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Vacuum Instability and New Constraints on Fermion Masses

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ABSTRACT

We show that in order for the physical vacuum, in the Weinberg-Salam model with one Higgs doublet, to be an absolute minimum (in the one-loop approximation), certain requirements on the fermion masses have to be met. Specifically, the quantity $\{\sum_i m_{f_i}^4\}^{1/4}$, where " $\sum_i$ " denotes the sum over fermions, is bounded from above by approximately 133.5 GeV ($\sin^2\theta_W = 0.25$) or 137.7 GeV ($\sin^2\theta_W = 0.2$).

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I. INTRODUCTION

It is now commonplace to describe hadronic elastic scattering as a diffractive shadowing of two spatially extended objects. In such a picture one expects a sharply peaked angular distribution whose width is reflective of the sizes of the particles being scattered. Without a theory for hadronic internal structure, quantitative calculations are not yet possible. However, for processes such as nucleus-nucleus elastic scattering where the structure of the colliding objects is known, detailed calculations can be carried out and compared with data.[1] From these analyses one can hope to abstract those features which might be relevant to hadron scattering. Chou and Yang[2] proposed an optical model for elastic scattering based on these considerations. In this model, the colliding hadrons are pictured as two spatially extended Lorentz contracted balls of hadronic matter, which propagate through each other. During the passage, various inelastic interactions take place, and the elastic amplitude is built up as the shadow of these inelastic interactions. The elastic amplitude at impact parameter b is given by an eikonal formula:

$$t_{el}^{AB}(b) = 1 - e^{-\langle\Omega(b)\rangle_{AB}} . \tag{1}$$

The eikonal function $\langle\Omega(b)\rangle_{AB}$ is assumed to be proportional to the overlap of the average matter densities $\langle\rho_A(b)\rangle$ and $\langle\rho_B(b)\rangle$ of the incident particles:

$$\langle\Omega(b)\rangle_{AB} = \kappa_{AB} \int d^2b' \langle\rho_A(\underset{\sim}{b}')\rangle\langle\rho_B(\underset{\sim}{b} - \underset{\sim}{b}')\rangle . \tag{2}$$

Model-independent analyses[1] of neutral-current data as well as the recent SLAC polarized electron scattering experiment[2] have revealed the remarkable fact that the only viable (and by far the simplest) $SU(2) \times U(1)$ gauge theory of electroweak interactions is the Weinberg-Salam model[3] (often referred to as the standard model). However, the data only tell us, so far, about the symmetry nature of the neutral current and its relative strength to the charged current. These striking facts, although in very good agreement with the standard model, are not sufficient[4] to prove its main ingredient, namely the spontaneously broken symmetry nature of gauge theories. Until one actually finds the Higgs boson(s) with couplings which are characteristic of spontaneously broken gauge theories (SBGT), one must look for indirect effects or requirements that follow from the intrinsic nature of SBGT. It is the purpose of this note to point out that one does indeed obtain non-trivial constraints on the fermion masses by just looking at the vacuum instability of the standard model with one Higgs doublet.[13]

From the pioneering works of Coleman, E. Weinberg and S. Weinberg,[5] we know that one-loop radiative corrections to the classical (tree) Higgs potential can drastically change the vacuum structure of the theory. If spontaneous symmetry breaking is to occur, certain relationships between various coupling constants of the theory have to be satisfied. These requirements turn into restrictions on the Higgs and fermion masses. More specifically, we obtain an upper bound (and under a special circumstance, even a lower bound) on the quantity $\{\sum_i m_{f_i}^4\}^{1/4}$, where "$\sum_i$" stands for the sum over fermions.

We restrict ourselves to the case of a single scalar doublet in the standard model. The zero- and one-loop contributions to the effective Higgs potential $V(\phi_c)$ are given by[5]

$$V(\phi_c) = -\frac{1}{2}\mu_R^2\phi_c^2 + \frac{\lambda}{4!}\phi_c^4 + \kappa\phi_c^4\left[\ln\left(\frac{\phi_c^2}{\langle\phi\rangle^2}\right) - \frac{25}{6}\right] \quad , \qquad (1)$$

where we have used the following renormalization conditions

$$\left.\frac{d^2V(\phi_c)}{d\phi_c^2}\right|_{\phi_c=0} = -\mu_R^2 \quad \left(\mu_R^2 > 0\right) \quad , \qquad (2)$$

$$\left.\frac{d^4V(\phi_c)}{d\phi_c^4}\right|_{\phi_c=\langle\phi\rangle} = \lambda \quad , \qquad (3)$$

and where $\kappa = (64\pi^2)^{-1}\left\{3\left(2g_{WH}^4 + g_{ZH}^4\right) + \lambda^2/4 - 4\sum_i g_{f_iH}^4\right\}$, $\phi_c^2 = \phi_c^\dagger\phi_c$. The constants g_{WH}, g_{ZH} and g_{f_iH} stand for the couplings of the Higgs boson to the $W^\pm$, Z bosons and the fermions respectively. They are given, in the standard model, by $g_{WH}^2 = g^2/4$, $g_{ZH}^2 = g^2/(4\sec^2\theta_W)$, where $g^2/8m_W^2 = G_F/\sqrt{2}$. We can rewrite Eq. (1) as follows

$$V(\phi_c) = -\frac{1}{2}\mu_R^2\phi_c^2 + \frac{\lambda_{int}}{4!}\phi_c^4 + \kappa\phi_c^4\ln\left(\phi_c^2/\langle\phi\rangle^2\right) \quad , \qquad (4)$$

where $\lambda_{int} = \lambda - 100\,\kappa$. For the one-loop approximation to be reliable, one needs $\left(\lambda, g^2, g_{f_iH}^2\right) << 1$, $\left(\lambda, g^2, g_{f_iH}^2\right) \times \ln\left(\phi_c^2/\langle\phi\rangle^2\right) << 1$. Since we are looking for an upper bound on the fermion masses, we will not neglect the contributions to $V(\phi_c)$ from fermion loops in Eq. (4).

The local minimum of $V(\phi_c)$ and the physical mass of the Higgs boson are defined by

$$\left.\frac{dV(\phi_c)}{d\phi_c}\right|_{\phi_c=\langle\phi\rangle} = 0 \quad , \qquad (5)$$

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So far we have obtained only a constraint on the Higgs boson mass as represented by the inequality (11). To get a condition on fermion masses, we need to take a closer look at the effective potential $V(\phi_c)$ as given by Eq. (4). We may ask the following question: is the local minimum given by Eq. (7) truly an absolute minimum? Under what conditions does that vacuum become unstable?

To answer the above questions, let us examine $V(\phi_c)$ carefully. From Eq. (4), one can see that for $\kappa < 0$, the effective potential $V(\phi_c)$ is unbounded from below for asymptotic values of ϕ_c. What $\kappa < 0$ means is $\sum_i g^4_{f_iH} > \frac{1}{4}\left\{3\left(2g^4_{WH} + g^4_{ZH}\right) + \lambda^2/4\right\}$. Repeating an argument due to Krive and Linde[7] who examined a simplified version of the σ-model, one can say that for $(g^2, \lambda) << g^2_{f_iH}$ ($\kappa < 0$), there is a certain value of ϕ_c, say $\tilde{\phi} > \langle\phi\rangle$, for which $g^2_{f_iH} \ln(\tilde{\phi}^2/\langle\phi\rangle^2) \simeq \lambda/g^2_{f_iH}$ or $g^2/(g^2_{f_iH}) << 1$, $\lambda \ln(\tilde{\phi}^2/\langle\phi\rangle^2) \simeq \lambda^2/g^4_{f_iH} << 1$, $g^2 \ln(\tilde{\phi}^2/\langle\phi\rangle^2) \simeq g^4/g^4_{f_iH} << 1$, and $V(\tilde{\phi}) < V(\langle\phi\rangle)$. In such a case the one-loop approximation is still reliable. One can see that the local minimum at $\langle\phi\rangle$ is unstable and is not the true vacuum of the theory. The true stable vacuum then occurs only at an asymptotically large value of ϕ_c, in which case one cannot rely on perturbation theory anymore. In Coleman's terms,[8] the local minimum at $\langle\phi\rangle$ is then a false vacuum.

Let us assume that the minimum at $\langle\phi\rangle$ is actually the true vacuum which we live in. We then have the condition $\kappa \geq 0$, which means that

$$\sum_i g^4_{f_iH} \leq \frac{1}{4}\left\{\frac{3}{16} g^4(2 + \sec^4\theta_W) + \frac{\lambda^2}{4}\right\} \quad . \tag{12}$$

With $m^2_{f_i} = g^2_{f_iH}\langle\phi\rangle^2$, $m^2_W = (g^2/4)\langle\phi\rangle^2$, $m^2_Z = (g^2/4)\langle\phi\rangle^2 \sec^2\theta_W$, $\langle\phi\rangle^2 = (\sqrt{2}G_F)^{-1}$, we can rewrite the constraint (12) as

$$\sum_i m_{f_i}^4 \leq \frac{3}{4} m_W^4 (2 + \sec^4 \theta_W) + \frac{\lambda^2}{32G_F^2} \quad , \tag{13}$$

where m_{f_i} is the mass of the i^{th} fermion. (13) is the basic constraint imposed on the fermion masses in the standard model with a single Higgs doublet.

For the validity of the one-loop approximation, one needs of course $\lambda << 1$. We shall indulge ourselves in letting $\lambda \leq 1$ as required for the validity of perturbation theory. Armed with this requirement we now distinguish two cases.

(a) $\kappa > 0$

The bound (13) now becomes

$$\sum_i m_{f_i}^4 < G_F^{-2} \left\{ \frac{3}{8} \pi^2 \alpha^2 \operatorname{cosec}^4 \theta_W (2 + \sec^4 \theta_W) + \frac{1}{32} \right\} \quad , \tag{14}$$

where we have used $m_W^2 = \left(\pi\alpha / \sqrt{2} G_F \right) \sin^{-2} \theta_W$, $\alpha \equiv e^2/4\pi$. We obtain

$$\left\{ \sum_i m_{f_i}^4 \right\}^{1/4} \begin{cases} < 133.5 \text{ GeV} & (\sin^2 \theta_W = 0.25) \\ < 137.7 \text{ GeV} & (\sin^2 \theta_W = 0.2) \end{cases} \quad . \tag{15}$$

Taking the masses of known quarks to be $m_u \sim 4$ MeV, $m_d \sim 7$ MeV, $m_c \sim 1.2$ GeV, $m_s \sim 150$ MeV, ?, $m_b \sim 4.6$ GeV, and also taking into account the masses of e, μ and τ, one can see that the upperbound (15) is rather insensitive to those "light" fermion masses. Therefore in (14), we can make the following replacement $\left\{ \sum_i m_{f_i}^4 \right\}^{1/4} \rightarrow \left\{ \sum_i m_{f_i}^4 \right\}^{1/4}_{\text{Heavy}}$, where the term "heavy" means that $\sum_i$ is to extend over fermions other than known ones. We will neglect the effects of strong interactions on the quark masses.

If there are heavy fermions or a large number of lighter ones which obey the bounds (15), one can see that the lower bound (11) on the Higgs mass can be significantly smaller than the Weinberg-Linde value which is just $(3\sqrt{2}G_F/16\pi^2) \times [\, m_W^4(2 + \sec^4\theta_W)\,]$ ($m_H > 5$ GeV for $\sin^2\theta_W \simeq 0.25$). Furthermore, as pointed out by some authors,[9] the vacuum at $\langle\phi\rangle \neq 0$ is a metastable one if $m_H^2 < 8\langle\phi\rangle^2\kappa$, for then the other minimum is at $\phi_c = 0$. As Linde[10] has shown, if the early Universe was in a metastable vacuum then for spontaneous symmetry breaking to occur, the Higgs mass has to obey $m_H > 260$ MeV, where the contributions from fermions to the effective potential have been neglected. If however heavy fermions do exist, we would expect the bound to be much lower than 260 MeV.[11]

(b) $\kappa = 0$

This case is interesting in its own right. The contributions from gauge boson, Higgs and fermion loops miraculously cancel each other. In this case, we have an equality in (13) and, for $0 \leq \lambda \leq 1$, the fermion masses obey

$$96.8 \text{ GeV} \leq \left\{ \sum_i m_{f_i}^4 \right\}^{1/4}_{\text{Heavy}} \leq 133.5 \text{ GeV } (\sin^2\theta_W = 0.25) \quad , \qquad (16)$$

$$106.6 \text{ GeV} \leq \left\{ \sum_i m_{f_i}^4 \right\}^{1/4}_{\text{Heavy}} \leq 137.7 \text{ GeV } (\sin^2\theta_W = 0.2) \quad , \qquad (17)$$

where again the contributions from known fermions to both upper and lower bounds are negligible.

The effective potential then takes the following familiar form

$$V(\phi_c) = -\frac{1}{2}\mu_R^2\phi_c^2 + \frac{\lambda}{4!}\phi_c^4 \quad . \qquad (18)$$

The <u>only</u> minimum occurs at $\langle\phi\rangle^2 = (6\mu_R^2)/\lambda$, and is <u>absolutely</u> stable. The Higgs mass is then given by

$$m_H^2 = \frac{\lambda}{3} <\phi>^2 \quad . \tag{19}$$

One can see from (19) that there no longer exists any restriction on the Higgs mass and it can be arbitrarily small if λ and μ_R^2 are themselves sufficiently small. If the Higgs boson is discovered to have an extremely small mass, one is tempted to conjecture that the case $\kappa = 0$ is what happens in nature and one should be ready for another generation of heavy fermions with masses in the range of 40–100 GeV.

Our discussions can be generalized to the case where we have more than one Higgs doublet. We suspect that the results presented here will not be much affected by the inclusion of many Higgs doublets. We also wish to point out that using partial wave unitarity at high energies, Chanowitz, Furman and Hinchliffe[12] have established upper bounds on quark and lepton masses which are significantly higher than the ones presented here. Their bounds are $(500/\sqrt{N})$ GeV and $(1.0/\sqrt{N})$ TeV for quarks and leptons separately with N being the number of flavor doublets.

The discovery of new flavors of quarks and leptons with high masses would be extremely important for our understanding of the nature of spontaneously broken gauge theories. One can look for these fermions either by the LEP machine (which is still under discussion), or indirectly through radiative corrections to low-energy processes such as the ones discussed by Veltman or Chanowitz et al.[12]

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Comparing this QCD picture with the parton model picture, we see that the valence partons which carry the charge and momentum are to be identified with the quarks at the ends of the tube, while the sea partons are to be identified with the glue contained in the tube. The transverse distribution of wee partons is not expected to be significantly different from that of the rest of the sea.[17]

In this picture, we expect the average matter distribution to have a steep central component compared to the charge distribution. Each configuration has matter distributed throughout the tube, whereas the charge is confined primarily to the ends, so that in doing the averages one expects more weight to be built up in the center for the matter than for the charge distribution. Such an argument depends, of course, on how each configuration is weighted, and this can be studied in models. One can also study to what extent this argument depends on the configurations being tube like.

A toy model can be easily constructed which illustrates the above ideas and their consequences. Our meson is modeled as a cylinder of vanishing radius a and varying length. The charge is located at the ends, and the neutral matter (glue) is distributed uniformly throughout the cylinder with a density ρ independent of the length $2R$ of the cylinder. The average matter density is then approximately

$$\rho(r) = \int d^3R\,|\Psi(R)|^2\theta(R-r)\theta(a-|r\sin\theta|) \tag{20}$$

up to a constant, where $\cos\theta = \mathbf{R}\cdot\mathbf{r}/Rr$, and $\Psi(R)$ is the wave function describing the amplitude for finding the valence quarks separated by a vector $\underset{\sim}{R}$. Since the valence quarks carry the charge, $|\Psi(R)|^2$ is the meson form factor; experimentally the Q^2 dependence is that of a monopole, which is

$$|\Psi(R)|^2 = \frac{e^{-R/R_0}}{R/R_0} \tag{21}$$

in coordinate space. Using this wave function, the density $\rho(r)$ can be analytically evaluated in the limit of small a:

$$\rho(r) = c\, e^{-r/R_0}\left\{\frac{1}{(r/R_0)^2} + \frac{1}{r/R_0}\right\} . \tag{22}$$

where c is a constant. The second term is proportional to the charge distribution; the first term is more singular as we expected. The matter distribution is more central since many more configurations have their centers overlapping than their ends. The comparison between charge and matter form factors is given in Fig.4.

There are obviously a number of important features left out of our toy model. The tube could have a width, and in QCD is expected to have one.[18] Also there may be configurations where the tube is more spherical than cylindrical, especially when the valence quarks are near each other. We have constructed simple models to take these effects into account and find the effect of including more three dimensional configurations is to increase the r.m.s. value of the matter distribution. This could have important consequences especially for baryons since one can imagine many more types of configurations than for mesons. Nevertheless it is still possible that the tube picture is at least approximately correct, even for baryons,[19] and could provide a basis for doing more detailed calculations.

Let us now consider elastic scattering in this QCD picture.[20] As emphasized, diffraction scattering depends on the average overall configurations. Since we have in mind a specific picture of which configurations are important we obtain some insight into the diffractive process. Now a configuration is labeled not only by a size and a shape, but also by a density of glue. There will be fluctuations in this density which give an important contribution to the inelastic diffraction cross section as has been argued previously.[10] What is new in our picture is that there also will be comparable contributions due to fluctuations in the size and shape.[21]

In this picture, the average eikonal will still be proportional to the overlap of the average matter distributions, which are estimated with the toy model. Since the matter distribution has a steep central component, so will the average eikonal. If this eikonal were used in the usual formula Eq.(1), then an incorrect elastic scattering amplitude would result. It is because of averaging over the fluctuations also for the elastic scattering amplitude that a consistent picture is possible. The importance of the fluctuations to elastic scattering is demonstrated by the large difference between our results and those which ignore such fluctuations. It is encouraging to see these large fluctuations can arise naturally in this QCD inspired picture of hadron structure. A serious test of those ideas would be to get a consistent phenomenological description of both elastic and inelastic diffraction data. This we have not yet attempted.

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21. We have studied the contribution to inelastic diffraction from the fluctuations in the size and shape and found some interesting results. This contribution is more peripheral than that from the density fluctuations, and likely to be concentrated to smaller masses. Several arguments suggest to us that diffractive resonance excitation is due to the size and shape fluctuations, and not to the density fluctuations.

FIGURE CAPTIONS

Fig.1. Eikonal functions deduced from proton-proton elastic scattering data at $\sqrt{s} = 53$ GeV (Ref.12). The curves are labeled by the corresponding total inelastic diffractive cross-section.

Fig.2. a) Proton's matter and charge form factors. The matter form factors $G_H^p(q^2)$ are obtained from the eikonal functions of Fig.1 through Eq.(17). The experimental data on the charge from factor $G_E^p(q^2)$ are from Ref.14.a(●) 14.b(□), 14.c(○) and 14.d(▲). The dipole parametrization Eq.(19) is also shown.

b) Proton's matter and charge distributions. The matter distributions $\rho_H^p(r)$ are obtained by Fourier transforming the matter form factor curves of Fig.2.a. The charge distribution corresponds to the dipole fit Eq.(19). All distributions are normalized to unity.

Fig.3. a-b) Two configurations of a meson, as seen in its rest frame. The valence quark and antiquark are the focii of a flux tube of color fields (gluons).

c-d) Transverse picture of a meson-meson collision, as seen in the infinite momentum frame. The transverse distribution of sea partons is assumed to be proportional to the corresponding color field density. Diffraction scattering is assumed to be due to wee-parton interactions. In case c) the matter overlap and correspondingly the wee-parton interaction probability is much larger than in case d. The impact parameter of the collision $\vec{B}$ is the same in both cases.

Fig. 4. Comparison between the matter and the charge form factors of a meson in the toy model described in the text.

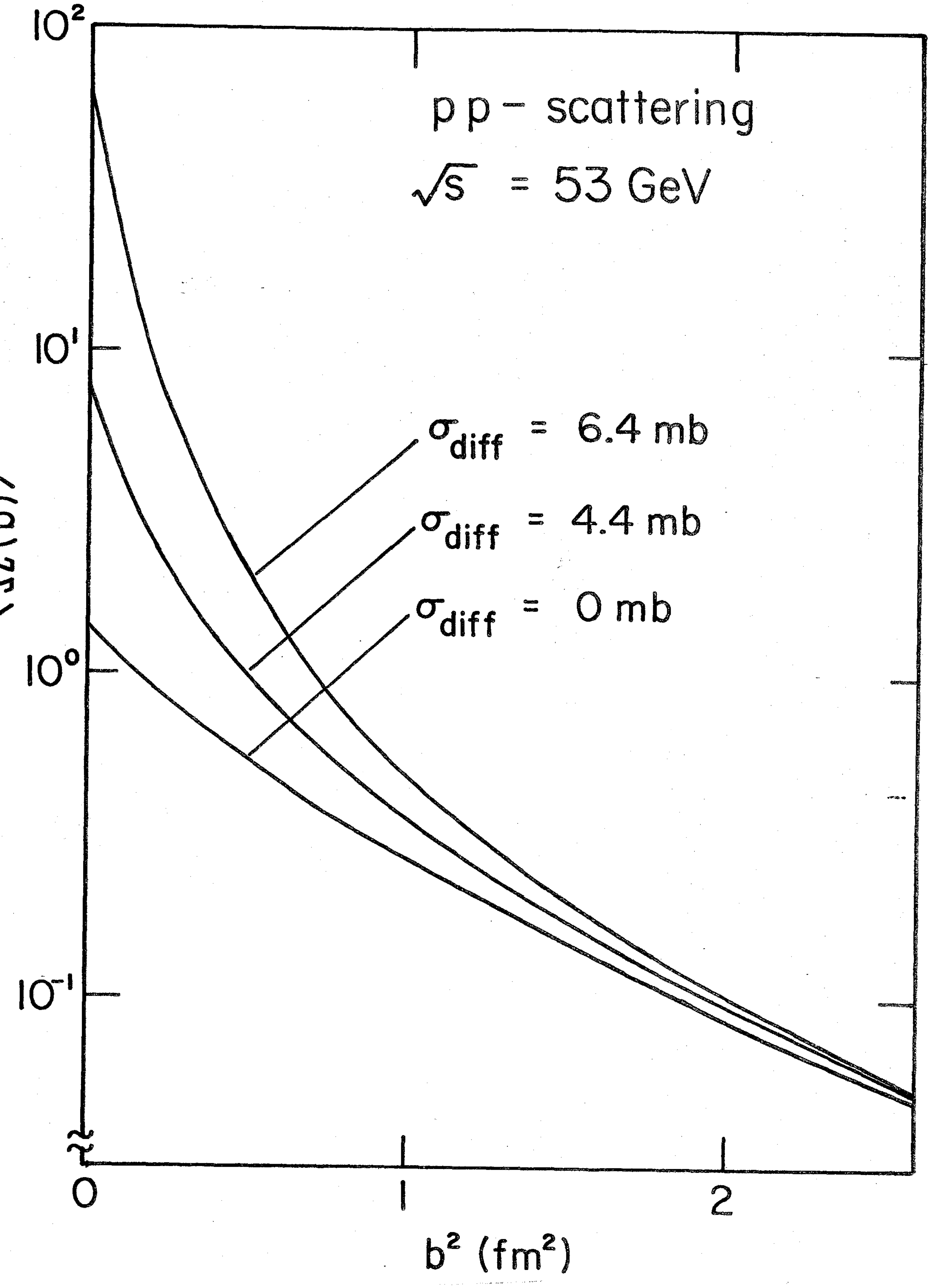

pp - scattering
√s = 53 GeV
σdiff = 6.4 mb
σdiff = 4.4 mb
σdiff = 0 mb
⟨Ω(b)⟩
10²
10¹
10⁰
10⁻¹
0
1
2
b² (fm²)

Fig. 1

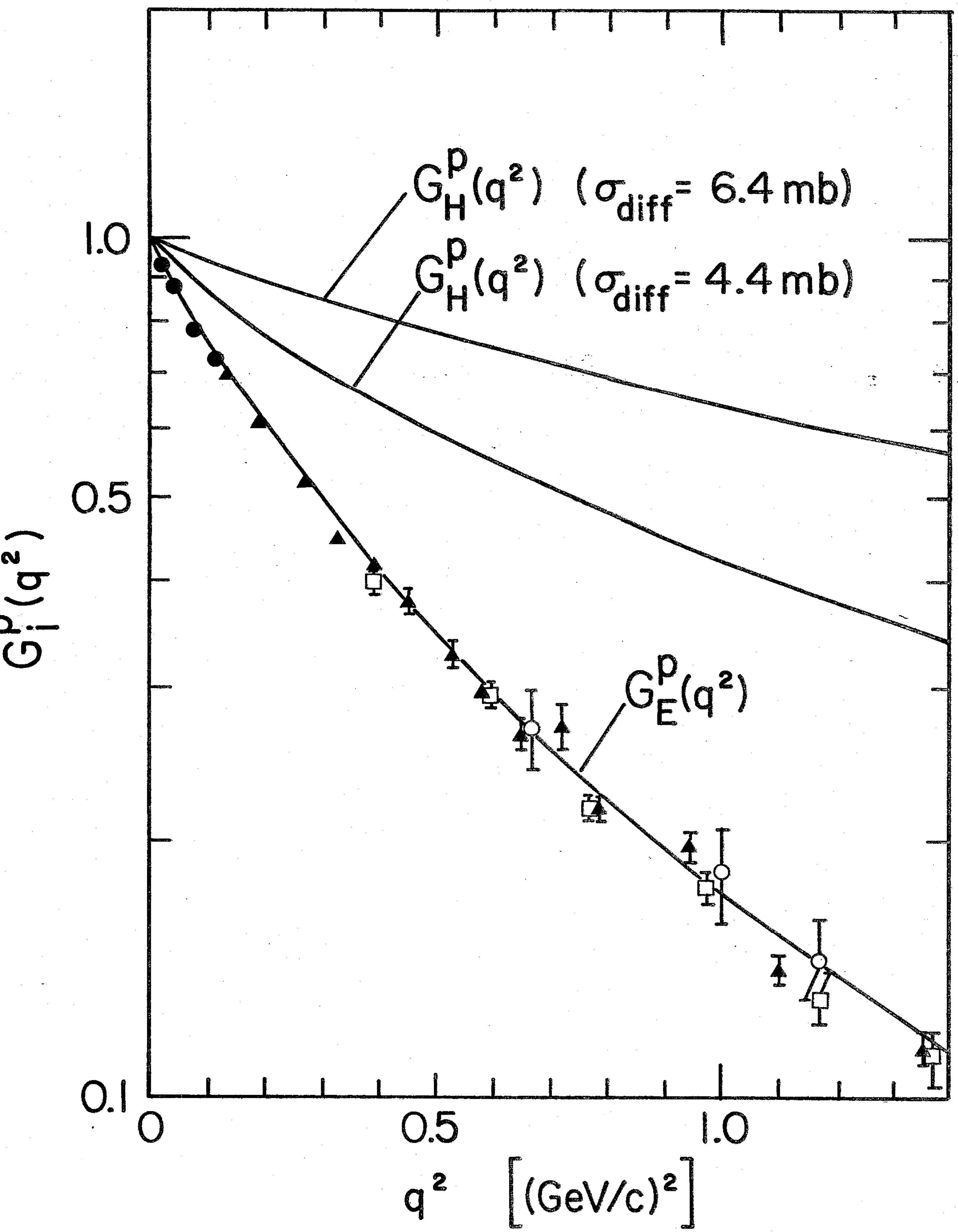
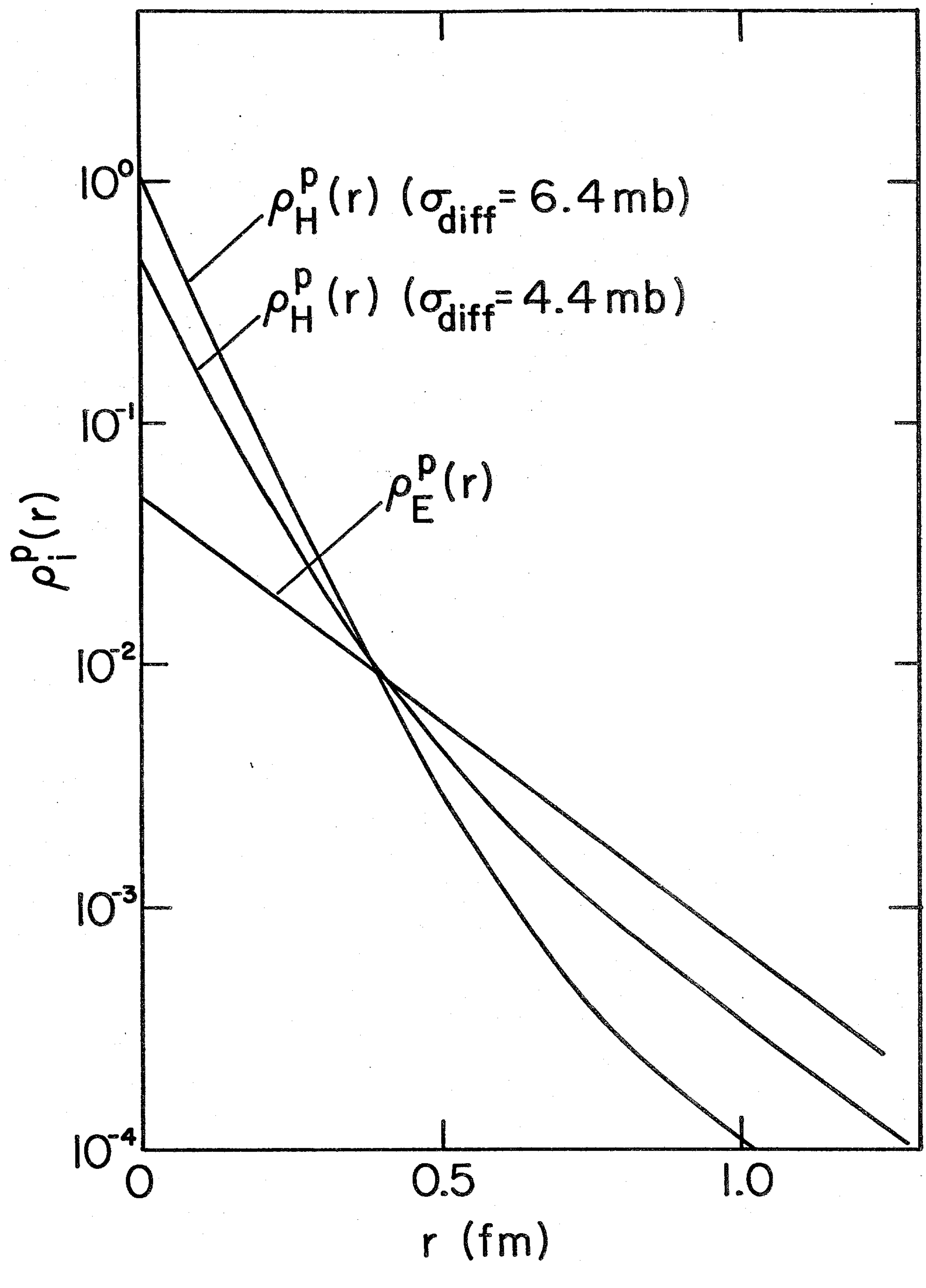

$G_H^p(q^2)$ (σ_{diff} = 6.4 mb)
$G_H^p(q^2)$ (σ_{diff} = 4.4 mb)
$G_E^p(q^2)$
1.0
0.5
0.1
$G_i^p(q^2)$
0
0.5
1.0
q^2 [(GeV/c)^2]

Fig. 2a



$\rho_H^p(r)$ (σ_{diff}= 6.4 mb)
$\rho_H^p(r)$ (σ_{diff}= 4.4 mb)
$\rho_E^p(r)$
$\rho_i^p(r)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.5
1.0
r (fm)

Fig. 2b

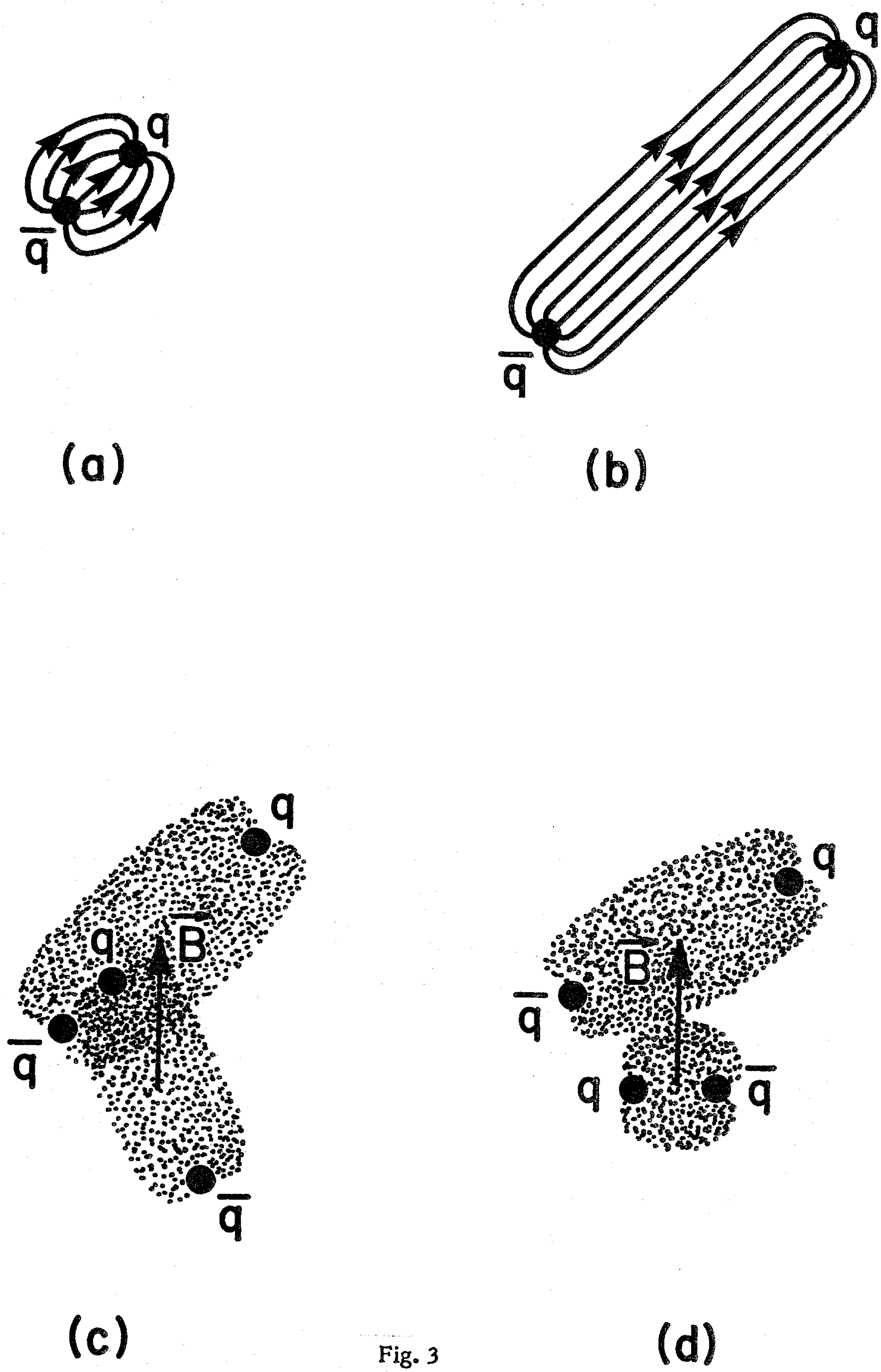
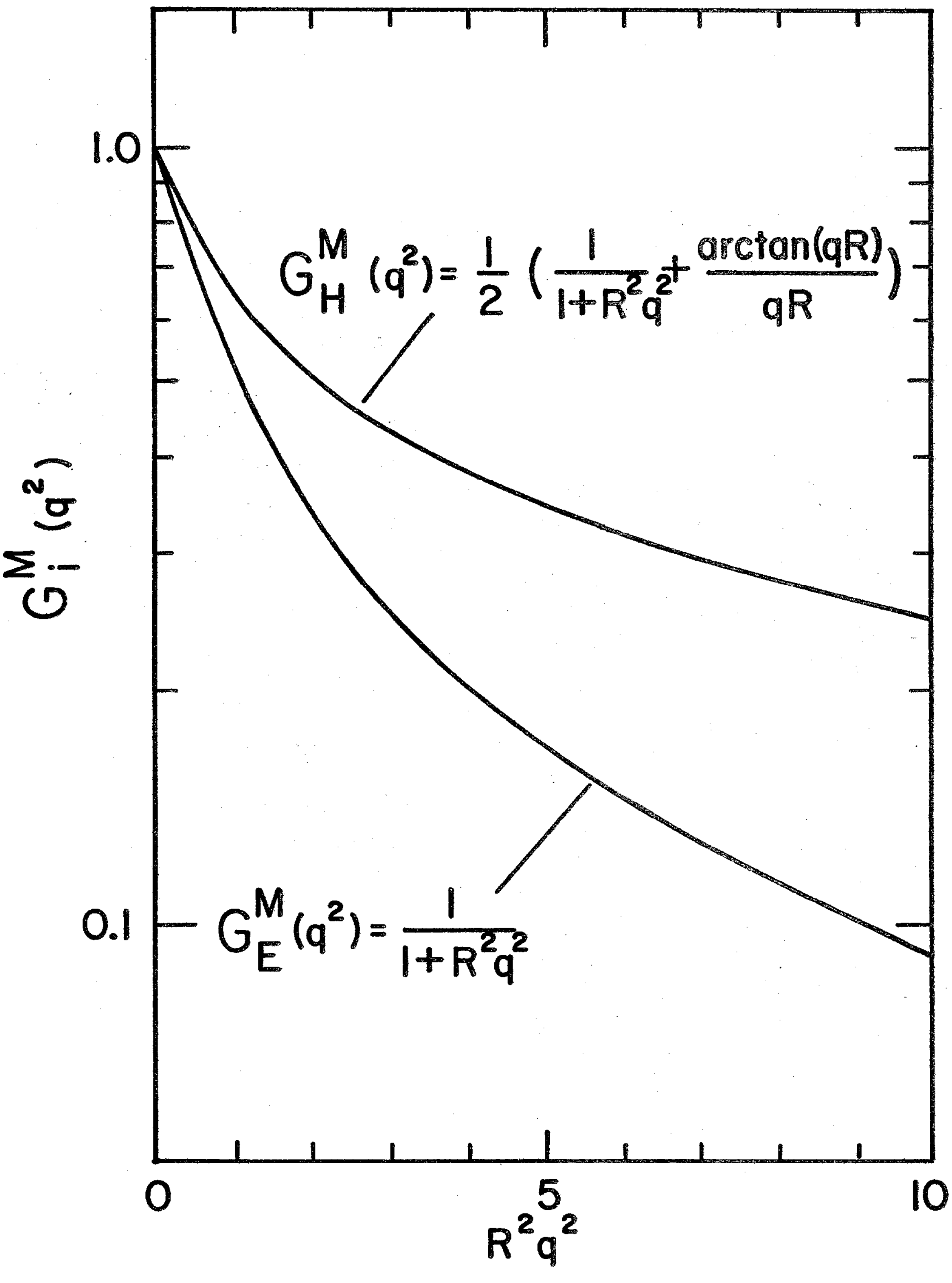


Fig. 3



1.0
0.1
0
5
10
R^2q^2
$G_i^M(q^2)$
$G_H^M(q^2) = \frac{1}{2}\left(\frac{1}{1+R^2q^2} + \frac{\arctan(qR)}{qR}\right)$
$G_E^M(q^2) = \frac{1}{1+R^2q^2}$

Fig. 4

Note added:

A weaker constraint is imposed on fermion masses if, instead of $\lambda \leq 1$, we require $(\lambda^2/256\pi^2) \leq 1$ or $\lambda \leq 16\pi$. This requirement coincides with that of Lee, Quigg and Thacker[12] who, using partial wave unitarity, found $m_H^2 \leq 8\pi\sqrt{2}/3G_F$ (or $m_H^2 \lesssim (1.0\ \text{TeV})^2$) with m_H^2 given by Eq. (19). In such a case, the absolute upper bound on $\{\sum_i m_{f_i}^4\}^{1/4}$ is now given by

$$\{\sum_i m_{f_i}^4\}^{1/4} < 873\ \text{GeV} \tag{20}$$

instead of the upperbounds found in (15), (16) and (17). Ignoring radiative corrections to the Higgs boson mass, one can rewrite the constraint (13), using Eq. (19), as

$$\{\sum_i m_{f_i}^4\}^{1/4} < \{\frac{3}{4} m_W^4 (2 + \sec^4\theta_W) + \frac{9}{16} m_H^4\}^{1/4} \tag{21}$$

The constraint (21) is then plotted in Fig. 1 as a function of the Higgs boson mass m_H.

We wish to thank Prof. J.D. Bjorken for valuable comments and for suggesting the idea of a mass plot as depicted in Fig. 1.

FIGURE CAPTION

Fig. 1: The allowed region (shaded area) of $\{\sum_i m_{f_i}^4\}^{1/4}_{\text{Heavy}}$ as a function of m_H (1.0 GeV is the absolute upper bound on m_H) for $\kappa \geq 0$ and for $\sin^2\theta_W = 0.2$.