Fermi National Accelerator Laboratory

FERMILAB-Conf-80/75-THY
August 1980

INVERSE SCATTERING AND THE Υ FAMILY

C. Quigg
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

Jonathan L. Rosner[+]
School of Physics and Astronomy, Univ. of Minnesota, Minneapolis, Mn 55455

ABSTRACT

A quarkonium potential is constructed with the help of masses and leptonic widths of the Υ(1S-4S) levels using the inverse scattering formalism. This potential agrees at all interquark separations beyond 0.06f with one constructed earlier from ψ and ψ', providing further evidence for flavor independence of the $Q\bar{Q}$ interaction. Comparison with other a priori potentials suggests that tests for a short-range Coulomb interaction (as predicted by QCD) will have to rely primarily on more precise values for $\Gamma(\Upsilon \to e^+e^-)$, on measurement of the 2S-2P spacing (predicted to be about 120 MeV for a short-range Coulomb-like interaction or in the inverse scattering formalism but about 150 MeV for an effective power-law potential), and on the discovery of heavier quarks.

INTRODUCTION

The nonrelativistic quark model, with Schrödinger dynamics, has some promise of describing the Υ family of resonances (reviewed in Ref. 1). A natural question is the form of the interquark interaction. The predictions of specific potentials with various degrees of theoretical motivation may be compared with spectroscopic data.[2-4] Scaling methods can show if a potential behaves like a power law $V(r) = A + Br^{\nu}$ in the region of interest.[5] The potential can be constructed directly from masses and leptonic widths with the help of the inverse scattering formalism.[6-8] A further step in this direction is described here.

We shall use the most recent parameters[1] of Υ(1S-4S) to construct a quarkonium potential which turns out remarkably similar to one constructed earlier from charmonium data.[6] Evidence for flavor independence of the quark-antiquark interaction is thereby extended beyond the range investigated earlier.[7] The predictions of this potential are compared with those of other a priori potentials.[2-4] It is found that most properties of the charmonium and Υ systems are sufficiently similar that tests for a QCD-motivated short-distance Coulomb singularity must be selected with some care. The masses of the lowest P-wave $b\bar{b}$ states, the leptonic width of the 1S Υ level, and properties of quarkonia heavier than the Υ family provide means for distinction among various potentials.

[+]Work supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-1764.

Presented at parallel session C7, 20th Int. Conf. on High Energy Physics, Madison, Wis., July 17-23, 1980.

METHODS AND RESULTS

A one-dimensional potential with a set of levels E_i may be constructed as follows. Choose an "ionization point" E_0 not far above the last level. If E_0 is chosen too high, a potential will be constructed with a large "energy gap"; it will consist of multiple buckets just as a solid (with energy gaps) also has multiple sites of attraction. Examples of this behavior have been demonstrated in some detail.[9] Let μ denote the reduced mass, and define

$$K_i = [2\mu\,(E_0 - E_i)]^{1/2} \tag{1}$$

Then there is a unique symmetric, reflectionless potential $V(E_0, \{K_i\}, x)$, approaching E_0 at $x = \pm\infty$, with just the indicated bound states.

The potential is required to be reflectionless to avoid the need for phase-shift information in the inverse scattering formalism.[6,10] (We don't have quark-antiquark phase shifts.) Moreover the inverse problem for reflectionless potentials is an algebraic one, while for any other situation it involves the solution of an integral equation.

The potential is taken to be symmetric with an eye to the three-dimensional S-wave problem. For $\ell = 0$, the Schrödinger equation

$$\left[- \frac{\nabla^2}{2\mu} + V(r) \right] \Psi(\vec{r}) = E \;\Psi(\vec{r}) \tag{2}$$

becomes

$$\left[- \frac{1}{2\mu} \frac{d^2}{dr^2} + V(r) \right] [r\,\Psi(r)] = E[r\,\Psi(r)], \tag{3}$$

a one-dimensional Schrödinger equation for $r\,\Psi(r)$. Note that $r\Psi(r)$ vanishes at r=0. Hence it can be viewed as an odd-parity eigenfunction $\psi_{2n}(r) = r\,\Psi_n(r)$ $(n=1,2,\ldots)$ in a symmetric potential $V(-r) = V(r)$. Here $n-1$ labels the number of nodes between r=0 and $r=\infty$. The corresponding quarkonium masses may be identified with the odd-parity energy levels: $M(nS) = E_{2n}(n=1,2,\ldots)$.

Let us solve the problem of N quarkonium levels. This leaves us with a collection of N unphysical levels $E_1, E_3, \ldots, E_{2N-1}$ and their corresponding even-parity wave functions $\psi_1, \psi_3, \ldots, \psi_{2N-1}$. In the absence of information about $E_1, E_3, \ldots$, we can specify instead the set of N values

$$|\psi'_{2n}(0)|^2 = |\Psi_n(0)|^2 = (16\,\alpha^2 e_Q{}^2)^{-1}(M(nS))^2\,\Gamma(nS \to e^+e^-) \qquad (n=1,\ldots,N), \tag{4}$$

where the last equality is from Ref. 11. These are related to the $\{E_i\}$ by simple combinatorial identities.[8] For example, when N=1, we have

$$|\psi_2'(0)|^2 = \frac{K_2}{2} \,|K_1^2 - K_2^2| \;, \tag{5}$$

which may be used to solve for K_1.

The quarkonium information we use[1] is summarized in Table 1.

Table 1. Properties of $^3S_1$ quarkonium levels.

$c\bar{c}$ level	Γ_{ee}, keV	$b\bar{b}$ level	Γ_{ee}
		$\Upsilon_1(9.46)$	Γ_1 = 1.0 - 1.3 keV
$\psi(3.095)$	4.8 ± 0.6	$\Upsilon_2(10.02)$	(0.45 ± 0.07) Γ_1
		$\Upsilon_3(10.35)$	(0.32 ± 0.06) Γ_1
$\psi(3.684)$	2.1 ± 0.3	$\Upsilon_4(10.57)$	(0.25 ± 0.05) Γ_1

The ψ and ψ' information were used in Ref. 6 to construct a potential which gave the correct value of the $c\bar{c}$ spin-averaged P wave mass $\bar{\chi}$ = 3.52 GeV, the correct Υ - Υ' spacing (0.56 GeV), and the correct Υ, Υ' leptonic widths. This was achieved with the choice E_0 = 3.8 GeV, m_c = 1.1 GeV. The potential is shown as the solid line in Fig. 1.

Previously[7] we used Υ, Υ' data to construct a potential which agreed with the solid curve in Fig. 1 out to the classical turning point of the Υ', but leveled off at 10.1 GeV for larger r. With four Υ levels it is now possible to extend the comparison to larger interquark separations. The results are the dashed and dotted curves in Fig. 1. The agreement with the charmonium potential is excellent. At very small distances ($r < 0.3\ \mathrm{GeV}^{-1}$ = 0.06 fm) there is some sensitivity to the exact value of $\Gamma(\Upsilon \to e^+e^-)$. We expect that if the potential really has a short-distance Coulomb singularity its reconstruction using Υ levels will be deeper than that based on the ψ levels, since the heavier quarks in the Υ can probe shorter distances.

A check has been performed on the Υ potentials to see if they can provide the correct charmonium level spacings. The result of solving the Schrödinger equation is $\psi'-\psi$ = (576, 574) MeV, ψ' - χ = (154, 152) MeV for $\Gamma(\Upsilon \to e^+e^-)$ = (1,1.25)keV.

The Schrödinger equation also can be solved for other $b\bar{b}$ (P-wave and D-wave) levels. The results are shown as the solid lines in Fig. 2. Also shown are the predictions of a logarithmic potential[3] and of a QCD-inspired potential[4] whose form in momentum space is

$$V(\vec{q}^{\,2}) = -\frac{4}{3}\alpha_s(\vec{q}^{\,2})\frac{4\pi}{\vec{q}^{\,2}} , \qquad (7)$$

$$\alpha_s^{-1}(\vec{q}^{\,2}) = \frac{11-\frac{2}{3}n_f}{4\pi}\ \ln\left(1+\frac{\vec{q}^{\,2}}{\Lambda^2}\right) , \qquad (8)$$

where a fit to charmonium gave Λ = 398 MeV.

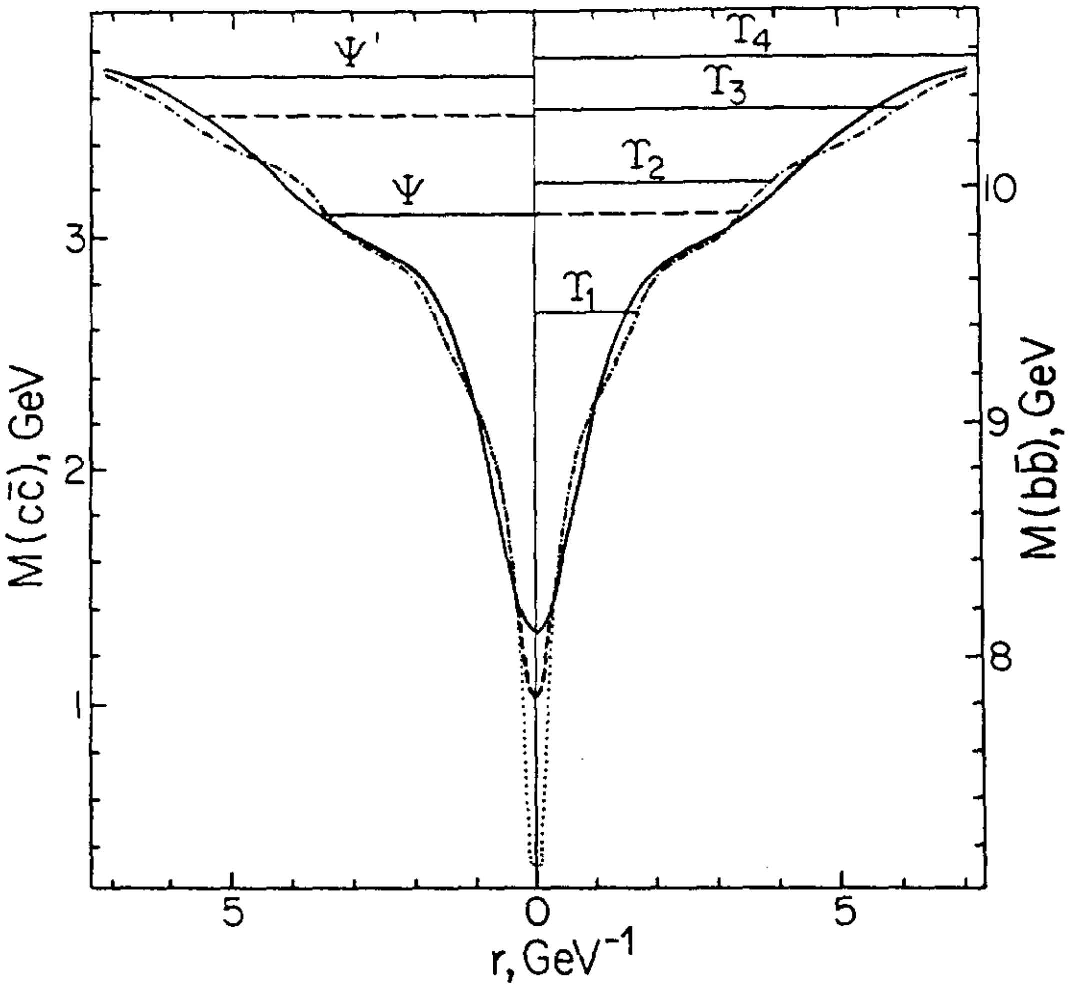

Fig. 1. Quarkonium potentials constructed by the inverse scattering method. Solid curve based on ψ, ψ' with E_0 = 3.8 GeV, m_c = 1.1 GeV. Dashed curve ($\Gamma(\Upsilon \to e^+e^-)$ = 1 keV) and dotted curve ($\Gamma(\Upsilon \to e^+e^-)_2$ = 1.25 keV) based on $\Upsilon_1, \ldots, \Upsilon_4$ with E_0 = 10.6 GeV, m_b = 4.5 GeV/c^2. Horizontal lines denote S wave levels (solid) and 2P levels (dashed) for charmonium (left) and upsilons (right).

The differences are not great. The Richardson potential is too "stiff" at large distances (as it is for charmonium) but this is not of serious concern to us. More important is the excess of predicted with respect to observed leptonic widths in the Richardson potential by factors of 1-1/2 to 2.[12] This would be evidence for important gluonic radiative corrections in the van Royen-Weisskopf formula (4). The expected form of these corrections if Coulomb binding dominates is

$$\Gamma_{ee} \sim |\Psi(0)|^2_{N.R.} \left[1 - \frac{16\alpha_s}{3\pi} \right], \qquad (9)$$

where N.R. denotes the non-relativistic limit, but opinions differ on

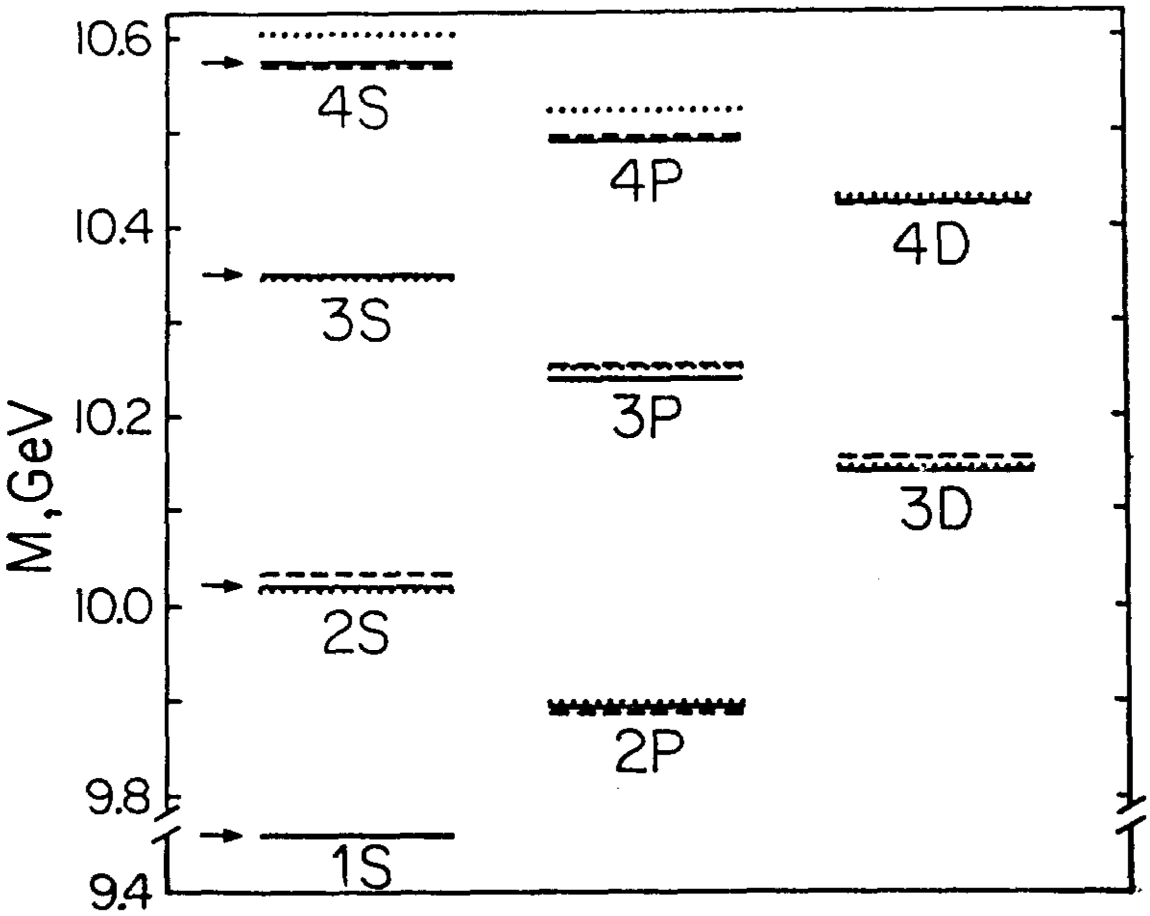

Fig. 2. Positions of S-, P-, and D-wave $b\bar{b}$ levels in several potentials. Solid lines: potentials constructed via inverse scattering (see Fig. 1). The average of predictions based on Γ_{ee} = 1.0, 1.25 keV is shown. Dashed lines: potential $V(r) = (0.715\ \mathrm{GeV})\ \ln(r/r_0)$. Dotted lines: Richardson potential (Eqs. (7), (8)). Arrows indicate positions of observed S wave Υ levels. The DESY mass scale (Υ = 9.46 GeV) is used.

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the magnitude of this correction in the presence of a confining potential.[13]

The most important distinction among the predictions of Fig. 2 is the 2S-2P spacing:

$$2S - 2P = \begin{cases} 120\text{-}130\ \mathrm{MeV}\,(\text{inverse}) \\ 150\ \mathrm{MeV}\ (\ V \sim \ln r) \\ 120\ \mathrm{MeV}\ (\ \text{Eqs. (7), (8)}) \end{cases} \qquad (10)$$

These differences may be crucial in observing monochromatic photons in $\Upsilon' \to \gamma\chi_b$. Even a smaller 2S-2P spacing is favored by a Coulomb + linear potential[2] in which the Coulomb strength does not decrease logarithmically at short distances.

We close by stressing the role of heavier quarks in deciding among potentials (e.g., $V \sim \ln r$ or Eqs. (7), (8).) For the former, one expects $\Gamma(nS \to e^+e^-) \sim m_Q^{-1/2} n^{-1}$. For a potential[14] not unlike (7), (8), the $m_Q^{-1/2}$ factor is absent but otherwise the leptonic widths behave very similarly. It is the overall scale of the leptonic widths (in particular $\Gamma(1S \to e^+e^-)$) that probes the deepest part of the quark-antiquark potential for any given quark mass.

CONCLUSIONS

We have constructed a quarkonium potential from four Υ levels using inverse scattering. A previous test of flavor independence of the interaction has been extended to larger interquark separations. The depth of this potential at very small distances is all that depends strongly on the absolute scale of Υ leptonic widths; otherwise the agreement with the charmonium potential is remarkable. Attention has been drawn to the importance of 1) $\Gamma(\Upsilon \to e^+e^-)$, 2) $M(\Upsilon') - M(\chi_b)$, and 3) leptonic widths of heavier quarkonia in distinguishing among various potentials.

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