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Introduction to Gauge Theories of the Strong, Weak, and Electromagnetic Interactions

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INTRODUCTION TO GAUGE THEORIES OF THE STRONG, WEAK, AND ELECTROMAGNETIC INTERACTIONS

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PROLOGUE

Arthur Wightman (1968) wrote in the thirteenth edition of the <u>Encyclopaedia Brittanica</u> that "Running through the theoretical speculation since World War II has been the idea that the observed particles are not really elementary but merely the states of some underlying single dynamical system. Such speculations had not led very far by the 1960s." The past dozen years have seen a revolution in the prevailing view of elementary particle physics. We now believe that a fundamental description of subnuclear physics must be based upon the idea that strongly-interacting particles (hadrons) are composed of quarks. Together with leptons, such as the electron and neutrino, quarks seem to be the elementary particles--at least at the present limits of resolution. It also appears that all of the fundamental interactions of the quarks and leptons are consequences of various gauge symmetries and may be attributed to the exchange of vector bosons.

The experimental support for this new point of view is multifarious and impressive, if largely circumstantial. It derives from the taxonomy of hadrons, the evidence for pointlike constituents within hadrons, the discovery of the quasiatomic spectra of the heavy mesons ψ/J and Υ, the successful prediction of charm, and the success of the Weinberg-Salam model with its implication of neutral weak currents. According to optimists, a grand synthesis of the strong,

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weak, and electromagnetic interactions is already at hand. A number of experiments are being mounted to search for the proton instability implied by specific "grand unified" theories. Already the first steps are being taken toward a super unification that incorporates gravitation.

These lectures are intended to provide an elementary introduction to the main ideas and consequences of gauge theories of the fundamental interactions. By elementary I mean that no great facility with the subtleties of field theory will be presupposed. Most of the important concepts and many of the experimental applications of gauge theories require no more than the ability to compute simple ("tree-graph") Feynman diagrams. Such computations will be stressed, at least in the Problems, because I believe one cannot possess the subject matter without them. On the other hand, higher-order corrections and the renormalization program will be mentioned only in passing.

In addition to keeping the mathematical level as low-brow as possible, I have tried to emphasize the basic concepts and to keep the organization and logic of the enterprise in plain view. The assumptions leading to the theories of current interest will also be set out in detail. From this approach it is hoped that there will emerge an appreciation of what has been accomplished in present-day theories and a recognition of their shortcomings; a feeling for what is elegant and what is artificial. In the end, of course, we wish to apply these theories not only to experments past, but also to experiments future. Undoubtedly the assiduous student will begin to develop an understanding of the great questions that lie before us and an instinct for incisive experimental thrusts!

The plan of these notes is as follows. Chapter 1 is devoted to a brief evocative review of current beliefs and prejudices that form the context for the discussion to follow. The idea of Gauge Invariance is introduced in Chapter 2, and the connection between conservation laws and symmetries of the Lagrangian is recalled. Non-Abelian gauge field theories are constructed in Chapter 3, by analogy with the familiar case of electromagnetism. The Yang-Mills theory based upon isospin symmetry is constructed explicitly, and the generalization is made to other gauge groups. Chapter 4 is concerned with spontaneous symmetry breaking and the phenomena that occur in the presence or absence of local gauge symmetries. The existence of massless scalar fields (Goldstone particles) and their metamorphosis by means of the Higgs mechanism are illustrated by simple examples. The Weinberg-Salam model is presented in Chapter 5, and a brief résumé of applications to experiment is given. Quantum Chromodynamics, the gauge theory of colored quarks and gluons, is developed in Chapter 6. Asymptotic freedom is derived schematically, and a few simple applications of perturbative QCD are exhibited. Details of the conjectured confinement mechanism are omitted. The strategy of "grand unified" theories of the strong, weak, and electromagnetic interactions is laid out in Chapter 7. Some properties and consequences of the minimal unifying group SU(5) are presented, and the gauge hierarchy problem is introduced in passing. The final chapter contains an essay on the current outlook: aspirations, unanswered questions, and bold scenarios.

Many of the topics addressed here are treated in far greater depth and detail in excellent summer school lectures or review articles, as well as the original literature. References will be made to these at appropriate points. It is my hope that, in addition to providing a self-contained introduction to gauge theories, these notes will make this valuable literature more accessible to the beginner.

1. ARTICLES OF FAITH

Today, many theorists are expressing in forceful terms their optimism that a grand synthesis of natural phenomena is at hand. To these visionaries, a unified description of the strong, weak, and electromagnetic interactions no longer seems a distant dream. Indeed there are those who argue that the unification has already been accomplished in principle, and that only gravitation remains to be incorporated.

What are the reasons for this unbounded confidence? Three important ingredients are the success of the quark model of hadrons, the remarkable triumphs of gauge theories of the weak and electromagnetic interactions, and the nonobservation of free quarks.

The quark model has long been known to provide a systematic basis for hadron spectroscopy. More recently we have come to appreciate the quark-parton picture as a quantitative phenomenology of deeply-inelastic lepton-hadron scattering. In the realm of electron-positron annihilations, the successful predictions of the pointlike character and the magnitude of the cross section for inclusive hadron production and of hadron jets at high c.m. energies have been impressive. Finally, the interpretation of high transverse momentum phenomena in hadron-hadron collisions in terms of hard scattering of pointlike constituents is extremely seductive.

The initial triumph of unified theories of the weak and electromagnetic interactions is aesthetic. In place of the serviceable low-energy phenomenology of the Fermi theory we now have an acceptable field theory which is renormalizable and unitary. The gauge theory solution to the unitarity problems of the Fermi theory is not unique—one can imagine Nature resorting to brute force techniques to enforce unitarity—and it has its price, which is the introduction of several new particles. The minimal (and therefore most appealing) set of hypothetical particles is composed of the intermediate vector bosons W^+, W^-, and Z^o, and a neutral Higgs scalar H, none of which has been observed. However, we do have some circumstantial evidence. The neutral current interactions mediated by Z^o have been found to occur with approximately the strength of the more classical charged current interactions. Their properties match in great detail the predictions of the Weinberg-Salam model.

We have come to terms with the nonobservation of free quarks by postulating that they are perpetually confined within hadrons. No thoroughly convincing mechanism of quark confinement has yet been devised, but it is widely held that quantum chromodynamics (or QCD), a gauge theory of colored quarks and gluons, will provide the solution. Because QCD is an asymptotically free theory, it would answer as well the old question, "How can quarks behave as free when they are bound up in hadrons?" Lastly, the success of the QCD-inspired quarkonium description of the J/ψ and Υ families of heavy mesons adds to the appeal of this confinement scheme.

Let us next briefly survey the fundamental constituents and elementary interactions as we now know them. The purpose of this section is to recapitulate very telegraphically what we think we know and why we believe what we believe.

A. Leptons

The leptons experience the weak and electromagnetic (not to mention gravitational) interactions, but not the strong interactions. All are spin-½ objects that are pointlike, which is to say structureless, at the current limits of resolution. The electron (511 keV/c^2), muon (106 MeV/c^2) and tau (1782 MeV/c^2) are all firmly established, as are the electron's neutrino (< 60 eV/c^2) and the muon's neutrino (< 650 keV/c^2). The tau's neutrino (< 250 MeV/c^2) is presumed to exist, although this has not been demonstrated directly. It is still a logical possibility, though not an appealing one, that $\nu_\tau \equiv \nu_e$.[1] Provided that ν_τ exists as a distinct, sequential lepton, a great deal is known about its interactions from the study of τ-decays. Specifically, it is known to couple left-handedly to τ with strength no less than 1/7 of the universal Fermi coupling. We shall assume that the $\tau - \nu_\tau$ coupling is indeed of universal strength. (One may hope that this will be confirmed in due course by a measurement of the τ lifetime; $\tau(\tau) \simeq 3 \times 10^{-13}$ sec. is expected.) Then the leptonic charged weak current can be described in terms of the weak-isospin doublets

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[1]Preliminary evidence against the $\nu_e - \nu_\tau$ identity has been presented by Hulth (1980).

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

where the subscript L denotes a left-handed, or V-A, structure. In other words, the charged current has the form[2] $\bar{\nu}_e \gamma_\mu (1 - \gamma_5) e$, etc. This structure leads as well to a correct description of the leptonic neutral weak current.

Notwithstanding this orderly pattern, many questions arise. Why are there three doublets of leptons? Will more be found? What is the pattern of lepton masses? Is lepton number conserved absolutely? Are the neutrinos exactly massless? Is the separate conservation of electron-number, muon-number, and tau-number an exact or only approximate statement? Do neutrino oscillations occur in nature?

Several recent experiments and many theoretical speculations bear on the last three points. A measurement of the end of the β-spectrum in tritium decay at the Institute for Theoretical and Experimental Physics in Moscow (Lyubimov, et al., 1980) yields a nonzero value for the mass of the electron's antineutrino: $M(\bar{\nu}_e) = (34.3 \pm 4)\ \mathrm{eV}/c^2$. This very interesting suggestion, which would carry important implications of a cosmological nature, requires independent confirmation. Other observations, including a measurement of $\bar{\nu}_e$-induced disintegration of the deuteron at the Savannah River Reactor (Reines, et al., 1980), have been adduced as evidence for neutrino oscillations, which would imply a mass difference among neutrinos and violation of electron-number conservation. All existing evidence requires an imaginative interpretation, so the case for neutrino oscillations (like that for a neutrino mass) remains unproved. But this is obviously an area to be watched with interest.

B. Quarks[3]

Quarks experience all of the known interactions: strong, weak and electromagnetic, and gravitational. Like the leptons, they are spin-½ particles which are pointlike at the current limit of resolution. Quarks were proposed as a means for understanding the basis of the SU(3) classification of the strongly-interacting particles

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[2]The Dirac algebra conventions are those of Bjorken and Drell (1964, 1965), except that $\bar{u}u = 2m$.

[3]For comprehensive reviews, see Lipkin (1973), Rosner (1974, 1980), Dalitz (1977), Hendry and Lichtenberg (1978), Greenberg (1978), Close (1979).

(hadrons). Ordinary mesons (those known to exist before November, 1974) occur only in SU(3) singlets and octets. The pseudoscalars include

$$\eta'(957) \text{ - singlet} \qquad \left.\begin{array}{cc} \pi^+\pi^0\pi^- & (140) \\ \eta & (549) \\ K^+K^0\overline{K}^0K^- & (497) \end{array}\right\} \text{octet} \quad .$$

Baryons occur only in octets, such as

$$\begin{array}{cc} p,\ n & (940) \\ \Lambda & (1115) \\ \Sigma^+,\ \Sigma^0,\ \Sigma^- & (1192) \\ \Xi^0,\ \Xi^- & (1315) \end{array}$$

and decimets, such as

$$\begin{array}{cc} \Delta^{++},\ \Delta^+,\ \Delta^0,\ \Delta^- & (1232) \\ Y_1^{*+},\ Y_1^{*0},\ Y_1^{*-} & (1385) \\ \Xi^{*0},\ \Xi^{*-} & (1530) \\ \Omega^- & (1672) \end{array} \quad .$$

No higher representations are indicated. This is a much more restrictive statement than the mere fact that SU(3) is a good classification symmetry, and it requires explanation.

The spectroscopy of "ordinary" particles can be summarized by the hypothesis (Gell-Mann, 1964; Zweig, 1964a, b, 1965) that there exists a fundamental triplet of quarks (up, down, strange), as shown in Fig. 1, and that mesons are composed of $q\bar{q}$:

$$\underset{\sim}{3} \otimes \underset{\sim}{3}^* = \underset{\sim}{1} \oplus \underset{\sim}{8} \quad ,$$

while baryons are composed of qqq:

$$\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = \underset{\sim}{1} \oplus \underset{\sim}{8} \oplus \underset{\sim}{8} \oplus \underset{\sim}{10} \quad .$$

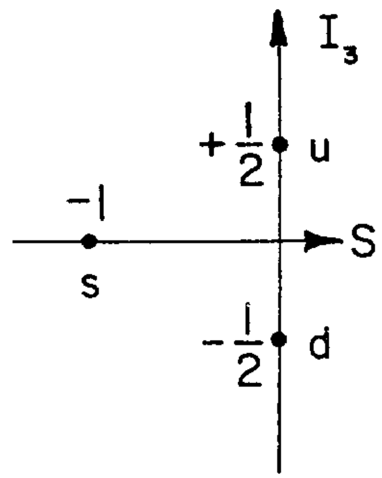

Fig. 1: Weight diagram for the fundamental ($\underset{\sim}{3}$) representation of SU(3).

These rules exhaust the representations seen prominently in Nature. It remains, of course, to understand why only these combinations of quarks and antiquarks are seen, or to discover under what circumstances more complicated configurations such as ($qq\bar{q}\bar{q}$ or $qqqq\bar{q}$ or 6q) might arise.

Since free quarks have not been isolated,[4] the properties of quarks are known rather indirectly but, we will argue, convincingly. Let us now discuss these in turn.

Quarks have baryon number 1/3; antiquarks have baryon number −1/3. This is evident from the fact that three quarks make up a baryon.

The quarks also carry fractional electric charge. The Gell-Mann-Nishijima formula for displaced charge multiplets,

$$Q = I_3 + \tfrac{1}{2}Y = I_3 + \tfrac{1}{2}(B + S) \quad ,$$

implies quark charges

[4]A recent review is that of Jones (1977). Some evidence for fractionally charged bulk matter has been presented by LaRue, et al. (1977, 1979, 1980).

$$e_u = 2/3 \quad , \quad e_d = e_s = -1/3 \quad .$$

The same assignments follow directly from examination of the baryon decimet:

$$\begin{aligned} \Delta^{++} &= uuu \\ \Delta^{+} &= uud \qquad \Omega^{-} = sss \\ \Delta^{0} &= udd \\ \Delta^{-} &= ddd \end{aligned}$$

A number of other tests of these assignments have been carried out. We recall three such tests.

Within the quark model, the leptonic decays of vector mesons, $V^0 \to \ell^+\ell^-$, proceed by the annihilation of a quark and antiquark into a virtual photon which disintegrates into the lepton pair, as shown in Figure 2. Apart from kinematical factors, the (reduced) rate for leptonic decay is proportional to the square of the quark charge--the strength of the $\gamma q\bar{q}$ coupling--and the probability for quark and antiquark to meet, which is given in a nonrelativistic description by $|\psi(0)|^2$, the square of the wavefunction at zero quark-antiquark separation.

$$\tilde{\Gamma}(V^0 \to \ell^+\ell^-) = \Gamma(V^0 \to \ell^+\ell^-) \times M_V{}^2 = K \cdot e_q{}^2 \cdot |\psi(0)|^2 \quad .$$

We assume that to first approximation the wavefunctions are the same for the light vector mesons $\rho(770)$, $\omega(784)$, $\phi(1019)$. It is then straightforward to compute the square of the effective quark charge for the three cases:

$$\begin{aligned} \rho^0 &= \tfrac{1}{\sqrt{2}}[u\bar{u} - d\bar{d}] \to e_q{}^2 = \left[\tfrac{1}{\sqrt{2}}\left(\tfrac{2}{3} + \tfrac{1}{3}\right)\right]^2 = \tfrac{1}{2} \quad ; \\ \omega^0 &= \tfrac{1}{\sqrt{2}}[u\bar{u} + d\bar{d}] \to e_q{}^2 = \left[\tfrac{1}{\sqrt{2}}\left(\tfrac{2}{3} - \tfrac{1}{3}\right)\right]^2 = \tfrac{1}{18} \quad ; \\ \phi^0 &= s\bar{s} \qquad\qquad\quad \to e_q{}^2 = \tfrac{1}{9} \quad . \end{aligned}$$

We therefore expect the reduced leptonic decay rates to be in the ratio

$$\tilde{\Gamma}(\rho^0) : \tilde{\Gamma}(\omega^0) : \tilde{\Gamma}(\phi) :: 9 : 1 : 2 \quad .$$

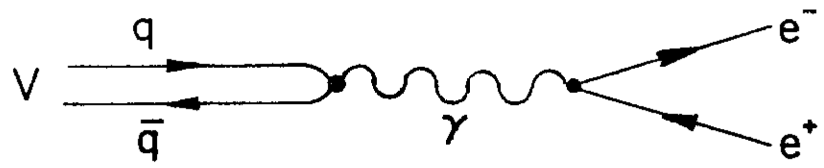

Fig. 2: Quark-model description of the decay of a neutral vector meson into a lepton pair.

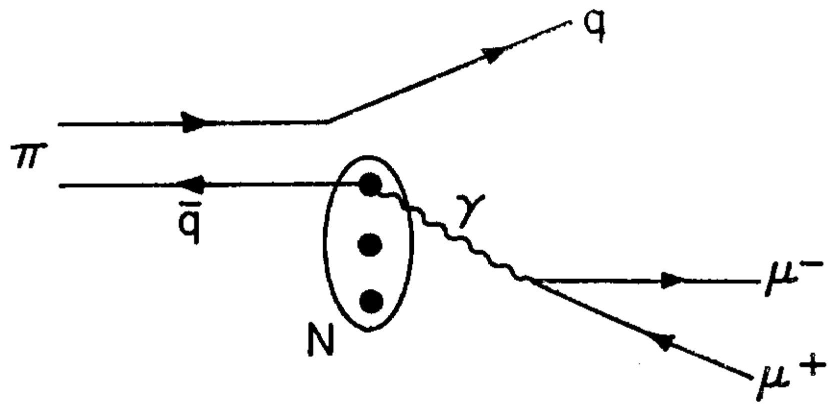

Fig. 3: The Drell-Yan process in $\pi N \rightarrow \ell^+\ell^-$ + anything.

Experimentally, the ratio is[5]

$$(8.7 \pm 2.9) : 1 : (2.8 \pm 0.8) \quad ,$$

which is in reasonable agreement, considering the crudeness of our approximations.

A second test has been made in the production of lepton pairs in pion-nucleon collisions. We regard this reaction as the annihilation of an antiquark from the pion with a quark from the nucleon, as illustrated in Fig. 3. The production of high-mass muon pairs has been studied in collisions of π^+ and π^- on Carbon

[5]See Particle Data Group (1980) for the decay rates and primary references.

$$\left.\begin{array}{l} \pi^- \ (\bar{u}d) \\ {}^{12}C \ (18u + 18d) \end{array}\right\} \quad \sigma \propto 18e_u^{\ 2} = 18 \times \frac{4}{9}$$

$$\left.\begin{array}{l} {}^{12}C \ (18u + 18d) \\ \pi^+ \ (u\bar{d}) \end{array}\right\} \quad \sigma \propto 18e_d^{\ 2} = 18 \times \frac{1}{9} \quad .$$

We therefore expect that

$$\frac{\sigma(\pi^- C \to \mu^+\mu^- + \ldots)}{\sigma(\pi^+ C \to \mu^+\mu^- + \ldots)} = 4 \quad ,$$

and this expectation has been confirmed by experiment as reviewed by Pilcher (1979).

A third test is provided by deeply-inelastic lepton-nucleon scattering. Taking out overall coupling strengths, the cross section $\sigma(\ell N \to \ell' + \text{anything})$ measures a "structure function" which is characteristic of the target.[6]

(i) The reaction $\nu T \to \mu^- + \text{anything}$ corresponds to the elementary process $\nu d \to \mu^- u$. Thus, the structure function $F_2(\nu T)$ counts the number of d-quarks in the target T.

(ii) The reaction $\bar{\nu} T \to \mu^+ + \text{anything}$ proceeds via the elementary interaction $\bar{\nu} u \to \mu^+ d$. Hence the structure function $F_2(\bar{\nu} T)$ counts the number of u-quarks in the target.

(iii) The reaction $eT \to e + \text{anything}$ is characterized by the elementary interaction $eq \to eq$. The structure function $F_2(eT)$ therefore counts the number of up-quarks in the target, weighted by $e_u^{\ 2} = 4/9$, plus the number of down-quarks in the target, weighted by $e_d^{\ 2} = 1/9$.

Consequently for an isoscalar target such as deuterium we expect

$$\frac{F_2(eD)}{F_2(\nu D) + F_2(\bar{\nu} D)} = \frac{3e_u^{\ 2} + 3e_d^{\ 2}}{6} = \frac{5}{18} \quad ,$$

[6] See the lectures by Perkins (1980), and the book by Feynman (1972).

which is again in close agreement with experiment.[7]

We also know that the quarks have spin-½. Clearly if baryons, which are fermions, are to be made of three identical constituents, the constituents must be fermions rather than bosons. In addition, the observed hadron spectrum corresponds to the objects which can be formed (according to our earlier rules for flavor properties) from $J^P = \frac{1}{2}^+$ quarks. It is straightforward to work out the level structure in the meson sector:

$$(q\bar{q}) \rightarrow J^{PC} = \underbrace{0^{-+},\ 1^{--}}_{L=0} \ ; \quad \underbrace{0^{++},\ 1^{++},\ 1^{+-},\ 2^{++},\ \ldots}_{L=1} \ .$$

This corresponds to the observed ordering of levels. Combinations of spin, parity, and charge conjugation such as $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}$, which cannot be formed from spin-½ quark-antiquark pairs, are not found in Nature. The analysis of baryon multiplets is similar but more tedious. Again the observed spectrum conforms to the quark model pattern.

In addition to this successful classification scheme, there are some dynamical tests of the quark spin. Consider the cross sections for absorption of longitudinal or transverse virtual photons in deeply-inelastic electron scattering. Working in the Breit frame of the struck quark, it is easy to see that a spinless quark can only absorb a longitudinal (helicity = 0) photon, since angular momentum conservation forbids the absorption of a transverse (helicity ± 1) photon, as may be seen in Fig. 4. Similarly, a spin-½ quark can absorb a transverse photon, but not a longitudinal photon (see problem 1). If quarks have spin-½, we therefore expect that

$$\sigma_{\text{Longitudinal}}/\sigma_{\text{Transverse}} = 0 \quad .$$

This is in schematic agreement with experiment (Perkins, 1980). A related test comes from the analysis of the angular distribution of hadron jets produced in electron-positron annihilations, which is observed to be identical to the production angular distribution of

[7] These predictions are shared by the Han-Nambu (1965) model of integrally-charged quarks, and it is notoriously difficult to draw distinctions (Lipkin, 1979; Chanowitz, 1975). Properties of the η(549) and η'(958) strongly favor the fractional charge assignment (Chanowitz, 1980).

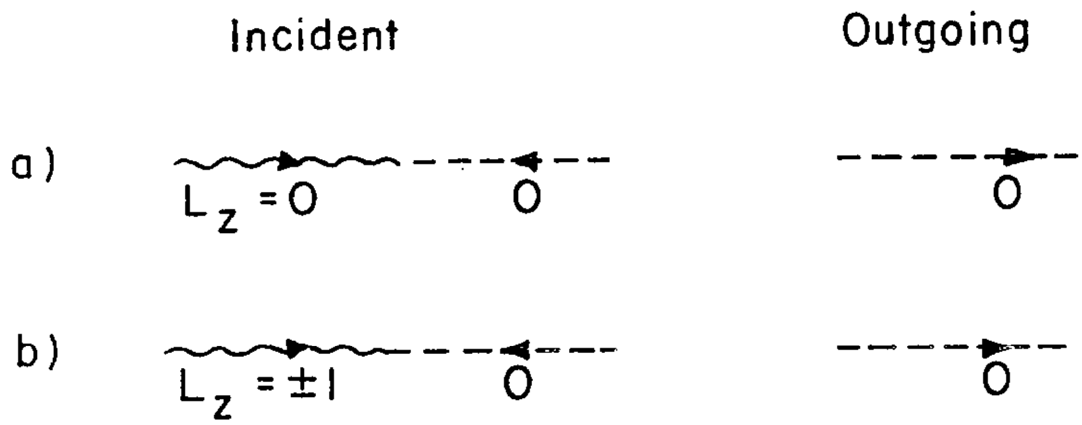

Fig. 4: (a) Absorption of a longitudinal photon by a spinless parton is allowed by angular momentum conservation. (b) Absorption of a transverse photon is forbidden.

$e^+e^- \to \mu^+\mu^-$. This implies (see problem 2) that the elementary process $e^+e^- \to q\bar{q}$ corresponds to the pair production of spin-½ objects.

Quarks have still another property, known as color. At first sight, the Pauli Principle seems not to be respected in the wavefunction for the Δ^{++}. This is a uuu state with spin = 3/2, isospin = 3/2, and angular momentum zero, which is a symmetric wavefunction of three identical fermions. Unless we are prepared to forgo the Pauli Principle or the quark model, it is necessary to invoke a new, hidden degree of freedom which permits the wavefunction to be antisymmetrized (Greenberg, 1964). In order that a three-quark wavefunction can be antisymmetrized, it is necessary that each quark flavor come in at least three distinguishable colors. If there were more than three quark colors, the unpleasant possibility of distinguishable (colored) species of proton would have to be faced, contrary to observation.

While the introduction of color would at first appear to be unnatural and artificial, subsequent observations have given strong support to the color hypothesis.[8] A number of observables are sensitive to the number of distinct species of quarks. As we have seen in Problem 2, the inclusive cross section for electron-positron annihilation into hadrons is described by the process $e^+e^- \to q\bar{q}$, where the quark and antiquark materialize with unit probability into the observed hadron jets. The ratio

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[8] For a recent review, see Greenberg and Nelson (1977).

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is then simply given by

$$R = \sum_{\substack{\text{quark}\\ \text{species}}} e_q^{\,2} \quad .$$

At c.m. energies between about 1.5 and 3.6 GeV, up, down, and srange quarks are kinematically accessible. In the absence of hadronic color we would therefore expect

$$R_0 = e_u^{\,2} + e_d^{\,2} + e_s^{\,2} = \frac{2}{3} \quad ,$$

whereas if each flavor comes in three colors, we should have

$$R_3 = 3\left[e_u^{\,2} + e_d^{\,2} + e_s^{\,2}\right] = 2 \quad .$$

Experiment decisively favors the color-triplet hypothesis, as shown in Figure 5.

A similar count of the number of distinguishable quarks of each flavor is provided by the branching ratios for decay of the tau-lepton. Within the quark model, τ decays may be described as shown in Fig. 6, namely by the decay of τ into ν_τ plus a virtual intermediate boson (W^-). The intermediate boson then disintegrates into all kinematically Cabibbo accessible fermion-antifermion pairs: $(e^-\bar{\nu}_e)$, $(\mu^-\bar{\nu}_\mu)$, $(\bar{u}d_\theta)$. The universality of charged-current weak interactions implies equal rates for each of these decays. Thus in the absence of color, we expect

$$B_o = \frac{\Gamma(\tau \to e^-\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \text{all})} = \frac{1}{3} \quad .$$

If quarks come in three colors, $(\bar{u}d_\theta)$ is augmented to $3(\bar{u}d_\theta)$, and we have

$$B_3 = \frac{1}{5} \quad .$$

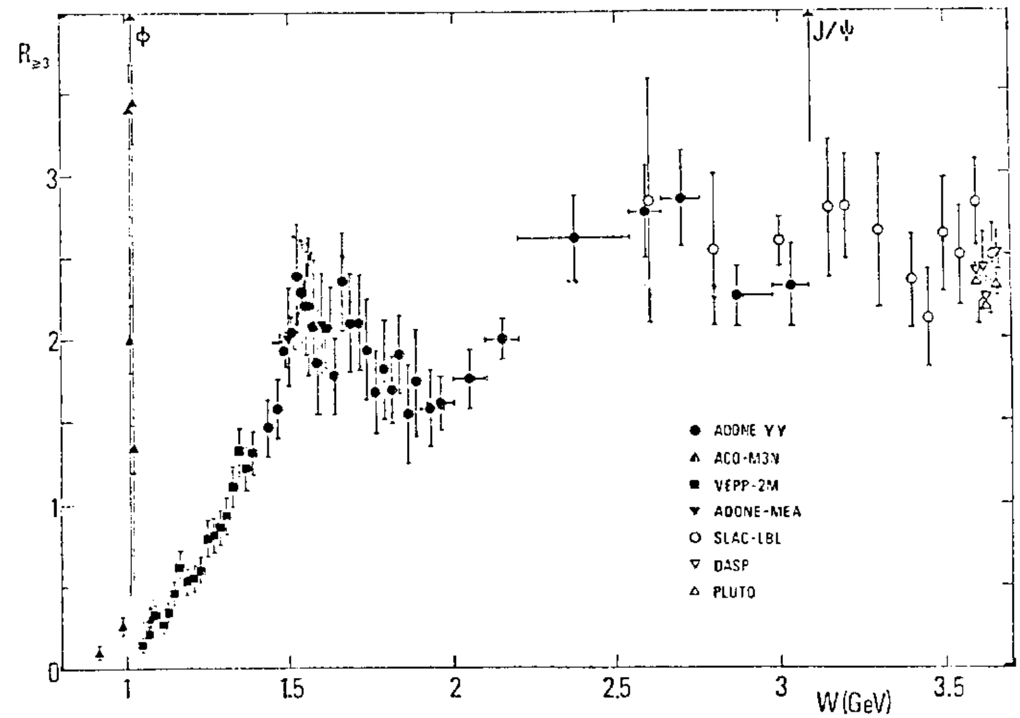

Fig. 5: The ratio $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ [from Spinetti, 1979].

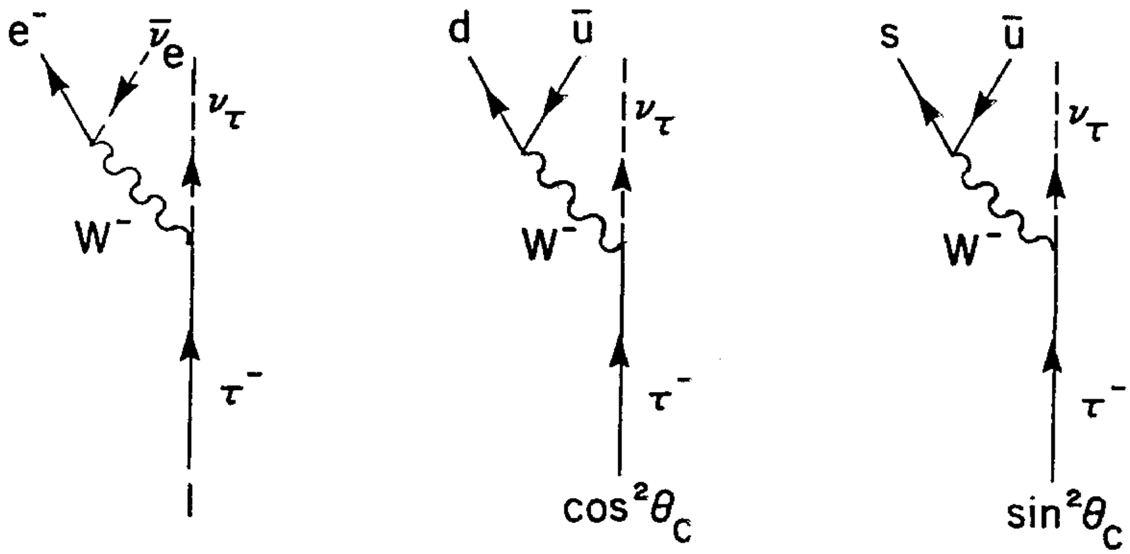

Fig. 6: Semileptonic decays (weight 1), and Cabibbo favored (weight $\cos^2\theta_C$) and suppressed (weight $\sin^2\theta_C$) nonleptonic decays of the τ-lepton.

The experimental branching ratio (Particle Data Group, 1980) is

$$B_{exp} = (17.44 \pm 0.85)\%$$

in accord with the color hypothesis.[9]

Finally let us notice that the quark model prediction for the decay $\pi^o \to \gamma\gamma$, the annihilation of quark and antiquark into two photons, is sensitive to the number of distinct quark loops that may contribute. This decay rate is also decisively in accord with the hypothesis that the light quarks u,d,s are color triplets.

Just as there are several doublets of leptons, different quark flavors have made themselves known to us through the strong interactions. Ordinary matter (protons, neutrons, pions) signals the existence of the up and down quarks. The strange particles (hyperons, kaons) were discovered in the associated production reactions

$$\pi N \to KY \quad ,$$

which implied the existence of a new additive quantum number dubbed strangeness, which is borne by the strange quark. Similarly, the quantum number called charm might well have been discovered through the observation of associated charmed-particle production in the reaction

$$\pi N \to \bar{D}C_o \quad ,$$

although it was in fact discovered by more indirect means.

The observation of hadrons with various internal quantum numbers (isospin, strangeness, charm,...) led to the use of SU(2,3,4,...) as symmetries of the strong interaction. These internal symmetries serve both for the classification of hadrons [cf. SU(3) and the evolution of the quark model] and for dynamical relations among strong interaction matrix elements. Given the very different masses of the nonstrange and strange particles (and hence of the nonstrange and strange quarks) it has been difficult to understand how SU(3) symmetry could arise dynamically. Furthermore, both SU(2) of isospin and SU(3) are excellent but not exact strong

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[9] A refined estimate for the leptonic branching ratio is 17.75% (Gilman and Miller, 1978; Kawamoto and Sanda, 1978).

interaction symmetries. Isospin invariance holds within a few percent and SU(3) is reliable within 10-20%.

How can this symmetry breaking be understood? The traditional view has been that SU(2), or isospin, violations are electromagnetic in origin. For example, the neutron-proton mass difference of 1.29 MeV/c^2 ("wrong" sign!) and the $\pi^+ - \pi^0$ mass difference of 4.60 MeV/c^2 are of a size to be consistent with an origin as electromagnetic perturbations. Similarly, deviations from exact SU(3) symmetry have been accounted for by mass differences, which are interpreted in terms of a "medium-strong" interaction[1] that transforms as a member of an SU(3) octet (specifically as λ_8).

But how do these mass splittings actually arise, and why do various quark flavors exist? Within the strong interactions, flavors do not appear to have any essential role, but only to contribute to a richness. In contrast, if we look to the weak interactions, the importance of being a flavor is more readily apparent. Because of the family patterns that are implied by the charged-current weak isospin doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

flavors appear to have specific roles to play. As we shall see shortly, the charmed quark was needed and had a function to fulfill in the weak interactions in advance of its discovery. On the other hand, charm and other new flavors seem merely to be tolerated by the strong interactions.

An evolving contemporary view[2] is that flavors are not fundamental from the point of view of the strong interactions. The elementary strong interactions are then "tasteless," and are sensitive only to the color charge of the constituents. According to this view, the breaking of flavor SU(N) symmetry is then ascribed to quark mass differences

$$m_u < m_d < m_s < m_c < m_b < \ldots$$

[1]This view is expounded in the reprint volume by Gell-Mann and Ne'eman (1964).

[2]Some representative discussion of the problem of quark masses may be found in Weinberg (1977) and Langacker and Pagels (1979).

and quark masses are thought to arise from the spontaneous symmetry breaking of the weak interactions.

This mode of thinking is at the moment not a complete explanation. We do not know why so many "fundamental" fermions should exist, or why the pattern of masses and mixing angles is what is seen. It is hoped that the further unification of elementary forces will provide at least partial answers to these and related questions.

C. The Fundamental Interactions

We now believe that the elementary interactions of the quarks and leptons can be understood as consequences of gauge invariances of the fundamental Lagrangian. These ideas will be derived in a logical sequence in the succeeding chapters. For the moment, let us simply recall some basic aspects of the familiar gauge theories and their consequences.

To review gauge theories of the weak and electromagnetic interactions, we consider the Weinberg (1967)-Salam (1968) theory of leptons in a world with two lepton "generations," represented by the weak isospin doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad .$$

The electromagnetic current is given schematically by

$$J_\sigma^{em} = -\bar{e}\gamma_\sigma e - \bar{\mu}\gamma_\sigma \mu \quad ,$$

and the charged weak current, indicated by the weak isospin doublets, is

$$J_\sigma^{(\pm)} = \sum_i \bar{\psi}_i \tau_\pm \gamma_\sigma (1 - \gamma_5) \psi_i \quad ,$$

where the composite spinors ψ_i represent

$$\psi_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad , \quad \psi_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

and the Pauli isospin matrices are

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad , \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .$$

In more explicit form, the charge-raising current is (see Problem 3)

$$J_\sigma^{(+)} = \bar{\nu}_e \gamma_\sigma (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\sigma (1 - \gamma_5) \mu \quad .$$

In such models the weak neutral current contains a piece which completes the weak isovector:

$$\begin{aligned} J_\sigma^{(3)} &= \tfrac{1}{2} \sum_i \bar{\psi}_i \tau_3 \gamma_\sigma (1 - \gamma_5) \psi_i \\ &= \tfrac{1}{2} \Big[\bar{\nu}_e \gamma_\sigma (1 - \gamma_5) \nu_e - \bar{e} \gamma_\sigma (1 - \gamma_5) e \\ &\quad + \bar{\nu}_\mu \gamma_\sigma (1 - \gamma_5) \nu_\mu - \bar{\mu} \gamma_\sigma (1 - \gamma_5) \mu \Big] \quad . \end{aligned}$$

In addition, the symmetry breaking entailed in weak-electromagnetic unification contributes to the weak neutral current a piece proportional to J_σ^{em}, with strength governed by a weak mixing angle θ_W. Thus, the neutral current is

$$\begin{aligned} J_\sigma^{(0)} &= J_\sigma^{(3)} - \sin^2 \theta_W J_\sigma^{em} \\ &= \tfrac{1}{2} \bar{\nu}_e \gamma_\sigma (1 - \gamma_5) \nu_e + L \bar{e} \gamma_\sigma (1 - \gamma_5) e \\ &\quad + R \bar{e} \gamma_\sigma (1 + \gamma_5) e + (e \to \mu) \quad , \end{aligned}$$

with

$$L = \sin^2\theta_W - \tfrac{1}{2} \quad , \quad R = \sin^2\theta_W \quad .$$

Notice that the electronic and muonic sectors remain disconnected, as required by the separate conservation of electron and muon number. We say that the leptonic neutral current is flavor conserving or diagonal in flavors.

What of the hadronic current? According to Cabibbo's picture of weak-interaction universality, the hadronic charged current is represented by the weak-isospin doublet

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad ,$$

where

$$d_\theta \equiv d \cos\theta_C + s \sin\theta_C$$

and θ_C is the Cabibbo angle (see Problem 4). In other words, the charge-raising weak current is

$$J_\sigma^{(+)} = \bar{u}\gamma_\sigma(1 - \gamma_5)d\cdot\cos\theta_C + \bar{u}\gamma_\sigma(1 - \gamma_5)s\cdot\sin\theta_C \quad .$$

We may ask why the hadron sector has an "extra," unused quark, that is why does the orthogonal combination

$$s_\theta = s\cos\theta_C - d\sin\theta_C$$

not appear in the weak current. In a similar vein, why are quarks and leptons not more symmetrical?

To investigate these questions further, let us form the Weinberg-Salam neutral current within the Cabibbo framework. It is

$$
\begin{aligned}
J_\sigma^{(0)} &= J_\sigma^{(3)} - \sin^2\theta_W J_\sigma^{em} \\
&= \tfrac{1}{2}\Big\{ \bar{u}\gamma_\sigma(1-\gamma_5)u - \bar{d}\gamma_\sigma(1-\gamma_5)d\cos^2\theta_C \\
&\quad - \bar{s}\gamma_\sigma(1-\gamma_5)s\sin^2\theta_C - \bar{s}\gamma_\sigma(1-\gamma_5)d\sin\theta_C\cos\theta_C \\
&\quad - \bar{d}\gamma_\sigma(1-\gamma_5)s\sin\theta_C\cos\theta_C \Big\} \\
&\quad - \sin^2\theta_W\Big\{ \tfrac{2}{3}\bar{u}\gamma_\sigma u - \tfrac{1}{3}\bar{d}\gamma_\sigma d - \tfrac{1}{3}\bar{s}\gamma_\sigma s \Big\} .
\end{aligned}
$$

Unlike the leptonic neutral current, the hadronic neutral current contains flavor-changing ($d \leftrightarrow s$) terms. This is experimentally unacceptable because of the stringent upper limit on the decay rate for $K^+ \to \pi^+ \nu\bar{\nu}$ and the small rate observed for $K_L \to \mu^+\mu^-$. It was shown by Glashow, Iliopoulos, and Maiani (1970) that lepton-hadron symmetry could be restored and the flavor changing neutral currents eliminated by the addition of a second weak-isospin doublet

$$
\begin{pmatrix} c \\ s_\theta \end{pmatrix}_L
$$

involving the charmed quark (Bjorken and Glashow, 1964). The hadronic neutral current now assumes the diagonal form

$$
\begin{aligned}
J_\sigma^{(0)} &= \tfrac{1}{2}\Big\{ \bar{u}\gamma_\sigma(1-\gamma_5)u + \bar{c}\gamma_\sigma(1-\gamma_5)c - \bar{d}\gamma_\sigma(1-\gamma_5)d \\
&\quad - \bar{s}\gamma_\sigma(1-\gamma_5)s \Big\} - \sin^2\theta_W J_\sigma^{em} .
\end{aligned}
$$

The discovery (Aubert, et al., 1974; Augustin, et al., 1974) of the family of ($c\bar{c}$) bound states known as psions and the observation (Goldhaber, et al., 1976; Peruzzi, et al., 1976) of charmed particles which decay according to the $(c, s_\theta)_L$ pattern constitute a striking confirmation of the GIM hypothesis.

Gauge theories also show considerable promise as descriptions of the strong interactions. Currently it is believed that quantum chromodynamics or QCD, a non-Abelian gauge theory of colored quarks interacting by means of massless, colored vector gluons, is the fundamental, underlying field theory of the strong interactions. The three quark colors are regarded as the generators of an $SU(3)_C$ color group, which is not to be confused with the flavor SU(3) of up, down, and strange quarks. The strong interactions among quarks are then mediated by an $SU(3)_C$ octet of colored gluons.

We shall argue that only color singlet objects may exist in isolation. Color confinement, as it is called, then provides an explanation of quark confinement: free quarks and free gluons will not be found. The arguments we have given in favor of color-triplet assignments for the familiar quarks do not imply that yet-to-be-discovered quarks must lie in the fundamental representation of $SU(3)_C$. Further guidance may come from specific grand unified theories. According to this view of the strong interactions, color is what distinguishes quarks from leptons. Since color plays the role of the (nonabelian) charge of the strong interactions and gluons are colored, they will interact among themselves by gluon exchange. We shall find in Chapter 6 that the strong coupling "constant" becomes small at very short distances. This property, which is known as asymptotic freedom may help to explain why quarks cannot be separated but behave within hadrons as if they are free particles.

Before leaving this introductory section, let us review the experimental evidence for the existence of gluons. It is basically of two kinds. First, from energy-momentum sum rules in lepton-nucleon scattering, we find that only about half of the momentum of a proton is carried by constituents which interact weakly or electromagnetically (i.e. by the quarks). Something else, electrically neutral and inert with respect to the weak interactions, must carry the rest. This role can be played by the gluons. Second, at c.m. energies exceeding 17 GeV, a fraction of electron-positron annihilations into hadrons display a three-jet structure instead of the familiar two-jet ($e^+e^- \to q\bar{q}$) structure (Mark-J Collaboration, 1979; PLUTO Collaboration, 1979; TASSO Collaboration, 1979; JADE Collaboration, 1979). This is interpreted as evidence for the process $e^+e^- \to q\bar{q}$ + gluon, in which the gluon is radiated from an outgoing quark leading to the spatial configuration depicted in Fig. 7.

With these concepts and aspirations as background, we now turn to the basic ideas that underlie gauge theories. The first of these is the notion of gauge invariance.

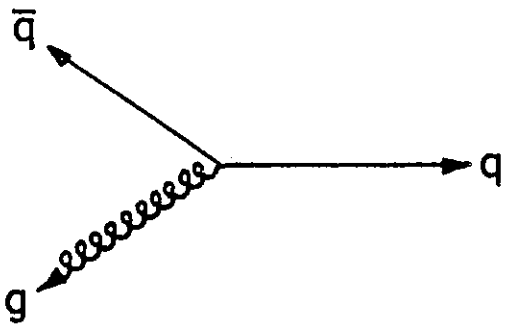

Fig. 7: Three-jet structure of the final state in the reaction $e^+e^- \to q\bar{q}g$.

2. THE IDEA OF GAUGE INVARIANCE

The concept of gauge invariance,[3] which (unlike Lorentz invariance) is a dynamical symmetry, arose from attempts by Hermann Weyl (1921) to unify gravity and electromagnetism through the use of a space-time dependent change of scale. Weyl's terminology, Eichinvarianz (Eich = gauge or standard) has survived although his initial attempts were unsuccessful.

Consider the change in a function f as we move from a point x_μ to $x_\mu + dx_\mu$. In a space with uniform scale, it is simply

$$x_\mu \to x_\mu + dx_\mu$$

$$f \to f + (\partial_\mu f)dx^\mu \quad .$$

But if in addition the scale, or unit of measure, changes from 1 at x_μ to $1 + S_\mu dx^\mu$ at $x_\mu + dx_\mu$, the value of the function at $x_\mu + dx_\mu$ becomes

$$\left(f + (\partial_\mu f)dx^\mu \right) \left(1 + S_\nu dx^\nu \right)$$

$$= f + (fS_\mu + \partial_\mu f)dx^\mu + \mathcal{O}(dx)^2 \quad .$$

To leading order in dx^μ, the increment in the function f is

$$(\partial_\mu + S_\mu)f \quad .$$

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[3]The history of gauge invariance has been reviewed by Yang (1975, 1977).

Weyl sought to identify S_μ with the vector potential A_μ of electromagnetism and thus to incorporate electromagnetism into a geometrical theory.

Let us see why this is incorrect. Recall from elementary quantum mechanics that the classical four-momentum p_μ goes over to the quantum-mechanical operator $-i\hbar\partial_\mu$. For a charged particle, the replacement is

$$p_\mu - \frac{e}{c} A_\mu \rightarrow -i\hbar(\partial_\mu - (ie/\hbar c) A_\mu) \quad .$$

We can therefore carry out Weyl's program if we identify

$$S_\mu = -(ie/\hbar c) A_\mu \quad ,$$

so that instead of investigating invariance of the laws of physics under a change of scale, we require invariance under a change of phase:

$$\left(1 - \frac{ie}{\hbar c} A_\mu dx^\mu \right) \approx \exp \left\{ \frac{-ie}{\hbar c} A_\mu dx^\mu \right\} \quad .$$

Following work by Fock (1927) and London (1927), Weyl began in 1929 to study invariance under this phase change, but retained the terminology, "gauge invariance."

A. Gauge Invariance in Classical Electrodynamics

The physical appeal of gauge invariance stems from the old observation[4] (Noether's Theorem) that to every continuous symmetry of the Lagrangian there corresponds a conservation law. This connection is reviewed in Problem 5. Let us first review the consequences of gauge invariance in classical electrodynamics.

Maxwell's equation for magnetic charge,

$$\nabla \cdot \underset{\sim}{B} = 0$$

invites us to write

$$\underset{\sim}{B} = \nabla \times \underset{\sim}{A} \quad ,$$

[4]A very readable and worthwhile discussion appears in Hill (1951).

where $\underset{\sim}{A}$ is the vector potential. This identification ensures that $\underset{\sim}{B}$ will be divergenceless, by virtue of the identity

$$\nabla \cdot (\nabla \times \underset{\sim}{A}) = 0 \quad .$$

If we add an arbitrary gradient to the vector potential

$$\underset{\sim}{A} \rightarrow \underset{\sim}{A} + \nabla\Lambda \quad ,$$

the magnetic field is unchanged, because

$$\underset{\sim}{B} = \nabla \times (\underset{\sim}{A} + \nabla\Lambda) = \nabla \times \underset{\sim}{A} \quad .$$

In similar fashion, the curl equation for the electric field

$$\nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} \quad ,$$

which can be rewritten as

$$\nabla \times \left(\underset{\sim}{E} + \frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t}\right) = 0 \quad ,$$

invites the identification

$$\underset{\sim}{E} + \frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} = -\nabla V \quad ,$$

where V is known as the scalar potential. In order that the electric field remain invariant under the shift

$$\underset{\sim}{A} \rightarrow \underset{\sim}{A} + \nabla\Lambda \quad ,$$

we must also require that

$$V \rightarrow V - \frac{1}{c}\frac{\partial \Lambda}{\partial t} \quad .$$

All of this can be expressed compactly in covariant notation. The electromagnetic field-strength tensor

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

built up from $A_\mu = (V, \underset{\sim}{A})$, is unchanged by the "gauge transformation"

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha \quad .$$

The fact that many different 4-vector potentials describe the same physics is a manifestation of the gauge invariance of classical electromagnetism.

The remaining Maxwell equations,

$$\nabla \cdot \underset{\sim}{E} = 4\pi\rho = -\nabla \cdot \dot{\underset{\sim}{A}} - \nabla^2 V$$

and

$$\begin{aligned} \nabla \times \underset{\sim}{B} &= \frac{4\pi}{c}\underset{\sim}{J} + \frac{1}{c}\dot{\underset{\sim}{E}} = \frac{4\pi}{c}\underset{\sim}{J} - \frac{1}{c}\ddot{\underset{\sim}{A}} - \frac{1}{c}\nabla \cdot \dot{\underset{\sim}{V}} \\ \parallel \quad & \\ \nabla \times (\nabla \times \underset{\sim}{A}) &= -\nabla^2 \underset{\sim}{A} + \nabla(\nabla \cdot \underset{\sim}{A}) \quad , \end{aligned}$$

(where $\dot{\underset{\sim}{A}} \equiv \partial \underset{\sim}{A}/\partial t$) correspond, in covariant form, to

$$\begin{aligned} \partial^\mu F_{\mu\nu} &= \partial_\nu(\partial^\mu A_\mu) - \partial^\mu \partial_\mu A_\nu \\ &= -4\pi J_\nu \quad . \end{aligned}$$

Two consequences are immediately apparent. First, the electromagnetic current J_ν is conserved:

$$\partial^\nu J_\nu = (-1/4\pi)\left\{\Box(\partial^\mu A_\mu) - \Box(\partial^\nu A_\nu)\right\}$$
$$= 0 \quad ,$$

where the d'Alembertian is $\Box \equiv \partial^\mu \partial_\mu$. Second, the wave equation in Lorentz gauge ($\partial^\mu A_\mu = 0$) and in the absence of sources ($J_\nu = 0$) becomes

$$\Box A_\nu = 0 \quad ,$$

which has the form of a Klein-Gordon equation for a massless particle.

We see in these familiar results a relationship between gauge invariance, current conservation, and massless vector fields.[5] Let us now attempt to understand these connections more precisely.

B. Phase Invariance in Quantum Mechanics

Suppose we knew the Schrödinger equation, but not the laws of electrodynamics. Would it be possible to derive (i.e. guess) Maxwell's equations from a gauge principle? The answer is yes! Let us trace the steps in the argument in detail.

A quantum-mechanical state is described by a complex Schrödinger wavefunction $\psi(x)$. Quantum-mechanical observables involve inner products of the form

$$\langle \mathcal{O} \rangle = \int \psi^* \mathcal{O} \psi$$

which are unchanged under a global phase rotation

$$\psi(x) \rightarrow e^{i\theta}\psi(x) \quad .$$

In other words, the absolute phase of the wavefunction cannot be observed and is a matter of convention.

[5] For more on the connection between global gauge invariance and current conservation, see Bjorken and Drell (1965), c. 11; Hill (1951); Abers and Lee (1973), §1; and Wigner (1967).

This raises the question: can we choose one phase convention in St. Croix and another in Batavia? Differently stated, can quantum mechanics be made invariant under local phase rotations

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \quad ?$$

We shall see that this can be accomplished, but at the price of introducing an interacting field that we will construct to be electromagnetic field.

The equations of motion always involve derivatives of ψ. But under gauge transformations these transform as

$$\partial_\mu\psi(x) \rightarrow e^{i\alpha(x)}\left[\partial_\mu\psi(x) + i(\partial_\mu\alpha(x))\psi(x)\right] \quad ,$$

which involves more than a mere phase change. The additional gradient-of-phase term spoils local phase invariance. Local gauge invariance may be attained, however, by the introduction of the electromagnetic field A_μ.

Consider local phase rotations of the form

$$\psi(x) \rightarrow e^{iq\theta(x)}\psi(x) \quad ,$$

where q is the electric charge of the particle specified by ψ, accompanied by the local gauge transformation

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\theta(x) \quad .$$

If the gradient ∂_μ is replaced by the gauge-covariant derivative

$$\mathscr{D}_\mu \equiv \partial_\mu - iqA_\mu \quad ,$$

it is easily verified that under local phase rotations

$$\mathscr{D}_\mu\psi(x) \rightarrow e^{iq\theta(x)}\mathscr{D}_\mu\psi(x) \quad .$$

Consequently quantities such as $\psi^*\mathscr{D}_\mu\psi$ are invariant under local gauge (phase) transformations. Moreover, the form of the coupling ($\mathscr{D}_\mu\psi$) between the electromagnetic field and matter is suggested (if not uniquely dictated) by local gauge invariance.

This example has shown the possibility of using local gauge invariance as a dynamical principle. With it as background, a more systematic treatment is now in order.

C. Phase Invariance in Field Theory

The basic object in field theories is the Lagrangian density $\mathscr{L}(\phi(x), \partial_\mu\phi(x))$, from which is constructed the classical action

$$\text{Action} \equiv \int_{-\infty}^{\infty} dt L(t) = \int d^4x\, \mathscr{L}(\phi(x), \partial_\mu\phi(x)) \quad .$$

The equations of motion of the fields follow from Hamilton's principle

$$\delta\int_{t_1}^{t_2} dt L(t) = 0 \quad ,$$

subject to the constraint that variations in the fields vanish at the endpoints. Satisfaction of Hamilton's principle is guaranteed by the Euler-Lagrange equations

$$\frac{\delta\mathscr{L}}{\delta\phi} = \partial_\mu \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)} \quad .$$

As an example, consider the Lagrangian for a free scalar field

$$\mathscr{L} = \tfrac{1}{2}\left[|\partial_\mu\phi|^2 - m^2|\phi|^2\right] \quad ,$$

from which the Euler-Lagrange equations lead to the Klein-Gordon equation

$$(\Box + m^2)\phi = 0 \quad .$$

A global gauge transformation on these fields,

$$\phi(x) \rightarrow e^{iq\theta}\phi(x) \qquad ,$$

$$\partial_\mu\phi(x) \rightarrow e^{iq\theta}(\partial_\mu\phi(x)) \qquad ;$$

leads to infinitesimal variations

$$\delta\phi = iq(\delta\theta)\phi \qquad ,$$

$$\delta(\partial_\mu\phi) = iq(\delta\theta)\partial_\mu\phi \qquad .$$

The statement of global gauge invariance is that such transformations leave the Lagrangian unchanged:

$$\delta\mathscr{L} = 0 \qquad .$$

By explicit computation we have

$$\begin{aligned}\delta\mathscr{L} &= \frac{\delta\mathscr{L}}{\delta\phi}\delta\phi + \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)}\delta(\partial_\mu\phi) + \frac{\delta\mathscr{L}}{\delta\phi^*}\delta\phi^* + \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi^*)}\delta(\partial_\mu\phi^*) \\ &= \left(\partial_\mu \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)}\right) iq(\delta\theta)\phi + \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)} iq(\delta\theta)\partial_\mu\phi - (\phi \rightarrow \phi^*) \\ &= i\delta\theta\partial_\mu\left[\frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)} q\phi - \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi^*)} q\phi^*\right] = 0 \qquad ,\end{aligned}$$

where the last step makes use of the equations of motion. Evidently we may identify the quantity in square brackets as a conserved current (density),

$$J^\mu = -iq\left[\frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi)}\phi - \frac{\delta\mathscr{L}}{\delta(\partial_\mu\phi^*)}\phi^*\right] \qquad ,$$

which satisfies

$$\partial_\mu J^\mu = 0 \qquad .$$

For the specific case of the massive scalar field theory, the conserved current

$$J^{\mu} = iq\phi^{*}\overleftrightarrow{\partial}^{\mu}\phi$$

is immediately recognizable as the electromagnetic current.

What are the consequences of <u>local</u> gauge invariance? The fields transform as

$$\phi(x) \rightarrow e^{iq\theta(x)}\phi(x) \quad .$$

Terms in the Lagrangian that depend only upon the fields are left invariant, just as before. There are no consequences beyond those of global gauge invariance. However, as we saw in our discussion of the Schrödinger equation, gradient terms transform as

$$\partial_{\mu}\phi(x) \rightarrow e^{iq\theta(x)}[\partial_{\mu}\phi(x)] + iq[\partial_{\mu}\theta(x)]e^{iq\theta(x)}\phi(x) \quad ,$$

which necessitate the introduction of a gauge-covariant derivative

$$\begin{aligned} \mathscr{D}_{\mu}\phi(x) &\equiv (\partial_{\mu} - iqA_{\mu}(x))\phi(x) \\ &\rightarrow e^{iq\theta(x)}\mathscr{D}_{\mu}\phi(x) \quad , \end{aligned}$$

provided that

$$A_{\mu}(x) \rightarrow A_{\mu}(x) + \partial_{\mu}\theta(x) \quad .$$

Again the requirement of local gauge invariance prescribes the form of the interaction between radiation and matter.

This time, let us look explicitly at the Dirac equation. The original Lagrangian

$$\mathscr{L}_{\text{free}} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$$

is replaced by

$$\begin{aligned}\mathscr{L} &= \bar{\psi}(i\gamma^\mu \mathscr{D}_\mu - m)\psi \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + q\bar{\psi}\gamma^\mu A_\mu \psi \\ &= \mathscr{L}_{\text{free}} + J^\mu A_\mu \quad ,\end{aligned}$$

where the (conserved) electromagnetic current has the familiar form

$$J_\mu = q\bar{\psi}\gamma^\mu\psi \quad .$$

We learned in our review of gauge invariance in classical electrodynamics that

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

is a locally gauge invariant quantity. A possible kinetic energy term is therefore $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, where the choice of normalization guarantees the correct, which is to say Maxwellian, equations of motion. Assembling all the pieces we therefore have

$$\mathscr{L}_{\text{QED}} = \mathscr{L}_{\text{free}} + J^\mu A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad .$$

A photon mass term would have the form

$$\mathscr{L}_\gamma = -\tfrac{1}{2}m^2 A^\mu A_\mu \quad ,$$

which obviously violates local gauge invariance because

$$A^\mu A_\mu \rightarrow (A^\mu + \partial^\mu\theta)(A_\mu + \partial_\mu\theta) \neq A^\mu A_\mu \quad .$$

Thus we find that local gauge invariance has led us to a massless photon. (We shall see in Chapter 4 how this conclusion may be evaded by the non-perturbative effects of spontaneous symmetry breaking.) The fascinating history of measurements of the photon mass is admirably reviewed in many places, including Goldhaber and Nieto (1971, 1976), Kobzarev and Okun (1968), and Jackson (1976). The best limit on the photon mass comes from the Pioneer 10 measurements of the magnetic field of Jupiter (Davis, et al., 1975). The upper limit at 90% confidence level is

$$M_\gamma < 4.5 \times 10^{-16} \mathrm{eV}/c^2 \quad ,$$

which corresponds to a modified Coulomb potential of the form

$$V \sim \frac{\exp\,(-r/r_o)}{r} \quad ,$$

with $r_o > 4.4 \times 10^5$ km. Subsequent space probes may be expected to improve this limit further.

D. Summary

We have now seen how global phase invariance leads to the existence of a conserved charge. The stronger requirement of local phase invariance requires the introduction of a massless gauge field A_μ and restricts the possible interactions of radiation with matter. The theory of electromagnetism (Quantum ElectroDynamics) is therefore the gauge theory of the group of phase transformations which is the Abelian group U(1). We shall next investigate the generalization of these ideas to non-Abelian groups. The resulting theories are known as non-Abelian gauge theories, or Yang-Mills theories.

3. NON-ABELIAN GAUGE THEORIES

In this chapter we undertake the extension of our ideas about local gauge invariance to gauge groups more complicated than the group of phase rotations.[6] We shall find that it is possible to enforce local gauge invariance by following essentially the same strategy as succeeded for electrodynamics. The principal difference (apart from algebraic complexity) will be the existence of interactions among the gauge bosons, which is a consequence of the non-Abelian nature of the gauge symmetry. As before, we proceed by example, developing the SU(2)-isospin gauge theory put forward by Yang and Mills (1954) and by Shaw (1955).

A. Motivation

The charge-independence of nuclear forces and many subsequent observations support the notion of isospin conservation in the strong interactions. What is meant by isospin conservation is that the laws of physics should be invariant under rotations in isospin space. Thus the neutron and proton must appear symmetrically. With electromagnetism "switched off," the difference between them is purely conventional, and so are their names. The ground-state of ^{4}He would imply the existence of two kinds of nucleons, just as the Δ^{++} demonstrated the need for three colors of quark.

The Lagrangian for free nucleons,

$$\begin{aligned}\mathscr{L}_o &= \bar{p}\left(i\gamma^\mu\partial_\mu - m\right)p + \bar{n}\left(i\gamma^\mu\partial_\mu - m\right)n \\ &= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \qquad ,\end{aligned}$$

with the composite spinor

$$\psi \equiv \begin{pmatrix} p \\ n \end{pmatrix} \qquad ,$$

leads to the Dirac equation. The free Lagrangian $\mathscr{L}_o$ has a global invariance under isospin rotations

---

[6]An excellent introduction, at a somewhat higher technical level, is given by Abers and Lee (1973). The problem of building gauge theories upon arbitrary gauge groups is addressed by Gell-Mann and Glashow (1961).

$$\psi \rightarrow \exp\left[\frac{i}{2}\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\right]\psi \qquad ,$$

where $\underset{\sim}{\tau} = (\tau_1, \tau_2, \tau_3)$ are the usual isospin (Pauli) matrices and $\underset{\sim}{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ is an arbitrary global gauge parameter. The invariance of the Lagrangian under global isospin rotations ($\delta\mathscr{L} \equiv 0$) implies the existence of a conserved isospin current density

$$\underset{\sim}{J}_\mu = \bar{\psi}\underset{\sim}{\tau}\gamma_\mu\psi \qquad .$$

In analogy with electromagnetism we are led to ask whether we can require that the freedom to name the two nucleon states be available independently at every space-time point. Can we, in other words, turn the global SU(2) invariance of the free field theory into local SU(2) invariance?

B. Construction

The construction of the theory proceeds just as in the Abelian case. If under a local gauge transformation the field transforms as

$$\psi(x) \rightarrow \psi'(x) = G(x)\psi(x) \qquad ,$$

with

$$G(x) \equiv \exp\left[\frac{i}{2}\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}(x)\right] \qquad ,$$

then the gradient transforms as

$$\partial_\mu\psi \rightarrow G(\partial_\mu\psi) + (\partial_\mu G)\psi \qquad .$$

To ensure the local gauge invariance of the theory, we first construct a gauge-covariant derivative

$$\mathscr{D}_\mu = I\partial_\mu - igB_\mu \qquad ,$$

where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

serves as a reminder that the operators are 2×2 matrices in isospin space. The object B_μ is the 2×2 matrix defined by

$$B_\mu = \tfrac{1}{2}\underset{\sim}{\tau} \cdot \underset{\sim}{b}_\mu \quad ,$$

where the three gauge fields are $\underset{\sim}{b}_\mu = (b_1, b_2, b_3)$, and g is the strong-interaction coupling constant.

The point of introducing the gauge fields and the gauge-covariant derivative is to obtain a generalization of the gradient which transforms as

$$\mathscr{D}_\mu\psi \rightarrow \mathscr{D}_\mu{}'\psi' = G(\mathscr{D}_\mu\psi) \quad .$$

Requiring this to be so will show us how B_μ must behave under gauge transformations. By explicit computation we have

$$\begin{aligned} \mathscr{D}_\mu{}'\psi' &= \partial_\mu\psi' - igB_\mu'\psi' \\ &= G(\partial_\mu\psi) + (\partial_\mu G)\psi - igB_\mu'(G\psi) \\ &\equiv G(\partial_\mu\psi) - igG(B_\mu\psi) \quad , \end{aligned}$$

which may be solved to yield the condition

$$-igB_\mu'G\psi = -igG(B_\mu\psi) - (\partial_\mu G)\,\psi \quad ,$$

which must hold for arbitrary values of the nucleon field ψ. Regarding the transformation condition as an operator equation and multiplying on the right by G^{-1}, we obtain

$$\begin{aligned} B_\mu' &= GB_\mu G^{-1} - \frac{i}{g}\,(\partial_\mu G)G^{-1} \\ &= G\left(B_\mu - \frac{i}{g}\,G^{-1}(\partial_\mu G)\right)G^{-1} \quad . \end{aligned}$$

While this transformation law may appear formidable at first sight, it has a very simple interpretation. Recall that in the case of electromagnetism the local gauge transformation was the phase rotation

$$G_{EM} = e^{iq\theta(x)} \quad .$$

A transcription of the general transformation law is therefore

$$\begin{aligned} A'_\mu &= GA_\mu G^{-1} - \frac{i}{q}(\partial_\mu G)G^{-1} \\ &= A_\mu - \frac{i}{q}\cdot iq(\partial_\mu\theta) = A_\mu + \partial_\mu\theta \quad , \end{aligned}$$

just as before. For the case of isospin gauge symmetry the meaning of the transformation condition is that B_μ is transformed by an isospin rotation plus a gradient term.

To this point in our construction of the gauge theory we have a Lagrangian given by

$$\begin{aligned} \mathscr{L} &= \bar{\psi}\left(i\gamma^\mu\mathscr{D}_\mu - m\right)\psi \\ &= \mathscr{L}_o - g\bar{\psi}\gamma^\mu B_\mu\psi \\ &= \mathscr{L}_o - \frac{g}{2}\underset{\sim}{b}_\mu\cdot\bar{\psi}\gamma^\mu\underset{\sim}{\tau}\psi \quad , \end{aligned}$$

namely a free Dirac Lagrangian plus an interaction term which couples isovector gauge fields to the (conserved) isospin current of the nucleons. To proceed further, we must construct a field-strength tensor and hence a kinetic energy term for the gauge fields.

In the case of electromagnetism we had

$$\begin{aligned} F_{\mu\nu} &= \frac{1}{iq}[\mathscr{D}_\mu, \mathscr{D}_\nu] \\ &= \partial_\nu A_\mu - iqA_\nu A_\mu - \partial_\mu A_\nu + iqA_\mu A_\nu \\ &= \partial_\nu A_\mu - \partial_\mu A_\nu - iq[A_\nu, A_\mu] \quad . \end{aligned}$$

The commutator of the two vector potentials vanishes for electromagnetism (because it is a theory based upon an Abelian gauge symmetry), and we recover the familiar classical definition.

For the SU(2)--and indeed general--gauge theory, we similarly define

$$F_{\mu\nu} = \frac{1}{ig}\,[\mathscr{D}_\mu, \mathscr{D}_\nu]$$

$$= \partial_\nu B_\mu - \partial_\mu B_\nu - ig[B_\nu, B_\mu] \quad .$$

Because the gauge symmetry is non-Abelian, the commutator of gauge fields does not vanish in general. It is easy to verify that $F_{\mu\nu}$ so defined is a locally gauge-invariant form and that

$$\mathscr{L}_{YM} = -\tfrac{1}{2} \underset{(\text{isospin})}{\text{trace}} F^{\mu\nu}F_{\mu\nu}$$

is an acceptable kinetic energy term. As in the Abelian case, no mass term of the form

$$-m^2 \underset{(\text{isospin})}{\text{trace}} B^\mu B_\mu$$

is compatible with local gauge invariance.

The final step in the construction of the gauge theory is to determine from the $F_{\mu\nu}F^{\mu\nu}$ term in the Lagrangian the interactions among gauge bosons. For the moment it will be sufficient to do this in a rather schematic fashion. In QED, only bilinear combinations of the gauge field occur in $\mathscr{L}_\gamma$. Thus we have only

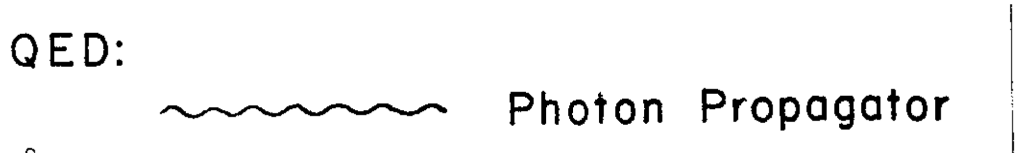


Fig. 8

This reflects the well-known fact that sourceless QED is a free (noninteracting) field theory. In the SU(2) gauge theory, in contrast, trilinear and quadrilinear terms also appear in $\mathscr{L}_{YM}$. We therefore have

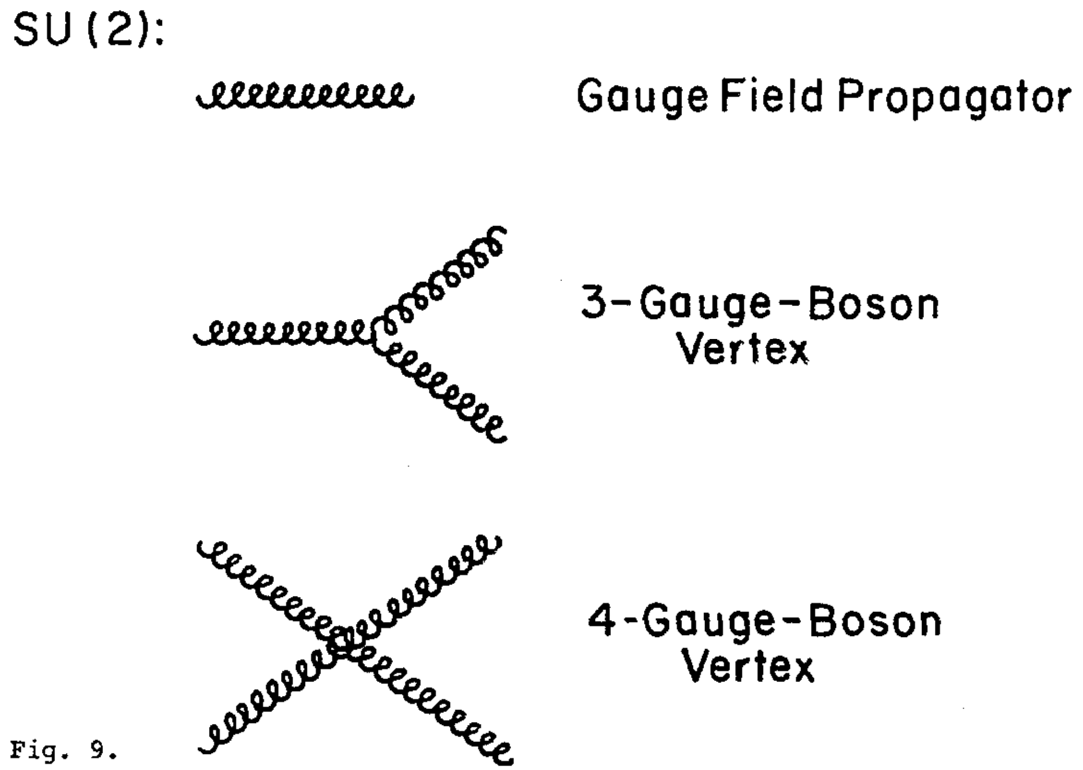


Fig. 9.

The presence of the interaction terms, or vertices, is a signal that sourceless Yang-Mills is a nonlinear (interacting) field theory. The gauge fields carry isospin and hence couple among themselves. The nonlinearity $[B_\nu, B_\mu]$ arises from the non-Abelian nature of the gauge group.

C. Conclusions

Several examples have shown how gauge principles may be used to guide the construction of theories. Global gauge invariance implies the existence of a conserved current. Local gauge invariance produces massless vector gauge bosons, prescribes (or at least restricts) the form of the interactions of gauge bosons with sources, and generates interactions among the gauge bosons, if the symmetry is non-Abelian (see Problem 6).

It is appealing to try to make use of observed symmetries of nature as gauge symmetries. This is indeed the course we shall follow in our later applications. However, the example of the Yang-Mills theory shows that success--in terms of agreement with experimental reality--is not assured in advance. Long-range forces between nucleons, mediated by massless vector quanta, are not observed. Therefore the Yang-Mills theory, based on the idea of isospin invariance (or flavor symmetry) as the strong-interaction gauge symmetry cannot be correct, as a specific theory of strong interactions. We could of course reinterpret the theory just constructed as a theory of weak interactions, based on the "weak-isospin" symmetry apparent (d'Espagnat and Prentki, 1962; Salam, 1962) in nuclear beta-decay. This interpretation too would founder on the prediction of massless gauge fields. Before returning to specific applications to the fundamental interactions we shall have to understand how to evade the prediction of massless gauge bosons while preserving the local gauge invariance of the Lagrangian. This is the subject of the next chapter.

4. SPONTANEOUS SYMMETRY BREAKING[7]

In this chapter, we distinguish among various types of symmetries: Poincaré invariance vs. internal symmetries, continuous vs. discrete symmetries, and exact vs. approximate symmetries. Among approximate symmetries, several different realizations are possible. The Lagrangian may have an imperfect (or explicitly broken) symmetry, or the Lagrangian may be symmetric but have a physical vacuum which does not respect the symmetry. In the latter case, the symmetry of the Lagrangian is said to be spontaneously broken.

Our concern here will be the conditions under which a symmetry is spontaneously broken and the consequences of spontaneous symmetry breaking. We shall find that if a theory has an exact, continuous symmetry which is not a symmetry of the physical vacuum, one or more massless particles, known as Goldstone bosons, must occur. If the spontaneously-broken symmetry is a local gauge symmetry, a miraculous interplay between the would-be Goldstone boson and the normally massless gauge bosons endows the gauge bosons with mass. The Higgs mechanism, by which this interplay occurs, is central to the current understanding of the intermediate bosons of the weak interactions.

A. The Idea of Spontaneously-broken Symmetries

The physical world manifests a number of apparently exact conservation laws which we believe reflect the operation of exact symmetries of nature. These include the conservation of energy and momentum, of angular momentum, and of electric charge. Among the so-called internal symmetries not explicitly related to Poincaré invariance are many useful approximate symmetries, such as isospin invariance, conservation of strangeness and charm, SU(3) invariance, etc. It is usual to treat these approximate symmetries by writing the Lagrangian as

$$\mathscr{L} = \mathscr{L}_{\text{symmetric}} + \delta\mathscr{L}_{\text{symmetry breaking}} \quad .$$

This form is particularly useful if $\delta\mathscr{L}_{SB}$ is small, in some sense, compared to $\mathscr{L}_S$, so that the symmetry-breaking may be treated as a perturbation. A familiar example is

$$\mathscr{L} = \mathscr{L}_{\text{strong}} + \mathscr{L}_{\text{EM}} \quad ,$$

[7] A comprehensive review of the material in this chapter is given by Bernstein (1974). See also Abers and Lee (1974). Dynamical mechanisms for spontaneous symmetry breaking were first discussed by Nambu and Jona-Lasinio (1961) and by Schwinger (1962a, b).

in which the strong-interaction Lagrangian is isospin invariant and responsibility for isospin violations is ascribed to the electromagnetic term $\mathscr{L}_{EM}$.

We saw by example in Chapter 2 how continuous symmetries of the Lagrangian lead to exact conservation laws. Approximate conservation laws may arise if the Lagrangian is imperfectly symmetric. It may also happen that the Lagrangian is exactly invariant under some symmetry, but the physical vacuum is not. This leads to exact conservation laws, but conceals the symmetry of the theory.

To see how this second situation may come about, let us consider a Lagrangian for a real scalar field ϕ which take the general form

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \quad .$$

How does the nature of the vacuum (and therefore of the particle spectrum) depend upon the effective potential $V(\phi)$? Suppose that the potential is an even function of ϕ,

$$V(\phi) = V(-\phi) \quad .$$

Then the Lagrangian is invariant under the parity transformation

$$\phi \rightarrow -\phi \quad .$$

To enumerate the possibilities, let us consider a potential of the form

$$V(\phi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}|\lambda|\phi^4 \quad .$$

The positive coefficient of the ϕ^4 term is chosen to ensure stability against large oscillations.

Two cases may now be distinguished. Case 1: $\mu^2 > 0$, corresponds to "ordinary" symmetry. With this choice, $V(\phi)$ has a unique minimum at $\phi = 0$, as shown in Fig. 10a, which corresponds to the vacuum state. The particle content of the theory is best examined in the Hamiltonian formalism. It is straightforward to make the transcription; more so in a single space dimension. We write

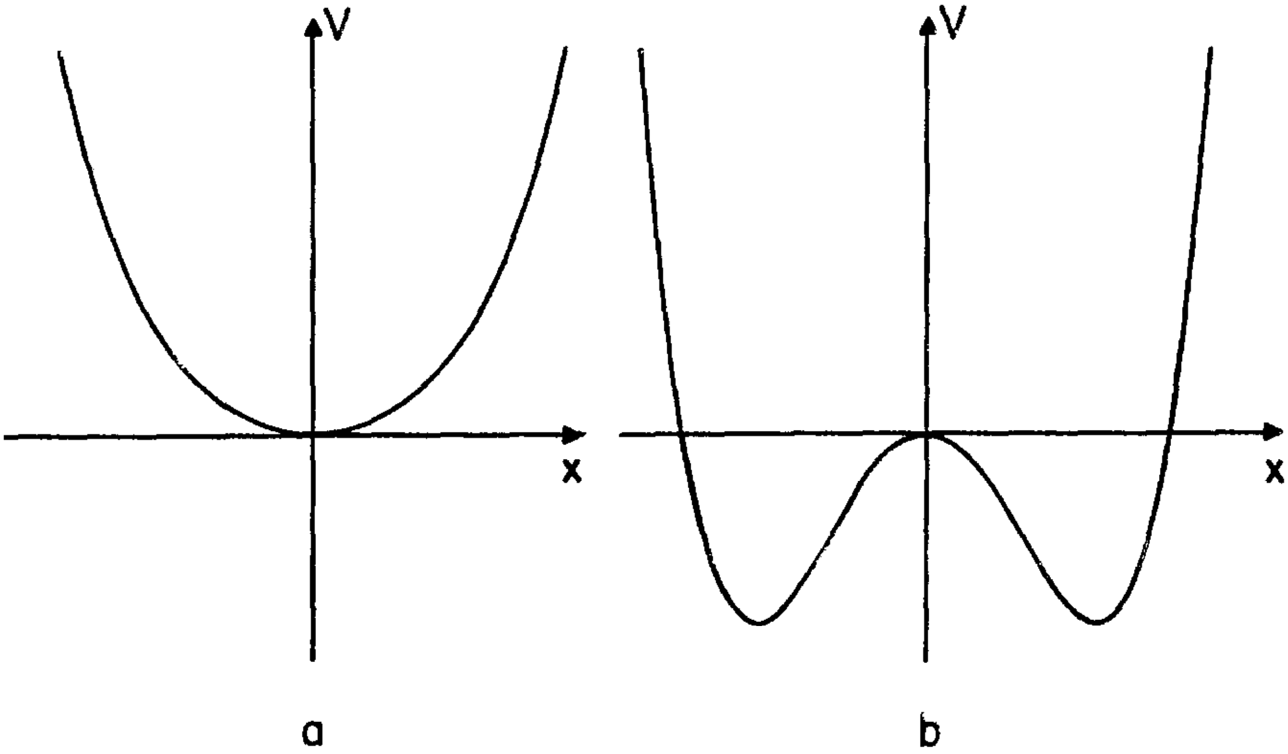


Fig. 10: (a) Ordinary effective potential with a unique minimum at x = 0. (b) Potential with a degenerate vacuum, corresponding to the case of spontaneously broken symmetry.

$$L = \int dx\, \mathscr{L}(x, t) = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial t} \right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x} \right)^2 - V \right] ,$$

and regard $\phi(x, t)$ as a canonical coordinate at each position x. It is convenient to divide space into cells of length ε labelled by x_i, with

$$\varepsilon = x_i - x_{i-1} .$$

Then we define

$$q_i \equiv \phi(x_i, t)$$

and identify the conjugate variable as

$$p_i \equiv dq_i/dt$$

In this notation, the Lagrangian becomes

$$L \rightarrow \varepsilon \sum_i \left\{ \frac{1}{2} p_i^{\,2} - \frac{1}{2\varepsilon^2} (q_i - q_{i-1})^2 - \frac{\mu^2}{2} q_i^{\,2} - \frac{|\lambda| q_i^{\,4}}{4} \right\},$$

and the Hamiltonian looks like

$$H = \varepsilon \sum_i \left\{ \frac{1}{2} p_i^{\,2} + \frac{1}{2\varepsilon^2} (q_i - q_{i-1})^2 + V(q_i) \right\} .$$

This discrete form shows field theory to be a very large collection of Schrödinger equations.[8] The minimum energy configuration is the one for which $q_i = q_{i-1}$ and $V(q_i) = 0$ for all coordinates q_i. The particle spectrum can be deduced by considering small oscillations around the vacuum $q_i = 0$ ($\phi = 0$), for which

$$H \simeq \varepsilon \sum_i \tfrac{1}{2} \{ p_i^{\,2} + \mu^2 q_i^{\,2} \} ,$$

which corresponds to the Hamiltonian of a free particle with $(\text{mass})^2 = \mu^2$.

<u>Case 2</u>: $\mu^2 < 0$, is the situation referred to as spontaneously-broken symmetry. The potential

$$V(\phi) = -\frac{1}{2}|\mu|^2 \phi^2 + \frac{|\lambda| \phi^4}{4} ,$$

shown in Fig. 10(b), has minima at

$$\phi = \pm \sqrt{\frac{-\mu^2}{|\lambda|}} \equiv \pm v ,$$

which correspond to two degenerate lowest-energy states. We may choose either of these (say, $\phi = v$) to be the vacuum. The parity transformation

$$\phi \rightarrow -\phi$$

[8] This formulation is developed at some length by Bjorken (1979).

is then a symmetry of the Lagrangian, but not of the vacuum state.

Define a shifted field

$$\phi' \equiv \phi - v \quad ,$$

so that the vacuum corresponds to $\phi' = 0$. In terms of the shifted field the effective potential is

$$V(\phi') = -\mu^2 \left[\phi'^2 + \frac{1}{v} \phi'^3 + \frac{1}{4v^2} \phi'^4 + \frac{\mu^2}{4|\lambda|} \right] \quad ,$$

and the Hamiltonian appropriate to small oscillations is

$$H \approx \varepsilon \sum_i \left\{ \frac{1}{2} p_i^{\,2} - \mu^2 q_i^{\,2} - \frac{\mu^4}{4|\lambda|} \right\} \quad .$$

This form represents the oscillations of states with $(\text{mass})^2 = -2\mu^2 > 0$, which do not manifest the symmetry of the original Lagrangian in any way.

This simple example has illustrated two important points. First, spontaneous symmetry breaking occurs when an exact symmetry of the Lagrangian is not respected by the vacuum. Second, spontaneous symmetry breaking is totally different in character from explicit symmetry breaking, in which the Lagrangian itself does not respect the symmetry.

B. Spontaneous Breaking of Continuous Symmetries

To make the leap to spontaneous breaking of continuous symmetries, let us consider the simple case of a Lagrangian for two scalar fields ϕ_1 and ϕ_2:

$$\mathscr{L} = \tfrac{1}{2} \left[(\partial^\mu \phi_1)(\partial_\mu \phi_1) + (\partial^\mu \phi_2)(\partial_\mu \phi_2) \right] - V(\phi_1^{\,2} + \phi_2^{\,2}) \quad ,$$

which is invariant under [U(1) or O(2)] rotations

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad .$$

As we have done before, we consider the effective potential

$$V(\phi^2) = \frac{\mu^2}{2}\phi^2 + \frac{|\lambda|}{4}(\phi^2)^2 \quad ,$$

where $\phi^2 \equiv \phi_1^2 + \phi_2^2$, and distinguish two cases.

Case 1: $\mu^2 > 0$ (ordinary symmetry). The vacuum occurs at $\phi_1 = \phi_2 = 0$, and for small oscillations the Lagrangian takes the form

$$\mathscr{L}_{\text{S.O.}} = \tfrac{1}{2}\left[(\partial^\mu\phi_1)(\partial_\mu\phi_1) - \mu^2\phi_1{}^2\right] + \tfrac{1}{2}\left[(\partial^\mu\phi_2)(\partial_\mu\phi_2) - \mu^2\phi_2{}^2\right] .$$

We recognize this immediately as the Lagrangian for two scalar particles of $(\text{mass})^2 = \mu^2$, which is to say a degenerate multiplet (doublet), in accord with our simplest expectations.

Case 2: $\mu^2 < 0$ (spontaneously broken symmetry). In this case, the absolute minimum of V occurs for

$$\phi_1{}^2 + \phi_2{}^2 = -\mu^2/|\lambda| \equiv v^2 \quad ,$$

which corresponds to a continuum of distinct vacuum states with identical energy. The degeneracy is a consequence of the O(2) symmetry of the potential. Let us choose as the physical vacuum state the configuration

$$\phi_1 = v \quad , \quad \phi_2 = 0 \quad .$$

(This can always be achieved by a suitable choice of coordinates.) Expanding about the vacuum configuration by defining

$$\phi_1' \equiv \eta = \phi_1 - v \quad ,$$

$$\phi_2' \equiv \xi = \phi_2 - 0 \quad ,$$

we obtain the Lagrangian for small oscillations

$$\mathscr{L}_{SO} \approx \frac{1}{2}\left[(\partial^{\mu}\eta)(\partial_{\mu}\eta) + (\partial^{\mu}\xi)(\partial_{\mu}\xi)\right] + \frac{\mu^4}{4|\lambda|}$$

$$+ \mu^2\eta^2 + 0 \bullet \xi^2 + \ldots \quad .$$

There are two particles in the spectrum. The η-particle, which corresponds to radial oscillations, has $(\text{mass})^2 = -2\mu^2 > 0$, while the ξ-particle, which corresponds to angular oscillations, is massless. The masslessness of ξ is a consequence of the O(2)-invariance of the Lagrangian, which means that there is no restoring force against angular oscillations. In contrast, the mass of the η-particle is a consequence of trying to displace the η against the restoring force of the potential.

This splitting of the spectrum is an example of Goldstone's Theorem (Goldstone, 1961; Goldstone, et al., 1962; Gilbert, 1964), according to which if a theory has an exact continuous symmetry of the Lagrangian which is not a symmetry of the vacuum, a massless particle must occur (see Problem 7).

C. The Higgs Mechanism

We next consider Lagrangians with spontaneously broken symmetries which also possess local gauge invariance. There will emerge a miraculous interplay between the massless gauge fields (such as made the Yang-Mills theory an unacceptable description of nucleons) and the massless scalar Goldstone particles (which are also uncommon in particle physics). The simplest example[9] of the Higgs phenomenon is provided by the Abelian Higgs model, a U(1)-invariant scalar field theory with an Abelian gauge field that describes the electrodynamics of charged scalars (Higgs, 1964a, 1967). The Lagrangian is

$$\mathscr{L} = \frac{1}{2}(\mathscr{D}^{\mu}\phi)^*(\mathscr{D}_{\mu}\phi) - \frac{\mu^2}{2}\phi^*\phi$$

$$- \frac{|\lambda|}{4}(\phi^*\phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad ,$$

where

$$\mathscr{D}_{\mu} = \partial_{\mu} - iqA_{\mu} \quad ,$$

$$F_{\mu\nu} = \frac{1}{iq}[\mathscr{D}_{\mu}, \mathscr{D}_{\nu}] \quad ,$$

[9] See the interesting review article by Linde (1979). Compare Problem 8.

and

$$\phi = \phi_1 + i\phi_2 \quad .$$

The Lagrangian is invariant under U(1) transformations

$$\phi \rightarrow \phi' = e^{i\theta}\phi \quad ,$$

and under the local gauge transformations

$$\phi(x) \rightarrow \phi'(x) = e^{iq\alpha(x)}\phi(x) \quad ,$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\alpha(x) \quad .$$

As usual, we have two cases, depending upon the parameters of the potential.

<u>Case 1</u>: $\mu^2 > 0$ (ordinary symmetry) leads to ordinary QED of charged scalars, with

1 massless photon

2 scalars ($\phi^\pm$) with $(\text{mass})^2 = \mu^2$.

<u>Case 2</u>: $\mu^2 < 0$ (spontaneously broken symmetry) requires a closer analysis. We must shift the fields to rewrite $\mathscr{L}$ in terms of displacements from the physical vacuum at $|\phi| = v$. Let

$$\phi = e^{i\xi/v}(v + \eta)$$

$$\approx v + \eta + i\xi \quad .$$

Then the Lagrangian appropriate for the study of small oscillations is

$$\mathscr{L}_{SO} = \tfrac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) + \tfrac{1}{2}(\partial^\mu\xi)(\partial_\mu\xi) - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$+ \mu^2\eta^2 + 0 \cdot \xi^2$$
$$- qvA_\mu(\partial^\mu\xi) + \frac{q^2v^2}{2} A^\mu A_\mu + \ldots \quad .$$

The η field has a $(\text{mass})^2 = -2\mu^2 > 0$, as expected. The gauge field A_μ appears to have acquired a mass, but is mixed up in the penultimate term with the seemingly massless ξ-field.

To see what is really going on, it is convenient to write the (ξ, A_μ) pieces as

$$\frac{q^2v^2}{2}\left(A^\mu - \frac{1}{qv}\partial^\mu\xi\right)\left(A_\mu - \frac{1}{qv}\partial_\mu\xi\right) \quad ,$$

a form which pleads for a gauge transformation corresponding to

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{qv}\partial_\mu\xi(x)$$
$$\phi \rightarrow \phi' = e^{-i\xi(x)/v}\phi(x) = v + \eta \quad .$$

Knowing that $\mathscr{L}$ is locally gauge invariant, we may return to the definition to compute

$$\mathscr{L}_{S.O.} = \frac{\mu^4}{4|\lambda|} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad .$$
$$+ \frac{q^2v^2}{2}A'_\mu A'^\mu + \frac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) + \mu^2\eta^2$$

The particle spectrum is now manifest:

- an η-field with $(\text{mass})^2 = -2\mu^2 > 0$;
- a massive vector field A'_μ, with $(\text{mass})^2 = q^2v^2 > 0$;
- no ξ-field.

Thanks to our choice of gauge, the ξ-particle has disappeared entirely! Where did it go? The gauge transformation

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{qv}\partial_\mu \xi$$

shows that what formerly was the ξ-field is reponsible for the longitudinal components of the A'_μ field. Before spontaneous symmetry breaking we had

$$\begin{array}{l} 2 \text{ scalars } (\phi^{\pm}) \\ \underline{+\ 2} \text{ helicity states of } A_\mu \\ 4 \text{ particle states} \end{array} \quad .$$

After spontaneous symmetry breaking, we are left with

$$\begin{array}{l} 1 \text{ scalar } (\eta) \\ \underline{+\ 3} \text{ helicity states of } A'_\mu \\ 4 \text{ particle states} \end{array} \quad .$$

It is commonly said that the massless photon "ate" the massless Goldstone boson to become a massive vector boson. The remaining massive scalar (η) is known as the Higgs boson. The gauge in which this became transparent is known as the unitary gauge (U-gauge), because only physical fields appear in the Lagrangian.

D. Spontaneous Breakdown of a Non-Abelian Gauge Symmetry

To approach the additional complications that attend the spontaneous breakdown of a non-Abelian gauge symmetry we choose as a useful prototype an SU(2) gauge theory and study scalar fields that make up the triplet representation:

$$\underset{\sim}{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad .$$

We shall construct a theory which is invariant under the gauge transformation

$$\underset{\sim}{\phi} \rightarrow \underset{\sim}{\phi}' = \exp\,[i\underset{\sim}{T}\cdot\underset{\sim}{\alpha}]\underset{\sim}{\phi} \qquad .$$

The exponential factor is a 3 × 3 matrix. The operator T_i generates isospin rotations about the i-axis, and satisfies the usual SU(2) algebra

$$[T_i, T_j] = i\varepsilon_{ijk}T_k \qquad .$$

It has the explicit form

$$T^i_{jk} = -i\varepsilon_{ijk} \qquad .$$

As usual the covariant derivative takes the form

$$\begin{aligned}\mathscr{D}_\mu &= I\partial_\mu - ig\underset{\sim}{T}\cdot\underset{\sim}{A}_\mu \\ &= \delta_{mn}\partial_\mu - g\varepsilon_{\ell mn}A^\ell_\mu \qquad ,\end{aligned}$$

and the Lagrangian is

$$\mathscr{L} = \tfrac{1}{2}(\mathscr{D}^\mu\underset{\sim}{\phi})^*\cdot(\mathscr{D}_\mu\underset{\sim}{\phi}) - V(\underset{\sim}{\phi}^*\cdot\underset{\sim}{\phi}) - \tfrac{1}{4}\underset{\sim}{F}_{\mu\nu}\cdot\underset{\sim}{F}^{\mu\nu} \qquad .$$

When $\underset{\sim}{\phi} = 0$ is a minimum of the effective potential V, we have an ordinary, isospin conserving, gauge-invariant Yang-Mills field theory. Of more interest to us at the moment is the spontaneously broken case, in which we choose the value of $\underset{\sim}{\phi}$ that minimizes $V(\underset{\sim}{\phi}^*\cdot\underset{\sim}{\phi})$ as

$$\underset{\sim}{\phi}_o = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \qquad .$$

We shift the fields and expand about the minimum configuration:

$$\underset{\sim}{\phi} = \exp\left\{\frac{i}{v}\,(\xi_1T_1 + \xi_2T_2)\right\}\begin{pmatrix} 0 \\ 0 \\ v+\eta \end{pmatrix} \qquad .$$

As in the Abelian Higgs model, we make use of gauge invariance and transform to U-gauge, by letting

$$\underset{\sim}{\phi} \rightarrow \underset{\sim}{\phi}' = \exp\left\{-\frac{i}{v}(\xi_1 T_1 + \xi_2 T_2)\right\}\underset{\sim}{\phi}$$

$$= \begin{pmatrix} 0 \\ 0 \\ v+\eta \end{pmatrix} \quad .$$

In the new gauge, the Lagrangian for small oscillations is

$$\mathscr{L}_{SO} \approx \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) + \mu^2\eta^2 - \frac{1}{4}\underset{\sim}{F}_{\mu\nu} \cdot \underset{\sim}{F}^{\mu\nu}$$
$$+ \frac{g^2v^2}{2}\left[A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu}\right] + \ldots \quad .$$

In this form the Lagrangian reveals that

- ξ_1 and ξ_2 have disappeared entirely, i.e. they have been "gauged away;"
- η has become a massive Higgs scalar with $(\text{mass})^2 = -2\mu^2 > 0$;
- the vector bosons corresponding to the (broken symmetry) generators T_1 and T_2 acquire $(\text{mass})^2 = -g^2\mu^2/\lambda > 0$;
- the Lagrangian (and the vacuum) remain invariant under T_3, and the corresponding gauge boson A^3_μ remains massless.

Again let us summarize what has happened to the theory. In the case of ordinary symmetry, we had

3 massive scalars $[(\text{mass})^2 = \mu^2]$

3 massless vector gauge bosons
(×2 helicity states)

9 degrees of freedom .

After spontaneous symmetry breaking, the particle spectrum consists of

1 Higgs scalar $[(\text{mass})^2 = -2\mu^2 > 0]$

1 massless gauge boson (×2 helicity states)

2 massive gauge bosons (×3 helicity states)

9 degrees of freedom .

In the absence of the local gauge symmetry, spontaneous symmetry breaking would have led to one massive scalar plus two Goldstone bosons.

5. THE WEINBERG-SALAM MODEL FOR LEPTONS

The examples of the preceding chapter have shown how spontaneous symmetry breaking can endow the gauge bosons with mass. This suggests a means for constructing a theory of the weak interactions which is based upon local gauge invariance. As usual, the choice of a gauge group is inspired by experiment but there is no guarantee that the theory will have acceptable consequences. In this instance, the first and in many ways simplest gauge theory of the weak and electromagnetic interactions, the Weinberg (1967)-Salam (1968) theory, gives an apparently successful account of all known data. It is this theory that we now construct.[1]

A. Structure of the Theory

We first consider only the electron and its neutrino, which form a left-handed "weak-isospin" doublet

$$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L ,$$

where

$$\nu_L = \tfrac{1}{2}(1 - \gamma_5)\nu ,$$

and

$$e_L = \tfrac{1}{2}(1 - \gamma_5)e .$$

Since the neutrino is apparently massless,

$$\nu_R = \tfrac{1}{2}(1 + \gamma_5)\nu = 0 ,$$

---

[1]For more extensive accounts, see Abers and Lee (1973), Bég and Sirlin (1974), Fradkin and Tyutin (1974), Taylor (1976). Some history of gauge models of the weak and electromagnetic interactions is reviewed by Coleman (1979), Hung and Quigg (1980), Glashow (1980), Salam (1980), and Weinberg (1980). Many technical issues are treated in the 1975 Les Houches Lecture Notes (Balian and Zinn-Justin, 1976).

so we designate only one right-handed singlet,

$$R \equiv e_R = \tfrac{1}{2}(1 + \gamma_5)e \quad .$$

This completes a description of the weak charged currents. To incorporate electromagnetism, we define a "weak hypercharge" Y. Requiring that the Gell-Mann-Nishijima relation

$$Q = I_3 + \tfrac{1}{2}Y$$

be satisfied leads to the assignments

$$Y_L = -1 \quad ,$$

$$Y_R = -2 \quad .$$

By construction, the weak isospin projection I_3 and the weak hypercharge Y are commuting observables,

$$[I_3, Y] = 0 \quad .$$

We now take the group of transformations generated by I and Y to be the gauge group SU(2) ⊗ U(1) of our theory. To construct the theory, we introduce gauge bosons

$$A^1_\mu, A^2_\mu, A^3_\mu \quad \text{for} \quad SU(2)$$

$$B_\mu \quad \text{for} \quad U(1) \quad .$$

The Lagrangian is written as

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{leptons}} \quad ,$$

where

$$\mathscr{L}_{\text{gauge}} = -\tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu} \quad ,$$

the field-strength tensors are

$$F^i_{\mu\nu} = \partial_\nu A^i_\mu - \partial_\mu A^i_\nu + g\varepsilon_{ijk}A^j_\nu A^k_\mu$$

and

$$f_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu \qquad ,$$

and

$$\mathscr{L}_{\text{leptons}} = \bar{R}i\gamma^\mu\left(\partial_\mu - \frac{ig'}{2}B_\mu Y\right)R + \bar{L}i\gamma^\mu\left(\partial_\mu - \frac{ig'}{2}B_\mu Y - \frac{ig}{2}\underset{\sim}{\tau}\cdot\underset{\sim}{A}_\mu\right)L \qquad .$$

The coupling constant for the U(1) gauge symmetry is chosen as $g'/2$, the factor of ½ being chosen to simplify later expressions, and the coupling constant for the SU(2) gauge group is called g.

This is not a satisfactory theory, for two reasons. It contains four massless weak gauge bosons (A^1, A^2, A^3, B^0), whereas nature has only one, the photon. In addition, the local $SU(2)_L$ invariance forbids an electron mass term. How can the theory be modified so there will be only one conserved quantity (the electric charge), and one massless gauge boson (the photon), and the electron will acquire a mass?

To accomplish these things, we introduce a complex doublet of (Higgs) scalars

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad ,$$

which transforms like an SU(2) doublet and must therefore have

$$Y_\phi = +1 \qquad .$$

We add to the Lagrangian a piece

$$\mathscr{L}_{\text{Higgs}} = (\mathscr{D}^\mu\phi)^\dagger(\mathscr{D}_\mu\phi) - V(\phi^\dagger\phi)$$

where as usual

$$V(\phi^\dagger\phi) = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{|\lambda|}{4}(\phi^\dagger\phi)^2 .$$

We are also free to add an interaction term which involves Yukawa couplings of the scalars to the fermions,

$$\mathscr{L}_{\text{inter}} = -G_e[\bar{R}\phi^\dagger L + \bar{L}\phi R] ,$$

which is symmetric under $SU(2)_L \otimes U(1)$ and has an admissible Lorentz structure.

Now let us imagine that $\mu^2 < 0$ and consider the consequences of spontaneous symmetry breaking. We choose

$$\langle\phi\rangle_o = \begin{pmatrix} 0 \\ v \end{pmatrix} ,$$

which breaks both $SU(2)_L$ and $U(1)_Y$, but preserves an invariance under the $U(1)_{EM}$ symmetry generated by the electric charge operator. Recall that a (would-be) Goldstone boson is associated with every generator of the gauge group that does not leave the vacuum invariant. The vacuum is left invariant by a generator $\mathscr{G}$ if

$$e^{i\alpha\mathscr{G}}\langle\phi\rangle_o = \langle\phi\rangle_o .$$

For an infinitesimal transformation, the left-hand side is

$$(1 + i\alpha\mathscr{G})\langle\phi\rangle_o .$$

Thus the condition for $\mathscr{G}$ to leave the vacuum invariant is

$$\mathscr{G}\langle\phi\rangle_o = 0 .$$

For the generators of $SU(2) \otimes U(1)$, we find

$$\tau_1\langle\phi\rangle_o = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0 \quad ,$$

$$\tau_2\langle\phi\rangle_o = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} -iv \\ 0 \end{pmatrix} \neq 0 \quad ,$$

$$\tau_3\langle\phi\rangle_o = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0 \quad ,$$

$$Y\langle\phi\rangle_o = +1\cdot\begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \quad ,$$

but

$$Q\langle\phi\rangle_o = \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_o = 0 \quad .$$

This is promising! Three of the original four generators are broken, but the linear combination corresponding to electric charge is not. The photon will therefore remain massless.

Next, we expand the Lagrangian about the minimum of V by writing

$$\phi = \exp\left\{\frac{-i\underset{\sim}{\xi}\cdot\underset{\sim}{\tau}}{2v}\right\}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \quad ,$$

and transforming at once to U-gauge:

$$\phi \to \phi' = \exp\left\{\frac{i\underset{\sim}{\xi}\cdot\underset{\sim}{\tau}}{2v}\right\}\phi = \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \quad ,$$

$$\underset{\sim}{\tau}\cdot\underset{\sim}{A}_\mu \to \underset{\sim}{\tau}\cdot\underset{\sim}{A}'_\mu \quad ,$$

$$B_\mu \to B_\mu \quad ,$$

$$R \to R \quad ,$$

$$L \to L' = \exp\left\{\frac{i\underset{\sim}{\xi}\cdot\underset{\sim}{\tau}}{2v}\right\} L \quad .$$

[Had we followed the usual procedure literally, we would have replaced the generator τ_3 in the shifted field ϕ by the combination $K = \frac{1}{2}(\tau_3 - Y)$ orthogonal to Q, which is strictly speaking the third broken generator. However, since $\tau_3 = K + Q$ and Q leaves the vacuum invariant, the effect is the same.]

We now may enumerate the consequences of spontaneous symmetry breaking. The Yukawa term in the Lagrangian becomes

$$\mathscr{L}_{\text{inter}} = - G_e v[\bar{e}_R e_L + \bar{e}_L e_R] + \ldots$$

$$= - G_e v\bar{e}e + \ldots \quad ,$$

so the electron has acquired a mass

$$m_e = G_e v \quad .$$

The Higgs term in the Lagrangian is

$$\mathscr{L}_{\text{Higgs}} = \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 + \frac{v^2}{4}\left[(g'B_\mu - gA_\mu^3)^2 + g^2\left[(A_\mu^1)^2 + (A_\mu^2)^2\right]\right] + \ldots \quad .$$

We see at once that the η-field has acquired a $(\text{mass})^2 = -2\mu^2 > 0$; it is the physical Higgs boson. If we define

$$W_\mu^{\pm} = \frac{A_\mu^1 \mp iA_\mu^2}{\sqrt{2}} \quad ,$$

the term proportional to $g^2 v^2$ is recognizable as a mass term for the charged vector bosons:

$$\frac{g^2 v^2}{4}\left[|W_\mu^+|^2 + |W_\mu^-|^2\right] \quad .$$

Thus the masses of the charged intermediate bosons are

$$M_W = |gv|/2 \quad .$$

Finally, defining the orthogonal combinations

$$Z_\mu = \frac{g'B_\mu - gA_\mu^3}{\sqrt{g^2 + g'^2}}$$

and

$$A_\mu = \frac{gB_\mu + g'A_\mu^3}{\sqrt{g^2 + g'^2}} \quad ,$$

we find that the neutral intermediate boson Z^o has acquired a mass

$$\begin{aligned} M_Z &= \sqrt{g^2 + g'^2}\, v/2 \\ &= \sqrt{1 + \frac{g'^2}{g^2}}\, M_W \end{aligned}$$

and that the A_μ field remains a massless gauge boson corresponding to the surviving $\exp\{iQ\theta(x)\}$ symmetry. We have achieved, at least schematically, the desired particle content--plus a massive Higgs scalar we didn't request.

Do the interactions also correspond to those in nature? The interactions among the W-bosons and leptons are of the form

$$\mathscr{L}_{W-\ell} = \frac{g}{\sqrt{2}}\left[\bar{\nu}_L\gamma^\mu e_L W_\ell^+ + \bar{e}_L\gamma^\mu \nu_L W_\mu^-\right] \quad ,$$

which is consistent with the familiar low-energy phenomenology provided we identify

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = 1/2v^2 \quad .$$

Similarly, the neutral-boson couplings to leptons are given by

$$\mathscr{L}_{0-\ell} = \frac{-gg'}{\sqrt{g^2 + g'^2}} A_\mu \bar{e}\gamma^\mu e$$

$$- \tfrac{1}{2}\sqrt{g^2 + g'^2} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$$

$$+ \frac{Z_\mu}{\sqrt{g^2 + g'^2}} \left(\frac{(g^2 - g'^2)}{2} \bar{e}_L \gamma^\mu e_L - g'^2 \bar{e}_R \gamma^\mu e_R \right) \quad .$$

Therefore we may indeed identify A_μ as the photon, provided that we set

$$\frac{gg'}{\sqrt{g^2 + g'^2}} = e \quad .$$

It is convenient to introduce a weak mixing angle θ_W and to parametrize

$$g' = g \tan \theta_W \quad ,$$

so that

$$g = e/\sin \theta_W \geq e \quad ,$$

$$g' = e/\cos \theta_W \geq e \quad ,$$

and

$$\sqrt{g^2 + g'^2} = g/\cos \theta_W \quad .$$

With these definitions, the connections between the $SU(2)_L \otimes U(1)$ gauge fields and the electroweak gauge fields are

$$B_\mu = A_\mu \cos \theta_W + Z_\mu \sin \theta_W \quad ,$$

$$A^3_\mu = A_\mu \sin \theta_W - Z_\mu \cos \theta_W \quad ;$$

or

$$Z_\mu = B_\mu \sin\theta_W - A_\mu^3 \cos\theta_W \quad ,$$

$$A_\mu = B_\mu \cos\theta_W + A_\mu^3 \sin\theta_W \quad ,$$

justifying the designation of θ_W as a weak mixing angle. Taken together, the coupling constant identifications lead to

$$\begin{aligned} M_W^2 &= g^2/4\sqrt{2}G_F = e^2/4\sqrt{2}G_F \sin^2\theta_W \\ &= \pi\alpha/\sqrt{2}G_F \sin^2\theta_W \\ &= (37.4\ \mathrm{GeV}/c^2)^2/\sin^2\theta_W \quad , \end{aligned}$$

and

$$M_Z^2 = M_W^2/\cos^2\theta_W \quad .$$

Notice that the dimensionless Yukawa coupling that endowed the electron with a mass is small:

$$G_e = 2^{3/4} m_e \sqrt{G_F} \approx 3 \times 10^{-6} \quad ,$$

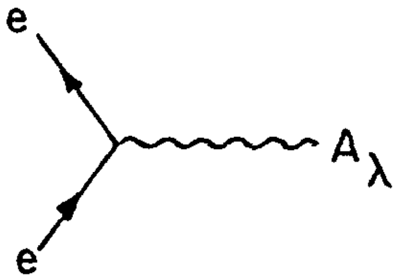
as well as arbitrary. The leptonic weak interactions are summarized by the Feynman rules for vertices given in Figure 11 (Fujikawa, et al., 1972).

B. Properties of the Gauge Bosons[2]

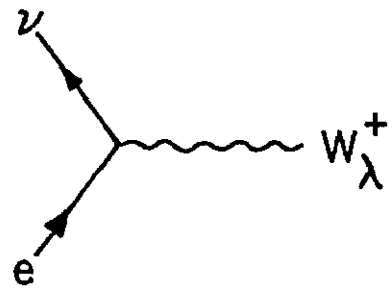
Within the Weinberg-Salam theory, there are definite predictions for the intermediate boson masses in terms of θ_W which may in principle be fixed in neutral current measurements. What can be said about the decays of the intermediate bosons? Let us begin by studying the leptonic decays of $W^\mp$.

In the W^- rest frame, the outgoing lepton momenta are

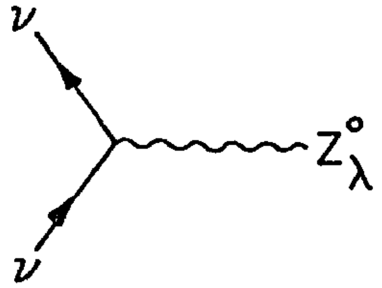
[2]A convenient summary appears in Quigg (1977). See Hung and Quigg (1980) for the evolution of the intermediate boson idea.



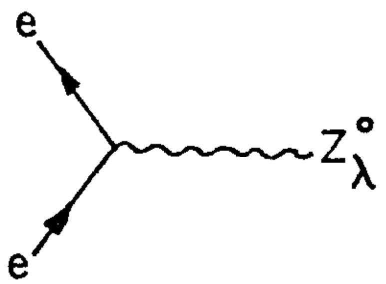
$$-ie\,\bar{e}\gamma_\lambda e$$



$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2}\bar{\nu}\gamma_\lambda(1-\gamma_5)e$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2}\bar{\nu}\gamma_\lambda(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2}\bar{e}\gamma_\lambda\left[c_V+c_A\gamma_5\right]e,$$

$$c_V = 4\sin^2\theta_W - 1,$$

$$c_A = 1$$

Fig. 11: Feynman rules for gauge boson interactions with leptons in the Weinberg-Salam model.

$$p = \frac{M_W}{2}(\sin\theta,\ 0,\ \cos\theta,\ 1)$$

$$q = \frac{M_W}{2}(-\sin\theta,\ 0,\ -\cos\theta,\ 1) \qquad ,$$

where the electron mass has been neglected, and the polarization of the decaying W^- is $\varepsilon_\mu \equiv (\underset{\sim}{\varepsilon},\ 0)$. The matrix element for the decay is

$$i\mathcal{M} = \bar{u}(e,\ p)\gamma^\mu(1 - \gamma_5)v(\nu,\ q)\varepsilon_\mu \cdot \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \qquad ,$$

so that

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \operatorname{tr}\left\{\not{\varepsilon}(1 - \gamma_5)\not{q}(1 + \gamma_5)\not{\varepsilon}^*\not{p}\right\}$$

$$= \frac{G_F M_W^2}{\sqrt{2}} \cdot 2 \operatorname{tr}\left\{(1 + \gamma_5)\not{\varepsilon}\not{q}\not{\varepsilon}^*\not{p}\right\}$$

$$= \frac{G_F M_W^2}{\sqrt{2}} \cdot 8 \left\{(\varepsilon \cdot q)(\varepsilon^* \cdot p) - (\varepsilon \cdot \varepsilon^*)(p \cdot q)\right.$$

$$\left. + (\varepsilon \cdot p)(\varepsilon^* \cdot q) + i\varepsilon_{\kappa\lambda\mu\nu}\varepsilon_\kappa q_\lambda \varepsilon^*_\mu p_\nu\right\} \qquad .$$

The decay rate cannot depend upon ε, so we first choose

$$\varepsilon = (0,\ 0,\ 1,\ 0) = \varepsilon^* \qquad ,$$

for which

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}} \sin^2\theta \qquad .$$

The differential decay rate is

$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 M_W} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta \qquad ,$$

and the partial decay rate

$$\Gamma(W^- \to e^-\bar{\nu}) = \int d\Omega \frac{d\Gamma}{d\Omega} = \frac{G_F M_W^3}{6\pi\sqrt{2}}$$

$$\approx \frac{23 \text{ MeV}}{\sin^3 \theta_W} .$$

For the value of the weak angle currently favored by experiments,[3] $\sin^2 \theta_W = 0.2$, we predict $M_W \approx 84 \text{ GeV}/c^2$ and $\Gamma(W \to e\nu) \approx 250$ MeV.

It is similarly straightforward to compute the decay angular distributions of intermediate bosons with transverse polarizations. The results are summarized in the Table:

helicity	$d\Gamma/d\Omega$
1	$\frac{G_F M_W^3}{32\pi^2\sqrt{2}} (1 - \cos\theta)^2$
0	$\frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$
-1	$\frac{G_F M_W^3}{32\pi^2\sqrt{2}} (1 + \cos\theta)^2$

These angular dependences are easily understood in terms of angular momentum conservation:

[3]Recent summaries, with extensive references to earlier work, include Abbott and Barnett (1978), Hung and Sakurai (1979); and Kim, et al. (1980).

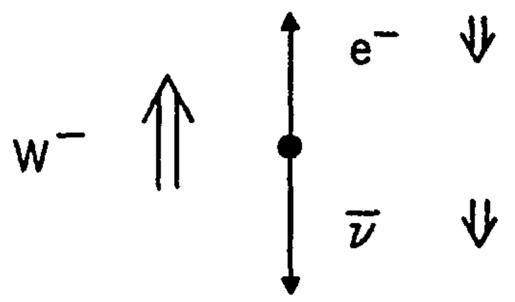


Fig. 12.

$\theta = 0$ is forbidden, but $\theta = \pi$ is OK. The situation is reversed for W^+:

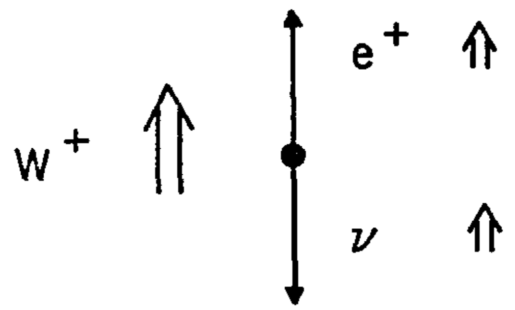


Fig. 13.

$\theta = 0$ is allowed, but $\theta = \pi$ is not. The positron tends to follow the direction of the W^+ polarization, while the electron avoids the direction of the W^- polarization. This is an example of C-violation in the weak interactions.

The leptonic decay rates of the neutral intermediate boson can be computed almost by transcription. We find

$$\Gamma(Z^o \to \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}} = \frac{11.4 \text{ MeV}}{\sin^3\theta_W \cos^3\theta_W} ,$$

$$\Gamma(Z^o \to e\bar{e}) = \Gamma(Z^o \to \nu\bar{\nu}) \times \left[(2\sin^2\theta_W)^2 + (1 - 2\sin^2\theta_W)^2\right] .$$

Thus, for $\sin^2\theta_W = 0.2$ we expect $M_Z \approx 94$ GeV/c^2, $\Gamma(Z^o \to \nu\bar{\nu}) \approx 180$ MeV, and $\Gamma(Z^o \to e\bar{e}) \approx 90$ MeV.

Extension of these calculations to other leptons and to non-leptonic decays can be made effortlessly after we have incorporated quarks and other lepton generations into the theory. The results are described in many places, and we merely refer to them.

C. Neutral Current Interactions

As a prelude to a very brief review of neutral current interactions, let us incorporate additional leptons into the Weinberg-Salam model. This is done merely by cloning the existing structure. We add further weak isospin doublets

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \cdots$$

and right-handed singlets

$$\mu_R \quad \tau_R \quad \cdots$$

with the same weak hypercharge assignments as

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad e_R .$$

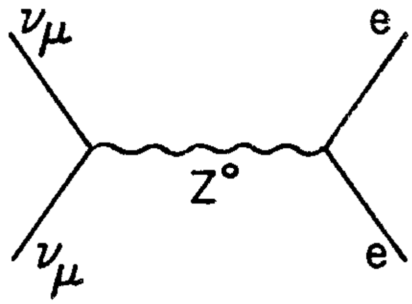
[By omitting right-handed neutrinos, we are continuing to regard the neutrinos as massless.] In addition, we include Yukawa interaction terms

$$\mathscr{L}_{\text{inter}} = -G_i\left[\bar{R}_i\phi^\dagger L_i + \bar{L}_i\phi R_i\right] ,$$

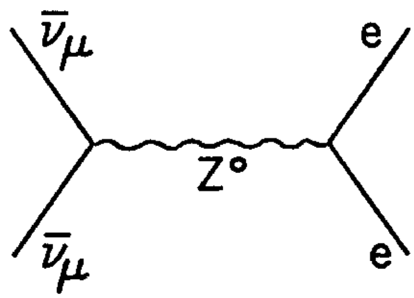
with i = e, μ, τ... This done, we obtain the same Feynman rules for the interactions of μ, ν_μ, τ, ν_τ, ... with γ, $W^\pm$, and Z^0 as we did for e and ν_e, and obtain mass terms for the charged leptons just as for the electron.

The leptonic neutral current interactions for which experiments may be contemplated are measurements of the cross sections for the neutrino-scattering reactions shown in Fig. 14 (see Problem 9),

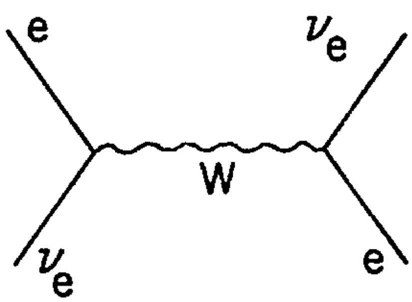
$\nu_\mu \; e \rightarrow \nu_\mu \; e$



$\bar{\nu}_\mu \; e \rightarrow \bar{\nu}_\mu \; e$



$\nu_e \; e \rightarrow e \; \nu_e$



$+ \; Z^\circ$

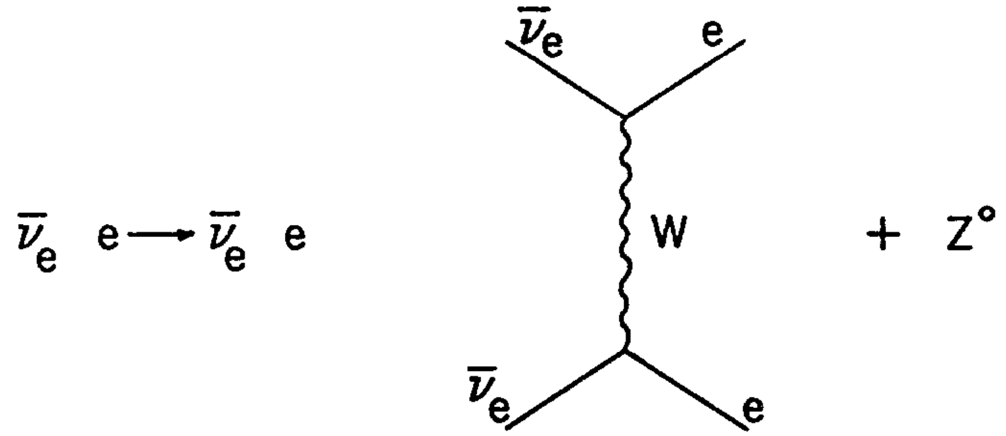


Fig. 14. Neutrino-electron scattering reactions in the Weinberg-Salam model.

together with the observation of $\gamma - Z^0$ interference in the reaction $e^+e^- \rightarrow \mu^+\mu^-$ illustrated in Fig. 15. The cross sections for neutrino-electron scattering ('t Hooft, 1971) are shown in Figure 16 as functions of the weak mixing angle.[4] Existing measurements are all consistent with the value

$$\sin^2\theta_W = 0.23 \pm 0.05 \qquad ,$$

which is in agreement with what is known from semileptonic processes.

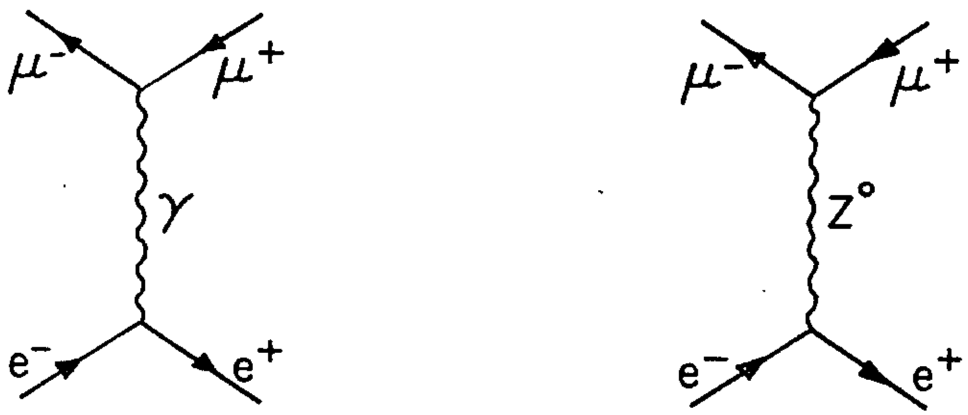


Fig. 15. Contributions to the reaction $e^+e^- \rightarrow \mu^+\mu^-$ in the Weinberg-Salam model.

[4]Details of the computations may be found in the lecture notes by Quigg (1976) and the monograph by Taylor (1976).

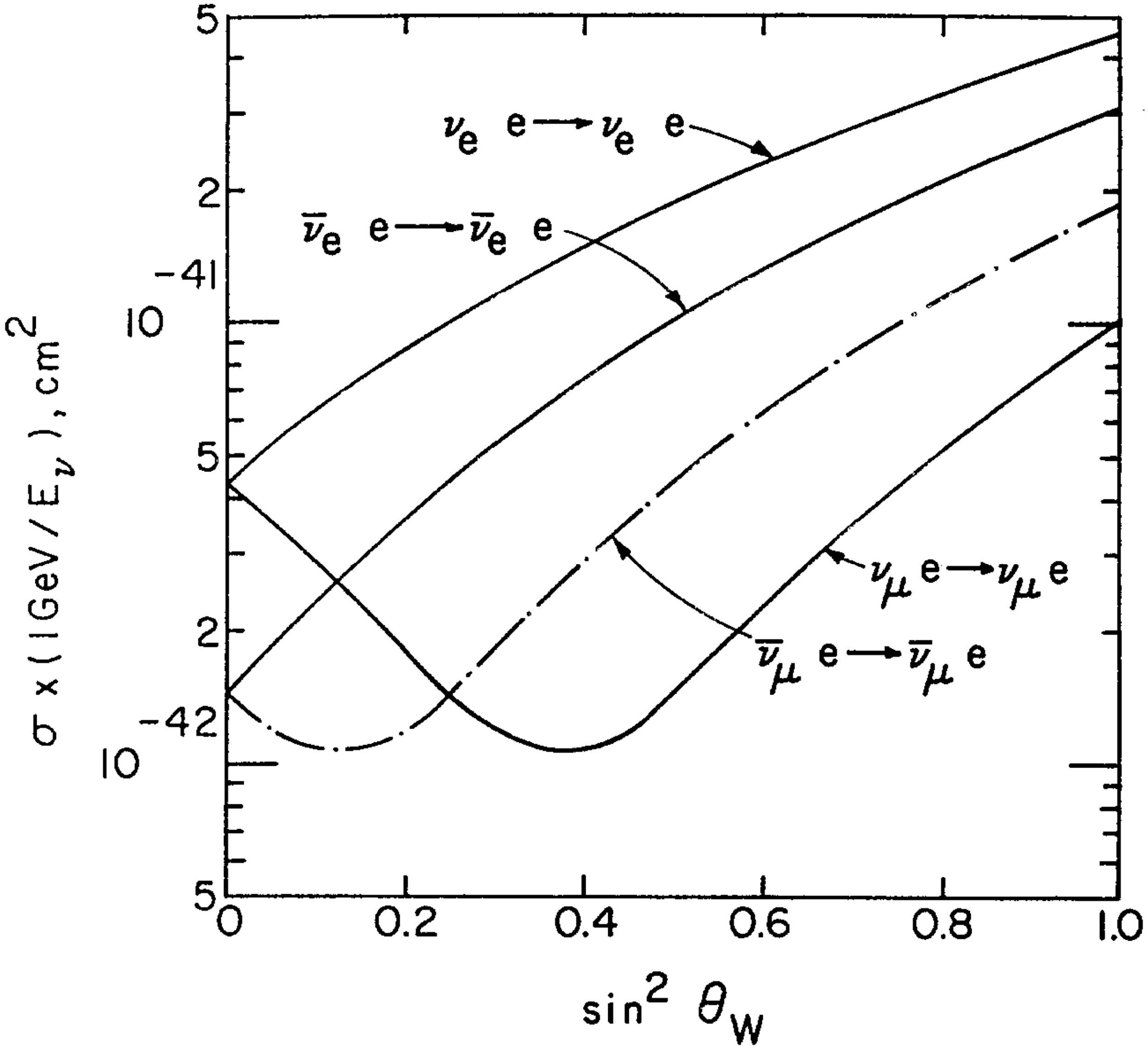


Fig. 16. Cross sections for neutrino-electron scattering in the Weinberg-Salam model.

D. Incorporating Hadrons

Extension of the Weinberg-Salam model to the hadronic sector is accomplished through the medium of the quark model. The necessity of enlarging the spectrum of quarks beyond u, d, and s has already been noted in Chapter 1, where the motivation for the Glashow-Iliopoulos-Maiani (1970) mechanism was reviewed. Rather than repeat here the arithmetic presented there, we shall simply read off from the structure of the Weinberg-Salam model of leptons the Feynman rules for the G-I-M scheme based on

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \quad u_R \quad d_R \quad c_R \quad s_R \quad ,$$

with

$$d_\theta = d \cos \theta_C + s \sin \theta_C \quad ,$$

and

$$s_\theta = s \cos \theta_C - d \sin \theta_C \quad .$$

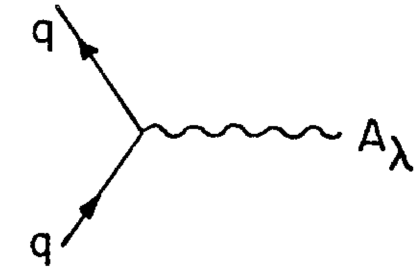
This construction yields the observed universality between leptonic and hadronic charged-current interactions, results in flavor-conserving neutral currents, and has an agreeable symmetry with the lepton sector based upon

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad e_R \quad \mu_R \quad .$$

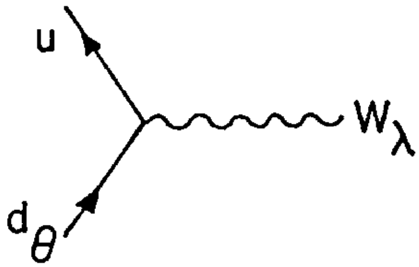
The Feynman rules for interactions between gauge bosons and quarks are presented in Figure 17.

The flavor-conserving neutral current property is easily generalized to the case of many quark generations. This is of more than academic interest because of the observation of the fifth quark (b-quark) in the Υ family of meson resonances, not to mention the existence of the charged lepton $\tau(1782)$. Suppose that there are n left-handed quark doublets

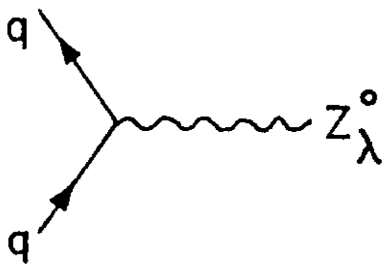
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \cdots \quad ,$$



$$i\, e_q\, \bar{q}\gamma_\lambda\, q$$



$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{u}\gamma_\lambda(1-\gamma_5)\,d_\theta$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{q}\gamma_\lambda(c_V + c_A\,\gamma_5)\,q$$

$$c_V = \tau_3 - 4e_q \sin^2\theta_W$$

$$c_A = -\tau_3$$

Fig. 17: Feynman rules for gauge boson interactions with quarks in the Weinberg-Salam model.

where the primes represent mixing among the charge-1/3 quarks. We write all of the quarks in terms of a composite (2n-component) spinor

$$\Psi = \begin{pmatrix} \vdots \\ t \\ c \\ u \\ \hline d \\ s \\ b \\ \vdots \end{pmatrix}$$

and express the charged current as

$$J_\lambda^{(+)} \sim \bar{\Psi}\gamma_\lambda(1 - \gamma_5)\mathscr{O}\Psi \qquad ,$$

where the $(2n \times 2n)$ matrix $\mathscr{O}$ is of the form

$$\mathscr{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$$

and U is the unitary, $(n \times n)$ matrix that describes quark mixing. The weak isospin contribution to the neutral current is

$$J_\lambda^{(3)} \sim \bar{\Psi}\gamma_\lambda(1 - \gamma_5)[\mathscr{O}, \mathscr{O}^\dagger]\Psi \qquad ,$$

but since

$$[\mathscr{O}, \mathscr{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad ,$$

the neutral current will be flavor diagonal.

As was the case for the leptons, fermion mass generation by the Higgs mechanism is both possible (which is a virtue) and completely *ad hoc* (which is not). Let us show this explicitly for the two-generation case. We assign

$$L_1 = \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad , \quad L_2 = \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

as $SU(2)_L$ doublets with weak hypercharge $Y_L = 1/3$, and

$$u_R, \quad d_R, \quad c_R, \quad s_R$$

as $SU(2)_L$ singlets with $Y(u, c) = 4/3$, $Y(d, s) = -2/3$. The Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$$

transforms as an SU(2) doublet with $Y = 1$, while the charge-conjugate doublet

$$\overline{\phi} = \begin{pmatrix} \overline{\phi}^o \\ -\phi^- \end{pmatrix}$$

is an SU(2) doublet with $Y = -1$. The most general gauge-invariant Higgs boson-fermion interaction is

$$\begin{aligned} \mathscr{L}_{\text{inter}} = \; & G_1[\overline{L}_1\overline{\phi}u_R + \text{h.c.}] + G_2[\overline{L}_1\phi d_R + \text{h.c.}] \\ & + G_3[\overline{L}_1\phi s_R + \text{h.c.}] + G_4[\overline{L}_2\overline{\phi}c_R + \text{h.c.}] \\ & + G_5[\overline{L}_2\phi d_R + \text{h.c.}] + G_6[\overline{L}_2\phi s_R + \text{h.c.}] \quad . \end{aligned}$$

The other conceivable terms, $\overline{L}_1\overline{\phi}c_R$ and $\overline{L}_2\overline{\phi}u_R$, vanish identically. Replacing ϕ by its vacuum expectation value

$$\langle\phi\rangle_o = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad ,$$

we obtain mass terms. The Yukawa couplings $G_1 \ldots G_6$ must be chosen so that u, d, s, c are mass eigenstates with the correct masses:

$$G_1 = m_u/v \quad , \quad G_2 = m_d \cos\theta_C/v \quad ,$$

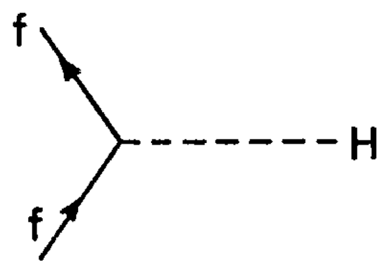
$$G_3 = m_s \sin\theta_C/v \quad , \quad G_4 = m_c/v \quad ,$$

$$G_5 = -G_2 \tan\theta_C \quad , \quad G_6 = G_3 \cot\theta_C \quad .$$

We are evidently at liberty to do this. Clearly any symmetry principle which relates the G_i may lead to connections between the quark masses and the Cabibbo (mixing) angle.

E. The Higgs Boson

With the G_i adjusted to reproduce the fermion mass spectrum, we may read off the Feynman rule for Higgs boson-fermion interactions:



$$\frac{-im}{v} = -im\,(G_F\sqrt{2})^{1/2}$$

Fig. 18.

The amplitude for Higgs decay into a fermion-antifermion pair is then simply

$$i\mathcal{M} = -im(G_F\sqrt{2})^{\frac{1}{2}}\bar{u}(p_1)v(p_2) \quad ,$$

where (neglecting the fermion masses)

$$p_1 = \tfrac{1}{2}M_H(0,\ 0,\ 1,\ 1) \quad ,$$

$$p_2 = \tfrac{1}{2}M_H(0,\ 0,\ -1,\ 1) \quad .$$

We then have

$$|\mathcal{M}|^2 = G_F m^2 \sqrt{2}\, \mathrm{tr}(\not{p}_1 \not{p}_2)$$

$$= \frac{4 G_F m^2 M_H^2}{\sqrt{2}} \quad ,$$

which implies an isotropic decay angular distribution

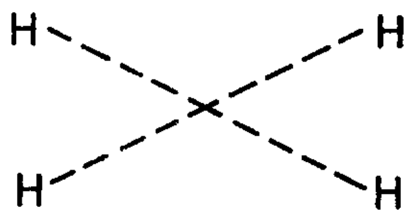
$$\frac{d\Gamma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 M_H} = \frac{G_F m_f^2 M_H}{16\pi^2\sqrt{2}} \quad ,$$

and a total decay rate

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \quad .$$

The dominant decay of a light ($M_H < 2M_W$) Higgs boson is therefore into pairs of the most massive fermion which is kinematically accessible.[5]

Nothing in the Weinberg-Salam theory specifies the mass of the Higgs boson, and nothing we have done depends in any direct way upon the value of this parameter. However the couplings of the Higgs scalar to gauge bosons are determined by the Higgs mass, as in the case of



$$\frac{-\lambda}{4} = \frac{G_F M_H^2}{2\sqrt{2}}$$

Fig. 19.

---

[5]The properties of a light Higgs boson are elaborated by Ellis, et al. (1976), and by Gaillard (1978).

Because of this, the overall consistency of the theory may imply some restrictions upon the Higgs mass.[6] The manner in which this comes about may be seen most simply by examining the behavior of the theory at high energies (Lee, et al., 1977a, b; Dicus and Mathur, 1973).

To begin, let us recall the familiar unitarity argument for the breakdown of the four-fermion theory. In the V-A theory with no intermediate bosons, the cross section for the reaction $\nu_\mu e \rightarrow \mu\nu_e$ is given by

$$\sigma(\nu_\mu e \rightarrow \mu\nu_e) = \frac{G_F^2 s}{\pi}\left[1 - \frac{(m_\mu^2 - m_e^2)}{s}\right]^2 \simeq \frac{G_F^2 s}{\pi} .$$

The angular distribution is isotropic,

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2}\left[1 - \frac{(m_\mu^2 - m_e^2)}{s}\right]^2 ,$$

which is to say that the scattering is purely s-wave.

Partial-wave unitarity constrains the modulus of the s-wave amplitude to be

$$|\mathcal{M}_0| < 1 ,$$

where the partial-wave expansion is

$$f(\theta) = \left(2\,\frac{d\sigma}{d\Omega}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{s}}\sum_{J=0}^{\infty}(2J+1)P_J(\cos\theta)\mathcal{M}_J$$

---

[6] A lower bound on M_H is derived by requiring that radiative corrections be controlled: Linde (1976); Weinberg (1976). An alternative approach to the heavy Higgs alternative has been pioneered by Veltman (1977a, b, 1980). Upper bounds on fermion masses are also implied.

and the explicit factor of two is to undo the spin average. This constraint is equivalent to the familiar restriction

$$\sigma < 4\pi/p_{CM}^2$$

for s-wave scattering. For this process,

$$\mathcal{M}_o = \frac{G_F s}{\pi\sqrt{2}} \left[1 - \frac{(m_\mu^2 - m_e^2)}{s} \right]$$

$$\approx \frac{G_F s}{\pi\sqrt{2}} \quad ,$$

which implies the unitarity constraint

$$\frac{G_F s}{\pi\sqrt{2}} < 1 \quad .$$

This means that the four-fermion theory can make sense only if

$$s < \pi\sqrt{2}/G_F \quad ,$$

which is to say that

$$\sqrt{s} < 617 \text{ GeV} \quad ,$$

or

$$p_{CM} < 309 \text{ GeV/c} \quad .$$

In gauge theories, the asymptotic growth of partial-wave amplitudes is regulated, and all amplitudes are *at worst* in logarithmic violation of partial-wave unitarity in lowest order. When the calculable higher-order corrections are applied, the amplitudes become properly finite. There is one possibly exceptional case: interactions in which the Higgs boson plays an important role. In gauge boson-gauge boson scattering, high-energy amplitudes are proportional to M_H^2. For example,

$$\mathcal{M}_0(HH \to HH) \underset{s\to\infty}{\sim} -\frac{1}{4}\frac{G_F M_H^2}{\pi\sqrt{2}} \quad .$$

For an elastic s-wave amplitude, the partial-wave unitarity restriction is

$$|\mathcal{M}_0| < 2 \quad ,$$

which is respected by the lowest-order amplitude only if

$$M_H^2 < \frac{8\pi\sqrt{2}}{G_F} \approx (1.75\ \text{TeV}/c^2)^2 \quad .$$

A systematic analysis of all channels (W^+W^-, Z^oZ^o, HZ^o, HH) leads to a refined "bound,"

$$M_H^2 < \frac{8\pi\sqrt{2}}{3G_F} \approx (1\ \text{TeV}/c^2)^2 \quad .$$

The meaning of this condition is that if $M_H \ll 1\ \text{TeV}/c^2$, the weak interactions remain weak at all energies (except near gauge boson masses) in the sense that tree diagrams are reliable; if $M_H > 1\ \text{TeV}/c^2$, partial-wave amplitudes for gauge boson scattering become large (i.e. weak interactions become *strong*). In the latter case, gauge boson interactions in the TeV regime may resemble hadronic interactions in the GeV regime.

F. Open Questions

In spite of (or because of) the spectacular phenomenological success of the Weinberg-Salam model, many questions present themselves. We close this chapter with a brief list.

- Is the Weinberg-Salam theory correct? Is it complete?

- Will the intermediate bosons be found with the expected properties?

- Will a Higgs boson be found? What are its properties?

- Are weak and electromagnetic interactions truly described by a gauge theory, or do we merely have the low-energy phenomenology characteristic of the Weinberg-Salam theory?

- What is the origin of fermion masses?
- What is the origin of generations?
- How does the mixing of quark flavors arise?
- What is the mechanism of CP violation?
- Are the neutrinos massless?
- What (if any) is the pattern of lepton mixing, and how does it arise?

6. QUANTUM CHROMODYNAMICS[7]

Having learned that local gauge invariance provides the key to understanding the weak and electromagnetic interactions, we turn our attention once again to the strong interactions. The work of Yang and Mills and of many others in the early 1960s showed that it is unlikely that a flavor symmetry (like isospin or SU(3)) will be the basis of a successful gauge theory. Furthermore, at what we currently perceive to be the constituent level of quarks and leptons, flavor has been seen to be an attribute of the weak interactions--rather than the strong. The property that distinguishes quarks from leptons is color, so it is natural to attempt to construct a theory based upon local color gauge symmetry.

The choice of a gauge group is guided by two empirical facts:

- the familiar quarks (u, d, s, c, b) are color triplets, but
- the known hadrons are color singlets.

An obvious candidate for the color gauge group is $SU(3)_C$, where the subscript C for color is to differentiate this symmetry from the approximate flavor SU(3) symmetry of the ordinary hadrons. This will be seen to be a felicitous choice.

The gauge bosons that will emerge in the theory are called gluons, because of their role in binding quarks together within hadrons. The couplings

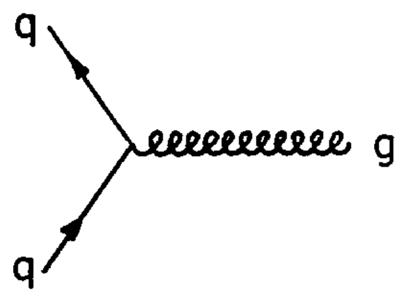


Fig. 20.

[7]A recent review has been given by Marciano and Pagels (1978). Various threads in the QCD tapestry are to be found in the papers by Bardeen, et al. (1973), Fritzsch, et al. (1973), Gross and Wilczek (1973b), and Weinberg (1973). See also the workshop proceedings edited by Frazer and Henyey (1979) and by Mahantappa and Randa (1980).

can exist for gluons that belong to the

$$\underset{\sim}{3} \otimes \underset{\sim}{3}^{*} = \underset{\sim}{1} \oplus \underset{\sim}{8}$$

representations of $SU(3)_C$. Color-singlet gluons would also couple to

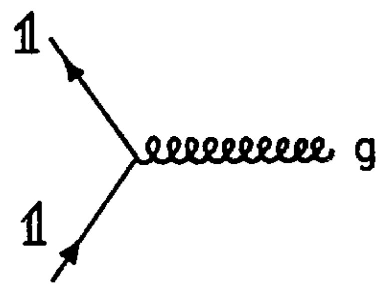


Fig. 21.

and would give rise to long-range forces between hadrons--the rock on which the Yang-Mills theory foundered--so we choose to exclude them from the theory.

If the quark colors are designated as red (R), blue (B), and green (G), the gluons may be represented conveniently as

$$\bar{R}B \quad \bar{R}G \quad \bar{B}R \quad \bar{G}R \quad \bar{B}G \quad \bar{G}B$$

$$\frac{\bar{R}R - \bar{B}B}{\sqrt{2}} \qquad \frac{\bar{R}R + \bar{B}B - 2\bar{G}G}{\sqrt{6}} \quad .$$

The last two are color-preserving forms which are orthogonal to the color-singlet combination

$$\frac{\bar{R}R + \bar{B}B + \bar{G}G}{\sqrt{3}} \quad .$$

The elementary interactions will be of the form

$$\text{Red quark} + \bar{R}B \text{ gluon} \rightarrow \text{Blue quark} \quad ,$$

etc.

A. Stability of Color Singlets

We wish to verify, or at least make it plausible, that the colorless (singlet) state is the configuration of lowest energy. This would account for the common occurrence of color singlet hadrons. To give a complete demonstration, it would be necessary to give a complete solution to the problem of hadron structure. Since this is beyond our means, we must be satisfied to examine the relative strengths of one-gluon-exchange interactions among quarks (Feynman, 1977). This style of investigation is referred to as a "maximally attractive channel" (MAC) analysis.

Consider first the scattering of two green quarks. This is mediated by a single diagram, namely by the exchange of a $(\bar{R}R + \bar{B}B - 2\bar{G}G)/\sqrt{6}$ gluon:

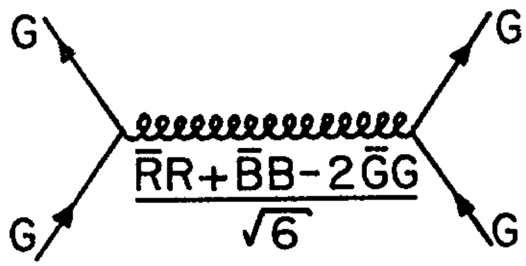


Fig. 22.

The interaction strength is proportional to

$$\left(\frac{-2}{\sqrt{6}}\right) \times \left(\frac{-2}{\sqrt{6}}\right) = +\frac{2}{3} ,$$

which is repulsive. As in electrostatics, like (color) charges repel.

The scattering of a blue quark and a green quark involves two interactions: a direct term

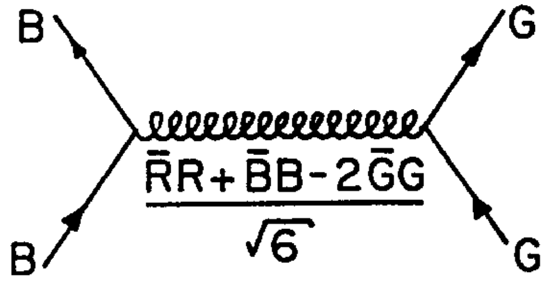


Fig. 23.

with strength

$$\left(\frac{1}{\sqrt{6}}\right)\left(\frac{-2}{\sqrt{6}}\right) = -\frac{1}{3} \quad ,$$

and an exchange term

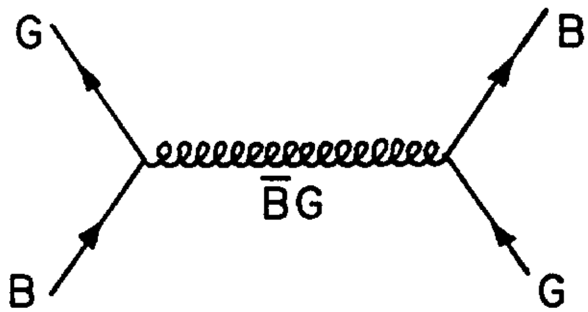


Fig. 24.

with strength

$$1 \times 1 = 1 \quad ,$$

which yields a total interaction strength of

$$-\frac{1}{3} + 1 = \frac{2}{3} \quad ,$$

which is again repulsive.

States symmetric and antisymmetric under color interchange may be classified as

$$\frac{BG \pm GB}{\sqrt{2}} = \begin{cases} S \\ A \end{cases} \quad .$$

What are the interaction energies in this basis? The direct term contributes $(S + A)(-1/3)$, while the exchange term contributes $(S - A)(1)$. Thus a symmetric configuration corresponds to an interaction strength

$$\mathcal{E}_S = 1 - 1/3 = 2/3 \quad ,$$

consistent with our result for GG scattering, and an antisymmetric combination corresponds to

$$\mathcal{E}_A = -1 - 1/3 = -4/3 \quad .$$

These results are $SU(3)_c$-invariant; they continue to hold under changes of labels R, B, and G.

To build up baryons, we sum over two-body quark-quark interactions. There are three possible three-quark configurations in color space.

- The totally symmetric color $\underset{\sim}{10}$ ($\square\square\square$) configuration, with interaction strength

$$\mathcal{E}_{10} = 3\mathcal{E}_{\text{direct}} + 3\mathcal{E}_{S(\text{exchange})} = 3 \cdot \mathcal{E}_S = +2 \quad ,$$

which is repulsive.

- The totally antisymmetric color singlet configuration, with interaction strength

$$\mathcal{E}_1 = 3\mathcal{E}_{\text{direct}} + 3\mathcal{E}_{A(\text{exchange})} = 3 \cdot \mathcal{E}_A = -4 \quad ,$$

which is attractive.

• The color $\underset{\sim}{8}$ $\left(\ydiagram{2,1}\right)$ state of mixed symmetry, with interaction strength

$$\mathscr{E}_8 = 3\,\mathscr{E}_{\text{direct}} + \mathscr{E}_{S(\text{exchange})} + \mathscr{E}_{A(\text{exchange})}$$

$$= 3(-1/3) + 1 - 1 = -1 \qquad ,$$

which is less attractive.

Of the three-quark states, it is the color-singlet configuration which is maximally attractive.

For a general configuration of n_q quarks with n_S symmetric and n_A antisymmetric pairs, we readily verify that

$$\mathscr{E} = (n_S - n_A) - \frac{1}{3}\binom{n_q}{2} \qquad .$$

With this result in hand, it is easy to investigate the interaction energy of more complicated configurations. Should a quark be bound to a (color singlet) proton? The proton-quark composite has a total energy

$$\mathscr{E}\left(\ydiagram{2,1,1}\right) = (1 - 3) - \frac{1}{3}\binom{4}{2} = -4 \quad ,$$

which is the same as the energy of a quark and proton in isolation:

$$\mathscr{E}\left(\ydiagram{1,1,1}\right) + \mathscr{E}(\ydiagram{1}) = -4 + 0 = -4 \qquad .$$

Thus there is no indication of appreciable binding. A similar examination of 5, 6,...-quark states reveals no energetic advantage to these configurations either. Thus, within the context of the one-gluon-exchange description, we have made it plausible that color singlet three-quark states are the most deeply bound multiquark states.

Let us now turn our attention to quark-antiquark states. As an example of a colored meson, consider a (color 8) $R\bar{G}$ state, for which the direct interaction

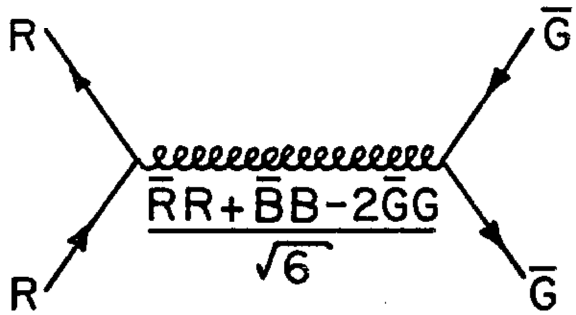


Fig. 25.

occurs with strength

$$\mathscr{E}_8 = \left(\frac{1}{\sqrt{6}}\right) \cdot (-)\left(\frac{-2}{\sqrt{6}}\right) = +\frac{1}{3} \quad ,$$

where the explicit minus sign is characteristic of an antiparticle vertex in a vector theory. The color 8 $q\bar{q}$ state is therefore expected to be unbound.

The analysis of colorless mesons is somewhat more involved. For a $(G\bar{G})$ state, there are three interactions possible:

- the color preserving

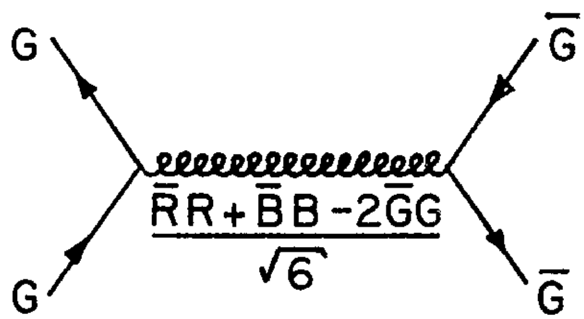


Fig. 26

which contributes

$$\mathscr{E}_i = \left(\frac{-2}{\sqrt{6}}\right)(-)\left(\frac{-2}{\sqrt{6}}\right) = -\frac{2}{3}$$

the $G\bar{G} \to R\bar{R}$ transition

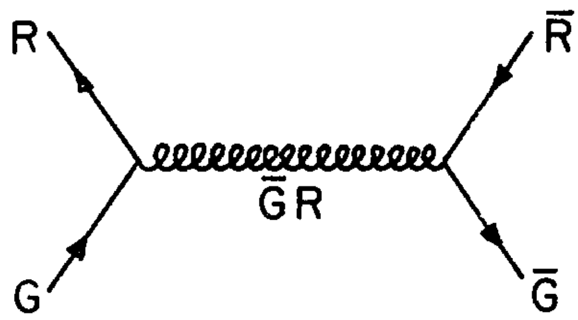


Fig. 27.

which contributes

$$\mathscr{E}_{ii} = (1)(-)(1) = -1 \quad ,$$

and the $G\bar{G} \to B\bar{B}$ transition

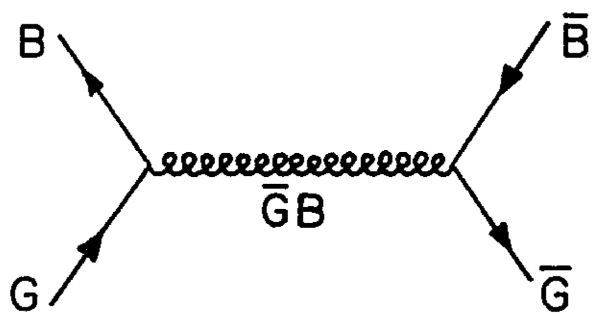


Fig. 28.

which contributes

$$\mathscr{E}_{iii} = (1)(-)(1) = -1 \quad .$$

The total energy of the color singlet state

$$\frac{R\bar{R} + B\bar{B} + G\bar{G}}{\sqrt{3}}$$

is given symbolically by

$$\mathscr{E}_1 = 3 \times \frac{1}{\sqrt{3}} |G\bar{G}\rangle \times \frac{1}{\sqrt{3}} (|R\bar{R}\rangle + |B\bar{B}\rangle + |G\bar{G}\rangle) \quad ,$$

where the factors represent (from right to left) a projection on the final color-singlet state, a projection on an initial $G\bar{G}$ state, and the effect of all the possible initial configurations. We recover simply

$$\mathscr{E}_1 = \mathscr{E}_i + \mathscr{E}_{ii} + \mathscr{E}_{iii} = -8/3 \quad ,$$

an attractive interaction.

The rather pedestrian analysis of this section has indicated that the color singlet qqq and $q\bar{q}$ states are likely to be the most stable configurations. This is pleasantly in accord with our knowledge of the properties of baryons and mesons, and lends support to our choice of $SU(3)_C$ as the gauge group for the strong interactions among quarks.

B. The QCD Lagrangian

With these preliminaries behind us, we may now formulate the gauge theory of color triplet quarks interacting by means of vector gluons which belong to the octet representation of $SU(3)_C$. The Lagrangian will have the standard Yang-Mills form, namely

$$\mathscr{L} = -\tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \bar{\psi}_\alpha(i\gamma^\mu \mathscr{D}^{\alpha\beta}_\mu - m\delta^{\alpha\beta})\psi_\beta \quad ,$$

where $(\alpha, \beta) = 1, 2, 3$ (or R, B, G) are color indices for the quark fields and $a = 1, 2, \ldots, 8$ is the gluon color label. The field strength tensor is given by

$$G^a_{\mu\nu} = \frac{1}{ig} [\mathscr{D}_\mu, \mathscr{D}_\nu]$$

where B^a_μ is the color gauge field (the gluon field) analogous to the isospin gauge field b_μ in the original Yang-Mills theory. The gauge-covariant derivative is a 3 × 3 matrix in color space,

$$\mathscr{D}_\mu = I\partial_\mu - \frac{ig}{2} \lambda^a B^a_\mu \quad ,$$

and λ^a are the eight 3 × 3 matrix representations of the $SU(3)_C$ octet.

These generators of SU(3) rotations are conventionally labeled

$$\lambda_1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ & R & B & G \\ \hline \bar{R} & 0 & 1 & 0 \\ \bar{B} & 1 & 0 & 0 \\ \bar{G} & 0 & 0 & 0 \end{array} \quad , \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} ,$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad , \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} ,$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad , \quad \lambda_8 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} .$$

In flavor SU(3), with (1, 2, 3) = (u, d, s), the matrices λ_1, λ_2, λ_3

correspond to the isospin generators τ_1, τ_2, τ_3. The λ-matrices have the following properties:

- $\mathrm{tr}\ (\lambda^a) = 0;$
- $\mathrm{tr}\ (\lambda^a \lambda^b) = 2\delta^{ab};$
- $[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c.$

The structure constants which are completely antisymmetric in the indices a, b, c are most easily evaluated as

$$f^{abc} = \frac{1}{4i}\ \mathrm{tr}\ (\lambda^c[\lambda^a, \lambda^b]) \quad .$$

The field-strength tensor can be written in terms of the structure constants as

$$G^a_{\mu\nu} = \partial_\nu B^a_\mu - \partial_\mu B^a_\nu - gf^{abc}B^b_\nu B^c_\mu \quad .$$

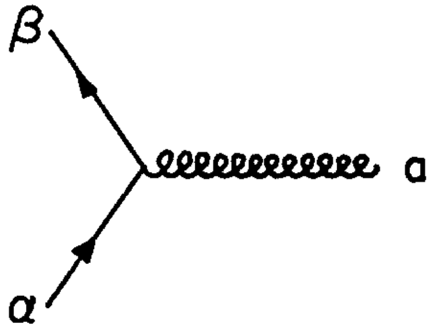
The quark-gluon interaction term contained within $\bar{\psi} i\gamma^\mu \mathscr{D}_\mu \psi$ is

$$\mathscr{L}_{\mathrm{inter}} = \frac{g}{2}\ \bar{\psi}\lambda^a \gamma^\mu B^a_\mu \psi \quad ,$$

in matrix notation with

$$\psi \equiv \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix} \quad .$$

This corresponds to the Feynman rule for the quark-gluon vertex



$$\frac{ig}{2}\,\lambda^{a}_{\alpha\beta}$$

Fig. 29.

Thus the force between two quarks is proportional to

$$\frac{1}{4}\sum_{a}\lambda^{a}_{\alpha\beta}\lambda^{a}_{\gamma\delta}$$

for the transition $\alpha + \gamma \rightarrow \beta + \delta$. This expresses in formal terms the heuristic discussion of the previous section, with the identifications

$$\bar{R}B = (\lambda_1 + i\lambda_2)/2 \quad ,$$

$$\frac{\bar{R}R - \bar{B}B}{\sqrt{2}} = \frac{\lambda_3}{\sqrt{2}} \quad ,$$

etc.

We shall assume provisionally that the local color gauge symmetry is exact. The infinite-range forces mediated by massless gluons give rise to some hope that quarks (and gluons) may be permanently confined. Should free quarks be found, we might later wish to consider the possibility that the symmetry is broken either spontaneously or explicitly (De Rújula, et al., 1978; Okun and Shifman, 1979).

C. Consequences of an Interacting Field Theory of Quarks and Gluons

Before proceeding to a specific study of the predictions of QCD, let us give a qualitative discussion of the implications of an interacting field theory of quarks and gluons. A convenient setting for this discussion is the deep-inelastic scattering of leptons from a nucleon target, in which a virtual photon (or intermediate boson) of $(\text{mass})^2 = -Q^2$ analyzes the target structure on a length scale characterized by $1/\sqrt{Q^2}$.

According to the parton model (Feynman, 1972), which ignores interactions among the quarks within a proton, the picture is rather simple. As Q^2 increases, the resolution becomes finer, and we are able to probe the elementary quark constituents of the proton, as shown in Figure 30. Once Q^2 is large enough for the quark to be

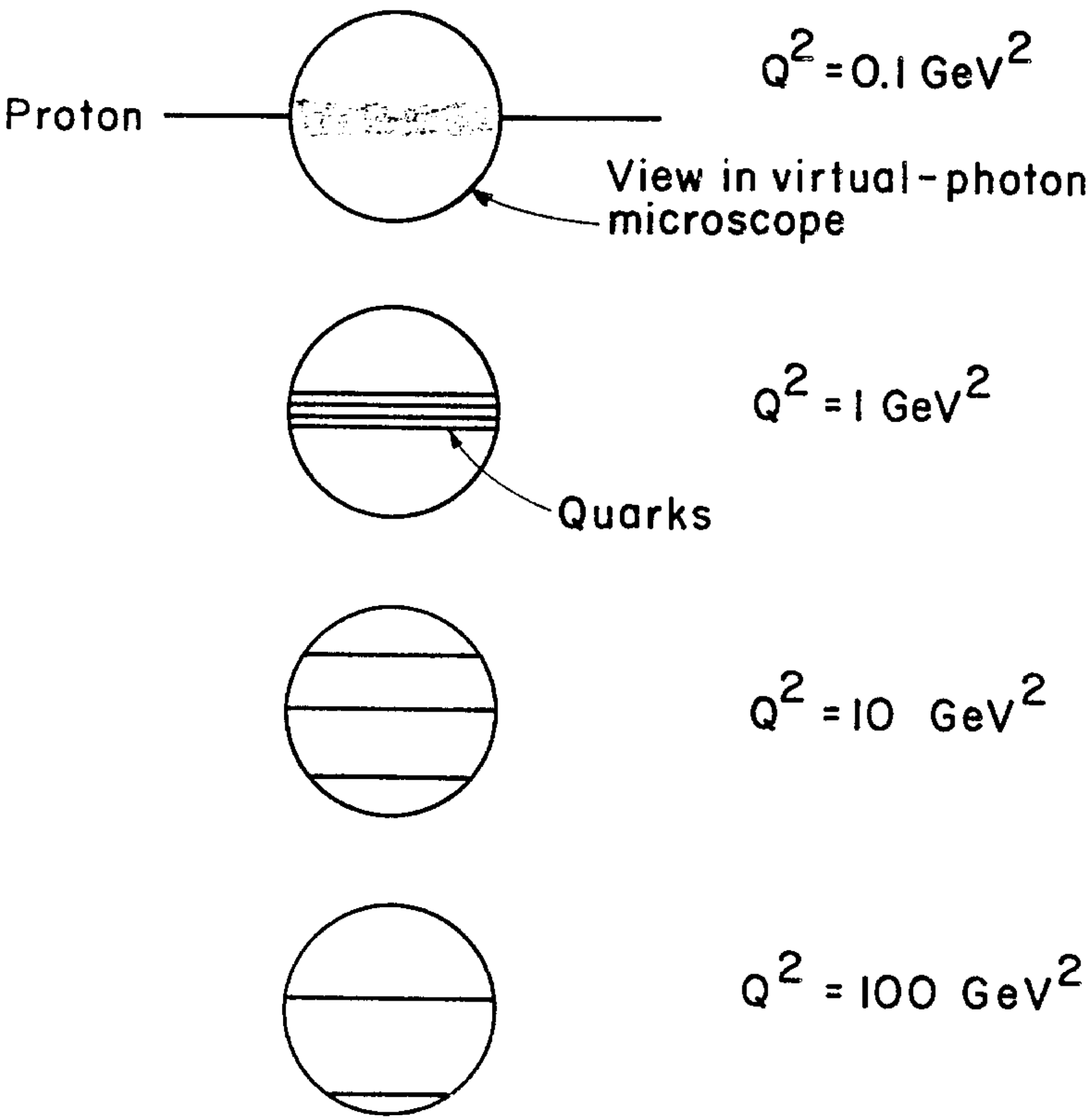


Fig. 30. Parton-model view of the proton.

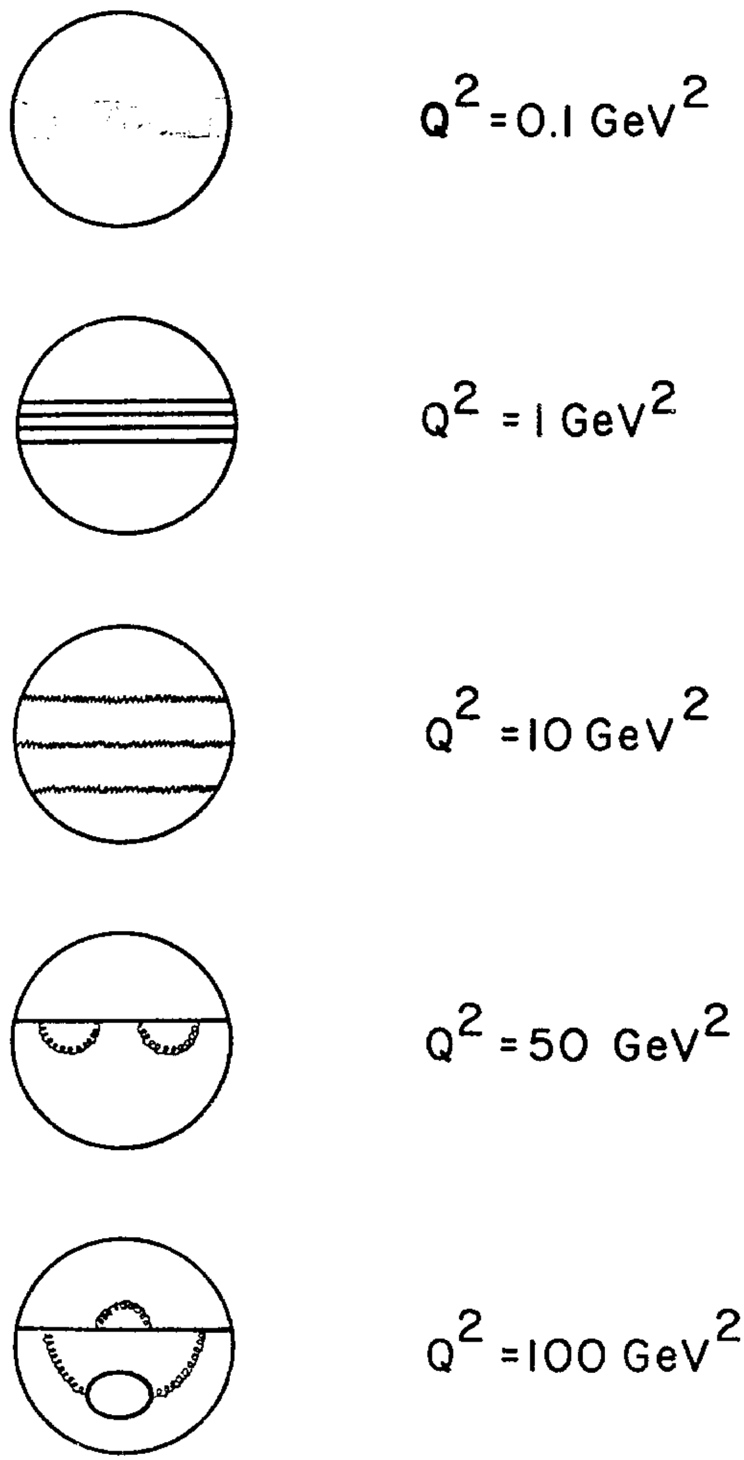


Fig. 31. Interacting-field-theory view of the proton.

resolved, no finer structure is seen. The quarks are structureless, have no size, and introduce no length scale. When Q^2 exceeds a few GeV^2, all fixed mass scales become irrelevant and the Q^2-dependence of structure functions can be determined by dimensional analysis.

In an interacting field theory, a richer picture is to be expected. As Q^2 increases beyond the magnitude required to resolve quarks, the quarks are found to have an apparent structure, which arises from the interactions mediated by gluons. This is indicated in Figure 31. The fluctuations shown there lead to scaling violations in deep-inelastic scattering. The structure functions $F(x, Q^2)$ measure the distribution of quarks in a fast-moving proton as a function of momentum fraction

$$x = p_{quark}/p_{proton} \quad .$$

As Q^2 grows, the structure functions undergo a characteristic evolution. For large x $(0.3 \lesssim x \lesssim 1)$, it becomes increasingly likely that a quark with momentum fraction x will be caught in mid-dissociation into components with $x_1 + x_2 = x$:

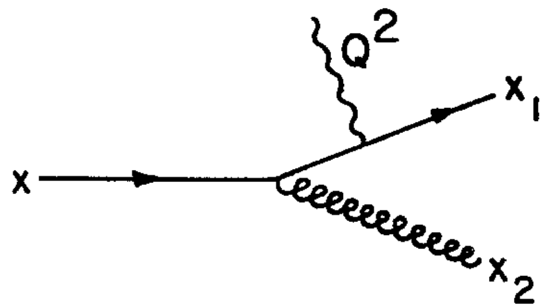


Fig. 32.

For small values of x << 1, the population of quarks and antiquarks will be enhanced by processes such as

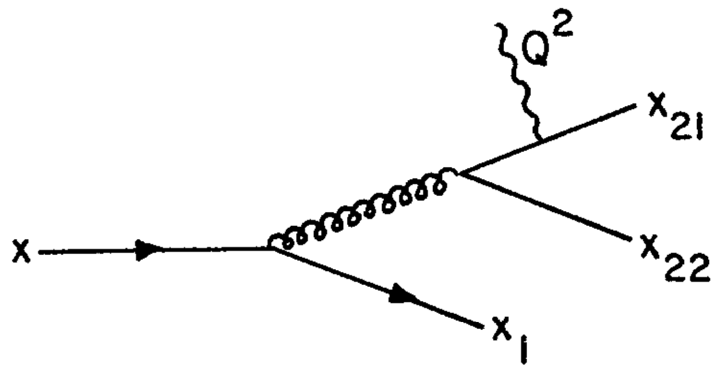


Fig. 33.

We therefore expect, in any interacting field theory, that as Q^2 increases the structure function will fall at large values of x and rise at small values of x, as shown in Figure 34. It remains for a quantitative analysis to show whether these effects are calculable in a given field theory (specifically in QCD). Furthermore, it is observed experimentally (Perkins, 1980) that effects of the kind we have discussed are *small*, which is to say that Bjorken (1967) scaling (by which is meant Q^2-independent structure functions) is an excellent approximation. Can this be explained?

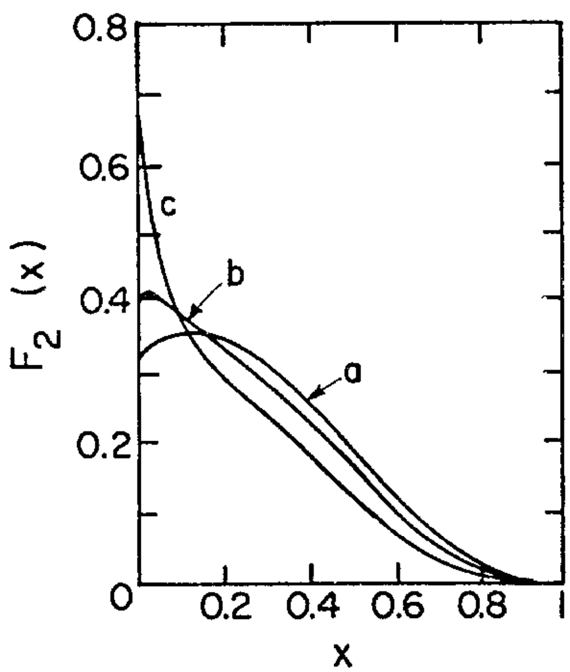


Fig. 34. Evolution of the proton structure function in an interacting field theory: (a) low, (b) medium, (c) high Q^2.

D. Charge Renormalization in QED and QCD

In interacting field theories, observables such as scattering amplitudes may be sensitive to higher-order corrections, in addition to Born diagrams. The modifications to lowest-order contributions are in general dependent upon kinematic variables. A convenient way of representing these modifications is by introducing a so-called "running coupling constant," i.e. an effective coupling that depends upon the kinematic circumstances.[8]

For example, in Quantum Electrodynamics (QED) the corrections to Coulomb's law introduced by the vacuum polarization diagram of Fig. 35 may be represented by the substitution

$$\alpha \to \alpha(Q^2) = \alpha(q_o^2)\left[1 + \frac{\alpha(q_o^2)}{3\pi}\log\left(\frac{Q^2}{q_o^2}\right)\right] .$$

Summation of higher-order corrections (retaining only the leading logarithms) shows this expression to be the first term in a power-series expansion of

$$\alpha(Q^2) = \frac{\alpha(q_o^2)}{1 - \frac{\alpha(q_o^2)}{3\pi}\log\left(\frac{Q^2}{q_o^2}\right)} ,$$

which is more conveniently written as

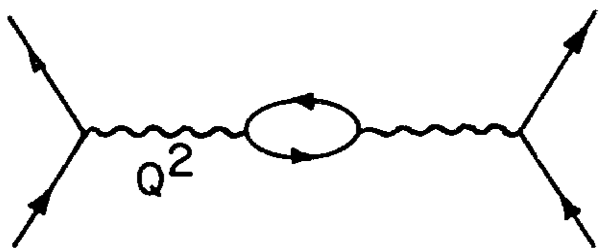


Fig. 35. Lowest-order contribution to the charge renormalization in QED.

[8] See, for example, the discussion in Bjorken and Drell (1964), c. 8.

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(q_o^2)} - \frac{1}{3\pi}\log\left(\frac{Q^2}{q_o^2}\right) .$$

Thus, given the value of the coupling constant at some arbitrarily selected momentum transfer q_o^2, one has a prediction for its evolution to arbitrary Q^2. This is sketched in Figure 36.

At shorter distances (or larger values of Q^2), the effective charge becomes larger. This phenomenon is a familiar one in classical electrodynamics. A test charge in a dielectric medium will polarize the medium as indicated in Figure 37. At any distance (larger than the molecular scale) from the test charge, the effective charge will by Gauss's law be smaller in magnitude than the test charge because of the opposite charge attracted by the test charge. Only at very short distances is the effective charge equal to the full magnitude of the test charge. Thus the QED vacuum is seen to behave as a polarizable medium.

In non-Abelian gauge theories such as QCD there are both close similarities to QED and also crucial differences. Coulomb's law for gluon exchange is modified by quark-antiquark vacuum polarization loops of Fig. 38 which modify the effective coupling in a way that can be read off from the QED calculation. For each quark flavor, the loop contribution is given by

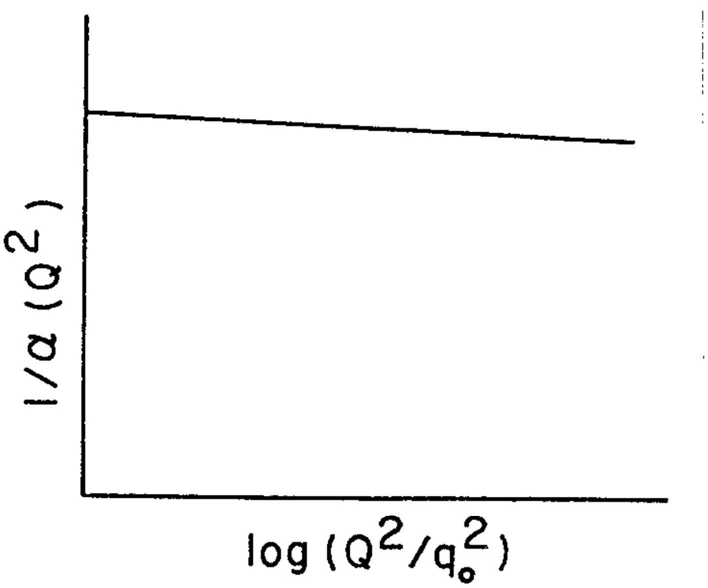


Fig. 36. Evolution of the running coupling constant in QED.

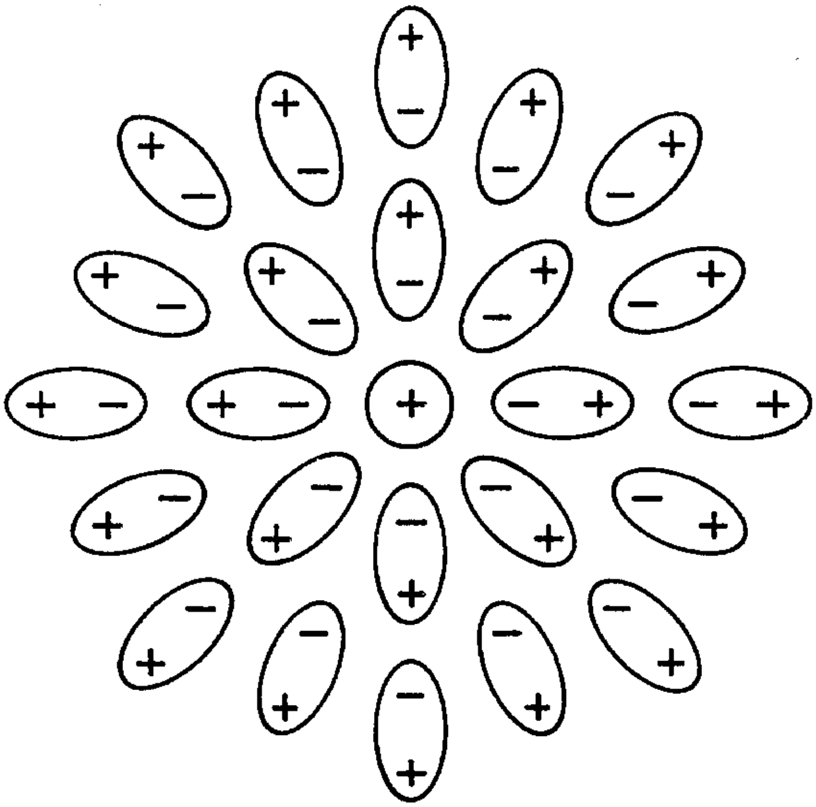


Fig. 37. Polarization of a dielectric medium by a test charge.

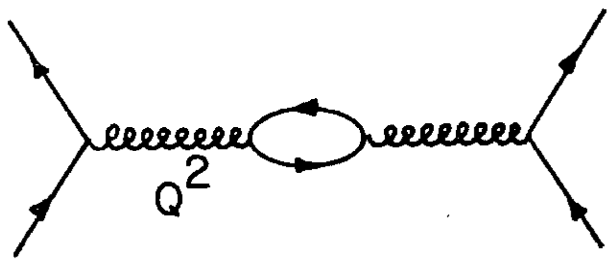


Fig. 38. Fermion-loop contribution to the charge renormalization in QCD.

$$"\mathrm{QED}" \times \mathrm{tr}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right) = "\mathrm{QED}" \times \frac{2\lambda^{ab}}{4}$$

$$= \tfrac{1}{2}"\mathrm{QED}" \quad .$$

Thus the contribution of quark loops to the evolution of the strong coupling constant is

$$\alpha_s(Q^2) - \alpha_s(q_o^2)\Big|_{\text{quarks}} = \frac{\alpha_s^2(q_o^2)}{6\pi}\, n_f \log\ (Q^2/q_o^2) \quad ,$$

where n_f is the number of quark flavors.

There are in addition gluon loops to be considered. The Feynman rules for gluon self-interactions are gauge dependent and so therefore is the diagram-by-diagram analysis of the contribution from gluon loops. A useful grouping is to separate the physically realizable intermediate states composed of "transverse" gluons (g_T)

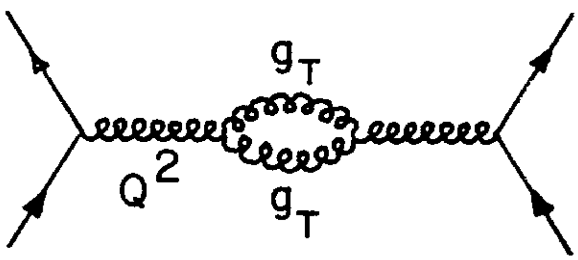


Fig. 39.

which give a contribution

$$\frac{\alpha_s^2(q_o^2)}{4\pi} \log\ (Q^2/q_o^2)$$

that differs from the quark loop contributions only by spin factors. The remaining loops, which correspond to virtual states that are not physically realizable because they include "Coulomb gluons" (g_C)

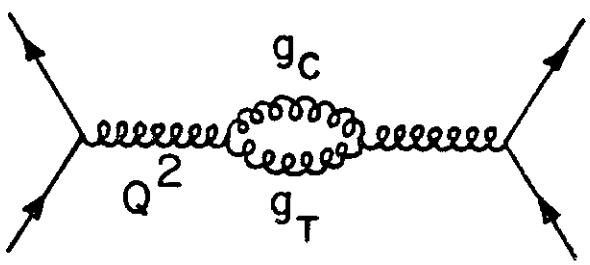


Fig. 40.

yield a contribution that is twelve times larger, and *opposite in sign*,

$$\frac{-3\alpha_s^2(q_o^2)}{\pi} \log (Q^2/q_o^{\ 2}) \qquad .$$

Taken together, the net contribution of the gluon loops is negative—characteristic of *anti*screening.

I know of no simple quantitative argument which explains how this comes about. The possibility of antiscreening can be understood in qualitative terms, as shown in Figure 41. Suppose our "test charge" is a blue quark at the origin, and the probe we employ to measure its charge is a $R\bar{B}$ gluon. It may happen that before the probe reaches the origin, the blue quark radiates a virtual $B\bar{G}$ gluon and thus fluctuates into a green quark--to which the probe is blind. Rather than being concentrated at the origin, the net color charge will thus be distributed throughout the gluon cloud. Therefore only by inspecting the test charge from long distances will one be able to measure its full effect. Apparently the first to notice the possibility of an antiscreening term in non-Abelian gauge theories was Khriplovich (1969).

In QCD, the combined effect of all the quark and gluon loops is to produce a running coupling constant which in leading logarithmic approximation is given by

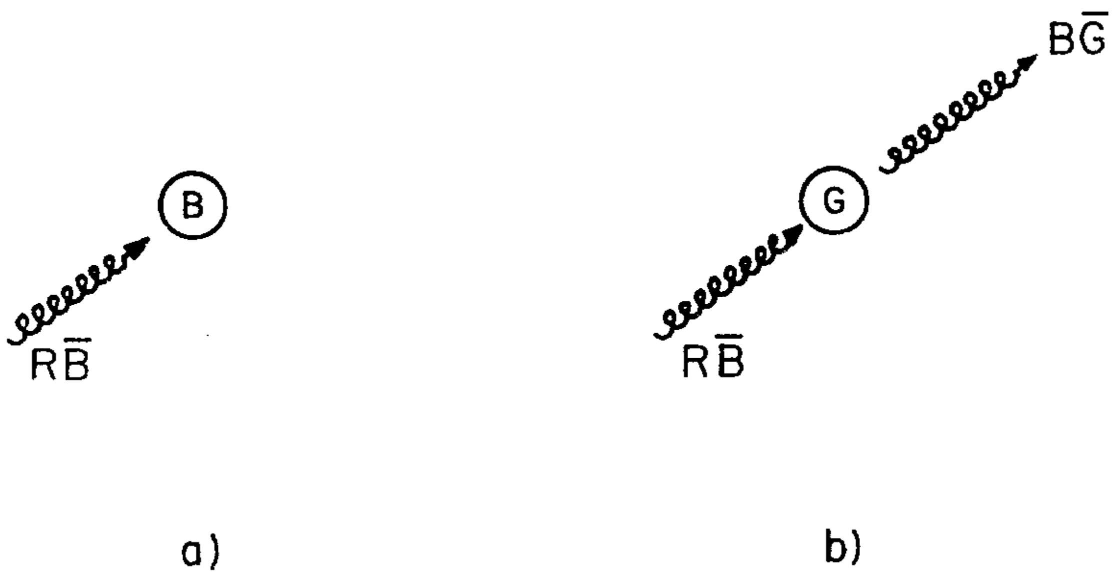


Fig. 41. (a) $R\bar{B}$ gluon probe incident on a blue quark may find (b) the blue charge dispersed as a result of vacuum fluctuations.

$$1/\alpha_s(Q^2) = 1/\alpha_s(q_o^2) + \frac{33 - 2n_f}{12\pi} \log (Q^2/q_o^2) \quad .$$

So long as the number of quark flavors does not exceed 16, the coefficient of the logarithm is positive and the effective coupling becomes smaller at large Q^2 or short distances. In other words, there is net antiscreening. The profound significance of this circumstance for a calculable theory of the strong interactions was recognized by Gross and Wilczek (1973) and Politzer (1973). The existence of a regime in which $\alpha_s(Q^2) \ll 1$ implies a realm in which QCD perturbation theory should be valid. This property of non-Abelian gauge theories is known as asymptotic freedom.[9] While it by no means justifies all the hypotheses of the parton model, it does make it plausible that at very short distances (i.e. when examined by very-high Q^2 probes) quarks may behave nearly as free particles within hadrons. As the sketch of the evolution of $1/\alpha_s(Q^2)$ in Figure 42 shows, the growth of the coupling at large distances indicates the existence of a domain in which the strong interactions become formidable. This strong-coupling regime undoubtedly is of key importance for quark (or color) confinement.

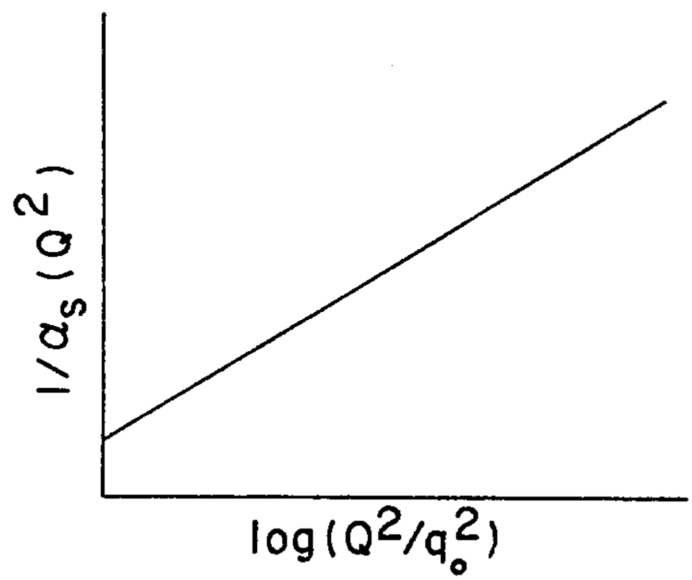


Fig. 42. Evolution of the running coupling constant in QCD.

[9] Reviews of asymptotic freedom include those by Politzer (1974), Peterman (1979), and Berestetskii (1976). Applications of perturbative QCD are emphasized by Buras (1980), Brodsky (1979), and Novikov, et al. (1978).

E. Perturbative QCD: An Example

The simplest illustration of QCD perturbation theory is the calculation of the cross section for e^+e^- annihilation into hadrons. In the parton model, this process is represented by the elementary transition illustrated in Fig. 43, which yields a cross section

$$\sigma_{\text{parton}}(s) = \frac{4\pi\alpha^2}{3s}\left[3 \sum_{\substack{\text{quark}\\ \text{flavors}}} e_q^{\,2}\Theta(s - 4m_q^{\,2})\right] ,$$

where the factor of three in the numerator is a consequence of quark color and the theta-function is a crude representation of threshold kinematics.

To the extent that α_s is small, it makes sense to compute the strong-interaction (QCD) corrections to the parton model in perturbation theory. The first-order radiative corrections, which are characterized by the diagrams of Fig. 44, yield (Jost and Luttinger, 1950; Appelquist and Georgi, 1973; Zee, 1973)

$$\sigma_1(s) = \sigma_{\text{parton}}(s)\left(1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2)\right) .$$

The strong-interaction corrections are

- Calculable (and free of infrared problems);

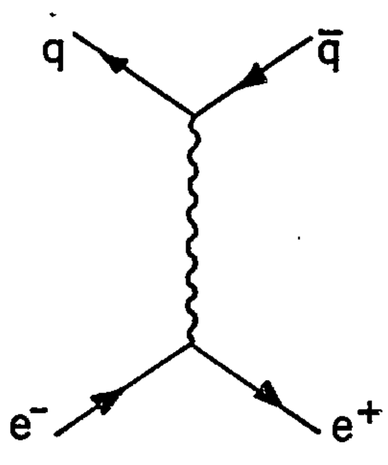


Fig. 43. Parton-model description of electron-positron annihilation into hadrons.

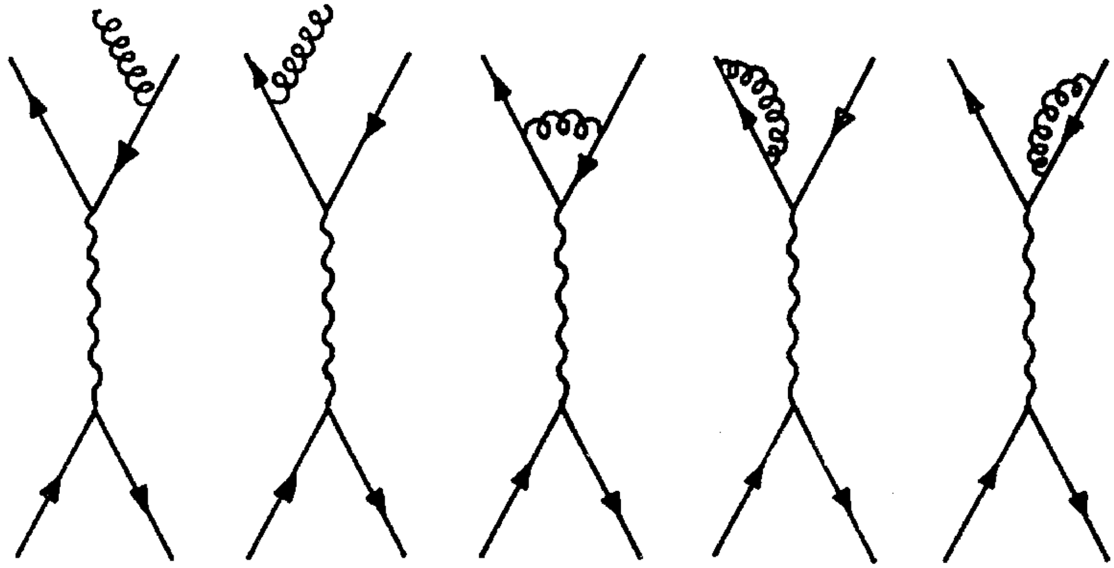


Fig. 44. Lowest-order strong-interaction corrections to electron-positron annihilations into hadrons.

• Small, in the asymptotically free region (and the $\mathcal{O}(\alpha_s^2)$ corrections (Chetyrkin, et al., 1979; Dine and Sapirstein, 1979; Celmaster and Gonsalves, 1980) are not enormous);

• Positive; and

• Decreasing with increasing s.

The effect of these corrections to the parton model is shown schematically in Figure 45, in which the quantity

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is plotted as a function of energy.

Although QCD prescribes the evolution of the strong coupling constant, it does not specify the magnitude of $\alpha_s(q_o^2)$. We are thus left to wonder at what value of s (or Q^2) will first-order perturbation theory be trustworthy. The value of $\alpha_s(q_o^2)$ must be determined experimentally, and the absence of a Thomson limit in QCD makes this a nontrivial task. Two estimates, which perhaps represent reasonable extremes, are shown in the Table.

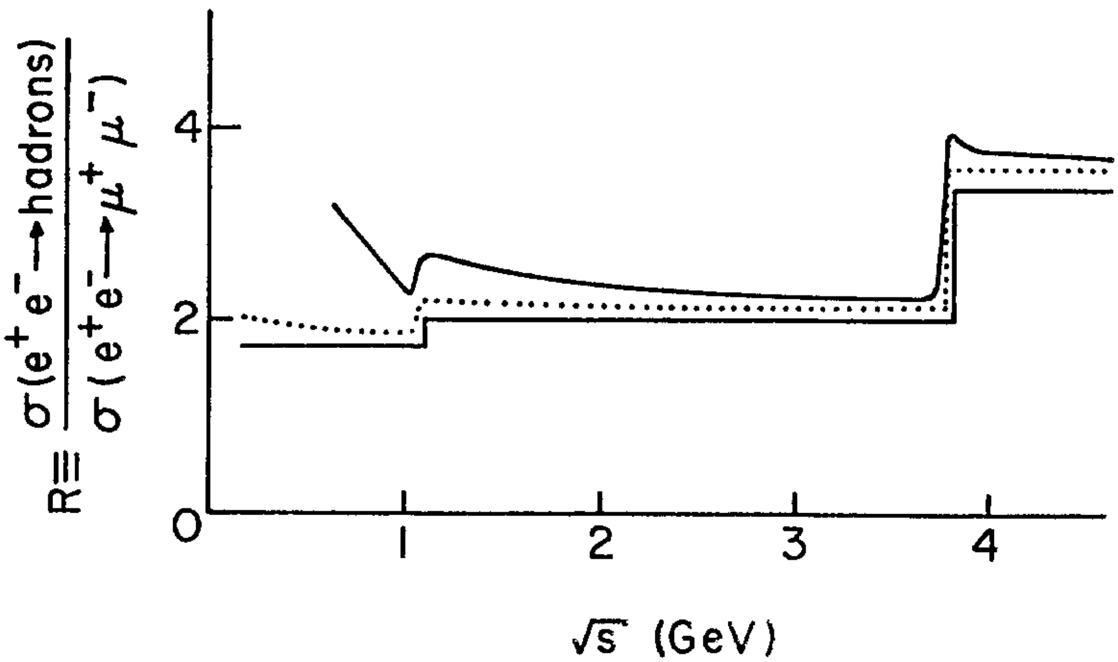


Fig. 45. Schematic behavior of the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ as a function of energy in the parton model (steps) and including lowest-order QCD corrections (smooth curve). The dotted curve corresponds to a smaller value of $\alpha_s(q_o^2)$.

s (GeV^2)	$\alpha_s(s)$ A	$\alpha_s(s)$ B
1	0.30	2.9
5	0.22	0.67
10 ψ	$\boxed{0.2}$	$\boxed{0.5}$
16 $c\bar{c}$	0.19	0.43
100 Υ	0.15	0.28
1000 PETRA/PEP	0.12	0.20

What do the data say?[1] There is as yet not a good test of the QCD corrections to $\sigma(e^+e^- \rightarrow \text{hadrons})$ because

- $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ is imperfectly subtracted;
- Systematic errors are $\sim$15%, which is also the size of the disagreement between various experiments;

[1] A recent assessment is given by Barnett, et al. (1980).

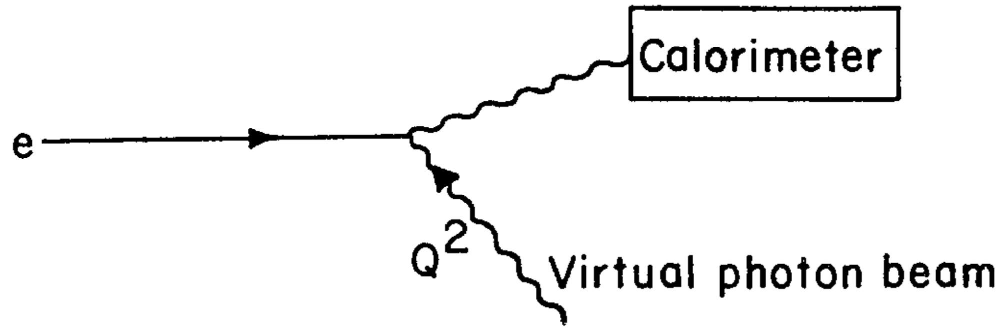


Fig. 46. Conceptual experiment to measure the momentum spectrum of electrons in a beam prepared as monochromatic.

• $\alpha_s(s)$ is not well known from other experiments; perhaps it can best be measured here.

F. Radiative Corrections to Deep-Inelastic Scattering

The evolution with Q^2 of deep-inelastic structure functions may also be analyzed in QCD perturbation theory. To make clear the logical structure, it is helpful to revert to QED and to consider a Gedankenexperiment to measure the momentum spectrum of electrons in a "monochromatic" beam, as shown in Fig. 46. The (perfect) calorimeter measures the energy of the backscattered photon and therefore determines the energy of the electron. If the momentum of the prepared beam is defined to be 1, then in zeroth order the momentum distribution in the beam is

$$\frac{dN}{dz} = N\delta(z - 1) \quad ,$$

where

$$z \equiv (\text{measured momentum})/(\text{prepared momentum}) \quad .$$

The virtual dissociation

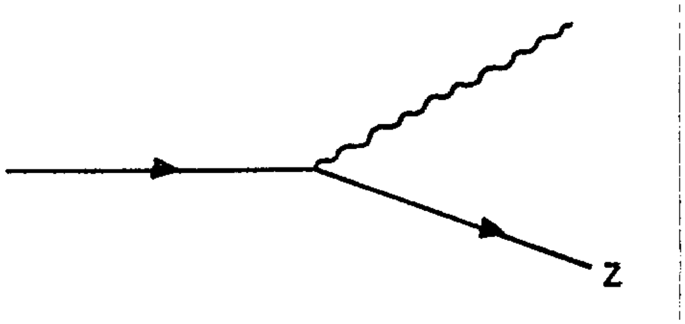


Fig. 47.

induces in the beam a component with $z < 1$. The sensitivity of the apparatus to these fluctuations is a function of q^2, as suggested by uncertainty principle arguments. The fluctuations are calculable in QED.

Define the parameter

$$\tau \equiv \log (Q^2/q_o^2) ,$$

and let

$$\frac{\alpha}{2\pi} P_{e\leftarrow e}(z)\, d\tau$$

represent the probability of finding an electron carrying a fraction z of the parent electron's momentum. Then if $e(z, \tau)$ is the number density of electrons observed in $(z, z + dz)$ by a probe with resolving power characterized by τ, it follows at once that

$$\frac{de}{d\tau}(z, \tau) = \frac{\alpha(\tau)}{2\pi} \int_x^1 dy \int_0^1 dz\, \delta(zy - x)\, e(y, \tau) P_{e\leftarrow e}(z)$$

$$= \frac{\alpha(\tau)}{2\pi} \int_x^1 \frac{dy}{y}\, e(y, \tau) P_{e\leftarrow e}(z) .$$

For an initial distribution

$$e(y, \tau) = N\delta(y - 1) ,$$

we recover

$$\frac{de}{d\tau}(x,\ \tau) = \frac{N\alpha(\tau)}{2\pi} P_{e\leftarrow e}(x) \quad .$$

By virtue of the same fluctuations, there are also photons in the beam. Let

$$\frac{\alpha}{2\pi} P_{\gamma\leftarrow e}(z)\, d\tau$$

be the probability of finding a <u>photon</u> carrying a fraction z of the parent electron's momentum, and let $\gamma(z,\ \tau)$ be the number density of photons observed in $(z,\ z + dz)$ by a probe with resolving power characterized by τ. We may imagine a Gedankenexperiment to observe these photons, as for example depicted in Fig. 48. If the source of the virtual photon probe is (for example) a nitrogen nucleus, the Gedankenapparat can be recognized as a surrogate for the development of electromagnetic showers in the atmosphere. Indeed, the theory of cascade showers (Rossi, 1952) has much in common with the present discussion. The evolution of the photon distribution is given by

$$\frac{d\gamma}{d\tau}(x,\ \tau) = \frac{\alpha(\tau)}{2\pi} \int_x^1 \frac{dy}{y}\, e(y,\ \tau) P_{\gamma\leftarrow e}(x/y) \quad .$$

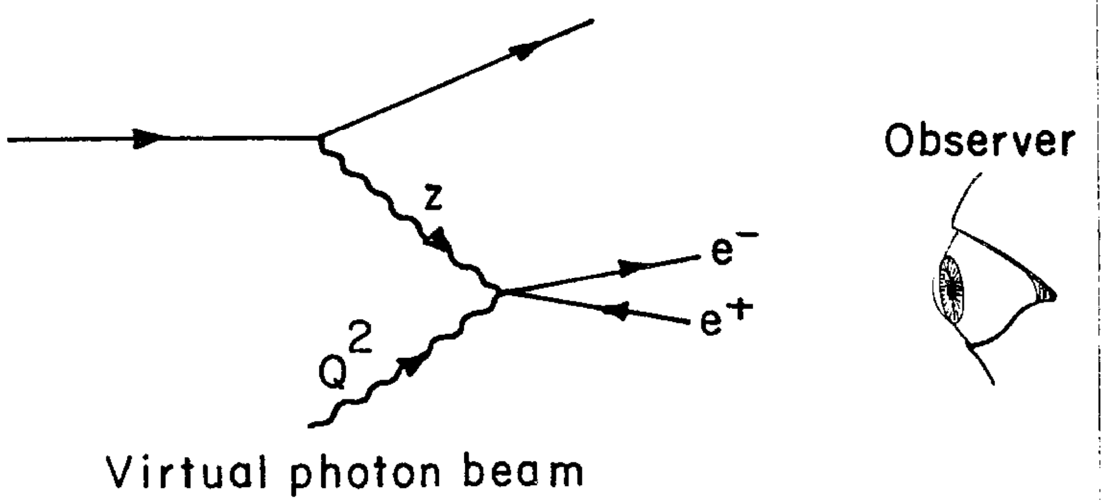


Fig. 48. Conceptual experiment to measure the momentum spectrum of photons in an electron beam prepared as monochromatic.

Of course, the photons may themselves fluctuate

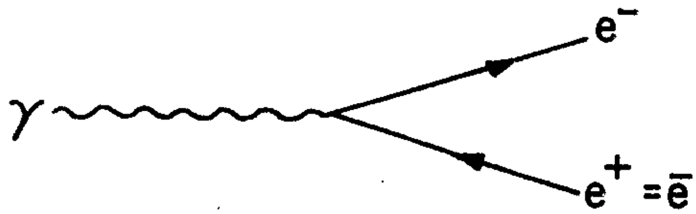


Fig. 49.

so we are led to define

$$\frac{\alpha}{2\pi} P_{e\leftarrow\gamma}(z)d\tau$$

as the probability of finding an electron (or positron) carrying a fraction z of the momentum of the parent photon.

The evolution of an electron distribution is now given by

$$\frac{de}{d\tau}(x, \tau) = \frac{\alpha(\tau)}{2\pi}\int_x^1 \frac{dy}{y}\left[e(y, \tau)P_{e\leftarrow e}(x/y) + \gamma(y, \tau)P_{e\leftarrow\gamma}(x/y)\right].$$

The induced positron component will obey an equation identical in form,

$$\frac{d\bar{e}}{d\tau}(x, \tau) = \frac{\alpha(\tau)}{2\pi}\int_x^1 \frac{dy}{y}\left[\bar{e}(y, \tau)P_{e\leftarrow e}(x/y) + \gamma(y, \tau)P_{e\leftarrow\gamma}(x/y)\right],$$

while the photon component evolves according to

$$\frac{d\gamma}{d\tau}(x, \tau) = \frac{\alpha(\tau)}{2\pi}\int_x^1 \frac{dy}{y} P_{\gamma\leftarrow e}(x/y)\left[e(y, \tau) + \bar{e}(y, \tau)\right] .$$

It is convenient to define moments of the distribution functions as

$$M_n(\tau) \equiv \int_0^1 dx\ x^{n-1} e(x,\ \tau) \quad ,$$

$$\bar{M}_n(\tau) \equiv \int_0^1 dx\ x^{n-1} \bar{e}(x,\ \tau) \quad .$$

The evolution of the moments is then easily computed. It takes a particularly simple form for the combination $M_n(\tau) - \bar{M}_n(\tau)$, for which

$$\frac{d}{d\tau}\,(M_n(\tau) - \bar{M}_n(\tau)) = \int_0^1 dx\ x^{n-1} \left[\frac{de}{d\tau}\,(x,\ \tau) - \frac{d\bar{e}}{d\tau}\,(x,\ \tau)\right]$$

$$= \frac{\alpha}{2\pi} \int_0^1 dx \int_x^1 \frac{dy}{y}\, P_{e\leftarrow e}(x/y)\, x^{n-1} \left[e(y,\ \tau) - \bar{e}(y,\ \tau)\right]$$

$$= \frac{\alpha}{2\pi} \int_0^1 d(\tfrac{x}{y}) \int dy\ y^{n-1}\left[e(y,\ \tau) - \bar{e}(y,\ \tau)\right](\tfrac{x}{y})^{n-1} P_{e\leftarrow e}(x/y)$$

$$= \frac{\alpha}{2\pi}\left[M_n(\tau) - \bar{M}_n(\tau)\right] A_n \quad ,$$

where

$$A_n \equiv \int_0^1 dz\ z^{n-1} P_{e\leftarrow e}(z) \quad .$$

Writing

$$\Delta_n(\tau) \equiv M_n(\tau) - \bar{M}_n(\tau) \quad ,$$

we have that

$$d \log\,(\Delta_n(\tau))/d\tau = \frac{\alpha(\tau)}{2\pi}\, A_n \quad .$$

If the effective coupling evolves as

$$\alpha(\tau) = \alpha(0)/[1 + b\alpha(0)\tau\ ] \quad ,$$

the differential equation is easily integrated to

$$\log\left(\frac{\Delta_n(\tau)}{\Delta_n(0)}\right) = \frac{\alpha(0)A_n}{2\pi}\int_0^\tau \frac{d\tau}{1+b\alpha(0)\tau}$$

$$= \frac{\alpha(0)A_n}{2\pi\alpha(0)b}\log\left(1+b\alpha(0)\tau\right)$$

$$= \frac{A_n}{2\pi b}\log\left(\frac{\alpha(0)}{\alpha(\tau)}\right) \quad .$$

We therefore predict that

$$\frac{\Delta_n(\tau)}{\Delta_n(0)} = \left[\frac{\alpha(\tau)}{\alpha(0)}\right]^{-A_n/2\pi b} \quad .$$

In QED, we have seen before that $b = -1/3\pi$, so that

$$\frac{\Delta_n(\tau)}{\Delta_n(0)} = \left[\frac{\alpha(\tau)}{\alpha(0)}\right]^{3A_n/2} \quad .$$

Thus, we have a prediction for the q^2-evolution of the moments and an especially simple prediction for

$$\frac{\log\left[\Delta_n(\tau)/\Delta_n(0)\right]}{\log\left[\Delta_k(\tau)/\Delta_k(0)\right]} = \frac{A_n}{A_k} \quad .$$

These specific forms are valid in first-order perturbation theory, although it is possible to incorporate higher-order corrections by iteration. The evolution of the moments is completely specified by the exponents A_n which may be calculated without reference to the electron and photon distribution functions. To describe the evolution of individual moments, rather than moment-by-moment ratios, it is necessary to know or determine the coupling constant $\alpha_s(0)$.

In fact, nothing of the procedure we have followed is specific to QED. The same method can be adapted to QCD, as was done by Altarelli and Parisi (1977), by identifying the electron, positron, and photon distributions as quark, antiquark, and gluon distributions, and allowing for the possibility of a gluon fluctuating into two gluons.

The so-called splitting functions $P_{i \leftarrow j}(z)$, which are to be computed in perturbation theory, satisfy some obvious sum rules. Fermion number conservation,

$$\int dx \left[\frac{dq^i}{d\tau}(x,\tau) - \frac{d\bar{q}^i}{d\tau}(x,\tau) \right] = 0$$

implies that

$$\int_0^1 dz\, P_{q \leftarrow q}(z) = 0 \qquad .$$

Here the superscript i has been introduced as a flavor index for the quarks. Momentum conservation,

$$\int_0^1 dx\, x \left[\sum_i \frac{dq^i}{d\tau}(x,\tau) + \sum_i \frac{d\bar{q}^i}{d\tau}(x,\tau) + \frac{dG}{d\tau}(x,\tau) \right] = 0$$

imposes two constraints:

$$\int_0^1 dz\, z \left[P_{q \leftarrow q}(z) + P_{g \leftarrow q}(z) \right] = 0 \qquad ,$$

and

$$\int_0^1 dz\, z \left[2n_f P_{q \leftarrow g}(z) + P_{g \leftarrow g}(z) \right] = 0 \qquad ,$$

where n_f denotes the number of quark flavors. In addition, momentum conservation at the elementary vertices requires a number of symmetry properties to hold for $z \neq 1$:

$$P_{q \leftarrow q}(z) = P_{g \leftarrow q}(1 - z) \qquad ,$$

$$P_{q \leftarrow g}(z) = P_{q \leftarrow g}(1 - z) \qquad ,$$

$$P_{g \leftarrow g}(z) = P_{g \leftarrow g}(1 - z) \qquad .$$

The computation of the splitting functions is described in detail in the paper by Altarelli and Parisi (1977). The procedure is straightforward:

- For $z \neq 1$, $P_{i\leftarrow j}$ is related to the square of an elementary matrix element.

- The strength of a $\delta(z - 1)$ term in $P_{i\leftarrow i}$, reflecting the possibility that there be no fluctuation, is fixed by imposing the integral constraint equations.

The resulting exponents for QCD are

$$A_n(q \leftarrow q) = \frac{4}{3}\left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n}\frac{1}{j} \right] ,$$

$$A_n(g \leftarrow q) = \frac{4}{3}\left[\frac{n^2 + n + 2}{n(n^2 - 1)} \right] ,$$

$$A_n(q \leftarrow g) = \frac{1}{2}\left[\frac{n^2 + n + 2}{n(n+1)(n+2)} \right] ,$$

$$A_n(g \leftarrow g) = 3\left[-\frac{1}{6} + \frac{2}{n(n-1)} + \frac{2}{(n+1)(n+2)} - 2\sum_{2}^{n}\frac{1}{j} - \frac{n_f}{9} \right] .$$

In any field theory, the splitting functions and hence the exponents are calculable in perturbation theory. A weak-coupling theory such as QED can be expected to give reliable results at low orders in perturbation theory. For the strong interactions, only an asymptotically free theory (such as QCD) presents any hope that low-order perturbation theory should be reliable. Of course, the value of Q^2 at which first-order results become trustworthy is not specified a priori. Specific numerical results for the $q\leftarrow q$ (nonsinglet) exponents in first-order QCD are: $A_1 = 0$ (by fermion number conservation), $A_2 = -1.78$, $A_3 = -2.78$, $A_4 = -3.49$, $A_5 = -4.04$, $A_6 = -4.50$, $A_7 = -4.89$. (See Problem 10.)

Professor Perkins has dealt at length in his lectures (Perkins, 1980) with the comparison of QCD and experiment. My remarks will therefore be quite brief. We have seen that the Q^2-evolution of the $q\leftarrow q$ (nonsinglet) moments is given by

$$\Delta_n(\tau) \quad = \quad \Delta_n(0)\left[\frac{\alpha_s(0)}{\alpha_s(\tau)}\right]^{A_n/2\pi b} \qquad .$$

In QCD, the running coupling constant is

$$\frac{\alpha_s(0)}{\alpha_s(\tau)} \quad = \quad 1 + \frac{\alpha_s(0)}{12\pi}\,(33 - 2n_f)\,\tau \qquad ,$$

which is to say that

$$b \quad = \quad (33 - 2n_f)/12\pi \qquad .$$

Hence the evolution of the moments becomes

$$\Delta_n(\tau) \quad = \quad \Delta_n(0) \div \left[1 + \frac{\alpha_s(0)\,(33 - 2n_f)\,\tau}{12}\right]^{6A_n/(33 - 2n_f)} \qquad .$$

It has become conventional to write

$$1/\alpha_s(q_o^2) \quad \equiv \quad \frac{(33 - 2n_f)}{12\pi}\log\,(\mu^2/\Lambda^2) \qquad ,$$

so that

$$1/\alpha_s(\tau) \quad = \quad 1/\alpha_s(0) + \frac{(33 - 2n_f)}{12\pi}\,\tau \qquad ,$$

is equivalent to

$$\alpha_s(Q^2) \quad \approx \quad \frac{12\pi}{(33 - 2n_f)\log\,(Q^2/\Lambda^2)} \qquad ,$$

which, it must be remembered, becomes nonsensical for $Q^2 \approx \Lambda^2$.

At issue are several points. The form of the moment-evolution equation is qualitatively valid. With $A_n < 0$ and $33 - 2n_f > 0$, Δ_n is correctly predicted to decrease with increasing Q^2. The quantitative reliability is more delicate to assess. Numerical fits determine, in principle at least, $\alpha_s(Q^2)$ or Λ^2. However there are a few ambiguities. These have to do with the self-consistency of first-order perturbation theory, the importance of nonleading-logarithmic corrections, and the choice of the effective number of quark flavors, in addition to the self-consistency of the fitting procedure itself. The moment-by-moment prediction is a much less differential test, but has the virtue of giving a single number. In this case, the Λ^2-ambiguity is absent--or at least hidden.

G. Status of QCD

Quantum Chromodynamics incorporates many of the observed systematics of the strong interactions in an elegant way that is in accord with currently-held theoretical prejudices. It promises calculability for the strong interactions in an unspecified asymptotically free regime. Some observables, such as

- $\sigma(e^+e^- \to \text{hadrons})$, and
- scaling violations in deep-inelastic scattering

hint that the domain of computability is not far away, and may already be accessible to experiment. Although there are many reasons for pessimism (which is sometimes known as realism), one may sensibly imagine a quantitative verification of the perturbative aspects of QCD within five years.

In the nonperturbative regime, which presumably has to do with confinement, there are numerous qualitative hints that color confinement may emerge. A detailed understanding of the hadron spectrum appears to require new mathematical inventions. Experimentally, it is most important to test the confinement *hypothesis* by searching for free quarks or for the signatures of unconfined color. It is not acceptable blithely to ignore the evidence for fractionally charged matter (LaRue, Phillips, and Fairbank, 1980), merely because the results seem unlikely or because they conflict with QCD orthodoxy. Sensitive negative searches for quarks continue to be interesting, and the convincing observation of free quarks would be revolutionary (see Lackner and Zweig, 1980).

7. GRAND UNIFICATION

A. Motivation[2]

With Quantum Chromodynamics and the Weinberg-Salam theory "in hand," what remains to be explained? In fact there are many observations which are explained only in part, or not at all, by the separate gauge theories of the strong and weak and electromagnetic interactions.

- The mixing parameter $\sin^2\theta_W$ is arbitrary, and there are three coupling constants: α_{EM}, α_s, $\sin^2\theta_W$. Could this number be reduced to two or one?

- Quarks and leptons both are spin-½, "fundamental" constituents. Are they connected in any way?

- The leptonic and hadronic charged weak currents are identical in form:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

  Why this pattern? How many quark and lepton generations are there?

- Why is $Q(e) + Q(p) \equiv 0$? Why is $Q(\nu) - Q(e) \equiv Q(u) - Q(d)$? Why is $Q(d) = (1/3)Q(e)$? Why is $Q(\nu) + Q(e) + 3Q(u) + 3Q(d) = 0$?

- Fermion masses and mixings are arbitrary. (Why) is the neutrino massless? Higgs couplings are arbitrary.

- The strengths of weak and electromagnetic interactions become comparable for $s \gg M_W^2$.

- It is conceivable that $\alpha_s(Q^2) \to \alpha_{EM}$ for very large values of Q^2.

- Gravitation is absent.

These observations provide motivations which fall into several categories. Some argue for a qualitative quark-lepton connection.

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[2] Cogent summaries are given by Harari (1978), Wilczek (1979), and Gaillard (1980).

Others inspire a more complete unification of weak and electromagnetic interactions, perhaps in the form of an additional gauge symmetry such as

$$G \supset SU(2)_L \otimes U(1) \quad ,$$

which would fix θ_W. Still others suggest a "Grand Unification" of weak, electromagnetic, and strong interactions. The energy at which $\alpha_s = \alpha_{EM}$ sets the scale at which grand unification is realized. This would automatically complete the unification of weak and electromagnetic interactions. Finally, it is possible to entertain a "Super Unification" which would include gravitation as well. Elaboration of this possibility will be deferred to a later St. Croix school.

One cannot fail to notice that both QCD and the Weinberg-Salam theory are <u>gauge theories</u>. It is therefore natural to base grand unification on a simple group $G \supset SU(3)_c \otimes SU(2) \otimes U(1)$, in order that there be a single coupling constant. The mass scale at which the symmetry is attained can be estimated from the evolution of the running couplings to be $\sim 10^{15}$ GeV. The gauge group G will contain extra gauge bosons beyond $W^{\pm}$, Z^0, γ, and gluons, which will carry both flavor and color properties. They are presumably very massive, because their effects are unfamiliar to us. One may speculate that all colored gauge bosons will be confined, along with the quarks and gluons.

Once grand unification is undertaken, there is no reason not to assign quarks and leptons to the same representation. Baryon and lepton conservation may be violated because exact conservation would require a massless, and unobserved, gauge boson. It is therefore likely that neither baryon number nor lepton number will be conserved exactly.

B. SU(5)

The minimal example of a grand unified theory is the SU(5) model introduced by Georgi and Glashow (1974). To analyze the structure of this model, it is helpful to refer to an $SU(2) \otimes SU(3)$ decomposition of the low-dimensioned representations of SU(5):

	SU(5)		(SU(2), SU(3))
	5	$\supset$	$(2, 1) \oplus (1,3)$
	5^*	$\supset$	$(2, 1) \oplus (1, 3^*)$
	10	$\supset$	$(2, 3) \oplus (1, 3^*) \oplus (1, 1)$
	10^*	$\supset$	$(2, 3^*) \oplus (1, 3) \oplus (1, 1)$
	$24 = 24^*$		$(1, 8) \oplus (2, 3) \oplus (2, 3^*) \oplus (3, 1) \oplus (1, 1)$

The first quark-lepton generation may be regarded as 15 left-handed (two-component) fermions, namely

2 leptons	(ν_e, e^-)	,
1 antilepton	e^+	,
6 quarks	$(u_R, u_B, u_G, d_R, d_B, d_G)$	,
6 antiquarks	$(\bar{u}_R, \bar{u}_B, \bar{u}_G, \bar{d}_R, \bar{d}_B, \bar{d}_G)$	.

These may be assigned to SU(5) representations as

$$5^* = \begin{pmatrix} \nu_e \\ e^- \\ \bar{d}_R \\ \bar{d}_B \\ \bar{d}_G \end{pmatrix}_{\text{left}}$$

and

$$10 = \begin{pmatrix} 0 & e^+ & d_R & d_B & d_G \\ -e^+ & 0 & u_R & u_B & u_G \\ -d_R & -u_R & 0 & \bar{u}_G & -\bar{u}_B \\ -d_B & -u_B & -\bar{u}_G & 0 & \bar{u}_R \\ -d_G & -u_G & \bar{u}_B & -\bar{u}_R & 0 \end{pmatrix}_{\text{left}}$$

Although it would have been pleasing to assign all the particles of the first generation to a single irreducible representation (as can be done (Ramond, 1980) in the closely analogous group SO(10)), there is nothing objectionable about this assignment. The assignments are guided by the tracelessness of the electric charge operator ($\Sigma Q_i = 0$), which is a generator of SU(5).

Constructing a gauge theory by the usual procedure, we encounter 24 gauge bosons:

γ	1
W^+, W^-, Z^o	3
gluons	8
$X^{\pm 4/3}$	$3 \times 2 = 6$
$Y^{\pm 1/3}$	$3 \times 2 = 6$

where the γ and Z^o acquire their ultimate identities only after the $SU(2) \otimes U(1)$ symmetry is broken down to $U(1)_{EM}$ (see Buras, et al., 1978). The fermion-gauge boson couplings present in the Lagrangian include those which occur in $SU(3)_c \otimes SU(2) \otimes U(1)$:

$$U(1) \qquad g_1 B_\mu \sum_{\substack{\text{fermion} \\ \text{species}}} \bar{f}\gamma^\mu Y f \qquad ,$$

$$\mathrm{SU(2)} \qquad g_2 \underset{\sim}{A}_\mu \cdot \left(\sum_{\text{quarks}} \bar{q}_L \gamma^\mu \underset{\sim}{T} q_L + \sum_{\text{leptons}} \bar{\ell}_L \gamma^\mu \underset{\sim}{T} \ell_L \right) ,$$

$$\mathrm{SU(3)}_C \qquad g_3 G^a_\mu \sum_{\text{quarks}} \bar{q} \gamma^\mu \lambda^a q ,$$

together with the couplings of the new leptoquark bosons:

$$g_4 X^i_\mu \left(\bar{d}^c_{iL} \gamma^\mu e_L + \varepsilon_{ijk} \bar{u}_{jL} \gamma^\mu u^c_{kL} + \bar{e}^+_L \gamma^\mu d_{iL} + \text{h.c.} \right) ,$$

$$g_5 Y^i_\mu \left(\bar{d}^c_{iL} \gamma^\mu \nu_L + \varepsilon_{ijk} \bar{u}_{jL} \gamma^\mu d^c_{kL} + \bar{e}^+_L \gamma^\mu u_{iL} + \text{h.c.} \right) ,$$

where (ijk) are color-triplet indices (RBG) and q^c denotes a charge-conjugate quark spinor. The coupling constants $g_1 \ldots g_5$ are all to be related at the unification mass by the group structure.

The new vertices in the theory, summarized in Fig. 50, mediate transitions such as proton decay, which proceeds via three elementary processes that change baryon number and lepton number by -1 and fermion number by -4. These are shown in Fig. 51. The intermediate bosons W^+, W^-, Z^0 acquire masses according to the usual spontaneous symmetry breaking procedure. The leptoquark bosons X and Y must be endowed with enormous masses by means of a similar scheme, for the theory to survive the existing bounds (Reines and Schultz, 1980) on the proton lifetime,

$$\tau(p) \quad > \quad 2 \times 10^{30} \mathrm{y} \times \Gamma(p \to \mu + X)/\Gamma(p \to \text{all}) .$$

The necessary symmetry breaking can be achieved in two steps. First, a real $\underset{\sim}{24}$ (of scalar fields) is introduced to break SU(5) down to $\mathrm{SU(3)}_C \otimes \mathrm{SU(2)} \otimes \mathrm{U(1)}$. At this step the X and Y leptoquark bosons acquire mass. Next, a complex $\underset{\sim}{5}$ (of scalar fields) is employed to break $\mathrm{SU(3)}_C \otimes \mathrm{SU(2)} \otimes \mathrm{U(1)}$ down to $\mathrm{SU(3)}_C \otimes \mathrm{U(1)}_{EM}$. This is the straightforward extension of the symmetry breaking in the Weinberg-Salam theory, in which a complex scalar SU(2) doublet breaks $\mathrm{SU(2)} \otimes \mathrm{U(1)}$ down to $\mathrm{U(1)}_{EM}$. The spontaneous symmetry breaking gives rise to many physical Higgs scalars. From the $\underset{\sim}{24}$, after twelve of their fellows have become the longitudinal components of massive X and Y vector bosons, there remain massive scalars with quantum numbers specified by

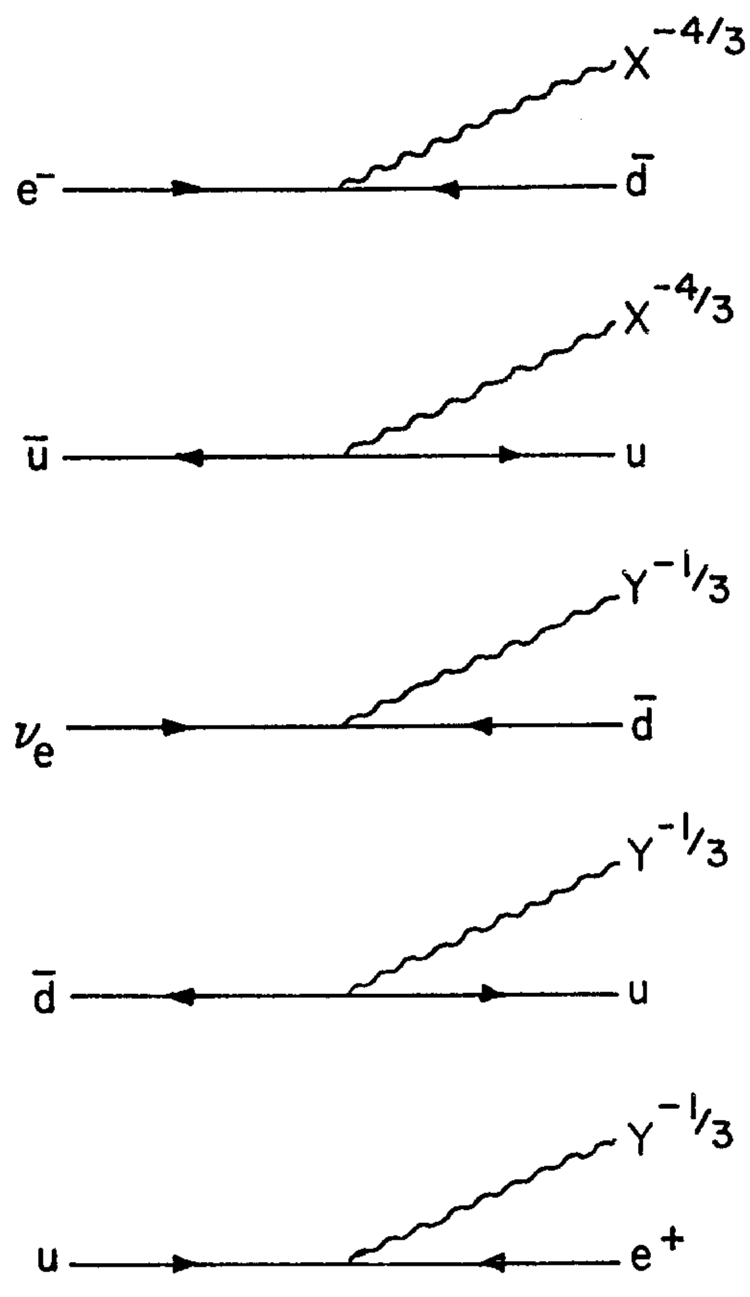


Fig. 50. New fermion-fermion transitions which appear in the grand unified theory SU(5).

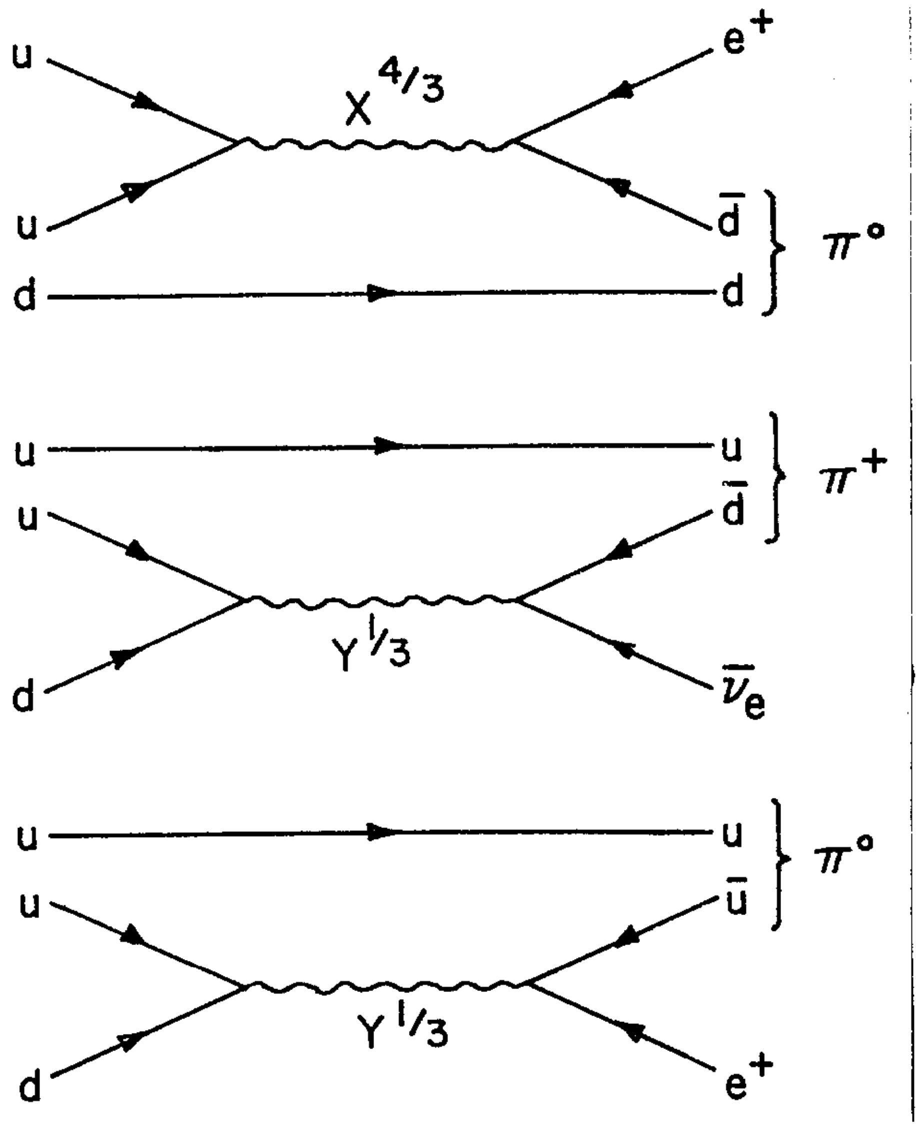


Fig. 51. Some mechanisms for proton decay in SU(5).

$$(1,\ 8) \oplus (1,\ 1) \oplus (3,\ 1) \quad ,$$

which may be hoped to have masses comparable with those of X and Y. From the complex $\underset{\sim}{5}$, only three fields are eaten by W^+, W^-, and Z^0. There remain as physical particles the normal (1, 1) Higgs scalar, still with unknown mass, plus

$$(1,\ 3) \oplus (1,\ 3^*) \quad ,$$

a color triplet $h^{\pm 1/3}$. These leptoquark Higgs bosons can mediate proton decay, by transitions analogous to those that are mediated by $Y^{\pm 1/3}$. It is therefore necessary to arrange locally gauge-invariant interactions among the $\underset{\sim}{5}$ and $\underset{\sim}{24}$ fields which will yield enormous masses for $h^{\pm 1/3}$.

The minimal SU(5) theory has numerous attractive features.

- It contains $SU(3)_C \otimes SU(2) \otimes U(1)$.
- The charged currents are V - A.
- The neutrino is automatically massless. (A virtue for the moment; otherwise see SO(10).)
- Charge is quantized.
- Masses of leptoquark bosons can be made large, but below the Planck mass $M_P \equiv \sqrt{\hbar c/G_{Newton}} = 1.22 \times 10^{19}$ GeV/c^2.
- The weak mixing angle, $\sin^2 \theta_W \approx 0.20$, in approximate agreement with experiment.
- Proton decay is possible, and may lead to an understanding of the apparent baryon excess in the universe (Sakharov, 1967; Yoshimura, 1978; Toussaint, et al., 1979).
- SU(5) provides an existence proof for grand unified theories, and seems to show that a unification of the strong, weak, and electromagnetic interactions can meaningfully be achieved without gravitation.

There are as well a number of problems to be faced.

- Each family or generation is reducible.
- Why do generations repeat? How many are there?
- Why are there $\sim$12 orders of magnitude between the mass scales at which the two symmetry breakings occur? Is it possible to maintain the result

$$M_W/M_X \lll 1$$

beyond lowest-order perturbation theory?

- Gravity is omitted.

• No insight is gained into the nature of fermion masses or mixing angles. CP-violation in the weak interactions does not arise gracefully.

At a minimum, grand unification reminds us that we do not understand baryon and lepton number conservation. It therefore becomes an experimental imperative to probe the soft spots in search of neutrino masses, violations of lepton number, and evidence for proton instability. For my part, I attach little significance to specific numerical predictions of grand-unified theories. To the extent that they set inviting targets for experiment, they are undoubtedly of inspirational value. But the path between grand-unifying gauge group and experimental tests is often long, winding, and slippery!

8. OMNE IGNOTUM PRO MAGNIFICO

I began these lectures by remarking upon the widespread belief that a grand synthesis of physical law is at hand. Our subsequent discussions have demonstrated the power of gauge principles as sources of the fundamental interactions. Gauge theories are renormalizable, may be asymptotically free, and have been known to agree with experiment. But how are we to choose a gauge group?

The minimal strategy of grand unified theories, as we have seen in Chapter 7, is to find a simple gauge group which contains the "well-established" color and flavor groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This is not a unique guiding principle. It nevertheless seems to many of my colleagues that the only interactions in Nature are those we know plus those required to complete the grand (or later, super) unification. By extensions of this reasoning, many conclude that a vast desert awaits us; that no interesting new phenomena will occur between the mass of the intermediate bosons of the weak interactions and the mass of the leptoquark bosons. History does not encourage such a bleak view, but it is fair to argue that our illustrious predecessors who erroneously thought the end was in sight did not have local gauge invariance beside them, to guide them.

I labor under different delusions: that there will be new surprises, new phenomena, and that everything we don't know will turn out to be wonderful. Although the grounds for this simple faith are largely neurochemical, it may have some basis in physics. Within the framework of gauge theories (for we know nothing else), who is to say that we have already noticed all the gauge symmetries relevant at moderate energies? Without experimental searches or a comprehensive understanding of the physical origin of gauge invariance, this is merely attractively economical speculation. That the known running coupling constants should meet in a single point is likewise the simplest, but not the only, possibility. I expect pleasant surprises!

In addition to the interaction problem, there is the problem of the fundamental fermions. We do not understand generations, masses, mixing angles, or CP-violation. The elementary Higgs boson realization, which we have discussed, has an unappealing arbitrariness and proliferation of parameters. Dynamical symmetry breaking schemes promise succor to those who would believe ours to be the only possible world. The right such scheme has not yet emerged. An alternative approach is to impute structure to the quarks and leptons, and to seek simple patterns at the next level of fundamental constituents. Still another is to hope that the inclusion of gravity will prove so restrictive as to compel the existence of the universe as we find it--no more, and no less.

Within the conventional framework, there remains the issue of color confinement, and of the hadron spectrum. It is my feeling that new mathematical inventions and perhaps new physical imagery will be required before a theoretical solution is in hand. On the experimental side, much remains to be learned about the hadronization of quarks and gluons, and about the hadron spectrum itself. The Tevatron and the new $\bar{p}p$ colliders will have much to say in answer to questions that we can now pose only vaguely.

I hope these lectures have communicated not only a few facts, but also a feeling that there is much to be done that is significant and exciting, that there are many opportunities to contribute to this numinous intellectual adventure in which we all share.

ACKNOWLEDGMENTS

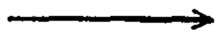
It is a pleasure to thank Tom Ferbel for issuing the challenge to prepare these lectures and for his energetic efforts to ensure the existence and success of the St. Croix School. The present text evolved from a course of lectures I gave at the University of Chicago in the Fall Quarter of 1979. The comments, questions, and occasional expressions of bewilderment of the students in that course have been extremely valuable. I am grateful to Prof. K. Chadan, Prof. J. Tran Thanh Van, and their colleagues in the Laboratoire de Physique Théorique et Hautes Energies, Université de Paris XI, Orsay for providing a tranquil but stimulating haven in which to accomplish some of the writing of these notes. My visit to Orsay was made possible by a generous grant from the French Ministry of Foreign Affairs. My colleagues Andrzej Buras and P.Q. Hung have been most helpful in suggesting pedagogically useful references and in offering advice on the presentation of specific topics. Mark Fischler, Chris Hill, and Hank Thacker also read sections of the text and helped to reduce the number of infelicities and blunders. To them and to the participants in the St. Croix Institute go my warm thanks. Finally, I am pleased to acknowledge the indispensable contributions of Angela Gonzalez, who drew the figures, and Trudi Legler, who typed the manuscript.

Problem 1. Analyze the absorption of a virtual photon by a spin-1/2 quark in the Breit frame (brick-wall frame) of the quark. Kinematics:

incident:

$(p_z, E) =$ $(Q, 0)$ $(-\frac{Q}{2}, \frac{Q}{2})$

outgoing:



$(\frac{Q}{2}, \frac{Q}{2})$

(a) Show that the squared matrix element for the absorption of a longitudinal photon vanishes.

(b) Compute the square of the matrix element for absorption of a photon with helicity = +1, i.e. a transverse photon.

(c) How would your result for a longitudinal photon differ if the incident quark and photon were not precisely (anti)collinear?

Problem 2.

(a) Compute the differential cross section $\frac{d\sigma}{d\Omega}$ and the total (integrated) cross section

$$\sigma \equiv \int d\Omega \frac{d\sigma}{d\Omega} \text{ for the reaction } e^+e^- \rightarrow \mu^+\mu^- \quad .$$

Work in the c.m. frame, and in the high-energy limit (where lepton masses may be neglected). Assume the colliding beams are unpolarized and sum over the polarizations of the produced muons.

(b) Look up the evidence for $q\bar{q}$ jets in the reaction $e^+e^-\to$hadrons. [G. J. Hanson, *et al.*, Phys. Rev. Lett. 35, 1609 (1975).] Now compute the differential cross section $d\sigma/d\Omega$ for the reaction $e^+e^-\to\mu^+\mu^-$, assuming the initial beams are transversely polarized. See also R.F. Schwitters, *et al.*, Phys. Rev. Lett. 35, (1975).

Refer to Bjorken and Drell or a similar textbook for help with the computation.

Problem 3.

Assume that the charged weak current has the left-handed form discussed in class, and that the interaction Hamiltonian is of the "current-current" form,

$$\mathcal{H}_W \sim JJ^\dagger + J^\dagger J \quad .$$

(a) Enumerate the kinds of interactions (i.e., terms in the Hamiltonian) that may occur in a world composed of the electron and muon generations

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad .$$

(b) List the leptonic processes which are consistent with the known selection rules but do not occur in $\mathcal{H}_W$. Example: $\nu_\mu e\to\nu_\mu e$.

Problem 4.

Now consider the interactions of a single lepton doublet

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

with a single quark doublet

$$\begin{pmatrix} u \\ d \end{pmatrix}_L .$$

(a) In the limit of large incident energy, and neglecting the electron mass, calculate the differential cross section

$$\frac{d\sigma_{cm}}{d\Omega}$$

and integrated cross section

$$\sigma = \int d\Omega_{cm}\, d\sigma/d\Omega_{cm}$$

for the reactions

$$\text{(i)} \quad \nu_e + d \to e^- + u$$

$$\text{(ii)} \quad \bar{\nu}_e + u \to e^+ + d \quad .$$

Assume that the quarks have a common mass, m_Q.

(b) Discuss the difference in the cross sections for (i) and (ii), and provide a physical explanation for it.

Problem 5.

Use the requirement that the Lagrangian be invariant under a continuous symmetry to deduce the conserved quantity corresponding to a transformation. Show that invariance under

$$\left.\begin{array}{r l} \text{(i)} & \text{translations in time} \\ \text{(ii)} & \text{translations in space} \\ \text{(iii)} & \text{spatial rotations} \end{array}\right\} \text{ implies conservation}$$

$$\text{of} \left\{\begin{array}{r l} \text{(i)} & \text{energy} \\ \text{(ii)} & \text{momentum} \\ \text{(iii)} & \text{angular momentum} \end{array}\right.$$

reference: E.L. Hill, Rev. Mod. Phys. 23, 253 (1953).

Problem 6. Derive the Yang-Mills Lagrangian for a scalar field theory in which the three real scalar fields correspond to the triplet representation of SU(2). The basic Lagrangian is

$$\mathscr{L} = \frac{1}{2}\left[(\partial_\mu \underset{\sim}{\phi})^2 - m^2 \underset{\sim}{\phi}^2\right] ,$$

with

$$\underset{\sim}{\phi} = \begin{pmatrix} \phi_+ \\ \phi_0 \\ \phi_- \end{pmatrix} .$$

Problem 7.

Analyze the spontaneous breakdown of a global SU(2) symmetry. Consider the case of three real scalar fields ϕ_1, ϕ_2, ϕ_3, which comprise an SU(2) triplet, denoted

$$\underset{\sim}{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} .$$

The Lagrangian density is

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \underset{\sim}{\phi}) \cdot (\partial_\mu \underset{\sim}{\phi}) - V(\underset{\sim}{\phi} \cdot \underset{\sim}{\phi}) ,$$

where as usual

$$V = \frac{\mu^2}{2} \underset{\sim}{\phi} \cdot \underset{\sim}{\phi} + \frac{\lambda}{4}(\underset{\sim}{\phi} \cdot \underset{\sim}{\phi})^2 .$$

Assume the potential has a minimum at

$$\langle \underset{\sim}{\phi} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} .$$

Then show that (1) the Lagrangian remains invariant under T_3; (2) The particles associated with T_1 and T_2 become massless

(Goldstone) particles; (3) The particle associated with T_3 acquires a mass $= \sqrt{-2\mu^2}$.

Problem 8. The Ginzburg-Landau Theory of Superconductivity provides a phenomenological understanding of the Meissner effect: the observation that an external magnetic field does not penetrate the superconductor. Ginzburg and Landau introduce an "order parameter" ψ, such that $|\psi|^2$ is related to the density of superconducting electrons. In the absence of an impressed field, expand the Free Energy of the superconductor as

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha|\psi|^2 + \beta|\psi|^4 \quad ,$$

where α and β are phenomenological parameters.

(a) Minimize $G_{\text{super}}(0)$ with respect to the order parameter and discuss the circumstances under which spontaneous symmetry breaking occurs. Compute $|\psi_0|^2$, the value at which $G_{\text{super}}(0)$ is minimized.

(b) In the presence of an external field H_e, a gauge-invariant expression for the free energy is

$$G_{\text{super}}(H_e) = G_{\text{super}}(0) + \frac{H_e^2}{8\pi} + \frac{1}{2m^*}\,\psi^*\left(-i\hbar\nabla - \frac{e^*}{c}\,\underset{\sim}{A}\right)^2\psi \quad .$$

[The effective charge e^* turns out to be 2e, because $|\psi|^2$ represents the density of Cooper pairs.] Derive the field equations that follow from minimizing $G_{\text{super}}(H_e)$ with respect to ψ and $\underset{\sim}{A}$. Show that in the weak-field approximation ($\nabla\psi\approx 0$, $\psi\approx\psi_0$) the photon acquires a mass within the superconductor.

Problem 9.

Compute the differential and total cross sections for $\nu_e e$ and $\bar{\nu}_e e$ elastic scattering in the Weinberg-Salam model. Work in the limit of large M_W, M_Z. The computation is done most gracefully by Fierz reordering one of the graphs.

Problem 10 . Using the Altarelli-Parisi approach and working to lowest order in perturbation theory, show that in a theory of colored quarks interacting by means of scalar gluons the non-singlet critical exponent is

$$A_n^{NS} = \frac{\alpha}{4\pi} C_2(R)\left[1 - \frac{2}{n(n+1)}\right] \quad .$$

where $C_2(R) = (N^2-1)/2N$ for $SU(N)_C$. Predict the slopes of

(i) M_5/M_3

(ii) M_6/M_4 .

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