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Strong and Weak CP Violation

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ABSTRACT

We study the renormalization of the QCD vacuum parameter θ which arises from CP violation in the weak interactions. In the Kobayashi-Maskawa extension of the Weinberg-Salam model to include six quarks the first renormalization of θ occurs in $O(\alpha^2)$ and is apparently $O(10^{-16})$. If we assume that $\theta = 0$ at some unknown "relaxation" scale μ_o, this renormalization makes a contribution to the neutron electric dipole moment which is probably $O(10^{-31}$ to $10^{-32})$ cm and smaller than the purely perturbative contribution. Infinite renormalization of θ may first occur in $O(\alpha^7)$, and we isolate a topological class of diagrams of this order which do indeed require infinite renormalization of θ. For any reasonable choice of the relaxation scale μ_o, the residual finite θ renormalization is much smaller than the first finite $O(\alpha^2)$ contribution. We finish with some remarks about θ renormalization in other weak interaction models of CP violation.

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1. INTRODUCTION

It used to be thought that QCD automatically possessed C, P and T invariance.[1] This was very fortunate, since the experimental limits on the intrinsic violation of these symmetries by the strong interactions are very stringent. But now instantons have been discovered,[2] and it has been pointed out that the QCD vacuum must have a non-trivial topological structure.[3] The actual vacuum requires an angle parameter θ for its characterization, and QCD violates CP invariance if $\theta \neq 0$. The best limit on CP violation by the strong interactions comes from the experimental upper limit on the neutron electric dipole moment. Taking this to be 3×10^{-24} cm[4] and using a recent estimate[5] that in QCD the dipole moment is $O(4 \times 10^{-16}\,\theta)$ cm, one feels the need to have $\theta < O(10^{-8})$. It is therefore very desirable to find within the QCD framework a natural reason either why $\theta = 0$ exactly, or else why θ is very small.

Peccei and Quinn[6] pointed out one way of getting $\theta = 0$ automatically, by imposing a new global chiral U(1) symmetry (to be called $U(1)_{PQ}$) on the QCD Lagrangian. Invariance under the associated chiral transformation can be used to demonstrate the equivalence of theories with different values of θ, which therefore presumably conserve CP as if $\theta = 0$. How to realize a $U(1)_{PQ}$ symmetry? One possibility would be that some quark, least unlikely to be the u, has zero bare mass. While not rigorously excluded phenomenologically, this solution has difficulty in fitting the observed π, K and baryon masses.[7,8] Another way of realizing a $U(1)_{PQ}$ symmetry which was proposed by Peccei and Quinn[6] involved choosing a non-minimal system of Higgs multiplets to give quark masses, with their couplings constrained to possess the required chiral symmetry. It was pointed out by Weinberg[9] and Wilczek[10] that this Peccei-Quinn model featured a very light pseudoscalar physical Higgs particle, called the axion a. In the simplest type of model the axion mass would probably be O(100 to 1000) MeV, and such a light axion now seems to be excluded experimentally. For example, high energy beam dump experiments[11] could produce the axion through its mixing with the π^0 and η, whose production cross-sections are reasonably well-known. The constraints on $a - \pi^0$ and $a - \eta$ mixing in the simplest model give a lower bound[12] for the production and interaction of the axion which is

$$\sigma(p + p \to a + X)\,\sigma(a + p \to X) \geq O(10^{-65})\ \mathrm{cm}^4 \quad . \tag{1.1}$$

This is about two orders of magnitude higher than the experimental upper limit[11,13,14] of $O(10^{-67})$ cm^4 which would apply to axions with masses $\lesssim 2$ MeV. A light axion could also be copiously produced by nuclear reactors,[9] and could generate deuterium break-up reactions ($a + D \to n + p$) or undergo "compton" scattering ($a + e \to \gamma + e$), neither of which have been seen. Unfortunately, theoretical calculations of the rate of axion production in nuclear reactions are not totally reliable.[9,14] Therefore, these negative observations are not conclusive, although they fuel our intuition that the axion does not exist, at least in the simple form originally proposed.

Regardless of the phenomenological situation, it is theoretically desirable to study alternative ways of keeping CP violation by the strong interactions within acceptable limits. A general way of doing this is to find a theory in which $\theta = 0$ naturally as a zeroth order approximation, but may get renormalized by a finite and small amount when higher order weak/electromagnetic effects are taken into account. This can be done in the context of weak gauge groups which are more extensive[15] than the minimal Weinberg-Salam model.[16] In view of this $SU(2) \times U(1)$ model's phenomenological success when compared with almost all experiments[17,18] except some in atomic physics[19] whose interpretation is cloudy, we will discard models not based on a $SU(2) \times U(1)$ gauge group. In this case the only degrees of freedom are in the representations and couplings of fermions and Higgs particles. In view of phenomenology and the natural flavor conservation conditions,[20] we will assume that all left-handed quarks and leptons are in doublets, and all right-handed quarks and leptons singlets as in the usual Weinberg-Salam model. In view of the success[17] of the $I = \frac{1}{2}$ Higgs rule for the strength of neutral current reactions, we will only use $I = \frac{1}{2}$ Higgs multiplets. But how many quark doublets (N_D) and how many Higgs multiplets (N_H)? With $N_D \geq 3$, as now seen experimentally, Kobayashi and Maskawa[21] pointed out that CP violation can be introduced into the weak coupling matrix U of the Weinberg-Salam model, and this needs only one Higgs multiplet.[22] There are models with $N_H > 1$ where Higgs exchanges are the dominant source of CP violation. One of these[23] turns out to have an unacceptably large renormalization of θ. Another[24] has an amount of θ renormalization which is on the borderline of acceptability, and which is also on the borderline as far as Higgs exchange contributions to $\Delta S = 2$ transitions are concerned. In this paper we are mainly concerned with θ renormalization in the Kobayashi-Maskawa[21] (KM) extension of the Weinberg-Salam model to include CP violation with 6 quarks and just one Higgs multiplet.[22] Our results can easily be extended if there are more than six quarks, and we will indicate where our analysis would be modified.

This model has the unattractive feature that no good rationale is known for setting $\theta \approx 0$ in any approximation. Furthermore, because CP is violated by dimension 4 terms in the Lagrangian, the renormalization of θ is expected to be infinite. We have nothing useful to say about the first of these problems. Instead, we analyze the amount of θ renormalization in the KM model assuming θ is fixed to have some value θ_o probably $\theta_o = 0$, when the theory is renormalized at some "relaxation" scale μ_o. This analysis gives some idea of the order of magnitude of θ to be expected in this model, if some "vacuum relaxation" mechanism could be found.[8] Our objectives are twofold: to ascertain in what order of weak and electromagnetic perturbation θ renormalization first occurs and how big it is, and to pinpoint the order of weak and electromagnetic perturbation in which an infinite renormalization first occurs.

In a free quark model, the renormalization of θ in the KM model would first occur in 6th order. However, when perturbative QCD effects are taken into account there is a first finite renormalization in 4th order of magnitude

$$\Delta\theta \approx \left(\frac{\alpha}{\pi}\right)^2 (s_1^2 s_2 s_3 \sin\delta) O\left(\frac{m_q}{m_W}\right)^4 \quad . \tag{1.2}$$

If there are just 6 quarks, the heaviest masses appearing in (1.2) can be $m_s{}^2 m_c{}^2$, so we would guess that (1.2) yields

$$\Delta\theta \approx O(10^{-16}) \quad . \tag{1.3}$$

Note however that if there are two more heavy quarks with mixing angles comparable to those for lighter quarks then one could have $m_b{}^2 m_t{}^2$ in (1.2) and $\Delta\theta = O(10^{-12})$. When combined with Baluni's estimate[5] of the magnitude of the neutron electric dipole moment for a given value of θ, the estimate (1.3) suggests a contribution to the dipole moment of the general order 10^{-31} to 10^{-32} cm (10^{-27} to 10^{-28} cm if there were two quarks with mass $>m_{t,b}$). This is much smaller than the present experimental limit,[4] and is smaller than the purely perturbative contribution in this model of CP violation.[25] We find a first (logarithmically) divergent contribution to θ renormalization in 14th order $O(\alpha^7)$. We exhibit a class of diagrams of $O(\alpha^7)$ with a topology which permits an infinite renormalization of θ, and demonstrate that the divergence is not cancelled when all these diagrams are added together. We cannot exclude the possibility that this divergence is cancelled by diagrams with some other topology, but we can argue that no divergence occurs before $O(\alpha^7)$, and see no reason why divergent contributions to θ should be totally absent. If we take the "relaxation"[8] scale μ_o to be less than $\exp(O(10^{18}))$ GeV, the residual finite renormalization of θ from these diagrams is less than the finite $\Delta\theta$ of equation (1.2), when θ is measured on the scale of the proton as in the neutron electric dipole moment. For plausible choices of the "relaxation" scale μ_o less than the Planck mass $= O(10^{19})$ GeV, the "infinite" contribution to $\Delta\theta$ is totally negligible.

Section 2 of this paper sets out a general formalism for calculating θ renormalization, including a renormalization group approach for regarding θ as an effective coupling constant[26] and calculating its value away from the "relaxation" scale μ_o. Section 3 looks at θ renormalization in low orders of weak and electromagnetic perturbation. It establishes that θ renormalization in the KM model indeed starts in $O(\alpha^2)$, whereas a free quark approximation neglecting perturbative QCD corrections would have suggested $O(\alpha^3)$. Section 4 develops the argument that an infinite renormalization of θ first occurs in $O(\alpha^7)$. It also exhibits a topological class of diagrams of this order whose logarithmic divergences in the manifestly renormalizable 't Hooft-Feynman gauge[27] do not cancel. The numerical significance of this infinite renormalization is discussed. Section 5 makes some remarks about alternatives to the KM model,[21,22] such as models[23,24] with multiple Higgs, and grand unified models of the strong, weak and electromagnetic interactions.[28] It concludes with some comments about the state of the art of CP violation.

2. GENERAL FORMALISM

As mentioned in the introduction, we will not address the question of specifying θ in the KM model.[21,22] We restrict ourselves to discussing its renormalization. We further restrict our analysis to the θ renormalization resulting from the necessity of redefining the quark mass matrix at each order in weak/electromagnetic perturbation theory.[9] If any other source of θ renormalization exists,[10] it is likely to have similar structure in the CP violating weak couplings, and its contribution to θ will hopefully not change the order of magnitude estimates we will present in this paper.

We write the inverse quark propagator as

$$S_o^{-1}(p) = \not{p} - M \tag{2.1}$$

with M the real and diagonal quark mass matrix at zeroth order in weak/electromagnetic perturbation theory. We define by $\Sigma(p)$ the sum of irreducible weak/electromagnetic diagram contributions to the quark propagator. It has the Dirac decomposition

$$\Sigma(p) = A\not{p}L + B\not{p}R + MCL + DMR \tag{2.2}$$

where hermiticity imposes the constraints

$$A = A^\dagger \quad , \quad B = B^\dagger \quad , \quad C = D^\dagger \quad . \tag{2.3}$$

Combining (2.1) and (2.2) we see that the full inverse quark propagator has the form

$$S^{-1}(p) = \not{p} - M - \Sigma(p)$$

$$= \not{p}(1 - A)L + \not{p}(1 - B)R - MC(1+C)L - (1 + C^{\dagger})MR \quad . \qquad (2.4)$$

The expression (2.4) must be subjected to wave function renormalizations

$$\psi_L = \frac{1}{\sqrt{1 - A}}\psi_L^{Ren} \quad , \quad \psi_R = \frac{1}{\sqrt{1 - B}}\psi_R^{Ren} \qquad (2.5)$$

which yield the renormalized propagator

$$S^{Ren^{-1}}(p) = \not{p}L + \not{p}R - \frac{1}{\sqrt{1 - B}}M(1+C)\frac{1}{\sqrt{1 - A}}L$$

$$-\frac{1}{\sqrt{1 - A}}(1 + C^{\dagger})M\frac{1}{\sqrt{1 - B}}R \quad . \qquad (2.6)$$

The renormalized quark mass matrix M^{Ren} is therefore just

$$M^{Ren} = \frac{1}{\sqrt{1 - B}}M(1+C)\frac{1}{\sqrt{1 - A}}L \quad . \qquad (2.7)$$

One now makes M^{Ren} (2.7) real and diagonal by unitary transformations on the left- and right-handed quark fields

$$\psi_L^{Ren'} = V_L\psi_L^{Ren} \quad , \quad \psi_R^{Ren'} = V_R\psi_R^{Ren} \qquad (2.7)$$

so that

$$M^{Ren} = V_R^{\dagger}M^{Ren'}V_L \quad . \qquad (2.8)$$

In this case one makes a net chiral transformation through a net angle

$$\delta\theta = \arg\det V_R{}^\dagger V_L$$

$$= \arg\det M^{Ren} \tag{2.9}$$

because the final renormalized quark mass matrix $M^{Ren'}$ is by definition real and diagonal. As pointed out by Weinberg,[9] the chiral transformation (2.9) causes a corresponding renormalization of the CP violating QCD vacuum parameter θ. Referring back to equation (2.7) and recalling (2.3) that A and B are hermitian matrices, we conclude that

$$\delta\theta = \arg\det (1 + C)$$

$$= \mathrm{Im}\,\mathrm{Tr}\ln(1 + C)$$

$$= \mathrm{Im}\,\mathrm{Tr}\, C + \ldots \quad . \tag{2.10}$$

Equation (2.10) summarizes the renormalization of θ in terms of the weak/electromagnetic perturbations (2.2) on the quark propagator.

So far we have not specified the prescription to be used for renormalizing the quark propagator. In weak/electromagnetic calculations it is common to renormalize on mass-shell. However, since we are interested in the purely strong interaction parameter θ, it is convenient to renormalize at an off-shell Euclidean momentum $p^2 = -\mu^2$, as is usually done in QCD calculations. We therefore specify the quark propagator and mass renormalization (2.6, 2.7, 2.8) by

$$S^{Ren^{-1}}(p)\Big|_{p^2=-\mu^2} = \not{p}L + \not{p}R - M^{Ren'}(\mu)(L+R) \quad . \tag{2.11}$$

Corresponding to this choice of renormalization prescription we have a renormalization group equation

$$\mu \frac{\partial\theta}{\partial\mu} = \beta_\theta\left(\alpha_s(\mu), \frac{m_q(\mu)}{\mu} \; ; \; \alpha, \frac{m_q(\mu)}{m_{Z,W,H}}\right) \tag{2.12}$$

just as one would have if θ were regarded as an effective coupling constant.[26] The function β_θ will be expanded as a power series in α, and has of course no zeroth order term, because the strong interactions by themselves do not renormalize θ. We therefore have

$$\mu \frac{\partial\theta}{\partial\mu} = \sum_{n=1}^{\infty} \alpha^n \beta_\theta^n\left(\alpha_s(\mu), \frac{m_q(\mu)}{\mu}, \frac{m_q(\mu)}{m_{Z,W,H}}\right) \quad . \tag{2.13}$$

Our philosophy will be to assume that some external power as yet unknown constrains θ to take some specific value θ_o at some scale μ_o. Presumably the constraint is $\theta_o = 0$, perhaps corresponding to some sort of "vacuum relaxation,"[8] though there seems to be no way of deducing this within the KM-QCD framework. We do not know the scale μ_o at which θ is fixed, but presume it to be somewhere between a typical weak scale O(100) GeV, and the Planck mass O(10^{19}) GeV, which is the largest fundamental mass scale known to us. Perhaps μ_o has something to do with the scale of grand unification of the strong, weak and electromagnetic interactions which may[28] be O(10^{15} to 10^{16}) GeV?

The best constraint on θ comes from the neutron electric dipole limit,[4] which applies to the value of θ on a typical strong interaction scale ~ 1 GeV:

$$\theta\,(\mu = 1\text{ GeV}) \lesssim 10^{-8} \qquad . \tag{2.14}$$

Our task will be to see whether the renormalizations $\beta_\theta{}^n$ of equation (2.13) are sufficiently small in the KM model that the constraint (2.14) is satisfied whatever the scale μ_0 at which $\theta = 0$. We should observe at the outset that in the KM model[21,22] CP is violated by the "hard" dimension 4 $q\bar{q}$-Higgs couplings, so that we would expect some diagrams to make divergent contributions to θ. They will however only make finite contributions to β_θ, and will therefore make a contribution to θ (1 GeV) which is proportional to ln (μ_0/1 GeV). This logarithm will not be too large ($\lesssim 1/\alpha$) if $\mu_0 \lesssim$ the Planck mass. We therefore feel that the presence of divergent contributions to θ renormalization in the KM is no more serious than its inability to predict θ in the first place.

3. LOW ORDER CONTRIBUTIONS TO θ RENORMALIZATION

In the following section we shall show using a manifestly renormalizable gauge[27] that a divergent contribution to $\Delta\theta$ first appears in 14th order. For studying lower order finite contributions it is more convenient[25] to use the unitary gauge where charged boson exchange projects out left-handed quarks so that any quark running between two W-vertices has as propagator $\not{p}\,(p^2 - m^2)^{-1}$. In sec. 3.1 we first estimate the weak-electromagnetic contribution to $\Delta\theta$ neglecting strong interaction corrections. In Sec. 3.2 we turn to strong interaction effects which alter substantially the order of magnitude of the estimate.

3.1. Free quarks

As discussed in section 2, we must calculate the helicity-flip contribution to the self energy, and evaluate the quantity

$$\Delta\theta = \text{Im Tr} \ln(1+C) = \text{Im}\left[\text{Tr } C - \frac{1}{2}\text{Tr } C^2 + \frac{2}{3}\text{Tr } C^3 + \ldots\right] \tag{3.1}$$

where C is the matrix defined in Eq. (2.2). It is generally a polynomial in the Cabibbo matrix U and its hermitian conjugate. Since $W^{\pm}$ emission is necessarily followed by $W^{\mp}$ emission, the general form of C is, for, say external (charge -1/3) catho-quarks,[29]

$$C = \alpha^n \int \frac{d^{4n}k}{(2\pi)^{4n}} \prod_i^N F_i(M, k)U^\dagger F_i'(\overline{M}, k)U \tag{3.2}$$

where $N \leq n$ is the number of W-loops and $\overline{M}$ (M) is the (diagonal) mass matrices for ano (catho) quarks. In order to get an imaginary contribution to Tr C^P, C^P must contain at least two W-loops since

$$u^*_{\alpha\beta} f(m_\beta) u_{\alpha\beta} f(m_\alpha) \tag{3.3}$$

is real. In addition, since C is a left-right transition operator, we cannot have a W attached to the outcoming quark line. Thus the only possible contributions are those of fig. 1, where the dotted line can be attached anywhere along the quark line. The lowest order non-vanishing contribution to (3.1) is therefore of order α^3, and will be of the form

$$\Delta\theta^3 = \left(\frac{\alpha}{\pi}\right)^3 \text{Im}\,(u^*_{\alpha\beta} u_{\gamma\delta} u^*_{\gamma\beta} u_{\alpha\delta}) f(m_\alpha, m_\gamma; m_\beta, m_\delta) \quad . \tag{3.3}$$

The expression (3.3) vanishes for $\beta = \delta$, so we may write:

$$\Delta\theta^3 = \left(\frac{\alpha}{\pi}\right)^3 \mathrm{Im}\,(u^*_{\alpha 1}u_{\gamma 1}u^*_{\gamma 2}u_{\alpha 2})[f(m_\alpha, m_\gamma; m_1, m_2) - (m_1 \leftrightarrow m_2)]$$

$$+ \text{cyclic permutations } (1, 2, 3) \quad .$$

if we have 3 quark doublets. Finally we may use unitarity:

$$u^*_{\alpha 3}u_{\gamma 3} = \delta_{\alpha\gamma} - u^*_{\alpha 2}u_{\alpha 2} - u^*_{\alpha 1}u_{\alpha 1}$$

to write

$$\Delta\theta^3 = \left(\frac{\alpha}{\pi}\right)^3 \mathrm{Im}\,(u^*_{\alpha 1}u_{\gamma 1}u^*_{\gamma 2}u_{\alpha 2}) \Big[\{f(m_\alpha, m_\gamma; m_1, m_2) - (m_1 \leftrightarrow m_2)\}$$

$$+ \text{cyclic permutations } (m_1, m_2, m_3)\Big] \quad . \qquad (3.4)$$

A similar antisymmetrization holds in α and γ. In order to get a non-vanishing result we require three propagator subtractions for each quark charge. This renders the Feynman integrals so highly convergent that the longitudinal parts of W propagators are scaled by quark masses

$$\frac{k_\mu k_\nu}{m_W^2} \to \frac{m_q^2}{m_W^2} \quad ,$$

and each W-loop is scaled by the W-mass:

$$(p^2 - m_W^2)^{-1} \to m_W^{-2} \quad .$$

The manifest order of magnitude of diagrams in Fig. 1 is therefore

$$\Delta\theta^3 \sim \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{m_q}{m_W}\right)^4 \times \left\{\begin{matrix} 1 \\ (m_q/m_Z)^2 \\ (m_q^{\,2}/m_W m_H)^2 \end{matrix}\right\} \text{ for } \left\{\begin{matrix} \gamma \\ Z \\ H \end{matrix}\right\} \text{ exchange} \quad . \tag{3.5}$$

On closer inspection however, one sees that the antisymmetrization (3.4) requires further subtractions in the W-propagators so that the γ- and Z-exchange diagrams are reduced by an additional factor $(m_q/m_W)^4$. We shall study these diagrams in some detail.

If C_n is the order α^n contribution to C, we have

$$\Delta\theta^3 = \mathrm{Im}\left[\mathrm{Tr}\, C_3 + \tfrac{1}{2}\,\mathrm{Tr}\,(C_1C_2 + C_2C_1) + \tfrac{2}{3}\,\mathrm{Tr}\, C_1^{\,3}\right] \quad . \tag{3.6}$$

In the unitary gauge C_1 and C_2 get no contribution from charged W's. The diagrams of Figs. 2(a) and (b) give

$$\Sigma_1(p) = A_1(p^2)\not{p}\, L \quad , \quad \Sigma_2(p) = A_2(p^2)\not{p}\, L \quad , \tag{3.7}$$

so there is no contribution from the second and third terms of Eq. (3.6). We need only consider Tr C_3 which receives contributions from the one particle irreducible diagrams of Figs. 3-5. Fig. 3 is symmetric in the interchange of β and δ; Fig. 4 is symmetric in the interchange of α and γ (for γ, Z exchange there is a mass insertion m_α; for H exchange there is a coupling factor m_α. These factor out of the definition of C_3, leaving only a mass dependence $(p_i^{\,2} - m_i^{\,2})^{-2}$ for each propagator, since the left projection operators in the W-couplings forbid any other mass insertions.) We are then left with the contribution of Fig. 5(a), which is expressed in terms of the one loop contribution to the vertex function, Fig. 6.

Consider first Z and γ exchange. If we define the vertex function of Fig. 5(b) by $i\Gamma_\rho$, the self energy parts of Fig. 6 (Figs. 6(a) and (b)) give a contribution (for external catho-quarks)

$$i\bar{\Gamma}_\rho^{\,a+b} = -i\frac{g^2}{2}\frac{2e}{3}\int\frac{d^4s}{(2\pi)^4}U^\dagger\gamma_\mu\left\{\frac{(\not{s}+\not{r})\gamma_\rho\not{s}+\gamma_\rho M^2}{D(s,\bar{M}^2)}A_1(s^2)\frac{s^2}{s^2-M^2}\right.$$

$$\left.+\frac{(s+r)^2}{(s+r)^2-M^2}A_1[(s+r)^2]\frac{(\not{s}+\not{r})\gamma_\rho\not{s}+\bar{M}^2\gamma_\rho}{D(s,\bar{M}^2)}\right\}\gamma_\nu UL\,F_{\mu\nu}(s-q)\;, \quad (3.8)$$

where A_1 is defined in Eq. (3.7); M is the relevant mass matrix and does not commute with A_1;

$$D(s,M^2) = [s^2-M^2][(s+r)^2-M^2] \quad ; \quad (3.9)$$

$$F_{\mu\nu}(p) = i\Delta_{\mu\nu}(p) = \frac{g_{\mu\nu}-p_\mu p_\nu/m_W^{\,2}}{p^2-m_W^{\,2}} \quad . \quad (3.10)$$

We want to extract the antihermitian part of $\bar{\Gamma}_\rho$ in order to get an imaginary contribution to the trace. Hermitian conjugation of Eq. (3.8) gives for $(\bar{\Gamma}_\rho^{\,a+b})^\dagger$ the same equation but with the replacement

$$s \leftrightarrow s+r$$

in the expression in brackets. Changing the integration variable to $s' = -s-r$, we obtain the expression (3.8) but now with $F(s-q)$ replaced by $F(s+q+r)$. Next consider the proper vertex parts of Fig. 6, Figs. 6(c) and (d), which we define by $\Gamma_\rho^{\,1}$. We get a contribution:

$$i\bar{\Gamma}_\rho^{\,c+d} = \frac{g^2}{2}\int\frac{d^4s}{(2\pi)^4}U^\dagger\gamma_\mu\frac{\not{s}+\not{r}}{(s+r)^2-\bar{M}^2}\Gamma_\rho^{\,1}(s,r)\frac{\not{s}}{s^2-\bar{M}^2}\gamma_\nu\,ULF_{\mu\nu}(s-q)\;, \quad (3.11)$$

where we have used the fact that the quark propagators are sandwiched between left projectors. Hermitian conjugation of Eq. (3.11) interchanges s and s + r in the propagators. Making the change of variables s' = -s - r as before, we find that the antihermitian part, $\overline{\Gamma}_\rho - \overline{\Gamma}_\rho^\dagger$ is proportional to

$$\Gamma_\rho^1(s, r)F_{\mu\nu}(s - q) + \Gamma_\rho^{(1)\dagger}(-s - r, r)F_{\mu\nu}(s + r + q) \quad .$$

The proper vertex parts of Fig. 6 have the form:

$$i\,\Gamma_\rho^{1(c)} = -\frac{e}{3}\frac{g^2}{2}\int \frac{d^4t}{(2\pi)^4}\gamma_\mu U \frac{[(\not s + \not t + \not r)\gamma_\rho(\not s + \not t) + M^2\gamma_\rho]}{D(s + t, M^2)}\gamma_\nu U^\dagger L F_{\mu\nu}(t) \quad (3.12a)$$

$$i\Gamma_\rho^{1(d)} = -e\frac{g^2}{2}\int \frac{d^4t}{(2\pi)^4}\gamma_\mu U\frac{(\not s + \not t)}{(s + t)^2}U^\dagger\gamma_\nu L F_{\mu\sigma}(t - r)F_{\nu\tau}(t)$$

$$\times \quad \{g_{\sigma\tau}(2t - r)_\rho - g_{\sigma\rho}(t - 2r)_\nu - g_{\tau\rho}(t + r)\} \quad . \quad (3.12b)$$

Making the change of variables t' = -t in Eq. (3.12a) and t' = -t + r in Eq. (3.12b), the substitution s → -s - r and taking the hermitian conjugate, we find

$$i\Gamma_\rho^1 = -i\,\Gamma_\rho^{(1)\dagger} \quad ,$$

so that the antihermitian part of $\overline{\Gamma}_\rho$ is in all cases proportional to

$$F_{\mu\nu}(s - q) - F_{\mu\nu}(s + r + q)$$

$$= \frac{(2s + r)\cdot(2q + r)}{[(s - q)^2 - m_W^2][(s + q + r)^2 - m_W^2]}\left(g_{\mu\nu} - \frac{(s - q)_\mu(s - q)_\nu}{m_W^2}\right)$$

$$+ \frac{\tfrac{1}{2}[(2s + r)_\mu(2q + r)_\nu + (2q + r)_\mu(2s + r)_\nu]}{m_W^2[(s + q + r)^2 - m_W^2]} \quad . \quad (3.13)$$

By inspection of the required antisymmetrization of the propagators, the integral is finite after all W-propagators are removed. Then to lowest order in ${m_W}^{-2}$, the integrals (3.8) and (3.11) take the form (nonvanishing contributions from A_1 and $\Gamma_\rho^{\,1}$ are order ${m_W}^{-4}$ and ${m_W}^{-2}$, respectively)

$$i\overline{\Gamma}_\rho = \frac{(2q+r)_\alpha}{{m_W}^6} \int \frac{d^4s}{(2\pi)^4} f_{\alpha\rho}(s,r) = i(2q+r)_\alpha \Gamma_{\alpha\rho}(r) \quad . \tag{3.14}$$

Inserting this expression for Γ_ρ into the diagram of Fig. 5(a), we get an expression of the form:

$$\Delta\theta^3 = -\frac{1}{{m_W}^6}\,\frac{e}{3} \int \frac{d^4r}{(2\pi)^4} \left\{ \frac{g_{\sigma\rho}}{r^2} \text{ or } i\Delta_{\sigma\rho}(r^2, {m_z}^2) \right\} \frac{\gamma_\rho}{(r+q)^2 - M^2} (2q+r)_\alpha \Gamma_{\alpha\rho}(r) \quad . \tag{3.15}$$

After reduction of the γ-matrices the integrand is necessarily a Lorentz scalar (terms like $[\not{r}, q]$ will vanish after r-integration) of the form:

$$\Delta\theta^3 = -\frac{e}{3{m_W}^6} \int \frac{d^4r}{(2\pi)^4} \frac{(2q+r)\cdot r}{(r+q)^2 - M^2} f(r^2) \quad . \tag{3.16}$$

But if we evaluate (3.16) on the mass shell of the external quark line $q^2 = M^2$, we have

$$(r+q)^2 - M^2 = 2r\cdot q + r^2$$

so that the external mass dependence disappears, and the result vanishes under antisymmetrization, so that we must retain higher order terms in $(q\cdot s,\ q\cdot r,\ q^2)/{m_W}^2$ in Eq. (3.13).

For Higgs exchange, the propagators appearing in the expressions analogous to (3.8) and (3.12) are linear rather than quadratic in momentum, and the result turns out to be proportional to the sum rather than the difference of the propagators in (3.13), so we do not gain any extra suppression factor. This is related to the CP property of the $\bar{q}qH$ vertex as we shall discuss in section 3.2. However there is an extra factor of $m_q{}^2/m_W{}^2$ from the Higgs coupling, so in the free quark model we get contributions

$$\Delta\theta^3 \sim \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{m_q}{m_W}\right)^6 \times \begin{cases} (m_q/m_W)^2 & (\gamma) \\ (m_q/m_W)^4 & (Z) \\ (m_q/m_H)^2 & (H) \end{cases} \quad . \tag{3.17}$$

3.2. Strongly interacting quarks

It is obvious that we can replace a photon line by a gluon line in the diagrams discussed in the preceding section (this would give a null result when the contributions of external ano- and catho-quarks are added if the dependence on $m_{\alpha,\gamma}$ were the same as on $m_{\beta,\delta}$; it is not). This gives us a contribution

$$\Delta\theta \sim \left(\frac{\alpha_s}{\pi}\right) \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_q}{m_W}\right)^8 \quad .$$

To the next order in strong interactions we can replace the free quark propagator

$$(\not{p} + \not{q} - M)^{-1}$$

by the dressed propagator

$$\frac{1}{\not{p} + \not{q} - M}\, \Sigma(p+q)\, \frac{1}{\not{p} + \not{q} - M}$$

giving a net q^2- dependence although the pole factors both disappear if we normalize on mass shell. Then we get a contribution

$$\Delta\theta^2 \sim \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_q}{m_W}\right)^6 \qquad .$$

Finally, we need not exchange only a single gluon. The dashed line in Fig. 5 can equally well represent a multigluon system. Let the black box in Figs. 5(b) and 6 represent a vertex with external left-handed quarks and some external boson system. The vertex can be represented by an effective operator in momentum space:

$$O^{\alpha\beta}(s, r) = \psi_L^{\alpha}(s + r)\gamma_\mu \psi_L^{\beta}(s) F_\mu (\text{Boson fields}) \qquad . \qquad (3.15)$$

A subtraction in the external W propagator in Fig. 5(b) will be necessary if

$$O(s, r) = O^{\dagger}(-s - r, r) \qquad .$$

For the quarks, this is equivalent to the CP operation which interchanges incoming and outgoing quarks. For single W-exchange, CP is a good symmetry of the vertex; then

$$O(s, r) = \pm O^{\dagger}(-s - r, r)$$

depending whether the boson system is even or odd under CP. For a single vector field:

$$F_\mu = A_\mu \quad , \quad CP = +1 \qquad .$$

For a single Higgs scalar:

$$F_\mu = r_\mu H \quad , \qquad CP = -1 \qquad .$$

(To lowest order in m_W^{-2}, the quark current operator is local and F_μ does not depend on s.) For a two gluon system, we can construct C-even or -odd states. For a symmetric two-gluon system, F_μ is identical to the quantity calculated[30] [see Eqs. A(8)-A(12) of Ref. 30] for the $\bar{s}d\gamma\gamma$ vertex. It is CP even because of Furry's theorem which allows only the axial part of the quark current to contribute, and because a symmetric digluon state is C-even. It may be that only the symmetric C-even part survives momentum integration. For three gluons, both the axial and vector parts of the quark current can contribute, and in either case we can form color singlet states which are C-even (F-couplings) or C-odd (D-couplings). Then we would expect to generate CP-odd vertices such as

$$F_\mu = \begin{cases} f_{ijk}\tilde{F}_{\mu\nu}{}^i F_{\nu\rho}{}^j F_{\rho\tau}{}^k k_\tau \\ \qquad\text{or} \\ d_{ijk}F_{\mu\nu}{}^i F_{\nu\rho}{}^j F_{\rho\tau}{}^k k_\tau \end{cases} \qquad (3.19)$$

where $F_{\mu\nu}{}^i$ is the gluon field tensor and k is some gluon momentum. From these diagrams we expect a contribution of at least

$$\Delta\theta^2 \sim \left(\frac{\alpha_s}{\pi}\right)^3 \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_q}{m_W}\right)^4 \qquad . \qquad (3.20)$$

When the expression (3.20) is put into the renormalization group equation (2.13) we find a contribution to β_θ of order

$$\beta_\theta{}^2 \ni \left(\frac{\alpha_s}{\pi}\right)^4 \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_q}{m_W}\right)^4 \qquad . \qquad (3.21)$$

When integrated over μ, this indicates that the α_s/π in equation (3.20) should be interpreted as α_s/π (1 GeV) ~ 1 when we seek to evaluate the θ(1 GeV) which we need to calculate[5] the neutron electric dipole moment. The quark mass factors in (3.21) must be $O(m_s^2 m_c^2)$, since the CP violation would vanish if any pair of quark masses were equal. The angle factors can be read off any 2 × 2 submatrix of the KM matrix.[21] The resulting contribution to θ is therefore

$$\Delta\theta^2 \approx \left(\frac{\alpha}{\pi}\right)^2 s_1^2 s_2 s_3 \quad \sin\delta \, \frac{m_s^2 m_c^2}{m_W^4} \approx 10^{-16} \quad . \tag{3.22}$$

We remark in passing that similar considerations should hold for the purely perturbative contribution to the neutron dipole moment. We can construct a vertex function of the type

$$F_\mu = F_{\mu\nu}^{e} \vec{F}_{\nu\rho} \cdot \vec{F}_{\rho\tau} k_\tau$$

when $F_{\mu\nu}^e$ is the electromagnetic field tensor and k the photon momentum. Setting k = 0 elsewhere in the diagram of Fig. 7 no longer symmetrizes the quark propagators as it did in the free quark case studied by Shabalin.[25] There might also be a one gluon-one photon vertex, antisymmetric in gluon and photon variables (the symmetric part would be relatively reduced by one factor of m_W^{-2}), but in any case we expect a nonvanishing contribution of order

$$\frac{D}{e} \sim m_{u,d} \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_q}{m_W}\right)^4 s_1^2 s_2 s_3 \sin\delta \quad . \tag{3.23}$$

In equation (3.23) the mass factors may be $(m_t^2 m_s^2)/m_W^4$ or $(m_b^2 m_c^2)/m_W^4$, resulting in a value for the neutron electric dipole moment $O(m_W^2/m_t^2$ or

m_W^2/m_b^2) larger than that estimated in ref. 22. We would therefore expect the purely perturbative contribution to electric dipole moment to be O(10^{-28} to 10^{-29}) cm.

4. INFINITE CONTRIBUTIONS TO θ RENORMALIZATION

We now want to ascertain in what order of perturbation theory divergent contributions to the imaginary part of the mass matrix, and hence infinite θ renormalization, first occur. We remind the reader that such (logarithmic) divergences appear inevitable, since CP invariance is broken by Lagrangian terms of dimension 4 in the KM model,[21,22] so that CP is not violated softly. To locate divergent contributions to $\Delta\theta$ it is convenient to use the manifestly renormalizable 't Hooft-Feynman gauge.[27] The Feynman rules for quark-antiquark-boson couplings in this gauge are shown in Fig. 8. We see that the same phase-ridden KM coupling matrix U[21] appears in the couplings of both the (transverse) $W^{\pm}$ and the unphysical Higgs $\phi^{\pm}$. Note also that there are quark mass factors in the coupling of the $\phi^{\pm}$ and ϕ^o, $\overline{\phi}^o$. To get some CP violation we need to exploit the differences in quark masses. To get divergent contributions to $\Delta\theta$ at most one of these quark mass factors may come from a mass insertion on a quark propagator, and the rest from the dimension 4 Lagrangian couplings in Fig. 8. As noted in section 2, we need at least four U or $U^{\dagger}$ matrices along the fermion line in order to get CP violation. To be in lowest feasible order, we will use only four such matrices. Also so as to be at the lowest feasible order, we can discard diagrams where the bosons emitted from the fermion line interact with each other. We are therefore led to contemplate diagrams where the fermion line is festooned with non-interacting bosons. The diagram will have a factor from the q-$\overline{q}$-boson vertices of the generic form

$$\mathrm{Tr}\left(U^{\dagger} m_a^{\ n_1}\, U m_c^{\ n_2}\, U^{\dagger} m_a^{\ n_3}\, U m_c^{\ n_4}\right) \tag{4.1}$$

and a logarithmic divergence which will be identical for diagrams of identical topology, but will in general differ for different topologies.

It is easy to satisfy oneself using the Feynman rules of Fig. 8 that the powers $n_1 \ldots n_4$ in the generic expression (4.1) must all be even, and there will be a phase and hence CP violation only if they are all ≥ 2. The expression (4.1) will be symmetric, and hence no phase CP violation will arise, from any of the following low order combinations of the n_i:

$$n_1 = n_2 = n_3 = n_4 = 2$$

$$n_1 = 4, \; n_2 = n_3 = n_4 = 2, \quad \text{and permutations thereof}$$

$$\left\{ \begin{array}{ll} & n_1 = n_3 = 4, \; n_2 = n_4 = 2 \\ \text{or} & n_2 = n_4 = 4, \; n_1 = n_3 = 2 \end{array} \right. . \tag{4.2}$$

The first combinations of the type (4.1) which might give a phase and hence CP violation are therefore

$$\mathrm{Tr}\left(U^\dagger m_a{}^4\, U m_c{}^4\, U^\dagger m_a{}^2\, U m_c{}^2\right) \tag{4.3a}$$

and

$$\mathrm{Tr}\left(U^\dagger m_a{}^2\, U m_c{}^4\, U^\dagger m_a{}^4\, U m_c{}^2\right) . \tag{4.3b}$$

We see from (4.3) that the lowest order in which a phase is potentially available is 12th order. To get a divergence in this order all the quark mass factors in (4.3) would have to come from Higgs couplings, and there would be no vector boson couplings. But for every diagram on a cathoquark[29] line giving an expression of type (4.3a) there will be a diagram on an anoquark[29] line giving an expression of type (4.3b). When we add these together, the phases will cancel and there will be no CP violation. To get something non-zero, we need to add to twelfth-order diagrams which yield expressions of the type (4.3) at least one U(1) boson line with at least one end on a right-handed fermion line so as to differentiate between ano- and cathoquarks. Therefore, the lowest order in which we may possibly find a logarithmically divergent contribution to θ renormalization is the 14th.

We should emphasize at this point that we cannot demonstrate that there is indeed a divergence in 14th order-this would require classifying and evaluating a very large number of diagrams of different topologies which would seem an impossible task. Instead, we will just look at the diagrams of one topological class which all have the same logarithmic divergence, and check whether their coefficients (4.3) sum to zero. Of course, even if the sum were non-zero, this would prove nothing since the logarithmic divergence in this class of diagrams could be cancelled by that in some other class. However, the presence of dimension 4 CP violation in the KM model leads us to believe that there will be a divergent contribution to $\Delta\theta$ somewhere, and that this analysis at least establishes a lower bound on where it may arise.

The generic topological class of fourteenth order diagrams we consider is indicated in Fig. 9. The ten diagrams of this type which have external cathoquark lines and the potentially CP violating coupling factors (4.3) are portrayed in Fig. 10. Notice that we have included diagrams with the mirror image of the configuration of Fig. 9. The coupling factors for these diagrams are

$$(a), (d) \} = - g_R^c g_L \, \mathrm{Tr}\left[U^\dagger m_a^2 U m_c^4 U^\dagger m_a^4 U m_c^2\right] \tag{4.4a}$$

$$\left.\begin{matrix}(b), (c)\\(e), (f)\\(g), (h)\\(i), (j)\end{matrix}\right\} = - g_R^c g_L \, \mathrm{Tr}\left[U^\dagger m_a^4 U m_c^4 U^\dagger m_a^2 U m_c^2\right] \quad . \tag{4.4b}$$

It is clear from the different number of diagrams of types (4.4a) and (4.4b) that when we add to Fig. 10 the corresponding diagrams with ano- and cathoquarks exchanged, there will be no cancellation of the logarithmic divergences because $g_R^c \neq g_R^a$. We conclude that in the 't Hooft-Feynman gauge[27] the generic diagrams illustrated in Fig. 9 give a net divergent contribution to $\Delta\theta$.

How serious is this divergence? It means that θ is not specifiable, but we know already that in the KM model with one Higgs multiplet that there is no way of setting $\theta = 0$ unless we make an arbitrary assumption. If we regard θ as an effective coupling constant,[26] as discussed in section 2, this divergence can be renormalized, for example by specifying that $\theta = 0$ when the theory is renormalized at some "relaxation" scale μ_o. Corresponding to this renormalization there is the renormalization group equation (2.12). The logarithmic divergence we have found makes an appearance in

$$\beta_\theta^7 \ni O\left(\frac{m_t^4 m_b^4 m_s^2 m_c^2}{m_W^{12}}\right) s_1^2 s_2 s_3 \sin\delta \tag{4.5}$$

if we just have 6 quarks. Integrating this from the relaxation scale to the scale of order 1 GeV relevant to the neutron electric dipole moment, we find

$$\Delta\theta^7 \approx \left(\frac{\alpha}{\pi}\right)^7 \left(\frac{m_t^4 m_b^4 m_s^2 m_c^2}{m_W^{12}}\right) s_1^2 s_2 s_3 \sin\delta \ln\left(\frac{\mu_o}{1\ \mathrm{GeV}}\right) \quad . \tag{4.6}$$

If we compare this with the first finite contribution (1.2):

$$\Delta\theta \approx \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{\tilde{m}_q}{m_W}\right)^4 s_1{}^2 s_2 s_3 \sin\delta \qquad (4.7)$$

we see that the residual finite renormalization after relaxation is less than the first truly finite piece (4.7) if (assuming 6 quarks, all with $m_q \lesssim 10^{-1}\, m_W$):

$$\ln\left(\frac{\mu_o}{1\ \mathrm{GeV}}\right) < \left(\frac{\alpha}{\pi}\right)^{-5} \left(\frac{m_W}{m_{t,b}}\right)^8 = O(10^{18}) \qquad . \qquad (4.8)$$

Hence the residual renormalization of θ after relaxation is negligible if

$$\mu_o < \exp\,(O(10^{18}))\mathrm{GeV} \qquad (4.9)$$

(or $\exp\,(O(10^{10}))$ GeV if there are quarks with masses $\sim m_{W,Z}$). If we conservatively assume that μ_o is less than the Planck mass, then the "infinite" contribution to θ is less than 10^{-32} if all quarks have masses $\leq 10^{-1}\, m_{W,Z}$. We conclude that the "infinite" θ renormalization in the KM model is not very large.

5. GENERAL REMARKS

Having made detailed studies of the KM model, we would now like to make some remarks about other models, and assess the significance of our results.

Weinberg[23] has proposed a model with ≥ 3 Higgs doublets in which CP is violated by dimension 4 terms in the Higgs potential and there is no natural way of imposing $\theta = 0$. The predominant violation of CP is through Higgs exchanges which generate an effective milliweak $\bar{q}q\bar{q}q$ interaction of strength

$$\sim \mathrm{Im}\ A\, m_{q_1} m_{q_2} \times (\text{generalized Cabibbo angles}) \qquad (5.1)$$

where m_{q_1} and m_{q_2} are generic quark masses, and

$$\mathrm{Im}\ A = O(G_F/m_H^{\ 2})$$

where m_H is a generic Higgs boson mass. Application to the K^o-$\bar{K}^o$ system suggests that

$$m_s m_c \frac{\mathrm{Im}\ A}{G_F} \approx 3 \times 10^{-3} \qquad (5.3)$$

corresponding via equation (5.2) as $m_H = O(15)$ GeV. In this model, the dominant second order contribution to θ renormalization comes from the diagram in Fig. 11. It makes a contribution which is of order

$$\Delta\theta^2 \sim m_q^{\ 2} m_H^{\ 2}\ \mathrm{Im}\ A \times \text{(generalized Cabibbo angle factors)} \quad . \qquad (5.4)$$

There is no particular reason why the angle factors in (4.4) should be small, so from (5.2) we find

$$\Delta\theta^2 = \sum_q O(m_q^{\ 2} G_F) \qquad (5.5)$$

where the summation sign indicates that contributions from all quarks must be added. Already one quark has a mass $\sim$5 GeV, and there is presumably at least one more with mass $\geq$7 GeV. We therefore believe that in the Weinberg[23] model

$$\Delta\theta^2 \geq O(10^{-3} \text{ to } 10^{-4}) \qquad (5.6)$$

which is phenomenologically unacceptable by comparison with the neutron electric dipole moment. When we examine the fourth order diagram of Fig. 12, we find a logarithmic divergence. The diagrams of Fig. 11 are finite because of an effective "subtraction" in the Higgs propagator. But the Weinberg multi-Higgs potential has sufficiently many degrees of freedom to allow the 3-Higgs vertex in Fig. 12 to destroy this delicate cancellation. This divergence is clearly more drastic than the 14th order divergence in the KM model.

Georgi[24] has proposed a model with 2 Higgs doublets in which CP is violated by "soft" dimension 2 terms in the Higgs potential. His model has the virtue that $\Delta\theta$ is finite to all orders because of the softness of CP violation. This model has $\Delta S = 2$ Higgs exchanges which necessitate $m_H \gtrsim (G_F^{-1/2})$ to be no larger than the observed K^o-$\bar{K}^o$ mixing. On the other hand the multi-Higgs couplings become strongly interacting if $m_H > G_F^{-1/2}$. The model is therefore committed[23] to $m_H = O(G_F^{-1/2})$, and the probable dominance of Higgs exchanges in the $\Delta S = 2$ K^o-$\bar{K}^o$ mixing, with aspersions therefore cast on the successful double W exchange calculation of the charmed quark mass.[30] CP violation arises from mass mixing Δ of the Higgs multiplets, which should be of order

$$\Delta \sim 10^{-3}\, m_H^2 \qquad . \tag{5.7}$$

The second order θ renormalization comes from Fig. 13 and is of order

$$\begin{aligned}\Delta\theta^2 &\simeq \sum_q \Delta\, m_q^2\, G_F/m_H^2 \\ &= \sum_q O(10^{-3}\, m_q^2 G_F) \qquad .\end{aligned} \tag{5.8}$$

By the same reasoning as before, the inclusion of top and bottom quarks suggests that (5.8) may imply

$$\Delta\theta^2 \gtrsim O(10^{-6} \text{ to } 10^{-7}) \qquad . \tag{5.9}$$

While not excluded, this renormalization is rather large compared with the phenomenological bound of $\theta \lesssim 10^{-8}$ deduced from the neutron electric dipole moment.[4,5]

Since the problems we have been discussing involve the interplay between the strong, weak and electromagnetic interactions, it is natural to ask what happens to θ and its renormalization in grand unified models which seek to unify all these interactions. One such model is the SU(5) model of Georgi and Glashow,[28] which is the minimal theory containing QCD and the Weinberg-Salam model.[16] It can easily accommodate the KM model[21] of CP violation, with one Higgs 5-plet and "hard" symmetry breaking in the dimension 4 $q\bar{q}$-Higgs couplings. In this model it is most convenient to compute the renormalization of the argument of the determinant of the $q\bar{q}$-H coupling matrix. This turns out to start being non-zero in fourth order, and possibly infinite in eighth order with a finite residual renormalization which is probably smaller than the fourth order contribution. The renormalization of θ therefore seems to be about the same in this model as in the KM version of the Weinberg-Salam model, though it should be studied more deeply.

The analysis of this paper has certainly not revealed a totally satisfactory resolution of the problem of CP violation via the strong interaction θ vacuum parameter when weak interaction effects are taken into account. The Georgi[24] model has a second order renormalization of θ which may be too large to be viable. The Kobayashi-Maskawa[21] model is at an aesthetic disadvantage because of the need to set $\theta = 0$ arbitrarily at some "relaxation" scale μ_o. On the other hand, given this assumption the finite renormalization (1.2) of θ is phenomenologically viable, and suggests a very small value for the neutron electric dipole moment. Clearly, more studies of possible patterns of CP violation in the $SU(2) \times U(1)$ Weinberg-Salam model are necessary, focussing in particular on the "relaxation" hypothesis and alternative Higgs structures.

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FIGURE CAPTIONS

Fig. 1: General form of $O(\alpha^3)$ contributions to θ renormalization in the Kobayashi-Maskawa model[21,22] with strong interactions neglected.

Fig. 2: Diagrams giving wave function renormalization in (a) 2nd order, and (b) fourth order.

Fig. 3: One particle irreducible diagrams contributing to C_3, but not to Im Tr C_3 because they are symmetric under $\beta \leftrightarrow \delta$.

Fig. 4: As for Fig. 3, except that they are symmetric under $\alpha \leftrightarrow \gamma$.

Fig. 5: Remaining 6th order contribution to C_3, (a) showing its decomposition (b) into a vertex function.

Fig. 6: One loop contributions to the vertex function in Fig. 5b.

Fig. 7: A typical QCD correction to the photon vertex relevant to the neutron electric dipole moment.

Fig. 8: Relevant vertices for $W^{\pm}$, Higgs and U(1) boson couplings in the manifestly renormalizable 't Hooft-Feynman gauge.[27] A common factor of $2^{1/4} G_F^{1/2}$ has been removed from the Higgs couplings.

Fig. 9: Generic topology of a class of divergent CP violating 14th order diagrams in the Kobayashi-Maskawa model.[21,22]

Fig. 10: Diagrams in the class of Fig. 9 which cause infinite renormalization of θ. The quark charges and helicities are indicated explicitly only in figure (a).

Fig. 11: Lowest order diagram renormalizing θ in the Weinberg[23] model of CP violation.

Fig. 12: Divergent 4th order contribution to θ renormalization in the Weinberg model of CP violation.

Fig. 13: Convergent second order contribution to θ renormalization in the Georgi[24] model of CP violation.

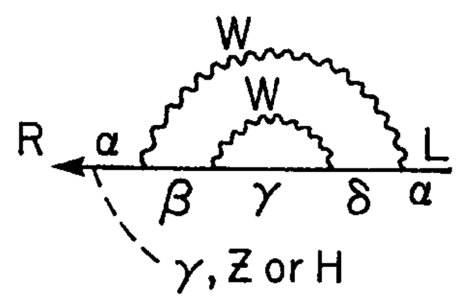


Fig. 1

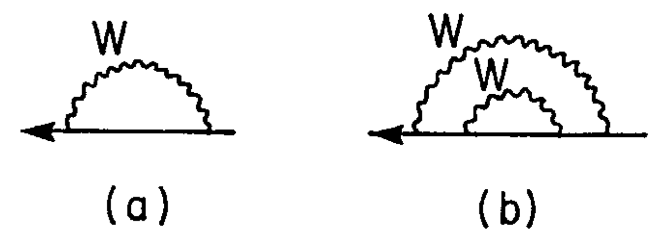


Fig. 2

Fig. 3

Fig. 4

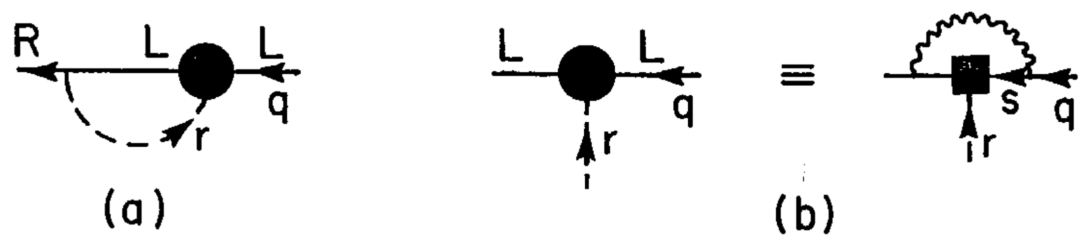


Fig. 5

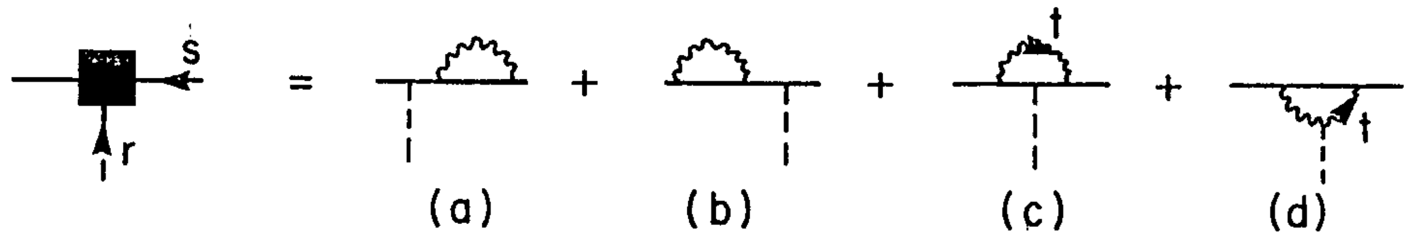


Fig. 6

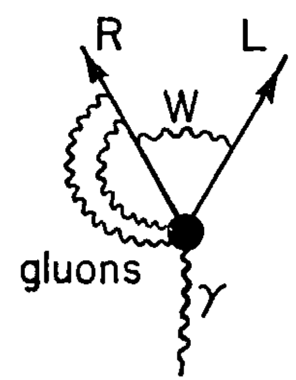
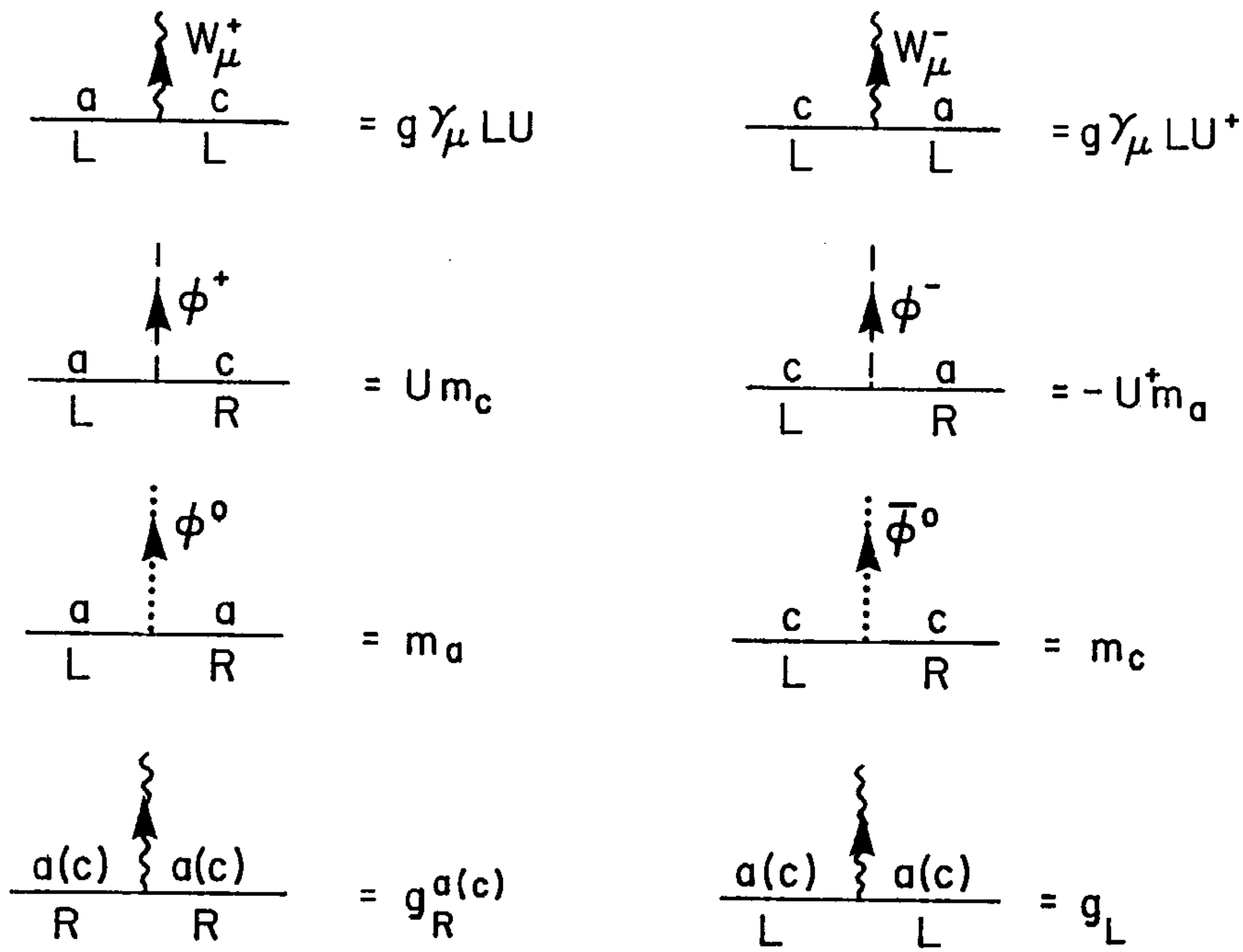
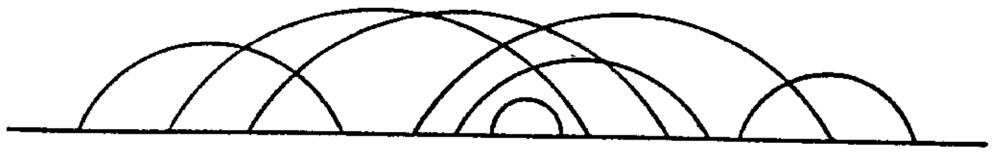


Fig. 7



W_μ^+
a c
L L
$= g\gamma_\mu LU$
W_μ^-
c a
L L
$= g\gamma_\mu LU^+$
ϕ^+
a c
L R
$= Um_c$
ϕ^-
c a
L R
$= -U^+m_a$
ϕ^0
a a
L R
$= m_a$
$\bar{\phi}^0$
c c
L R
$= m_c$
a(c) a(c)
R R
$= g_R^{a(c)}$
a(c) a(c)
L L
$= g_L$

Fig. 8



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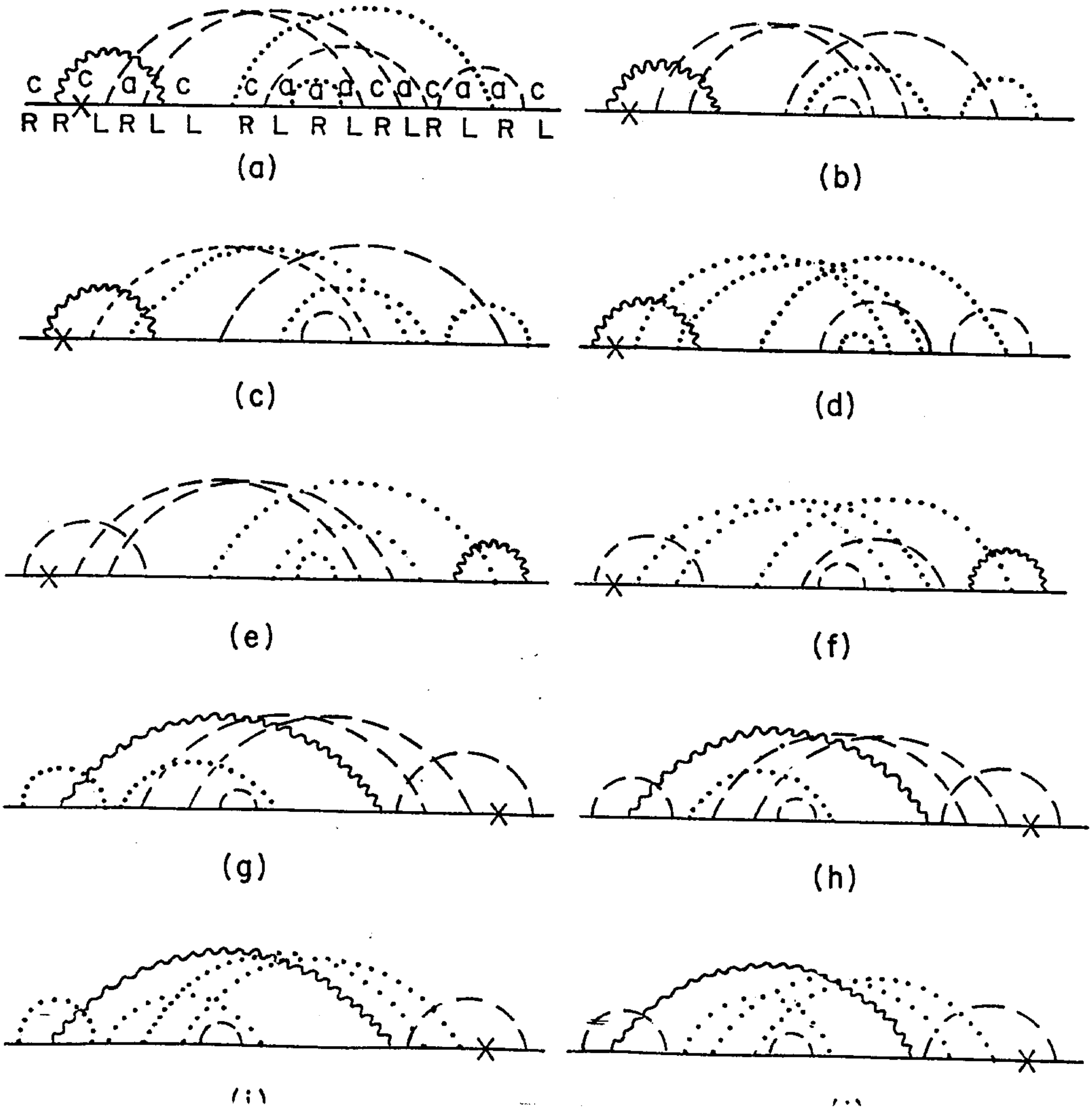

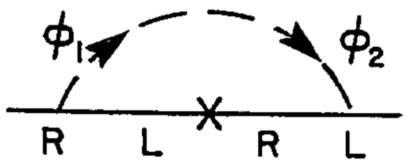

Fig. 11

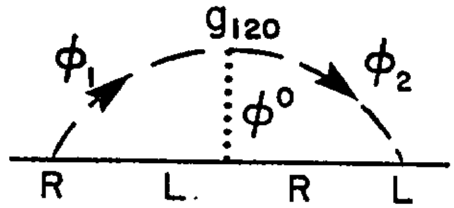

Fig. 12

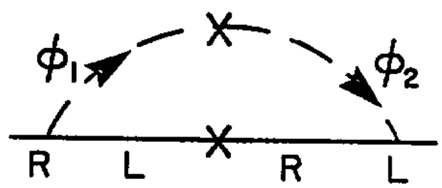

Fig. 13

ACKNOWLEDGMENTS

One of us (J.E.) thanks the DAMTP, Cambridge and Fermilab theory groups, and the other (M.K.G.) thanks the SLAC theory group for kind hospitality while this work was being done. We acknowledge useful discussions with V. Baluni, R.J. Crewther, M. Peshkin, J. Prentki, M. Veltman and S. Weinberg.

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FIGURE CAPTIONS

Fig. 1: General form of $O(\alpha^3)$ contributions to θ renormalization in the Kobayashi-Maskawa model[21,22] with strong interactions neglected.

Fig. 2: Diagrams giving wave function renormalization in (a) 2nd order, and (b) fourth order.

Fig. 3: One particle irreducible diagrams contributing to C_3, but not to Im Tr C_3 because they are symmetric under $\beta \leftrightarrow \delta$.

Fig. 4: As for Fig. 3, except that they are symmetric under $\alpha \leftrightarrow \gamma$.

Fig. 5: Remaining 6th order contribution to C_3, (a) showing its decomposition (b) into a vertex function.

Fig. 6: One loop contributions to the vertex function in Fig. 5b.

Fig. 7: A typical QCD correction to the photon vertex relevant to the neutron electric dipole moment.

Fig. 8: Relevant vertices for $W^{\pm}$, Higgs and U(1) boson couplings in the manifestly renormalizable 't Hooft-Feynman gauge.[27] A common factor of $2^{1/4}G_F^{1/2}$ has been removed from the Higgs couplings.

Fig. 9: Generic topology of a class of divergent CP violating 14th order diagrams in the Kobayashi-Maskawa model.[21,22]

Fig. 10: Diagrams in the class of Fig. 9 which cause infinite renormalization of θ. The quark charges and helicities are indicated explicitly only in figure (a).

Fig. 11: Lowest order diagram renormalizing θ in the Weinberg[23] model of CP violation.

Fig. 12: Divergent 4th order contribution to θ renormalization in the Weinberg model of CP violation.

Fig. 13: Convergent second order contribution to θ renormalization in the Georgi[24] model of CP violation.

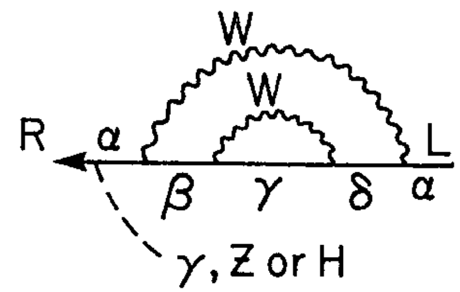


Fig. 1

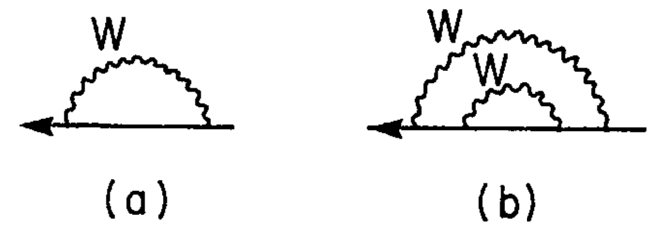


Fig. 2

Fig. 3

Fig. 4

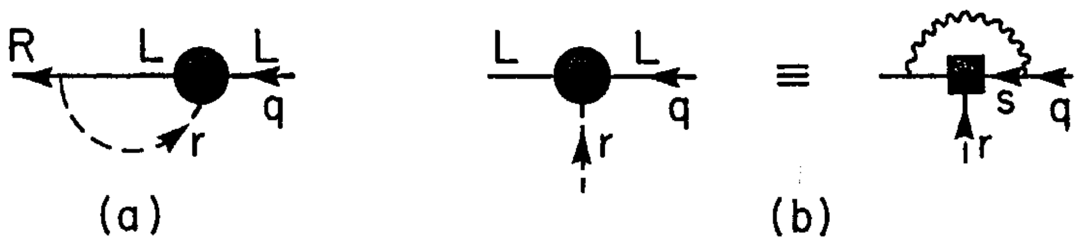


Fig. 5

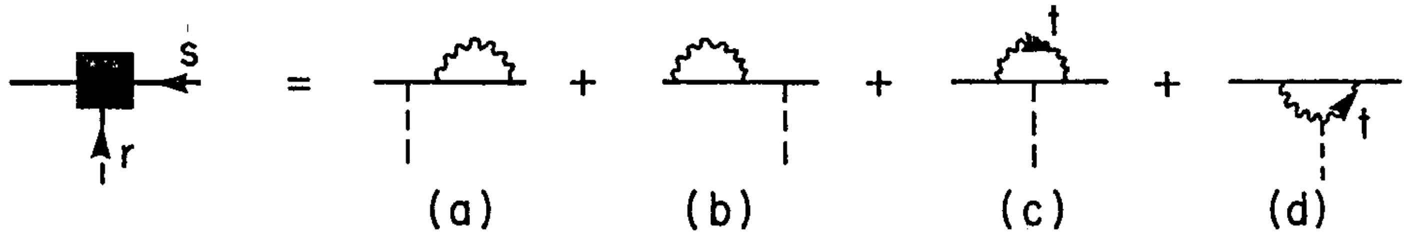


Fig. 6

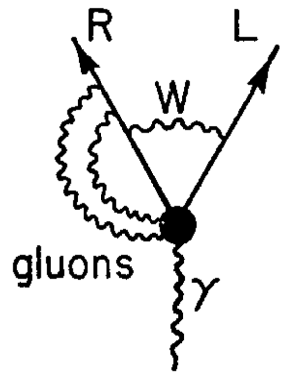
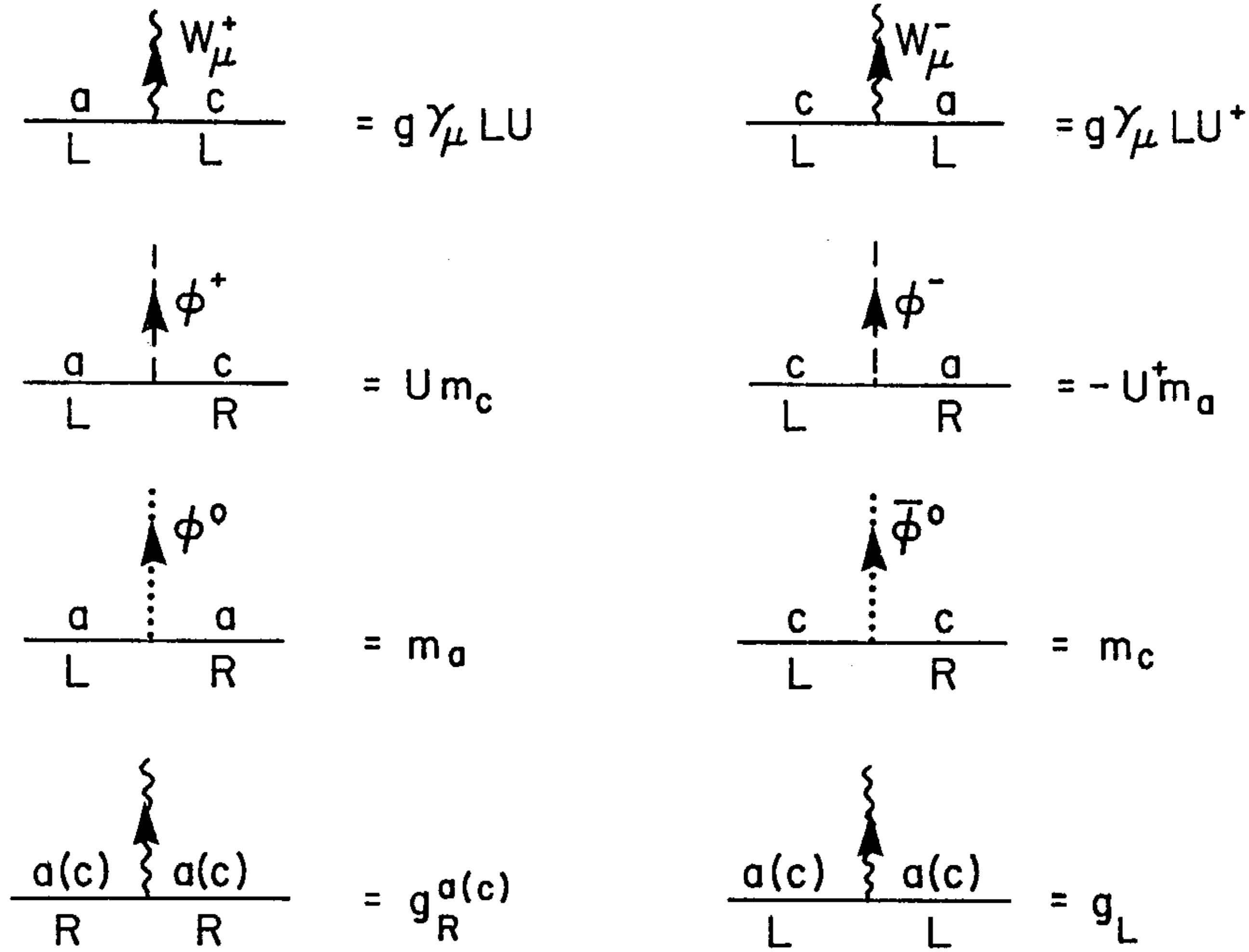
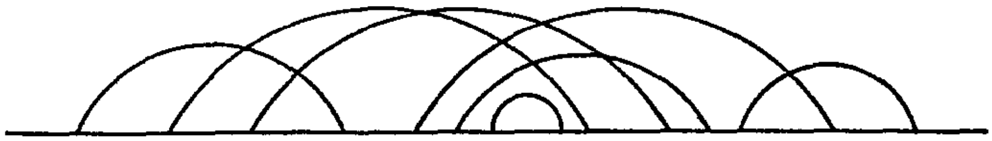


Fig. 7



W$_\mu^+$
a
c
L
L
= g γ_μ LU
W$_\mu^-$
c
a
L
L
= g γ_μ LU$^+$
ϕ^+
a
c
L
R
= U m$_c$
ϕ^-
c
a
L
R
= − U$^+$m$_a$
ϕ^0
a
a
L
R
= m$_a$
$\bar{\phi}^0$
c
c
L
R
= m$_c$
a(c)
a(c)
R
R
= g$_R^{a(c)}$
a(c)
a(c)
L
L
= g$_L$

Fig. 8



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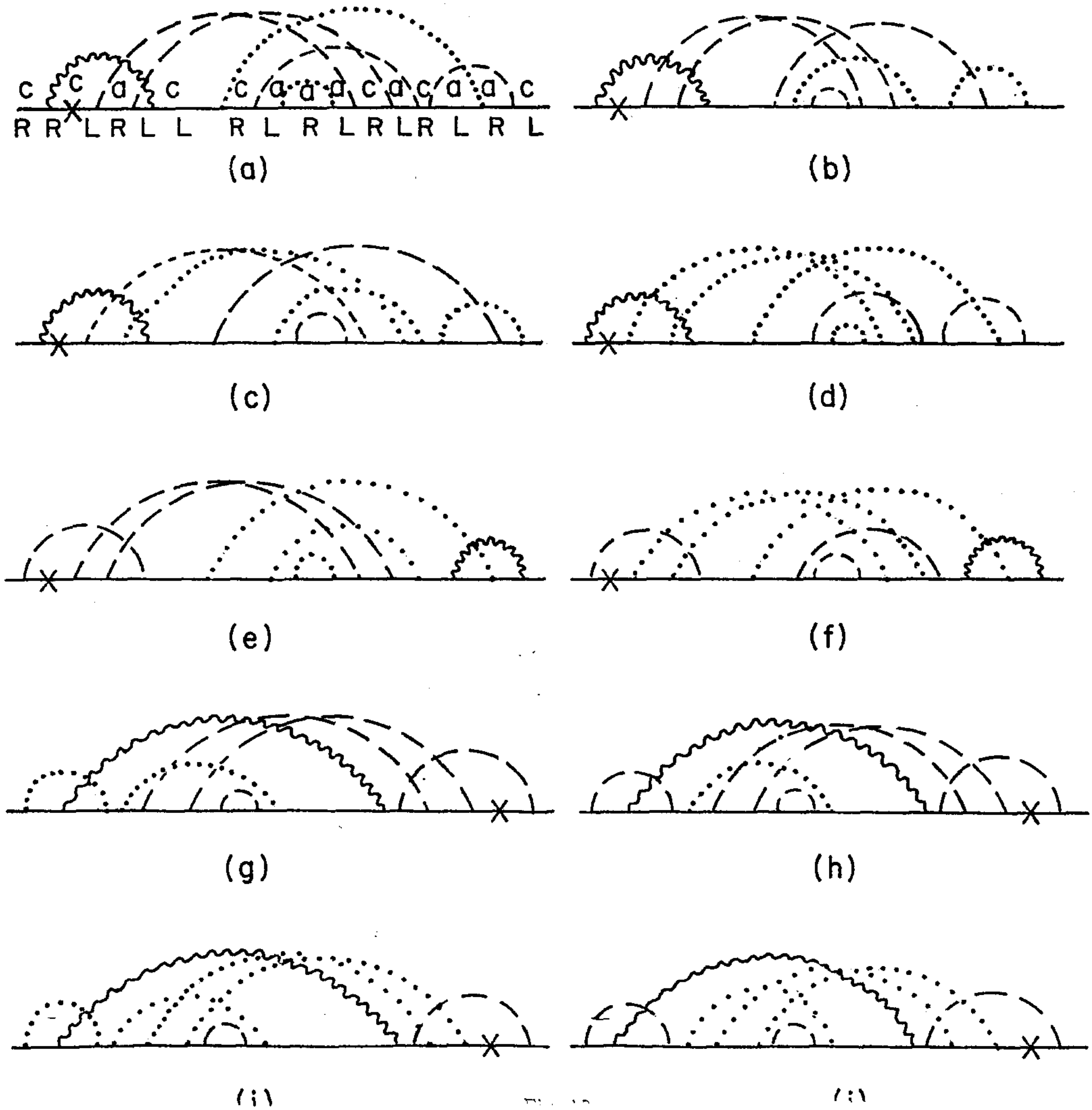

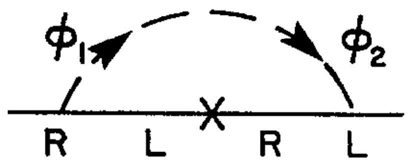

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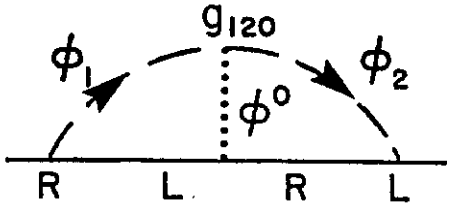

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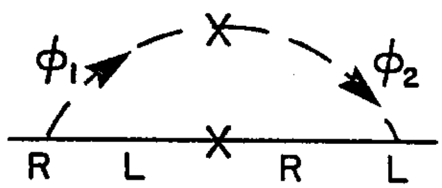

Fig. 13