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SU(5) WITHOUT SU(5): WHY B-L IS CONSERVED AND BARYON NUMBER NOT IN UNIFIED MODELS OF QUARKS AND LEPTONS*

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The existence of an SU(2) x U(1) classification[1] for two completely different types of particles which seem to belong together in common multiplets motivates a search for a higher symmetry to unify the two and be <u>the</u> non-Abelian gauge theory of the World.[2] Today SU(5) and higher groups containing SU(5) are the main candidates.

But not so long ago there was another SU(2) x U(1) classification for two completely different types of particles, strange and nonstrange, which seemed to belong together in common multiplets. This SU(2) x U(1) of isospin and strangeness motivated a search for a higher symmetry to unify the two. The SU(3) gauge theory called the eightfold way brought strange and nonstrange particles into unified multiplets and was believed to be <u>the</u> non-Abelian gauge theory of the world. Today the unification of strange and nonstrange particles into flavor SU(3) remains, but it is no longer a candidate

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for a gauge theory. The ρ, ω and K^* are no longer an octet of gauge bosons and flavor SU(3) has been revealed to be an accidental symmetry based upon our incomplete knowledge of the number of flavors.

Will history repeat itself with quark-lepton unification and SU(5)? To prepare for such a repetition of history we examine quark-lepton unification without assuming higher symmetries or more general gauge theories. In particular we look for properties generally attributed to SU(5) models which are already present without the assumption of SU(5).

Conservation of baryon number seems to be violated in any model which introduces bosons that transform a quark into a lepton or vice versa. Yet a proton can absorb a pion or a kaon and be transformed into a neutron or a hyperon without causing global non-conservation of electric charge or strangeness in strong interactions. Thus the mere existence of bosons which can change a fermion quantum number is not sufficient for violation of a conservation law. There must be some additional reason why the new lepto-quark bosons introduced in unified models[1,2] do not carry baryon number to give a global conservation law in the same way that pions and kaons carry and conserve charge and strangeness.

A mysterious conservation of B-L appears in these models[3,4] that violate baryon number conservation. This has the consequence that protons decay into antileptons, rather than into leptons; i.e. into positrons and positive muons, rather than electrons or negative muons. Many of these models are based on a higher symmetry group which contains SU(5).

To understand the origin of the baryon-number nonconservation, the tendency to conserve B-L, and the SU(5)-like properties which arise in these models, we search for model-independent constraints on the S-matrix which are already present in view of known symmetries. The approach is one which I learned from Prof. Giulio Racah--brute force. Those who are familiar with Racah's elegant papers but did not know him personally do not realize that he often found his elegant general results by first grinding through a large number of calculations of individual cases. He would carefully note any systematics in his numerical results, use these to conjecture general principles, and then prove the theorems. We follow this approach and begin by looking at the quantum numbers of the observed particles for any regularities which explain the peculiar conservation laws and violations. At this stage it is pure numerology. But when systematics are found, it becomes more than numerology.

The brute force problem which we pose is to examine the most general S matrix for the thirty quarks and leptons in the first generation, commonly classified in the 5, 5*, 10 and 10* representations of SU(5), under the assumptions only of conservation of electric charge and weak isospin. We might consider writing a computer program to print out all allowed S-matrix transitions, classifying them according to whether they conserve or violate B or B-L. However, by playing around with our numerology, we find a quicker way to do this without a computer. All the desired selection rules and conservation laws are easily demonstrated with the aid of the following unorthodox linear combinations of well known quantum numbers[5,6]

$$\kappa = 3(B-L) - H \tag{1a}$$

$$P = \frac{3(B-L)-H}{4} - \frac{3(Q-I_3)}{5} = \frac{\kappa}{4} - \frac{3}{5}(Q-I_3) \tag{1b}$$

$$\beta = \frac{3(B-L)-H}{4} - (Q-I_3) = P - \frac{2}{5}(Q-I_3) \tag{1c}$$

where B is the baryon number, L the lepton number, H the helicity, defined as +1 and −1 respectively for right and left handed states, Q the electric charge and I_3 the third component of the weak isospin. These quantum numbers are defined to be additive for a multifermion state.

The motivation for introducing these quantum numbers (1) is seen in the simple structure of their eignevalues for the fifteen quark and lepton states of the first generation and their fifteen antiparticles listed in Table I. The pentality quantum number P has the following remarkable properties:

1. Although it is a linear combination of two terms with quarter-integral and tenth-integral values, the values of P for the first generation fermions are only fifth-integral.

2. The total pentality of a multifermion state cannot change by a fifth-integral amount in any transition which conserves Q and I_3. Thus the pentality is an additive quantum number with fifth-integral eigenvalues which is allowed to change only by integral amounts in any transition which conserves Q and I_3. This gives the pentality selection rule

$$\delta P = \delta \beta = 0,\ 1,\ 2\ \ldots\ldots \tag{2a}$$

This immediately implies that

$$\delta \kappa = 4n\ , \tag{2b}$$

where n is an integer. Since δH and $\delta(B-L)$ are both even integers, the selection rule (2b) requires that $\delta H/2$ and $\delta(B-L)/2$ must either be both even integers or both odd integers. Thus transitions with odd and even helicity flips have different selection rules for $B-L$.

For odd-helicity-flip transitions,

$$\delta H = 2(2n+1) \Rightarrow \delta(B-L) = 2(2n'+1) \tag{2c}$$

where n and n' are integers. <u>Thus $B-L$ cannot be conserved in odd helicity flip transitions</u>. For helicity conserving and even flip transitions

$$\delta H = 4n \Rightarrow \delta(B-L) = 4n' \tag{2d}$$

where n and n' are integers. In four point functions $\delta H = 0$, 2 or 4 and $\delta(B-L) = 0$ or 2, since $\delta(B-L) = 4$ can be achieved only in a transition between two leptons and two antileptons which cannot conserve Q and I_3. <u>Thus $B-L$ is conserved in all</u>

Table 1. Quantum Numbers of First Generation Fermions

Fermion	H	Q	I_3	κ	P	B	β	λ	μ	ν
$\bar{d}_R$	1	+1/3	+1/2	-2	-2/5	-1/3	-1/3	-1/5	0	-1
e^-_R	1	-1	0	-4	-2/5	0	0	-1/5	1	-5
u_R	1	+2/3	0	0	-2/5	+1/3	-2/3	-1/5	-1	+3
$\bar{u}_R$	1	-2/3	-1/2	-2	-2/5	-1/3	-1/3	-1/5	0	-1
e^+_L	-1	+1	0	+4	2/5	0	0	+1/5	-1	+5
$\bar{u}_L$	-1	-2/3	0	0	2/5	-1/3	2/3	+1/5	1	-3
u_L	-1	+2/3	+1/2	+2	2/5	+1/3	+1/3	+1/5	0	1
d_L	-1	-1/3	-1/2	+2	2/5	+1/3	+1/3	+1/5	0	1
e^+_R	1	+1	+1/2	+2	1/5	0	0	+3/5	0	3
$\bar{\nu}_R$	1	0	-1/2	+2	1/5	0	0	+3/5	0	3
d_R	1	-1/3	0	0	1/5	+1/3	+1/3	+3/5	1	-1
e^-_L	-1	-1	-1/2	-2	-1/5	0	0	-3/5	0	-3
ν_L	-1	0	+1/2	-2	-1/5	0	0	-3/5	0	-3
$\bar{d}_L$	-1	+1/3	0	0	-1/5	-1/3	-1/3	-3/5	-1	+1
Right Handed Neutrinos										
$\bar{\nu}_L$	-1	0	0	+4	1	0	1	1	1	1
ν_R	+1	0	0	-4	-1	0	-1	-1	-1	-1

helicity-conserving and double-flip four point functions which conserve Q and I_3. This includes all simple boson exchange diagrams without derivative couplings, since these have either flip or nonflip couplings, and must have the same at both vertices. A single-flip transition $\delta H = 2$ arises only in the exchange of an object which flips helicity at one vertex and not at the other.

The "quasibaryon number" β (1c) is conserved in all processes which conserve Q, I_3, H and $B-L$ and in particular in all four point functions with no helicity change, e.g. vector boson exchange, which conserve electric charge and weak isospin. However, β is seen from Table I to be exactly equal to the baryon number for all quark and lepton states[5] except the right-handed up quark u_R and its conjugate left-handed $\bar{u}_L$. For any system of quarks and leptons

$$\beta = B - n(u_R) + n(\bar{u}_L) \qquad (3)$$

where u denotes any quark with electric charge $+2/3$. This result (3) holds for any number of standard generations of quarks and leptons having massless left-handed neutrinos and no right-handed neutrinos.

For all processes where β is conserved, which includes all vector boson exchanges, B is conserved as long as $n(u_R) - n(\bar{u}_L)$ does not change. This includes the standard $SU(2) \times U(1)$ model of weak interactions where the u_R couples only to neutral currents which cannot change flavor. However, B must be violated by any helicity conserving boson exchange with a non-trivial coupling to u_R which changes it into something else which is not a u_R and with an additional coupling to any other particle. Four-point functions exist in which this boson is exchanged between a u_R and another object and the u_R is changed to something else and not restored at the other vertex. Then $n(u_R) - n(\bar{u}_L)$ changes and B must be violated if β is conserved.

Thus conservation of weak isospin is inconsistent with conservation of baryon number in processes mediated by a gauge boson which couples to the u_R quark in a non-trivial manner and allows a u_R quark to decay into something else. Baryon number, weak isospin and electric charge cannot all be conserved in any process having the form

$$u_R \to X + G \to X + Y + Z$$

where G is a gauge boson, and X, Y and Z can be any quark or lepton state except a u_R. Any charge-conserving process which allows a u_R to decay into three fermions via an intermediate vector boson state must violate either weak isospin or baryon number.

Almost any model which introduces new particles must admit $\delta H = 0$ four-point functions in which the only external particles are first generation quarks and leptons. Including other generations with identical values of these quantum numbers does not change these conclusions. Unless these new processes treat u_R and $\bar{u}_L$ in the same trivial way as the standard model of SU(2) x U(1), they must necessarily violate baryon number conservation. This is the basic reason why baryon number nonconservation must arise in any such unification models. It is already required by consistency with charge and weak isospin conservation and the observed quantum numbers of the first generation of quarks and leptons. No further gauge theory is needed. This also explains why attempts to gauge baryon number would encounter inconsistencies.

These results show that in any gauge theory based on a group which includes SU(3) x SU(2) x U(1), baryon number can be conserved in gauge boson exchange only under one of the two following conditions:

1. The u_R and $\bar{u}_L$ states are classified in singlet representations of the gauge group. In that case they can only couple to U(1) generators and $n(u_R) - n(\bar{u}_L)$, β and B are all conserved. This occurs in the standard model of weak interactions.

2. Any gauge boson which couples non-trivially to u_R and

changes it into something else has only this coupling and the analogous coupling in other generations. In this case $n(u_R) - n(\bar{u}_L)$ defines a conserved "flavor symmetry" quantum number and is conserved like strangeness in strong interactions. Since gauge bosons are always classified in the adjoint representation of some group G, a boson which can change a u_R into another particle will have no other coupling only if the u_R is classified in the fundamental representation of G and the remaining particles in the first generation are either singlets in G or in the same fundamental representation as u_R. In this way an exchanged gauge boson which couples nontrivially to u_R couples at the two vertices with conjugate step operators in the fundamental representation of G and conserves $n(u_R) - n(\bar{u}_L)$. The gauge symmetry group may either be G itself or the direct product of G and another group. One example is $SU(n)_R$ in which n of the right handed fermions are classified in the fundamental representation of SU(n) and all the remaining particles are singlets. The case $n = 2$ which corresponds to a right-handed isospin acting only on quarks would satisfy this condition. The right-handed electron would be a singlet in this SU(n).

Note that these conditions follow from the properties of the four-point function in which only the observed particles appear as external particles. This is easily extended to the case where other particles exist; e.g. a right-handed neutrino, by restricting our attention only to those four-point functions where these additional particles do not appear as external particles. The couplings of gauge bosons to these additional particles play no role in the four-point functions under consideration and can be disregarded. Thus, for example, the case of a right-handed isospin discussed above could include a classification of the right-handed electron and neutrino in a doublet if right-handed neutrinos exist, since the transitions between e^-_R and ν_R do not occur in the four-point functions considered, and the e^-_R behaves like a singlet in the space of these four-point functions.

The unification schemes proposed do not generally fit either of these two conditions and are thus forced to violate baryon number conservation in four-point functions mediated by gauge boson exchange. However, extensions of SU(3) x SU(2) x U(1) involving the direct product of this group with groups generated by right-handed currents can conserve baryon number.

We now classify all three types of helicity amplitudes occurring in four point functions.

A. Helicity conserving processes: $\delta H = 0$, $\delta P = \delta\kappa = \delta\beta = \delta(B-L) = 0$. The processes allowed by the penality rule (2a) have the form

$$(P_1,P_2) \rightarrow (P_1,P_2) \tag{4a}$$

where P_i denotes any state having the eigenvalue $P = P_i$, together with all other processes obtained from (4a) by crossing.

B. Double-flip processes: $\delta H = \pm 4 = -\delta\kappa$, $\delta P = \pm 1 = \delta\beta$, $\delta(B-L) = 0$. The only processes allowed by the pentality selection rule (2a) have the form

$$(2/5,2/5) \rightarrow (-2/5,1/5) \tag{4b}$$

together with all other processes obtained from (3b) by crossing.

C. Single-flip processes: $\delta H = \pm 2$. The only processes allowed by the pentality selection rule (2a) have $\delta P = \pm 1 = \delta\beta$, $\delta(B-L) = \pm 2$ and the form

$$(1/5,1/5) \rightarrow (-1/5,-2/5) \tag{4c}$$

together with all other processes obtained from (3c) by crossing.

All these results obtained from the pentality selection rule which assumed only conservation of Q and I_3 can now be seen to be also SU(5) results. The eigenvalues of P are seen in Table I to label representations of SU(5). All the states in a given representation of SU(5) have the same eigenvalue of P. Thus P turns out to be an SU(5) invariant, although P is defined in terms of B and L which are completely outside of SU(5) and there is no group-theoretical reason connecting P with SU(5). The "pentality" quantum number P is seen to have the property analogous to triality for SU(3); it has the eigenvalues $\pm(1/5)$ for the fundamental representations 5 and 5* of SU(5) and the eigenvalues $\pm(2/5)$ for the 10 and 10* representations which are built from two fundamental representations. Note that the fifth-integral quantum numbers arise naturally in this description, without invoking any SU(5), and that the selection rule (2a) that the fifth-integral pentality number P can only change by an integral amount is also derived without SU(5).

The relations (5) are just the four point couplings allowed by SU(5). The $\delta H = 0$ transitions (4a) have the form

$$m \times n \rightarrow m \times n \tag{5a}$$

where m and n denote any representation of SU(5) among the 5, 5*, 10 and 10*. The $\delta H = \pm 4$ and ± 2 transitions (4b) and (4c) correspond respectively to the SU(5) couplings

$$10 \times 10 \rightarrow 10^* \times 5 \tag{5b}$$

$$5 \times 5 \rightarrow 5^* \times 10^* \tag{5c}$$

Our approach here is to note these connections with SU(5) as a guide to our formulation, but to keep everything completely independent of any symmetry assumptions beyond global $SU(2) \times U(1)$. Thus all the results obtained for four-point functions involving quarks and leptons hold in any generalization of the standard model which keeps charge and weak isospin conservation regardless of how many additional Higgses, Schmiggses or Technicrats are introduced. The SU(5)-like results must therefore be present in any such formulation regardless of whether or not these symmetries are explicitly assumed.

We now examine the three types of helicity amplitudes given in Eqs.(4) in more detail.

A. Helicity conserving $\delta H = 0$ transitions.

These can be characterized by the selection rules $\delta H = \delta P = \delta\kappa =$

$\delta\beta = \delta(B-L) = 0$. They conserve B-L and β, and therefore conserve baryon number if $n(u_R) - n(\bar{u}_L)$ does not change, but must violate baryon number conservation if it does change. These processes are the only ones of the three types that can be mediated by a Yukawa coupling via a helicity conserving vector boson or by any boson classified in the adjoint representation of SU(5). Note that the adjoint representation contains bosons which change $n(u_R) - n(\bar{u}_L)$.

B. Double flip $\delta H = \pm 4$ transitions.

These can be characterized by the selection rules $\delta H = \pm 4 = -\delta\kappa$; $\delta P = \mp 1 = \delta\beta$; $\delta(B-L) = 0$. They conserve B-L but have $\delta\beta = \mp 1$. They violate baryon number conservation unless $n(u_R) - n(\bar{u}_L)$ changes by one unit in the proper direction to match the change in β. These processes have the form (5b) in SU(5). They cannot be mediated by exchange of a boson classified in the adjoint representation of SU(5) but could be mediated by exchange of a particle such as a scalar Higgs which flips helicity and is classified in a 5 or 5* representation. Equation (5b) shows that in an SU(5) description, such a four point function can only go via intermediate states in the 5, 5*, 45 or 45* representations.

C. Single-flip $\delta H = \pm 2$ transitions.

These can be characterized by the selection rules $\delta H = \pm 2 = \delta(B-L)$; $\delta P = \pm 1 = \delta\beta$, $\delta\kappa = \pm 4$. They violate everything, including B-L and B. However, they corresponding to the SU(5) couplings (6c) and can only arise in an SU(5) formalism via an intermediate boson classified in the 10 or 10* representations with derivative couplings. Thus as long as no Higgses or other particles classified in the 10 or 10* are introduced with derivative couplings to quarks or leptons, these processes do not occur and B-L is conserved.

We now list explicitly all processes which violate baryon conservation and examine the implications for proton decay.

A. $\delta H = 0$ transitions (4a).

These are the only ones which can go via vector exchange. The quasi-baryon number β is conserved, and $\delta B \neq 0$ requires the disappearance of a u_R quark or its equivalent under crossing. Thus the most general $\delta B \neq 0$, $\delta H = 0$ process has the form:

$$u_R + X \rightarrow Y(P = -2/5) + X' \qquad (6)$$

where fermions X and X' must have the same eigenvalues of P.

The only candidates for Y are the companions of u_R in the $P = -2/5$ multiplet (the 10* of SU(5)) which are the e_R^-, $\bar{u}_R$ and $\bar{d}_R$. The e_R^- is immediately excluded, because the transition $u_R \rightarrow e_R^-$ changes electric charge by $\delta Q = -5/3$, and can only be balanced by the reciprocal transitions $e^- \rightarrow u$ which restores baryon conservation. The transitions $u_R \rightarrow \bar{u}_R$ and $u_R \rightarrow \bar{d}_R$ both involve a change in quasi-baryon number $\delta\mathcal{B} = +1/3$, and are related to one another by a weak isospin reflection. They must be balanced by a transition $X(\text{quark}) \rightarrow X'(\text{lepton})$ to conserve $\mathcal{B}$. The only quark-lepton transitions which can balance the $\delta Q = -4/3$ of the $u_R \rightarrow \bar{u}_R$ transition are the $d \rightarrow e^+$ transitions.

The allowed baryon-number violating processes with $\delta H = 0$ are therefore

$$u_R + d_R \rightarrow \bar{u}_R + e_R^+ \qquad (7a)$$

$$u_R + d_L \rightarrow \bar{u}_R + e_L^+ \qquad (7b)$$

together with their weak isospin reflections,

$$u_R + d_R \rightarrow \bar{d}_R + \bar{\nu}_R \qquad (7c)$$

$$u_R + u_L \rightarrow \bar{d}_R + e_L^+ \; . \qquad (7d)$$

This immediately gives the result that the lepton emitted in nucleon decay must be either positive or an antineutrino, but cannot be negative, nor a left handed neutrino.

The pairs of transitions (7a-7c) and (7b-7d) related by isospin reflection are required to be equal. However, there is no relation between the two pairs at this level, as they involve different $SU(2) \times U(1)$ multiplets. In SU(5) these pairs are related only at the level of a gauge theory by the universal couplings of gauge bosons. At the global SU(5) level with independent Yukawa couplings for different SU(5) multiplets the two pairs involve the couplings of the 10 and 5 of SU(5) respectively to the bosons classified in the 24. These couplings are independent if there is no higher symmetry like SO(10) or gauge condition which relates them.

However, isospin relations alone are sufficient to predict that the positron decay modes are stronger than the neutrino decay modes, since the neutrino decay (7c) is equal to one of the three positron decay modes (7a) by isospin, and there are two additional positron decays (7b) and (7d).

These results can be expressed as nucleon decays by adding a spectator u or d quark to the equation to give a nucleon on the left hand side and a pion on the right hand side. From (7a) and (7b) we obtain

$$p \to e^{+} + \pi^{\circ} \tag{8a}$$

$$n \to e^{+} + \pi^{-} \tag{8b}$$

From (7c) we obtain

$$p \to \bar{\nu}_R + \pi^{+} \tag{8c}$$

$$n \to \bar{\nu}_R + \pi^{\circ} \tag{8d}$$

From (7d) we obtain (8a) again.

B. Double-flip $\delta H = 4$ processes (3b).

These are most conveniently listed in the crossed representation with three $P = 2/5$ particles going into one with $P = 1/5$ and looking at each allowed value of the total electric charge. The complete set of $\delta H = 4$ processes are

$$Q = +1 \qquad e^{+}_L + u_L + \bar{u}_L \to e^{+}_R \qquad \delta B = \delta L = 0 \tag{9a}$$

$$u_L + u_L + d_L \to e^{+}_R \qquad \delta B = \delta L = -1 \tag{9b}$$

$$Q = 0 \qquad e^{+}_L + d_L + \bar{u}_L \to \bar{\nu}_R \qquad \delta B = \delta L = 0 \tag{9c}$$

$$u_L + d_L + d_L \to \bar{\nu}_R \qquad \delta B = \delta L = -1 \tag{9d}$$

$$Q = -\frac{1}{3} \qquad d_L + u_L + \bar{u}_L \to d_R \qquad \delta B = \delta L = 0 \tag{9e}$$

$$e^+_L + \bar{u}_L + \bar{u}_L \to d_R \qquad \delta B = \delta L = +1 \tag{9f}$$

These always conserve B-L, but have $\delta\beta = -1$ and violate baryon number conservation unless they involve a single u_R or $\bar{u}_L$ in which case the baryon number always turns out to be conserved. For each charge there is one $\delta B = 0$ and one $\delta B = \delta L = \pm 1$ process. Here also nucleons decay only into e^+ and $\bar{\nu}$.

C. Single-flip $\delta H = \pm 2$ processes (3c).

These are also conveniently listed by electric charge in the crossed representation with three $P = 1/5$ particles going into one with $P = -2/5$.

$$Q = -1 \qquad d_R + d_R + d_R \to e^-_R \qquad \delta B = -1, \delta L = +1 \tag{10a}$$

$$Q = -\frac{2}{3} \qquad d_R + d_R + \bar{\nu}_R \to \bar{u}_R \qquad \delta B = -1, \delta L = +1 \tag{10b}$$

$$Q = +\frac{1}{3} \qquad e^+_R + d_R + d_R \to \bar{d}_R \qquad \delta B = -1, \delta L = +1 \tag{10c}$$

$$Q = +\frac{2}{3} \qquad e^+_R + \bar{\nu}_R + d_R \to u_R \qquad \delta B = 0, \delta L = +2 \tag{10d}$$

These all have $\delta(B-L) = -2$ and $\delta\beta = -1$. All violate lepton conservation. Processes (10d) which conserve baryon number have $\delta L = 2$. The others all have $\delta B = \delta L = -1$ and give nucleon decays into e^- and ν.

We thus have demonstrated the following SU(5) like properties of four-point functions without explicit assumptions of SU(5).

1. All selection rules obtained from full SU(5) symmetry are already required by conservation of electric charge and weak isospin.

2. The four point functions are naturally classified into three types with different helicity structures, namely helicity conserving, double flip and single flip. This classification is in one-to-one correspondence with SU(5) classifications.

3. The violation of the conservation laws of B and B-L can be stated very generally and simply:

A. All helicity conserving and double flip amplitudes conserve B-L and allow nucleon decays only to antileptons. All single-helicity-flip amplitudes have $\delta(B-L) = \pm 2$ and allow nucleon decays only to leptons.

B. Baryon number conservation is violated in helicity conserving amplitudes only in those amplitudes which can be transformed by crossing to the decay of a right-handed up quark u_R into two antiquarks and an antilepton. All other helicity conserving amplitudes conserve B. Thus B is conserved in the standard model of weak interactions where the u_R couples only via neutral currents and cannot change into another state.

C. The role of the u_R is reversed in double-flip transitions. All double-flip transitions which involve the creation or annihilation of a single u_R or $\bar{u}_L$ conserve baryon number. All double-flip transitions which do not involve u_R or $\bar{u}_L$ or which create or annihilate a pair of them violate baryon number conservation.

4. The three types of four-point functions have simple interpretations in terms of the SU(5) and Lorentz quantum numbers of exchanged bosons which could give rise to these couplings if they are coupled to the fermions with Yukawa couplings.

Some of these results have been previously obtained using specific models[3] or symmetries,[4] particularly results on the conservation of B-L. Our results are more general, being model independent and assuming no higher symmetries. We also point out for the first time the close connection between the helicity structure of the amplitude and conservation of B and B-L.

All these results were rigorously obtained without any SU(5) symmetry assumptions. Yet one may be suspicious of hidden connections because the particular linear combinations (1) chosen ad hoc turned out to have SU(5) properties. We therefore investigate the properties of such linear combinations in more detail and look for possible implicit connections between them and higher symmetries.

In any scheme which unifies quarks and leptons and places them into common multiplets, quantum numbers are needed like generalizations of hypercharge which have the same eigenvalues for a large number of states including both quarks and leptons. The quantum number P has only two pairs of equal and opposite eigenvalues $\pm 1/5$ and $\pm 2/5$ for all sets of 30 states in the first three generations. We now examine all possible combinations of the quantum numbers labeling quantities conserved in $SU(2) \times U(1)$ to look for other linear combinations satisfying the requirement that only two pairs of equal and opposite eigenvalues should appear.

Since baryon number and electric charge are both third-integral for quarks and integral for leptons, a linear combination of these two must be found which is not third-integral in order to have a common eigenvalue for quarks and leptons. The combination $Q - I_3$ is used to give the same eigenvalue for both members of an isospin doublet. The most general linear combination $B - x(Q - I_3)$ has the eigenvalues $(1/3) - (1/6)x$ for the left handed quark doublet and $(1/3) - (2/3)x$ and $(1/3) + (1/3)x$ for the right handed u and d quarks respectively. With three independent eigenvalues for the quarks alone, without considering antiquarks and leptons, these can satisfy our requirement of two pairs of eigenvalues only if two of the three eigenvalues are equal and opposite. This occurs only for three values of x; namely $x = 4/5$, 2 and -4. With these values we define

$$\lambda = (B-L) - \frac{4}{5}(Q-I_3) = (4P + H)/3 \tag{11a}$$

$$\mu = (B-L) - 2(Q-I_3) = -2I_{3R} \tag{11b}$$

$$\nu = (B-L) + 4(Q-I_3) = 5\tilde{\lambda} \tag{11c}$$

where the additional term $-L$ has been added to keep the same eigenvalues for quarks and leptons. The quantum number λ is seen to be simply related to the pentality P. The quantum number μ is seen to be identical with the right handed isospin recently introduced by Marshak and Mohapatra.[7] The number ν is seen to be $5\tilde{\lambda}$ where $\tilde{\lambda}$ is the eigenvalue of λ for the isospin mirror state, $u \leftrightarrow d$ and $e \leftrightarrow \nu$.

This condition already suggests one of the general features found in unfiication schemes, the classification of quarks and antiquarks in the same multiplet. The condition that two quark states have equal and opposite eigenvalues of an additive quantum number which is constant in a multiplet gives quarks and antiquarks the same eigenvalue of this quantum number.

Values of λ, μ and ν are also given in Table I. But λ which is related to the pentality P is the only one with fractional eigenvalues. Thus the fifth-integral eigenvalues are a natural result of the SU(2) x U(1) classification of quarks and leptons and the condition restricting the eigenvalues to only two pairs. The normalization of the operators (11) is chosen to keep the coefficient of B–L equal to unity. Thus all three are conserved if B–L is conserved and conserved modulo 2 if B–L is not conserved. The fractional eigenvalues makes conservation modulo 2 a much more serious constraint for λ than for μ and ν.

The mysterious relation between the fifth-integral eigenvalues of the operator λ and SU(5) is clarified by noting that these eigenvalues have a simple interpretation in the SO(10) classification when the right handed neutrino is included which has an eigenvalue of -1. They are just the eigenvalues of the U(1) generator[4] which appears in the SU(5) x U(1) subgroup of SO(10) in the spinorial representation normally used to classify the quarks and leptons. If the operator λ is identified with this U(1) generator in SO(10), the classification and all the quantum numbers in Table I are evident. However, B and L are not defined in the usual SO(10) description, and the physical meaning of this U(1) generator is not obvious. There may be a deep underlying significance to the fact that the eigenvalues of λ are the same as those of the U(1) generator for the 16 dimensional spinorial representation in which the quarks and leptons are classified. This does not necessarily require that the operator as defined by Eq.(11a) should also be equivalent to this U(1) generator for other representations in which Higgs or technicolor particles might be classified.

We thus see an underlying SU(5)-like structure in the quantum numbers of the existing particles. This seems to go beyond the well known result that they just "happen to fit" into two complete irreducible representations of SU(5) and their conjugates. There are also SU(5)-like properties of the four-point S-matrix as well. The tantalizing question still remains whether these properties indicate a basic underlying symmetry or merely an accidental symmetry like flavor SU(3).

The first indication that all was not well with the flavor SU(3) gauge theory was ω-ϕ mixing. Two inequivalent representations of SU(3) were degenerate for reasons completely outside SU(3) and mixed badly to give states far from SU(3) eigenstates. This should not happen in the gauge theory of the world. Such mixing can test any higher symmetry, but cannot yet test SU(5) because there are no pairs of particles like ω and ϕ classified in inequivalent representations of the symmetry group and allowed to mix by the existing conservation laws. For SU(5) two inequivalent representations are needed with the same pentality. The The observed quarks and leptons are classified in the 5, 5*, 10 and 10* representations of SU(5) which all have different values of pentality and whose states cannot mix without violating the conservation laws for the known additive quantum numbers of electric charge, weak isospin and color.

The absence of a pair of states like ω-ϕ which can mix makes unambiguous tests of SU(5) very difficult and explains why so many SU(5)-like results are obtainable without SU(5). Since SU(5) is of rank 4, the same as SU(3) x SU(2) x U(1), all the additive conserved quantum numbers in SU(5) are already in SU(3) x SU(2) x U(1). Thus imposing SU(5) invariance can give no new selection rules based on additive quantum numbers. Since pentality is determined by the eigenvalues of the additive quantum numbers, the pentality selection rule is already implied by SU(3) x SU(2) x U(1). Non-trivial selection rules based on the non-Abelian quantum numbers of SU(5) are possible only when representations other than 5, 5*, 10 and 10* are present, so that couplings exist which are forbidden by SU(5) but allowed by pentality, and mixing can occur. As long as no other representations of SU(5) are present, the only kinds of SU(5) predictions which are not already present without SU(5) are relations between processes like (7a) and (7b) which depend upon SU(5) Clebsches. These are not easily tested.

These results are easily extended to treat any n-point function. A detailed analysis is given in Ref.5.

We now attempt to understand these results more clearly by looking for an even simpler derivation. The necessity for B violation and the mysterious conservation of B-L has been explained by a combination of SU(2) x U(1) selection rules, the classification of the known particles and the limitations of four point functions. We simplify this argument further and show that the selection rules (2c) and (2d) which led to the results for B-L conservation and B violation follow from the simple selection rule that the total number of particles with half-integral weak isospin can only change by an even number in a transition which conserves weak isospin.[6] This selection rule can be written

$$\delta(N_{\frac{1}{2}}+\bar{N}_{\frac{1}{2}}) = 2n \tag{12a}$$

where $N_{\frac{1}{2}}$ and $\bar{N}_{\frac{1}{2}}$ are the total number of particles and of antiparticles respectively with isospin $\frac{1}{2}$. Since $2\bar{N}_{\frac{1}{2}}$ is manifestly an even integer, the selection rule (12a) is equivalent to

$$\delta(N_{\frac{1}{2}}-\bar{N}_{\frac{1}{2}}) = 2n \ . \tag{12b}$$

In the standard models for quarks and leptons, all left-handed particles and right-handed antiparticles have weak isospin ½ and all right-handed particles and left-handed antiparticles have weak isospin zero. Thus the selection rule (12b) can be rewritten in terms of the numbers N_R, N_L, $\bar{N}_R$ and $\bar{N}_L$ of right-handed and left-handed particles and antiparticles.

$$\delta(N_L - \bar{N}_R) = 2n \ . \tag{13}$$

If the only relevant particles are quarks with $B = 1/3$, $L = 0$ and leptons with $B = 0$, $L = 1$,

$$N_L - \bar{N}_R = (3B + L - H)/2 \tag{14}$$

where H is defined to be $+1$ for a right-handed particle and -1 for left-handed.

Substituting Eq.(14) into Eq.(13) gives

$$\delta(3B + L - H)/2 = 2n \ . \tag{15a}$$

Since $2\delta B$ and $-\delta H$ are both even numbers, they can be subtracted from the selection rule (15a) to give

$$\delta(B - L - H) = 4n = \delta\kappa - 2\delta(B-L) \tag{15b}$$

This selection rule can also be states as the conservation of BL-parity[6] defined by

$$\pi_{BL} = (-1)^{(B-L-H)/2} \ . \tag{15c}$$

Since $\delta(B-L)$ is an even integer, the selection rules (15b) and (2b) are equivalent. Thus all results obtained from (2b) follow from the simple assumption (12a).

The conservation of B-L and nonconservation of B are seen to result from the weak isospin selection rule (12a) and the standard weak isospin classification of quarks and leptons. These results hold in any model for four-point functions mediated by gauge bosons if weak isospin is conserved. For other n-point functions the selection rules (2c) and (2d) or the equivalent conservation of

π_{BL} or the selection rule (15b) hold in all transitions which conserve weak isospin. For example, the six-point function needed to change a neutron into an antineutron for neutron oscillations[7] has $\delta(B-L) = 2$ and must have an odd number of helicity flips to conserve weak isospin. This transition is therefore forbidden if there are only vector boson exchanges which conserve H. In models with vector bosons and Higgs scalars, the six-point function describing $n \to \bar{n}$ must have an odd number of Higgs-fermion vertices; i.e. there must be at least one Higgs exchange between a fermion and a boson.

The results are easily generalized if new kinds of particles are introduced which have half-integral isospin. Eqs.(13) and (14) can be extended to include these new particles to give new selection rules for processes in which these particles are created or absorbed but weak isospin is still conserved.

New bosons such as gauge vector or Higgs bosons can also be included by giving them the B, L and H quantum numbers defined by any fermion pair state to which they are coupled. If B, L and H are not conserved, these quantum numbers may not be unique and can depend upon which fermion pair state is chosen. However, the value of π_{BL} and the selection rule (15) will be independent of the particular classification if Q and I_3 are conserved in all couplings of these bosons.

The selection rule (15b) will be violated if either the conservation of weak isospin or the conventional classification breaks down. This can occur, for example, if:

1. Dynamical mechanisms are introduced which violate weak isospin conservation. Mass terms which couple left and right-handed states are an example of such mechanisms. However, these must occur non-trivially within the n-point function, not only as corrections to give finite masses to external particles. Furthermore, the weak isospin breaking must give rise to transitions with $\delta I = 1/2$ in order to violate the selection rule (12) and its consequence (15b). Breaking described by integral δI does not affect

these conclusions; e.g. giving unequal masses to the W and Z bosons. Mass terms are generally small in these models, since particle masses are small on the scale of the grand unification mass.

2. New exotic external particles are introduced with the opposite correlation between helicity and weak isospin; e.g. a left-handed quark or lepton which is a weak isospin singlet. However, the above derivation breaks down only for transitions where such particles appear explicitly as external particles, not in loops which conserve weak isospin.

REFERENCES

1. H. L. Georgi and S. L. Glashow, Phys. Rev. Letters 32, 438 (1974).
2. S. L. Glashow, Harvard Preprint HUTP-79/A059.
3. S. Weinberg, Phys. Rev. Letters 43, 1566 (1979).
4. F. Wilczek and A. Zee, Phys. Rev. Letters 43, 1571 (1979) and Physics Letters 88B, 311 (1979).
5. H. J. Lipkin, Fermilab Preprint Pub-80/37-THY, to be published in Nuclear Physics B.
6. H. J. Lipkin, Phys. Rev. Letters 45, 311 (1980).
7. R. E. Marshak and R. N. Mohapatra, Physics Letters 91B, 222 (1980); Phys. Rev. Letters 44, 1316 (1980).