Fermi National Accelerator Laboratory

FERMILAB-Conf-80/73-THY
August 1980

QUANTUM CHROMODYNAMICS AND DEEP-INELASTIC SCATTERING†

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ABSTRACT

Moments of deep-inelastic structure functions, parton distributions and parton fragmentation functions are discussed in the context of Quantum Chromodynamics with particular emphasis put on higher order corrections. A brief discussion of higher twist contributions is also given.

INTRODUCTION

It is an experimental fact* that the deep-inelastic structure functions depend on $x = (Q^2)/2\nu$ and Q^2; i.e., Bjorken scaling is violated. Quantum chromodynamics (QCD) predicts scaling violations in deep-inelastic scattering** with the pattern consistent with the experimental findings. In comparing QCD predictions with the experimental data one can either work with the moments of structure functions or with the structure functions themselves. Quite gener lly QCD predictions for the moments of the simplest (non-singlet, NS) structure functions can be written as follows

$$M_n^{NS}(Q^2) = \int_0^1 dx\, x^{n-2} F^{NS}(x,Q^2)$$

$$= \sum_{\substack{t=2 \\ \text{even}}} \frac{A_n^{(t)}}{[Q^2]^{t-2}} \left[\alpha(Q^2)\right]^{d_n^{(t)}} \left[1 + R_n^{(t)} \frac{\alpha(Q^2)}{4\pi} + \ldots\right] \qquad (1)$$

'ere the sum runs over various twist ($\underline{t}$) contributions: Leading twist (t=2), twist four (t=4) and so on. Furthermore, $\alpha(Q^2)$ is the effective strong interaction coupling constant and $d_n^{(t)}$, $R_n^{(t)}$ and $A_n^{(t)}$ are numbers to be discussed below.

The expressions like (1) are rather formal and it is often convenient to cast them in a form of parton model formulas in which case the basic elements are the effective Q^2 dependent parton distributions and elementary parton cross-sections. In the case of semi-inclusive deep-inelastic scattering also the concept of the effective Q^2 dependent fragmentation functions is introduced.

*See the contributions to the Session "eN, μN and νN Interactions" and references therein.

**For recent reviews see refs. 1 and 2.

†Invited talk presented at the XXth International Conference on High Energy Physics, Madison, Wisconsin - July, 1980.

Here we shall discuss three topics:

i) Moments of the structure functions in the leading twist approximation ($t=2$ in Eq. 1) and with the next to leading order corrections taken into account ($R_n^{(2)}$ in Eq. 1).

ii) Parton distributions and parton fragmentation functions beyond the leading order in $\alpha(Q^2)$.

iii) Higher Twist ($t>2$) Contributions.

Our discussion includes the latest developments as well as some results obtained by various authors since the Tokyo Conference.

BASIC FORMULAE

In QCD and in the leading twist approximation the moments of any structure function are given as follows

$$M_n(Q^2) \equiv \int_0^1 dx\ x^{n-2} F(x,Q^2) = \sum_{i=NS,S,G} A_n^i(\mu^2) C_n^i\left(\frac{Q^2}{\mu^2},g^2\right) \quad . \quad (2)$$

Here $A_n^i(\mu^2)$ are the hadronic matrix elements of non-singlet (NS), singlet (S) and gluon (G) operators and C_n^i are the corresponding coefficient functions in the Wilson operator product expansion. Furthermore g is the renormalized quark-gluon coupling constant and μ^2 is the subtraction scale at which the theory is renormalized. The important property of Eq. (2) is the factorization of non-perturbative pieces $A_n^i(\mu^2)$ from perturbatively calculable coefficient functions $C_n^i(Q^2/\mu^2,g^2)$.

Specializing Eq. (2) to non-singlet structure functions, and using renormalization group equations for $C_n^{NS}(Q^2/\mu^2,g^2)$ one obtains

$$M_n^{NS}(Q^2) = A_n^{NS}(\mu^2)\exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} dg' \frac{\gamma_n^{NS}(g')}{\beta(g')}\right]\cdot C_n^{NS}(1,\bar{g}^2(Q^2)) \quad (3)$$

$$= A_n^{NS}(\mu^2)\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(\mu^2)}\right]^{d_n^{NS}}\left[1+\frac{\bar{g}^2(Q^2)-\bar{g}^2(\mu^2)}{16\pi^2} Z_n^{NS}\right]\cdot\left[1+\frac{\bar{g}^2(Q^2)}{16\pi^2}B_n^{NS}\right] \quad (4)$$

where terms of order $\bar{g}^4$ have been neglected.

Furthermore

$$d_n^{NS} = \frac{\gamma_n^0}{2\beta_0} \quad ; \quad Z_n^{NS} = \frac{\gamma_n^{(1)}}{2\beta_0} - \frac{\gamma_n^0}{2\beta_0^2}\beta_1 \quad , \quad (5)$$

In obtaining Eqs. (4) and (5) the following expansions for the anomalous dimensions (γ_n^{NS}), β functions and the coefficient function $C_n^{NS}(1,\bar{g}^2)$ have been used:

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$$= A_n^{NS}(\mu^2)\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(\mu^2)}\right]^{d_n^{NS}} \left[1 + \frac{\bar{g}^2(Q^2) - \bar{g}^2(\mu^2)}{16\pi^2} Z_n^{NS}\right] \cdot \left[1 + \frac{\bar{g}^2(Q^2)}{16\pi^2} B_n^{NS}\right] \quad (4)$$

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In obtaining Eqs. (4) and (5) the following expansions for the anomalous dimensions (γ_n^{NS}), β functions and the coefficient function $C_n^{NS}(1, \bar{g}^2)$ have been used:

$$\gamma_n^{NS}(\bar{g}) = \gamma_n^0 \frac{\bar{g}^2}{16\pi^2} + \gamma_n^{(1)} \frac{\bar{g}^4}{(16\pi^2)^2} \quad , \tag{6}$$

$$\beta(\bar{g}) = -\beta_0 \frac{\bar{g}^3}{16\pi^2} - \beta_1 \frac{\bar{g}^5}{(16\pi^2)^2} \quad , \tag{7}$$

and

$$C_n^{NS}(1,\bar{g}^2) = 1 + \frac{\bar{g}^2}{16\pi^2} B_n^{NS} \quad . \tag{8}$$

Finally the Q^2 evolution of $\bar{g}^2(Q^2)$ is given as follows

$$\frac{\bar{g}^2(Q^2)}{16\pi^2} \equiv \frac{\alpha(Q^2)}{4\pi} = \frac{\left[1-(\beta_1/\beta_0^2)\ln\ln(Q^2/\Lambda^2)/\ln(Q^2/\Lambda^2)\right]}{\beta_0 \ln(Q^2/\Lambda^2)} \tag{9}$$

with Λ being the famous QCD scale parameter. The parameters γ_n^0, $\gamma_n^{(1)}$, β_0, β_1 and B_n^{NS} have been calculated by at least two groups: in refs. 3 and 4, 5 and 6, 7 and 8, 9 and 10, and 11 and 12, respectively.

To proceed further one can use either formal approach or intuitive approach.

In the formal approach one proceeds as follows. Since the left-hand side of Eq. (4) does not depend on μ^2 the r.h.s. of this equation can be put in the following form

$$M_n^{NS}(Q^2) = A_n^{NS}\left[\alpha(Q^2)\right]^{d_n^{NS}} \left[1 + \frac{\alpha(Q^2)}{4\pi} R_n^{NS}\right] \quad , \tag{10}$$

with

$$R_n^{NS} = Z_n^{NS} + B_n^{NS} \quad . \tag{11}$$

and A_n^{NS} being independent of μ^2. Note that Eq. (10) represents just the twist two (t=2) contribution to Eq. (1).

In the intuitive approach setting $\mu^2=Q^2$ in Eq. (4) one obtains

$$M_n^{NS}(Q^2) = q_n^{NS}(Q^2) \cdot \sigma_n^{NS}(\bar{g}^2) \quad . \tag{12}$$

Here

$$q_n^{NS}(Q^2) \equiv A_n^{NS}(Q^2) = A_n^{NS}(\mu^2)\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(\mu^2)}\right]^{d_n^{NS}}\left[1 + \frac{\bar{g}^2(Q^2)-\bar{g}^2(\mu^2)}{16\pi^2} Z_n^{NS}\right] \quad (13)$$

can be interpreted as the moments of an effective, Q^2 dependent non-singlet parton distribution $q^{NS}(x,Q^2)$ (e.g., valence quark distribution), and

$$\sigma_n^{NS}(\bar{g}^2) = 1 + \frac{\bar{g}^2(Q^2)}{16\pi^2} B_n^{NS} \quad (14)$$

may be regarded as the elementary parton cross-section. We shall now discuss these two approaches in more detail.

FORMAL APPROACH

We just list the most important properties of Eqs. 10 and 11.

1) $\gamma_n^{(1)}$ and B_n^{NS} depend on the renormalization scheme[5] used to calculate these quantities. This renormaliation prescription dependence of $\gamma_n^{(1)}$ and B_n^{NS} cancels in Eq. 11) if these quantities are calculated in the same scheme; i.e., the combination

$$\frac{\gamma_n^{(1)}}{2\beta_0} + B_n^{NS} \quad (15)$$

is renormalization prescription independent.

2) The parameters R_n^{NS} depend on the definition of $\alpha(Q^2)$.[13,11] If $\alpha(Q^2)$ is redefined to $\alpha'(Q^2)$ with

$$\alpha(Q^2) = \alpha'(Q^2) + r\left[\alpha'(Q^2)\right]^2 \qquad r - \text{const.} \quad (16)$$

then the expansion parameters in R_n^{NS} in Eq. (10) are changed to

$$\left[R_n^{NS}\right]' = R_n^{NS} + 4\pi r d_n^{NS} \quad . \quad (17)$$

Of course the final answer for $M_n^{NS}(Q^2)$ is independent of the definition of $\alpha(Q^2)$ since each change of the expansion parameters R_n^{NS} is compensated by the corresponding change of the values of $\alpha(Q^2)$ or equivalently values of Λ extracted from experiment. This is illustrated by the following example.

3) For the $\overline{MS}$[11] and Momentum Subtraction (MOM)[14] schemes, which have been discussed widely in the literature, Eqs. (16) and (17) read as follows

$$\alpha_{MOM} = \alpha_{\overline{MS}}\left[1 + 1.55\beta_0 \frac{\alpha_{\overline{MS}}}{4\pi}\right] \quad , \qquad (16')$$

and

$$\left[R_n^{NS}\right]_{\overline{MS}} = \left[R_n^{NS}\right]_{MOM} + \beta_0\,[1.55]\,d_n^{NS} \quad . \qquad (17')$$

Numerically we have:

i) $2<\left[R_n^{NS}\right]_{\overline{MS}}<16$ and $-4\leq\left[R_n^{NS}\right]_{MOM}\leq 2$ for $2\leq n\leq 8$ with both $\left[R_n^{NS}\right]_{\overline{MS}}$ and $\left[R_n^{NS}\right]_{MOM}$ increasing monotonically with n.

ii) If $\Lambda_{\overline{MS}}=0.30$ GeV then the MOM scheme with $\Lambda_{MOM}=0.55$ GeV leads to essentially indistinguishable results for $M_n^{NS}(Q^2)$ for $Q^2\geq 10$ GeV^2 The corresponding values of $\alpha(Q^2)$ are $\alpha_{MOM}=0.32$ and $\alpha_{\overline{MS}}=0.24$ at $Q^2=10$ GeV^2.

iii) The quantity $1 + (\alpha(Q^2)/4\pi)R_n^{NS}$ in Eq. (10) varies for $2\leq n\leq 8$ and $Q^2=10\text{GeV}^2$ from 1.05 to 1.31 for $\overline{MS}$ scheme and from 0.92 to 1.08 for MOM scheme. Since in the leading order the quantity in question is equal to 1 we observe that MOM scheme seems to lead to a better expanssion in α that $\overline{MS}$ scheme. An opposite conclusion would be reached in the case of $1/\ln Q^2$ expansion.

4) One may think for a while that there is no point in doing next to leading and higher order calculations since at the end one can anyhow change the size of various terms in the expansion by redefining α. The point is however that by doing consistent higher-order calculations in various processes, such as deep-inelastic scattering, $e^+e^-\to$hadrons, photon-photon scattering etc. one can meaningfully compare QCD effects in these processes using one universal effective coupling constant $\alpha(Q^2)$ extracted, e.g., from deep-inelastic data. By studying higher order corrections to various processes one can find a universal definition of α for which the QCD perturbative expansions are behaving well. Such studies can be found in refs. 15,16,21. One finds that schemes with $\alpha_{\overline{MS}}\leq\alpha_i\leq\alpha_{MOM}$ lead to acceptable expansions. Another method for finding the optimal scheme for α has also been recently suggested.[17]

5) One can study properties of R_n which are independent of the definition of α.[18,19] One is so called Λ_n scheme.[13,11,18] Here one rewrites Eq. (10) as follows

$$M_n^{NS}(Q^2) = \tilde{A}_n^{NS}\left[\frac{1}{\ln(Q^2/\Lambda_n^2)}\right]^{d_n^{NS}}\left[\frac{1-(\beta_1/\beta_0^2)\ln\,\ln(Q^2/\Lambda^2)/\ln(Q^2/\Lambda^2)}{\beta_0\ln(Q^2/\Lambda^2)}\right]^{d_n^{NS}} \qquad (18)$$

with

$$\Lambda_n = \Lambda \exp\left[\frac{R_n^{NS}}{2\beta_0 d_n^{NS}}\right] \quad . \tag{19}$$

The n dependence of Λ_n is independent of the definition of α_S(see eq. 17 and 19). Λ_n increases roughly by factor 2 and 3 for F_2^{NS} and F_3 structure functions respectively if n is varied from n=2 to n=8. In the leading order Λ is independent of n. The n dependence of Λ_n as given by Eq. (19) is in a very good agreement[20] with experimental data indicating the importance of next-to-leading-order corrections. Other quantities which are independent of the definition of α can be found in refs. 18 and 19.

6) As already stated in connection with the n dependence of Λ_n, the next-to-leading-order corrections to the Q^2 evolution are different for F_2^{NS} and F_3 structure functions. In the leading order F_2^{NS} and F_3 have the same Q^2 evolution. (R_n^{NS} depends on the structure function considered—see point 15 below.)

7) There are several new effects related to next-to-leading-order corrections. Most of them are small. They are summarized in ref. 21.

This completes the listing of the main properties of Eq. (10). One should also mention that,

8) The next-to-leading-order corrections to the singlet structure functions are known.[11,12]

9) Some of the next-to-leading-order corrections to deep-inelastic scattering on polarized targets have been calculated in ref. 22.

Finally:

10) It has been suggested in Ref. 23 to use the moments

$$B_{M,N}(Q^2) = \frac{(M+N+1)!}{M!N!}\int_0^1 x^n(1-x)^M F(x,Q^2)dx \quad , \tag{20}$$

rather than the moments of Eq. (1), which for $N \gtrsim 4$ are mostly sensitive to x>0.5. With increasing M the moments of Eq. (20) become sensitive to small values of x and are particularly well suited for the study of gluon and sea distributions which are concentrated at small values of x.

INTUITIVE APPROACH

11) The novel feature of parton distributions beyond the leading order is that they can be defined in various ways. Two definitions have been discussed in the literature. They are as follows.

<u>Definition A.</u>[24] Moments of parton distributions are defined by the matrix elements of local operators normalized at Q^2. This is

the definition of Eq. (13) which constitutes the Q^2 evolution equation for so defined parton distributions.

Definition B.[25] The full higher order correction to F_2^{NS} is absorbed into the definition of parton distributions. Eqs. (12-14) are replaced by

$$M_n^{NS}(Q^2) = \left[q_n^{NS}(Q^2)\right]' \cdot \left[\sigma_n^{NS}(\bar{g}^2)\right]' \quad , \tag{12'}$$

with

$$\left[q_n^{NS}(Q^2)\right]' = A_n^{NS}(\mu^2)\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(\mu^2)}\right]^{d_n^{NS}}\left[1 + \frac{\bar{g}^2(Q^2)-\bar{g}^2(\mu^2)}{16\pi^2} R_n^{NS}\right], \tag{13'}$$

R_n^{NS} given by Eq. (11), and

$$\left[\sigma_n^{NS}(\bar{g}^2)\right]' = \begin{cases} 1 & \text{for } F_2^{NS} \\ 1 + \dfrac{\bar{g}^2(Q^2)}{16\pi^2}\left[B_n^{NS}\right]_3 - \left[B_n^{NS}\right]_2 & \text{for } F_3 \end{cases} \quad . \tag{14'}$$

It should be remarked that since Z_n^{NS} and B_n^{NS} are separately renormalization prescription dependent so are the parton distributions of def. A. On the other hand the parton distributions defined by (13') are renormalization prescription independent.

12) Also fragmentation functions can be defined in various ways.[26] If we consider the process $e^+e^- \to h$ + anything then the moments (in the z variable) of relevant cross-sections take for the non-singlet contributions the form of Eqs. (12-14) and (12'-14') with the following replacements

$$q_n^{NS}(Q^2) \to D_n^{NS}(Q^2) \quad , \quad A_n^{NS}(\mu^2) \to V_n^{NS}(\mu^2)$$

$$Z_n^{NS} \to \left[Z_n^{NS}\right]_T \quad , \quad B_n^{NS} \to \left[B_n^{NS}\right]_T \quad . \tag{21}$$

Here $D_n^{NS}(Q^2)$ and $V_n^{NS}(\mu^2)$ are the moments of a non-singlet fragmentation function and the time-like cut vertices[27] respectively. Furthermore the index T stands for "time-like". In order to keep uniform notation we shall in the following use $[Z_n^{NS}]_S$, $[B_n^{NS}]_S$ for Z_n^{NS} and B_n^{NS} of Eqs. (13) and (14) with the index "S" standing for "space-like".

13) In the $\overline{MS}$ scheme one finds,[21] for $2\leq n\leq 8$

$$\left[Z_n\right]_{S,T} = \begin{cases} 1.5 - 2.5 & \text{for S} \\ 0.5 - 1.5 & \text{for T} \end{cases} \tag{22}$$

The inequality $\left[Z_n\right]_S \neq \left[Z_n\right]_T$ expresses the violation[6,28] of the Gribov-Lipatov relation beyond the leading order ($\left[\gamma_n^{(1)}\right]_S \neq \left[\gamma_n^{(1)}\right]_T$) as opposed to $\left[\gamma_n^{(0)}\right]_S = \left[\gamma_n^{0}\right]_T$. For large n $\left[Z_n\right]_S \to \left[Z_n\right]_T \sim \ln n$.

Furthermore both $B_{n\ T}$ and $B_{n\ S}$ increase with n as $(\ln n)^2$ with[6,26,29]

$$\left[B_n\right]_T \underset{\text{Large } n}{\approx} \left[B_n\right]_S + \frac{8}{3}\pi^2 \tag{23}$$

We are now in a position to compare both definitions of parton distributions and fragmentation functions.

14) i) Evolution Equations for parton distributions and parton fragmenation functions are essentially the same in the case of the def. A and are different in the case of def. B due to the substantial difference in the values of $\left[B_n\right]_S$ and $\left[B_n\right]_T$.

ii) Furthermore the evolution equations in question are in the case of def. A essentially the same as leading order equations ($\left[Z_n\right]_T, \left[Z_n\right]_S$ are small) except for the modified evolution of the effective coupling constant (see Eq. 9). The evolution equations in the case of def. B differ substantially at large n (Large x) from the corresponding leading order equations due to the large values of $\left[B_n\right]_S$ and $\left[B_n\right]_T$ at large n and due to the non-trivial behavior $(\ln n)^2$ of these parameters.

iii) Whereas the input distributions or structure functions ($A_n^{NS}(\mu^2=Q_0^2)$) at some $Q^2=Q_0^2$ in the case of the def. B will be for F_2^{NS} the same as in the leading order (i.e., the data does not change) the input distribution in the def. A will differ considerably at low Q^2 and large x from those used in the leading order phenomenology. The reason is that B_n's differ considerably from 1 for low Q^2 and large n.

Of course the final results for the structure functions should be independent of any particular definition since the differences in the parton distributions and parton fragmentation functions are compensated by the corresponding differences in the parton cross-sections. A detailed study of the effects discussed here has been done in ref. 30. It turns out that in the range $5\leq Q^2\leq 200$ GeV^2 and $0.02\leq x\leq 0.8$ one can find simple parametrizations for both definitions of parton distributions which represent to a high accuracy Eqs. (13) and (13'). These parametrizations are of the form of leading order parametrizations of ref. 31, i.e.,

$$x\, q^{NS}(x,Q^2) \sim x^{\eta_1(\bar{s})} (1-x)^{\eta_2(\bar{s})} \tag{24}$$

$$\eta_i(\bar{s}) = \eta_i^{(0)} + \eta_i' \bar{s} \ ; \ \bar{s} = -\ln\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right] \ . \tag{25}$$

In accordance with points ii) and iii) one has

$$\left[\eta_i^0\right]_{LO} = \left[\eta_i^0\right]_B \neq \left[\eta^0{}_i\right]_A \tag{26}$$

$$\left[\eta_i'\right]_{LO} \approx \left[\eta_i'\right]_A \neq \left[\eta_i'\right]_B$$

where the indices LO, A, B stand for leading order, definition A and definition B, respectively. For instance $[\eta_2^0]_B=2.71$ and $[\eta_2^0]_A=3.40$ whereas $[\eta_2']_A=0.76$ and $[\eta_2']_B=1.5$.

15) On the level of structure functions themselves two main properties of the next-to-leading-order corrections are worthwhile mentioning

i) If $\Lambda_{\overline{MS}}$ is chosen so that $\Lambda_{\overline{MS}}=\Lambda_{LO}$, where Λ_{LO} is the scale in the leading order expression($B_n^{NS}=0, Z_n^{NS}=0$) then a stronger increase (decrease) of structure functions at small (large) values of x is predicted by next-to-leading-order corrections relative to LO predications. If $\Lambda_{\overline{MS}}$ is decreased so that scaling violations for x>0.4 are similar to those predicted by leading order formulae still some additional increase due to next to leading order corrections is seen at small x.

ii) This increase at small x is more pronounced for F_3 then F_2^{NS}.[30]

There is some indication that this additional increase in F_3 at small x has been seen in the data.[32] Detailed comparison should however also include charm production effects[33] in F_3 which are of order $O\left((m_c^2-m_s^2)/Q^2\right)$ with m_c and m_s being charm and strange quark mass, respectively.

It should also be remarked that in refs. 20, 34 fits of structure functions to the data have been made with the general conclusion that the next-to-leading-order corrections improve the agreement of QCD with the data.

Final message to our experimental colleagues: In ref. 20,30,34 and 35 simple inversion methods of moments of structure functions or parton distributions have been developed. Therefore, the analysis of structure functions beyond the leading order should be now as easy as in the leading order.

HIGHER TWISTS

At low values of Q^2 one has to worry in addition to logarithmic scaling violations about power-like scaling violations. In QCD they are represented by higher twist contributions; the terms in Eq. (1) with t>2. Let us summarize what is known at present about these contributions

16) There are many operators of a given twist>2 contributing to Eq. (1) and consequently there are many unknown non-perturbative parameters $A_n^{(t)}$ (t>2) which have to be extracted from the data. This makes the phenomenology of higher twist contributions very complicated.[36] The situation might be considerably simplified in certain regions of phase-space (e.g., $x \to 1$) and for particular cross-sections in which case one can identify and calculate the dominant higher-twist contributions.[37]

17) The anomalous dimensions of some of the twist four (t=4) operators have been calculated in ref. 38. The two novel features as compared with $d_n^{(2)}$ are as follows. $d_n^{(4)}$ can be negative as opposed to $d_n^{(2)} \geq 0$. Furthermore, whereas $d_n^{(2)} \sim \ln n$ for large n. The $d_n^{(4)}$ may increase linearly with n. These two features indicate that the structure of logarithmic corrections to higher twist contributions might be much more complicated than in the case of the leading twist. It would be interesting to study numerically these effects.

18) Phenomenologically one can study the effects of higher order twist contributions in deep-inelastic scattering by using "QCD motivated" parametrizations of the terms t>2 in Eq. (1). Such an analysis has been done one year ago by Abbott and Barnett[39] who found that the deep-inelastic data can be fit by higher twist contributions alone. Their combined analysis of twist 2 and higher twist contributions indicated that the value of the parameter Λ is strongly dependent on the size of higher twist contributions. If the latter increase the Λ decreases. Recent analyses of Duke and Roberts[40] and Pennington and Ross[41] who combine all the existing data show however that the best fits to the data can be obtained if the higher twist contributions are small. Similar conclusion has been reached in ref. 42. Even if higher twist contributions may appear to be of little importance in the analysis of deep-inelastic structure functions for $Q^2 > 5$ GeV^2 and $x < 0.8$, they may be and they probably are important for $x \to 1$. This appears to be the case as discussed in ref. 37.

SUMMARY

A. Leading twist QCD with next-to-leading-order corrections taken into account is in a good agreement with experimental data for $x \leq 0.8$. However, more phenomenology of next-to-leading-order corrections (in particular for fragmentation functions) is needed. Recall that QCD predicts[29] non-trivial x and z non-factorization in semi-inclusive deep-inelastic scattering. Further tests of these predictions are of interest.

B. For large n or large x the next-to-leading-order corrections are large and still higher order corrections are probably non-negligible. Ways of including these higher order corrections have been suggested.[43]

C. For $x \to 1$, or $z \to 1$ higher twist effects are probably important.[37] These effects can also be important in longitudinal structure functions.[44]

D. Finally there is the outstanding question of calculating the x dependence of structure functions at fixed value of Q^2. This has been addressed in the context of specific models in refs. 42 and 45.

I would like to thank Dennis Duke for discussions.

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