{U}{2} \log\ (2/\sqrt{3}\,) = 53 \text{ MeV} \tag{3a}$$

$$\left[m_i(\text{bar}) - m_i(\text{mes})\right]_{\text{exp}} = \frac{M(N) + M(\Delta)}{6} - \frac{3M(\rho) + M(\pi)}{8} = 54.5 \pm 1.5 \text{ MeV} \tag{3b}$$

where $U = 733$ MeV is the strength of the Quigg-Rosner logarithmic potential[5,6], determined by fitting the $\Psi - \Psi'$ splitting. The effect of this small correction on the quark mass difference in Eq.(1a) is shown to be negligible. But the effect on the quark mass ratio (m_s/m_u) used in Eq.(1c) is appreciable, and explains why the prediction analogous to (1c) using meson masses is not successful.

The ratio of meson mass differences analogous to the baryon masses in Eq.(1c) should be set equal to the quark mass ratio (m_s/m_u) in mesons, corrected for the meson-baryon difference (3a). We thus obtain a second new relation by correcting the baryon values $m_s = 513$ MeV and $m_u = 366$ MeV determined from baryon magnetic moments,

$$\frac{m_s(\text{mes})}{m_u(\text{mes})} = \frac{(-M_p/3\mu_\Lambda) - 53}{(M_p/\mu_p) - 53} = 1.62 = (M_{\rho^\circ} - M_{\pi^\circ})/(M_{K^{*+}} - M_{K^+}) = 1.61 \pm 0.01 \tag{3c}$$

This should be compared with the old baryon relation which is a rearrangement of Eq.(1c)

$$\frac{m_s(\text{bar})}{m_u(\text{bar})} = \frac{-M_p/3\mu_\Lambda}{M_p/\mu_p} = 1.53 = (M_{\Delta^+} - M_p)/(M_{\Sigma^{*+}} - M_{\Sigma^+}) = 1.53 \pm 0.01 \ . \tag{3d}$$

The essential physics underlying these two new predictions is that the effective quark mass parameter is smaller in mesons than in baryons because the stronger quark-antiquark force in the mesons gives stronger binding than in baryons and brings the interacting pairs closer together where the potential is stronger. This increased binding appears not only as a reduction in the total mass but also in the quark mass parameter which determines the spin splittings via the color magnetic interaction

We now derive these results explicitly by calculating the zero point energy in meson and baryon systems in the quasinuclear model of Refs.(1,2) as the ground state expectation value of the Hamiltonian for a system of n particles interacting with a two-body color exchange logarithmic potential

$$E_0 = \langle H \rangle = \left\langle \sum_i m_i + \sum_i \frac{p_i^2}{2m_i} + \sum_{i>j} U k_{ij} \log(r_{ij}/r_0) \right\rangle . \tag{4}$$

where k_{ij} is a color factor. The rest mass contribution to the energy is included, but the non-relativistic expression for the kinetic energy is used. The spin-dependent contribution has been averaged out to give the zero point energy for the appropriate spin averages of the hadron masses used in Eqs.(1) and (3). Evaluating the color factors and using the virial theorem gives a result valid for any n-body color-singlet bound state of quarks and antiquarks[1,2] with complete symmetry between the n constituents.

$$E_o(n) = n\left[m + \frac{U}{4} + \frac{1}{2} U \langle \log(r/r_0) \rangle_n \right] \tag{5}$$

To the extent that the variation in $\langle \log r \rangle$ from one hadron to another can be neglected, the zero point energy and the hadron masses are proportional to n, giving the familiar "quark counting" 3/2 ratio for baryon to meson masses. The relation (3a) is just the correction to this 3/2, determined from the difference in $\langle \log(r) \rangle$ between mesons and baryons. The value $2/\sqrt{3}$ comes from the assumption that r scales like $(p^2)^{-1/2}$ between mesons and baryons and using the scaling factor for p^2 from the virial theorem in Refs.(1,2).

The change in effective quark mass (3) between mesons and baryons is independent of quark flavor and cancels in any flavor-dependence relation analogous to (1) between hadrons like ρ, Δ, ϕ or Ω in which all constituents have equal mass. Corrections to (1) from zero point energies can arise in hadrons like K, K^* or Λ which contain both u and s-quarks. To estimate these corrections we use the Feynman-Hellmann theorem for the change in the mass of a hadron h with a change δm_i in the mass of quark i,

$$\delta M_h = \langle \partial H/\partial m_i \rangle_h \delta m = \left[1 - \langle (t_i/m_i) \rangle_h\right] \delta m_i \tag{6}$$

where t_i is the kinetic energy of particle i. This can be simplified by using the virial theorem and substituting

$$t_i = t_i^{rel}(n-1)m_j/[m_i + (n-1)m_j] \tag{7a}$$

$$\delta M_h = \left[1 - n\langle t_i^{rel}\rangle/\langle T\rangle \left\{ \frac{1}{m_i} - \frac{1}{m_i + (n-1)m_j} \right\} (U/4)\right]\delta m_i \tag{7b}$$

where T is the total kinetic energy and t_i^{rel} is the kinetic energy of the relative motion of particle i with respect to the n-1 other particles which are assumed to have equal masses denoted by m_j.

An exact relation for $\langle t_i^{rel}\rangle/\langle T\rangle$ in the baryon case is obtainable only by solving the three-body problem. However, good approximate estimates and upper and lower bounds for the corrections to Eq.(1) are obtained by use of the relation

$$\langle T\rangle \approx (n-1)\langle t_i^{rel}\rangle = \left(n-1+\frac{m_i}{m_j}\right)\langle t_i\rangle \tag{8}$$

This relation (8) is exact for the meson case and for a symmetric baryon state with equal contributions from all quarks to the baryon kinetic energy. Substituting (8) into (7) and integrating between limits i and f where all quarks initially have equal mass m_i and the mass of one quark is changed to m_f gives

$$\begin{aligned} M_h(f) - M_h(i) &= (m_f-m_i) - \left(\frac{nU}{4(n-1)}\right)\log\left[nm_f/[m_f+(n-1)m_i]\right] \\ &= (m_f-m_i) - \left(\frac{nU}{4(n-1)}\right)\log\left[1+(n-1)(m_f-m_i)/\left(m_f+(n-1)m_i\right)\right] . \end{aligned} \tag{9}$$

This result is exact for mesons but holds in the baryon case only for equal masses. However, the relation (8) should not change very much with mass in view of the stabilizing effect of the log potential which holds kinetic energies fixed when masses change. In any case an <u>increase</u> in m_i should change the equipartition of kinetic energies in the direction to <u>decrease</u> t_i^{rel} and thus <u>increase</u> δM_h in Eq.(7). The approximate result for baryons from Eq.(9) is thus also a lower bound. Substituting $n=2$ and $n=3$ into Eq.(9) then gives for the correction to Eq.(1)

$$\left[M_b(f) - M_b(i)\right] - \left[M_m(f) - M_m(i)\right] \geq (U/8)\log\left[\frac{16m_f(m_f+2m_i)^3}{27(m_f+m_i)^4}\right] \tag{10a}$$

A rigorous upper bound on $M_b(f)$ is obtained by using solutions for the equal mass case as trial variational wave functions. This gives

$$\left[M_b(f) - M_b(i)\right] - \left[M_m(f) - M_m(i)\right] \le (U/4)\log\left[1 + \frac{(m_f - m_i)^2(5m_f + 4m_i)}{27(m_f + m_i)^2 m_f}\right] \tag{10b}$$

Setting $U = 733$ MeV and $m_f/m_i = m_s/m_u \approx 3/2$ in Eqs.(10) gives bounds of about ±2 MeV. This is consistent with the experimental validity of the relation (1), and demonstrates that although zero-point energies of the order of 200 MeV were neglected in the original derivation, the error introduced is only of order one per cent of the neglected zero point energy because of the scaling properties between mesons and baryons.

Since the relation (1) has now been shown to hold with $m_s - m_u$ interpreted as an effective quark mass including zero point energy, the empirical validity of the relations (2) suggests that the same effective quark mass enters into the magnetic moment. The contribution of a constant scalar potential has been shown to contribute to this effective mass,[7] and the bag models[8] suggest that the zero point kinetic energy also contributes for the case of no potential. To generalize these results we consider a Dirac particle bound in an external potential which varies in space and has both a Lorentz scalar component S and a Lorentz four-vector component V and with a weak external magnetic field $\vec{H}$ from a vector potential $\vec{A}$. The Dirac equation for this system is

$$\left[\vec{\alpha}\cdot(\vec{p} - e\vec{A}) + \beta(m + S) + V - E\right]\Psi = 0 \tag{11}$$

Squaring the equation and rearranging terms leads to an exact equation resembling the nonrelativistic Schroedinger equation,

$$\left[(E - V + m + S)^{-1}\left\{(p - eA)^2 + \vec{\alpha}\cdot\left[\vec{p}, (\beta S+V)\right] - e\hbar\vec{\sigma}\cdot\vec{H}\right\} + V + m + S\right]\Psi = E\Psi \quad (12)$$

The usual nonrelativistic treatment obtains a factor 1/2m in both the kinetic energy and the magnetic moment by neglecting V, S, and E–m relative to m. We consider the full relativistic equation (12) and examine the dependence of the energy on the $\vec{\sigma}\cdot\vec{H}$ term treated as a perturbation.

Let us define an effective unperturbed Hamiltonian H_0, energy E_0 and wave function Ψ_0 by

$$H_o = (E_o - V + m + S)^{-1}\left\{(\vec{p} - e\vec{A})^2 + \vec{\alpha}\cdot\left[\vec{p}, (\beta S+V)\right]\right\} + V + m + S \quad (13a)$$

$$H_o\Psi_o = E_o\Psi_o \quad (13b)$$

Substituting Eq.(13) into Eq.(12) gives

$$(E - H_o)\Psi = (E - V + m + S)^{-1}\left\{(E_o - E)(H_o - V - m - S) - e\hbar(\vec{\sigma}\cdot\vec{H})\right\}\Psi \ . \quad (14)$$

This equation is still exact. Taking the scalar product of Eq.(14) with Ψ_o and keeping only terms to first order in the small quantities $\vec{\sigma}\cdot\vec{H}$ and $E-E_o$ then gives

$$\langle 2(E_o - V + m + S)^{-1}(E_o - V)(E - E_o)\rangle = -e\hbar\langle(E_o - V + m + S)^{-1}\vec{\sigma}\cdot\vec{H}\rangle \quad (15)$$

We define the "effective quark mass" m_{eff} by the relation

$$E - E_o = -e\hbar(\vec{\sigma}\cdot\vec{H})/2m_{eff} = -\vec{\mu}\cdot\vec{H} \quad (16)$$

From Eqs.(15) and (16)

$$m_{eff} = E_o - \langle V \rangle - \delta V \tag{17a}$$

where

$$\delta V = \frac{\langle (V - \langle V \rangle)(V - S)(E_o - V + m + S)^{-1} \rangle}{(E_o + m)\langle (E_o - V + m + S)^{-1} \rangle} . \tag{17b}$$

For a Lorentz scalar potential, Eq.(17) with $V = 0$ is the generalization of the previous results[7,8] that both the zero point kinetic energy and the zero point potential energy are included in the effective mass parameter appearing in the magnetic moment. For the case of a Lorentz vector potential, the potential energy does not contribute in the approximation where V is neglected. This is in agreement with the observation that the magnetic moment of an electron is unchanged by the electrostatic potential of a Van-de-Graaf accelerator and its motion is described by non-relativistic equations if its velocity is small, even though it may be bound by a potential many times its rest energy.

We therefore conclude that the quark mass difference in Eq.(1) can be interpreted as a difference between effective quark masses which include zero point kinetic and potential energies. If the confinement potential which determines the binding energy is Lorentz scalar, the same effective quark mass also appears in the magnetic moments and justifies the use of Eqs. (2).

The expression (17a) can be rewritten to exhibit an expansion in powers of (v/c),

$$m_{eff} = (m + \langle S \rangle) + (T + S - \langle S \rangle) - \delta V \tag{18a}$$

where the kinetic energy T is defined as

$$T \equiv E_o - S - V - m \,, \tag{18b}$$

The terms m and $\langle S \rangle$ are of order unity. T and $S - \langle S \rangle$ are of order $(v/c)^2$, and δV is of order $(v/c)^4$.

These results provide new insight into the successes of the nonrelativistic quark model. Its two basic features are: (1) No parameters are determined from first principles; all are adjusted to fit data. (2) Effects apparently neglected in the model are not small; however, their principal contributions are absorbed by a renormalization of the values of the parameters. That such effects can be renormalized away is non-trivial. The underlying physics is shown in Eqs. (1-3) and (17). That interaction energies can be absorbed in defining <u>the same effective quark mass</u> for both mesons and baryons follows from the properties of the color exchange force and would not hold for other forces. This result is implicitly contained in Nambu's old mass formula[1,2,9,10] for systems of quarks and antiquarks interacting via an octet of colored gauge vector gluons, which gave masses proportional to the number of constituents, even when the interactions were included. The relations between the "effective mass parameters" appearing in the magnetic moment, the kinetic energy and the total energy of a bound Dirac particle are inherent properties of the Dirac equation, and suggest that the confining potential must transform under Lorentz transformations like a scalar rather than a vector potential.

The coupling of a constituent quark to the electromagnetic current is determined by a single coupling constant which by gauge invariance is forced to be the electric charge of the quark. This is not true for the weak couplings, where no constraint relates axial vector and vector couplings, even though the basic theory has $g_V = g_A$ for bare or "current" quarks. The success of the prediction $\mu_p/\mu_n = -3/2$ and the failure of the prediction $(g_A/g_V)_N = 5/3$ are both natural in this approach, since the g_A/g_V prediction involves using coupling constants from first principles for constituent quarks and neglecting the necessary renormalization of parameters.

One remaining open question is the justification of the empirical success of the Hamiltonian (3) with effective two-body forces and no effective three-body forces for baryons. The successes of the new relations between properties of mesons and baryons as well as of old relations between baryon masses based on only two-body effective forces[11] indicate that the additional complications resulting from the three-body nature of the baryon are somehow negligible. Although there have been attempts to interpret properties of baryons as more than an assembly of three quarks with two-body interactions, there is as yet no convincing evidence for anything extra in the baryon.

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