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Quantum Mechanics and Quarkonium: An Introductory Review

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ABSTRACT

The spectra of the ψ and Υ families of heavy mesons are interpreted in the framework of nonrelativistic potential models. Recent data on the upsilons are summarized, and their impact is assessed.

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I. INTRODUCTION

It is particularly satisfying to be able to apply elementary techniques to the solution of topical problems, because simplification frequently provides the basis for insight. In this lecture, I will describe a number of techniques of nonrelativistic quantum mechanics which have proved useful in the analysis of the ψ and Υ families of heavy particles. For each technique I will cite illustrative examples instead of giving an exhaustive review, but extensive references to the literature will be given for other applications.

In Section II, I will give a brief review of ψ spectroscopy and the Schrödinger equation approach. The most recent Fermilab data on the Υ family are summarized in Section III. Selected examples of phenomenological applications of nonrelativistic quantum mechanics are given in Section IV, where the impact of very recent data from DORIS is also discussed. Some mention of topics not otherwise covered is made in Section V.

II. THE SCHRÖDINGER EQUATION APPROACH TO ψ SPECTROSCOPY

We believe[1] that hadrons are composed of quarks which, if not permanently confined, are at least very difficult to liberate. The nonobservation of free quarks[2] suggests that the binding force is formidable, but the success of the parton model[3] in describing hard scattering processes argues that quarks behave as quasi-free within hadrons. Although it has not been proved to yield quark confinement, the non-Abelian gauge theory of colored quarks and gluons known as quantum chromodynamics (QCD)[4] promises to explain this paradoxical circumstance through the property of asymptotic freedom.[5] Asymptotic freedom refers to the fact that in QCD, the strong interaction becomes feeble at large momentum transfers (short distances) so that quarks are weakly bound at small separations, but feel an increasingly strong restoring force at large separations.

On the basis of asymptotic freedom arguments, it was anticipated[6] that bound states of heavy quarks might be described by a nonrelativistic analog of the bound e^+e^- system, positronium.[7] As a cultural aside, I show the spectrum of positronium (Ps) in Fig. 1. The ground state, a favorite textbook example,[8] is split by the hyperfine interaction into the $J^{PC} = 1^{--}$ orthopositronium and $J^{PC} = 0^{-+}$ parapositronium components with lifetimes that differ by a factor of 1120.

The experimentally-studied spectrum of the ψ, or charmonium, family is exceedingly rich,[9] as shown in Fig. 2. Below the charm threshold lie the very narrow states $\psi(3095)$, $\psi'(3684)$, $\chi(3415)$, $\chi(3510)$, and $\chi(3550)$ for which quantum numbers are rather firmly established. Above charm threshold lie the discrete vector states

$\psi(3772)$ and $\psi(4414)$, and a jumble of levels around 4.1 GeV. In addition, there are suggestions of states which may be pseudoscalar: X(2830), $\chi(3455)$, X(3600),[10] none of which is clearly established in my opinion. The spectroscopic notation for the psion states as bound states of a charmed quark and charmed antiquark is given in Fig. 3.

In broad terms, the description of the psions as atomic levels of a nonrelativistic $(c\bar{c})$ system bound by a static potential has met with great success.[11,12] The spectrum looks like a nonrelativistic level scheme, and there have been some predictive triumphs. The principal challenges to the ingenuous model have primarily to do with spin-orbit and hyperfine splittings. To do justice to the quantitative difficulties and proposed resolutions would take us too far afield.[13] It will be enough to know that the Schrödinger equation approach does very well on the generalities of the charmonium system, and that the nonrelativistic approximation should be much better for families composed of heavier quarks.

III. NEW DATA ON THE Υ FAMILY: HADRON COLLISIONS[14]

At the Tokyo Conference, the Columbia-Fermilab-Stony Brook Collaboration (E288) presented[15] extensive new data on the reaction

$$p + N \to \mu^+\mu^- + \text{anything} \tag{3.1}$$

at 400 GeV/c, which add significantly to our knowledge of the upsilon complex. They have now accumulated more than 9,000 upsilon events under various running conditions. Figure 4 shows the results from a high-intensity run which lasted from November, 1977 to April, 1978. The dimuon invariant mass resolution for this run was $\Delta m/m \simeq 2.2\%$. The upsilon peak contains about 7000 events. No new families of more massive particles are apparent in these data, which have a significant sensitivity out to about 14 GeV/c^2. After subtracting a smooth (exponential) continuum, the experimenters arrive at the upsilon signal displayed in Fig. 5. The curves show a three resonance fit to the data, which yields[16]

$$\Upsilon(9.45 \pm 0.07\ GeV/c^2),\quad \sigma = (0.273 \pm 0.006)pb;$$
$$\Upsilon'(9.98 \pm 0.02),\quad \sigma = (0.073 \pm 0.006)pb;$$
$$\Upsilon''(10.35 \pm 0.01),\quad \sigma = (0.027 \pm 0.005)pb.$$

Further running was done in a high-resolution ($\Delta m/m \simeq 1.5\%$) mode, in the course of which about 500 upsilons were detected. This signal is shown in Fig. 6, where the Υ-Υ' separation is clearly visible. The curves again show a three resonance fit, this time with the parameters

$$\Upsilon(9.49 \pm 0.02\ GeV/c^2),\quad \sigma = (0.27 \pm 0.02)pb;$$
$$\Upsilon'(10.09 \pm 0.05),\quad \sigma = (0.09 \pm 0.02)pb;$$
$$\Upsilon''(10.46 \pm 0.06)\quad \sigma = (0.022 \pm 0.012)pb.$$

A combined analysis of all the Columbia-Fermilab-Stony Brook data is summarized in Table I. Comparison of the two-peak and three-peak fits leads to the conclusion that the statistical significance of the Υ'' is about four standard deviations. The third column shows the results of a fit which assumes the Υ and Υ' masses measured at DESY (of which more in §IV), for which the statistical significance of Υ'' increases to eleven standard deviations. A striking feature of these measurements and fits is the similarity between the excitation energies of Υ' and Υ'' and those of the corresponding structures ($\psi'(3684)$, $\psi(4028??)$) in the charmonium system. I will return to this point below.

A second group working at Fermilab, the Michigan-Northeastern-Washington-Tufts Collaboration[17] has accumulated 15,000 upsilons in a large-acceptance magnetized iron spectrometer, also studying reaction (3.1). Their data are shown before and after continuum subtraction in Figs. 7 and 8. The price for wide acceptance is poor resolution ($\Delta m/m \simeq 6\%$), so this experiment cannot add incisively to upsilon spectroscopy. It is, however, in a good position to study production characteristics of the Υ family.

Other hadron-induced dilepton experiments have been summarized by Lederman.[14] For the moment, they are limited to the observation of tens of events in the upsilon region and will not influence our view of the spectrum.

IV. PHENOMENOLOGICAL APPLICATIONS OF THE SCHRÖDINGER EQUATION

Let us now consider some consequences of the idea that nonrelativistic quantum mechanics is a valid framework for the analysis of the heavy mesons. We make two crucial assumptions, which are plausible but cannot be completely justified a priori. The first is that the nonrelativistic approximation is a good one. (We shall see that this does not lead to serious embarrassments.) The second is that the same potential describes the interactions of all flavors of massive quarks. This is very much in the spirit of QCD, provided all the quarks we deal with are color triplets.[18] Instead of comparing experimental results with the predictions of specific potential models, we shall investigate what experiment can tell us about the properties of the potential.

The level spacing of the upsilons, as indicated by the CFS experiment, is depicted in Fig. 9. It is striking that the intervals, which are also given in Table I, so closely resemble those of the ψ family:

$$\psi'(3684)-\psi(3095) = 0.589\ \mathrm{GeV}/c^2;$$

$$\psi''(4028??)-\psi(3095) = 0.933\ \mathrm{GeV}/c^2.$$

It is interesting to idealize this similarity by asking what form of potential will give equal level spacings, independent of the quark mass. This leads us to examine[19] the dependence of level spacings and other dimensionful quantities upon the reduced quark mass μ, for potentials of the form

$$V(r) = a\ r^{\varepsilon} \quad . \tag{4.1}$$

We first show that for potentials of the form (4.1) the scale of level spacings is given by

$$\Delta E \propto \mu^{-\varepsilon/(2+\varepsilon)} \quad . \tag{4.2}$$

In the reduced radial Schrödinger equation (for $u(r) = rR(r)$, where the Schrödinger wavefunction is $\Psi(\underset{\sim}{r}) = R(r)Y_{\ell m}(\theta,\phi)$)

$$\frac{-u''(r)}{2\mu} + \left[\frac{\ell(\ell+1)}{2\mu r^2} + V(r) - E\right]u(r) = 0 \tag{4.3}$$

with $\hbar = c = 1$, define the dimensionless parameter

$$\rho = \mu^p\ m_0^{\ 1-p}\ r \quad . \tag{4.4}$$

Here m_0 is a constant with dimensions of mass and the power p will be specified below. With the replacement

$$u(r) \equiv w(\rho) = w(\mu^p m_0^{\ 1-p}\ r) \quad , \tag{4.5}$$

we have

$$u''(r) = \mu^{2p} m_0^{\ 2(1-p)} w''(\rho) \quad , \tag{4.6}$$

so that

$$-\tfrac{1}{2}\mu^{2p-1}\ m_0^{\ 2(1-p)} w''(\rho) + \tfrac{1}{2}\mu^{2p-1}\ m_0^{\ 2(1-p)} \ell(\ell+1) w(\rho)/\rho^2 + \left[a\ \rho^{\varepsilon}\mu^{-\varepsilon p}\ m_0^{\ \varepsilon(p-1)} - E\right] w\ (\rho) = 0 \quad . \tag{4.7}$$

We now set

$$2p-1 = -\varepsilon p \tag{4.8}$$

and divide (4.7) by $\mu^{2p-1}\ m_0^{\ 2(1-p)}$, obtaining

$$-\tfrac{1}{2}w''(\rho) + \ell(\ell+1)w(\rho)/2\rho^2 + \left[\frac{a\rho^\varepsilon}{m_0^{1+\varepsilon}} - \frac{E}{\mu^{2p-1}m_0^{2-2p}}\right]w(\rho) = 0 \ . \tag{4.9}$$

We have now isolated the μ-dependence in the term $E/\mu^{2p-1}m_0^{2-2p}$; thus the scale of energy level spacings is given by $\Delta E \sim \mu^{2p-1}$. Solving (4.8) for $p = 1/(2+\varepsilon)$, we obtain eq. (4.2).

For level spacings which are independent of the quark mass,

$$\Delta E \propto \mu^0 \quad , \tag{4.10}$$

eq. (4.2) indicates that V(r) must vary more slowly than any power of r, i.e., that $\varepsilon=0$. Repeating the arguments above for the special case of $\varepsilon=0(p=\tfrac{1}{2})$, one finds that the potential

$$V(r) = C \ln(r/r_0) \tag{4.11}$$

is unique in giving level spacings which are strictly independent of quark mass.[20] With the parameters chosen to reproduce the ψ-ψ' splitting, the logarithmic potential

$$V(r) = (0.73 \text{ GeV}) \ln(r \cdot 1 \text{ GeV}) \tag{4.12}$$

gives an unexpectedly good account of psion spectroscopy. On balance, it is as successful as the Coulomb + linear form motivated by QCD.

According to eq. (4.4), quantities with dimensions of length L scale as

$$L \sim \mu^{-1/(2+\varepsilon)} \quad . \tag{4.13}$$

The size of a bound state with given quantum numbers is such a quantity. The matrix elements of electric and magnetic multipole operators scale as

$$\langle n'|Ej|n\rangle \sim L^{j} \tag{4.14}$$

and[21]

$$\langle n'|Mj|n\rangle \sim L^{j-1}/\mu \quad . \tag{4.15}$$

Since the radiative widths are given by

$$\Gamma(Ej \text{ or } Mj) \sim p_\gamma^{\,2j+1}|\langle n'|Ej \text{ or } Mj|n\rangle|^2 \quad , \tag{4.16}$$

and

$$p_\gamma \sim \Delta E \sim \mu^{-\varepsilon/(2+\varepsilon)} \quad , \tag{4.17}$$

we find

$$\Gamma(Ej) \sim \mu^{-[2j(1+\varepsilon)+\varepsilon]/(2+\varepsilon)} \tag{4.18}$$

and

$$\Gamma(Mj) \sim \mu^{-[2j(1+\varepsilon)+(3\varepsilon+2)]/(2+\varepsilon)} \quad . \tag{4.19}$$

For potentials weaker near r=0 than a Coulomb potential, i.e., for $\varepsilon > -1$, the relative importance of higher multipoles decreases as μ increases. For the logarithmic potential, electric dipole transition rates scale as $\Gamma(E1) \propto 1/\mu$.

Probability densities $|\Psi(\underset{\sim}{r})|^2$ have dimensions of inverse volume L^{-3} so scale as

$$|\Psi(\underset{\sim}{r})|^2 \sim \mu^{3/(2+\varepsilon)} \quad . \tag{4.20}$$

Such quantities are of interest, for example, in the decays of massive vector mesons $\mathscr{V}$ which are $^3S_1$ bound states of a quark and antiquark, for which[22]

$$\Gamma(\mathscr{V} \to \ell^+\ell^-) = 16\pi\alpha^2 e_Q{}^2 |\Psi(0)|^2/M(\ \ )^2 \quad , \tag{4.21}$$

where e_Q is the quark charge and $M(\mathscr{V})$ is the vector meson mass. For $\varepsilon > -1$, the scale of $M(\mathscr{V})$ will itself be set by μ for the low-lying levels.[23] Consequently we find

$$\Gamma(\mathscr{V} \to \ell^+\ell^-) \sim \mu^{-(1+2\varepsilon)/(2+\ \varepsilon)}, \quad \varepsilon \geq -1 \quad . \tag{4.22}$$

The ratios of radiative to leptonic widths are of concern for massive states:

$$\Gamma(Ej)/\Gamma(\mathscr{V} \to \ell^+\ell^-) \sim \mu^{(1-2j)(1+\varepsilon)/(2+\varepsilon)}, \quad \varepsilon \geq -1 \quad ; \tag{4.23}$$

$$\Gamma(Mj)/\Gamma(\mathscr{V} \to \ell^+\ell^-) \sim \mu^{-(1+2j)(1+\varepsilon)/(2+\varepsilon)}, \quad \varepsilon \geq -1 \quad . \tag{4.24}$$

Since $j \geq 1$, the exponents in (4.23) and (4.24) are both negative for $\varepsilon > -1$. Hence leptonic decays will dominate over radiative transitions as the quark mass increases. Choosing $\varepsilon \simeq 0$, as suggested by the equality of ψ and Υ mass splittings, we expect

$$\frac{\Gamma(\Upsilon' \xrightarrow{E1} \chi_b \gamma)}{\Gamma(\Upsilon' \longrightarrow \ell^+\ell^-)} \Bigg/ \frac{\Gamma(\psi' \xrightarrow{E1} \chi_c \gamma)}{\Gamma(\psi' \longrightarrow \ell^+\ell^-)} \propto \sqrt{\frac{m_c}{m_b}} \approx (0.5 \text{ to } 0.6). \tag{4.25}$$

The dependence of $|\Psi(0)|^2$ upon the quark mass provides another measure of the effective power-law form of the potential. Unless the potential is exactly power-behaved, the effective power determined in this manner need not be the same as the effective power implied by the level spacings, because the wavefunction at the origin probes shorter distances.[24] Likewise, the effective power inferred from the leptonic widths may be different for different radial excitations. The leptonic widths of ψ and ψ' have been measured in several experiments. I take as representative the values measured at SPEAR,[25]

$$\begin{aligned} \Gamma(\psi \rightarrow e^+e^-) &= 4.8 \pm 0.6 \text{ keV}, \\ \Gamma(\psi' \rightarrow e^+e^-) &= 2.1 \pm 0.3 \text{ keV}. \end{aligned} \tag{4.26}$$

Recently, the Υ and Υ' have been observed at the storage ring DORIS in e^+e^- annihilations into hadrons.[26-28] Let us take a few moments to review these new results. A first run in the upsilon region was made using the PLUTO and DASP detectors. The visible cross sections in these detectors are shown in Fig. 10. After removal of the PLUTO detector (in preparation for experiments to be done at the new storage ring PETRA) a second run was undertaken using the DASP detector and the DESY-Heidelberg NaI-Pb Glass detector. The results of this second run are shown in Fig. 11. These experiments localize the mass of the upsilon as

$$M(\Upsilon) = 9.46 \pm 0.01 \text{ GeV}/c^2 \quad , \tag{4.27}$$

and give direct evidence for the extreme narrowness of the resonance. Measurements of the leptonic width (subject to the plausible assumption that $\Gamma_{ee} \ll \Gamma_{total}$) are summarized in Table II. In addition, the PLUTO and DASP2 Collaborations have been able to quote rough values for the leptonic branching fraction, from measurements of the reaction

$$e^+e^- \to \mu^+\mu^- \quad . \tag{4.28}$$

These data, which are reproduced in Table III, permit an estimate for the total width of Υ, namely

$$(90\% \text{ C.L.})\ 25 \text{ keV} \leq \Gamma_{total} \leq 110 \text{ keV (1 std. dev.)}. \tag{4.29}$$

In a subsequent run at higher energies, the DASP2 and DESY-Heidelberg Collaborations observed the Υ' in e^+e^- annihilations into hadrons. These data are shown in Fig. 12. Preliminary values for the parameters of Υ' are given in Table IV.

As we shall see shortly, the DESY experiments make compelling the conclusion that the charge of the (fifth) quark which is the constituent of Υ is $e_Q = -1/3$. With this knowledge, we can extract the values of $|\Psi(0)|^2$ for Υ and Υ', and use (4.20) to explore the shape of the potential. The wavefunctions at the origin of the n=1 and n=2 levels of ψ and Υ are displayed in Fig. 13. Because the masses of the c-quark and the fifth quark are poorly known, I have indicated a plausible range for the quantity m_Q/m_c. With such a limited set of data, this exercise can only be illustrative. The effective powers of the potential deduced from these data are shown in table V. Obviously they do not reliably fix the potential form, but with more precise data from still more families of heavy mesons, we may hope to find this analysis more incisive.

Within a quarkonium family, the principal-quantum-number dependence of observables is another source of information about the nature of the potential. With the aid of an intermediate result,

$$|\Psi(0)|^2 = \frac{\mu}{2\pi}\left\langle\frac{dV}{dr}\right\rangle \tag{4.30}$$

(which is derived by direct computation from the Schrödinger equation), it is straightforward to compute[19] in WKB approximation that for power-law potentials of the form (4.1)

$$\begin{aligned} |\Psi_n(0)|^2 &\sim n^{2(\varepsilon-1)/(2+\varepsilon)} , \qquad \varepsilon>0 \\ |\Psi_n(0)|^2 &\sim n^{(\varepsilon-2)/(2+\varepsilon)} , \qquad -2<\varepsilon<0 \quad . \end{aligned} \tag{4.31}$$

For $\varepsilon\to 0$ both expressions imply that $|\Psi_n(0)|^2$ should behave as n^{-1} for large n. This is precisely the behavior found in Ref. 20 for the potential $V(r)\sim\ln r$.

I plot in Fig. 14 the values of $|\Psi_n(0)|^2$ for $\psi(3095)$, $\psi(3684)$, and $\psi(4414)$, which I regard as 1S, 2S, and 4S or 5S levels of the charmonium system. Blithely applying the semiclassical result (4.31) to the power-law fits shown in Fig. 14, we find

$$\begin{aligned} \varepsilon &= 0.01\pm 0.14 \quad \text{(4S assignment)}; \\ \varepsilon &= 0.05\pm 0.13 \quad \text{(5S assignment)}. \end{aligned} \tag{4.32}$$

A similar exercise for the Υ and Υ' leads to a best-fit of the form

$$|\Psi_n(0)|^2 \sim n^{-(1.64\pm 0.43)} \quad , \tag{4.33}$$

which corresponds to

$$\varepsilon = -0.48\pm 0.25 \quad . \tag{4.34}$$

We thus have a suggestion[12] that the potential may be steeper at short distances than at intermediate distances, a tendency compatible with the indications in Table V and Fig. 13.

As was the case for our discussion of the experimental mass dependence of $|\Psi(0)|^2$, this is purely an illustrative application. We look forward to data which will permit the use of eqs. (4.31) within their justified range of large n.

Another important semiclassical result concerns the number of $^3S_1$ $Q\bar{Q}$ states which lie below the threshold for Zweig-allowed decays (into $Q\bar{q}$ + $\bar{Q}q$ pairs, where q is a light quark).[29] It is a remarkable general result that the number n of such narrow states is

$$n \sim a(m_Q/m_c)^{\frac{1}{2}} \quad . \tag{4.35}$$

The coefficient a appears to be close to 2. This last deduction follows from the $c\bar{c}$ system, in which the second level ψ' lies just below charm threshold. We therefore expect three or four narrow upsilon states, as earlier suggested by Eichten and Gottfried[30] on the basis of a specific potential model.

The key to the result (4.35) is the observation[30,19] that although the dynamics of the $Q\bar{q}$ system cannot be expected to yield to a nonrelativistic approach, the dependence upon m_Q of the lowest (1^1S_0) mass becomes simple as m_Q becomes large. The $(Q\bar{q})$ mass depends upon m_Q in three ways: the additive contribution of m_Q; the hyperfine $({}^3S_1 - {}^1S_0)$ splitting which decreases monotonically ($\sim m_Q{}^{-1}$ in specific models); and the binding which has a feeble dependence through reduced mass effects. Thus the quantity

$$\delta(m_Q) \equiv 2m(\text{lowest } Q\bar{q} \text{ state}) - 2m_Q \qquad (4.36)$$

is expected to approach a finite limit δ_∞ as $m_Q \to \infty$. Furthermore, it is likely that $\delta(m_c)/\delta_\infty \approx 1$.

Let us now consider the $Q\bar{Q}$ system. It will be convenient to set the zero of energy at $2m_Q$. The threshold for decay of a $Q\bar{Q}$ state into $Q\bar{q} + \bar{Q}q$ is then $\delta(m_Q)$ above the zero point. Let V(r) be any potential which binds $Q\bar{Q}$ states rising at least $\delta(m_Q)$ above $2m_Q$. Any infinitely-rising confining potential satisfies this condition. Then the number n of bound states lying <u>below</u> $\delta(m_Q)$ is given in the semiclassical approximation by

$$\int_0^{r_0} dr \left[m_Q(\delta(m_Q) - V(r))\right]^{\frac{1}{2}} \simeq (n-\tfrac{1}{4})\pi \quad , \tag{4.37}$$

where r_0 is the point at which $\delta(m_Q) = V(r_0)$, which becomes independent of m_Q as $m_Q \to \infty$. Dividing both sides of (4.37) by $m_Q^{\frac{1}{2}}$, we obtain the result (4.35).

The result (4.35) is achieved in different ways for potentials of different shapes. For potentials $V \sim r^{\varepsilon}$ the level spacing behaves as $m^{-\varepsilon/(2+\varepsilon)}$. Thus the increase of $n(m_Q)$ as m_Q increases is achieved by packing levels more closely for $\varepsilon > 0$, and pushing them deeper into the well for $\varepsilon \leq 0$.

A different approach to quarkonium quantum mechanics is made attractive by the fact that in potential theory it is possible to prove useful theorems. I will discuss but a single example here. Since leptonic widths can be expressed as

$$\Gamma(\mathscr{V}\to e^+e^-) = \frac{4\alpha^2 e_Q^{\ 2}}{M(\mathscr{V})^2} \, m_Q \left\langle \frac{dV}{dr} \right\rangle \quad , \tag{4.38}$$

combining (4.21) and (4.30), it is possible to relate leptonic widths in the Υ family to those in the ψ family if the quark-mass dependence of $\left\langle \frac{dV}{dr} \right\rangle$ is known. This has been done[31] for potentials with $V' \geq 0$ and $V'' \leq 0$. A general proof that

$$\frac{\partial}{\partial m}\left(\left\langle \frac{dV}{dr} \right\rangle\right) \geq 0 \tag{4.39}$$

has been given for n=1. The inequality also holds for any n for power-law potentials (with $V'' < 0$)[31,19] and for any potential with $V' \geq 0$, $V' \leq 0$ in WKB approximation.[32] This gives rise to a set of lower bounds

$$\left.\begin{array}{l}\Gamma(\Upsilon\rightarrow e^+e^-) \geq 2.6\ \mathrm{keV}\cdot e_Q^{\,2}\ , \\ \Gamma(\Upsilon'\rightarrow e^+e^-) \geq 1.4\ \mathrm{keV}\cdot e_Q^{\,2}\ ,\end{array}\right\} \qquad (4.40)$$

based on the conservative assumption that $m_Q/m_c > 2.6$, and leptonic widths for ψ and ψ' one standard deviation less than the central values. The lower bounds which correspond to $e_Q = -1/3$, $+ 2/3$ are indicated in Fig. 15. Also shown are a number of predictions derived from specific potential models,[33] assuming $e_Q = -1/3$. All the models shown imply values of $\Gamma(\Upsilon'\rightarrow e^+e^-)$ which are too small to be interpreted in terms of $e_Q = +2/3$. The measurement of $\Gamma(\Upsilon\rightarrow e^+e^-)$ is not expected to be so decisive because the wavefunction of the Υ at the origin probes a new region of the potential, not yet restricted by charmonium. The DORIS measurements of the upsilon leptonic widths are plotted in Fig. 15 as well. They provide decisive evidence that the new quark has charge $-1/3$, since the measured value for $\Gamma(\Upsilon'\rightarrow e^+e^-)$ lies below 0.63 keV.

These examples illustrate the richness of quarkonium quantum mechanics, which permits us to make meaningful predictions and to draw secure inferences from experiment. The concluding section of this report is devoted to a bibliography of applications not discussed in detail.

V. A BRIEF GUIDE TO THE LITERATURE

Here I list, for the interested student, a number of topics in quarkonium quantum mechanics and some accessible references. In many cases, the literature can be traced from the papers cited here.

Scaling laws of the Schrödinger equation are reviewed by Quigg and Rosner, Ref. 19.

General theorems on the order of levels bound in potentials have been proved by A. Martin, Phys. Lett. 67B, 330 (1977) and by H. Grosse, Phys. Lett. 68B, 343 (1977).

The number of narrow vector states in a quarkonium family is discussed by Eichten and Gottfried, Ref. 30, and by Quigg and Rosner, Ref. 29.

The principal-quantum-number dependence of $|\Psi(0)|^2$ has been investigated by A. Martin, Phys. Lett. 70B, 194 (1977), who derived an inequality on $|\Psi_2(0)|^2/|\Psi_1(0)|^2$. Results valid in the semiclassical limit are given by Quigg and Rosner, Ref. 19 and Phys. Rev. D17, 2364 (1978), and by V. Gupta and R. Rajaraman, Tata Institute preprint (1978).

A theorem on the quark-mass-dependence of $|\Psi(0)|^2$ was proved by Rosner, Quigg, and Thacker, Ref. 31. General conditions for the validity of the result were further explored by Leung and Rosner, Ref. 31.

Semiclassical sum rules and related applications of "Q^2-duality" are explored by Quigg and Rosner, Phys.

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Quantum mechanical sum rules in the Thomas-Reiche-Kuhn style have been applied to charmonium by J.D. Jackson, Phys. Rev. Lett. 37, 1107 (1976).

An inequality on quark mass splittings, $m_b - m_c \geq 3.29$ GeV/c^2, has been established by H. Grosse and A. Martin, CERN-TH.2513.

Applications of the inverse scattering method to confining potentials have been begun by H.B. Thacker, C.Quigg, and J.L. Rosner, Phys. Rev. D18, 274, 287 (1978), and extended by H. Grosse and A. Martin, CERN-TH.2523.

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FOOTNOTES AND REFERENCES

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[7]For a general review of positronium, see M. Deutsch, Adv. Exp. Phys. 4, 63 (1975). A review of hyperfine structure is given by G.T. Bodwin and D.R. Yennie, Phys. Rep. 43C, 267 (1978).

[8]See, for example, J.M. Jauch and F. Rohrlich, Theory of Photons and Electrons (Reading, Mass.: Addison-Wesley, 1955), §12-6.

[9]Summaries of charmonium spectroscopy are given by G.J. Feldman and M.L. Perl, Phys. Rep. 33C, 285 (1977), and B. Wiik and G. Wolf, DESY Report No. 78/23. See also Particle Data Group, Phys. Lett. 75B, 1 (1978), for references on individual states.

[10]Preliminary evidence for a few examples of the cascade $\psi(3684) \rightarrow \gamma + X(3600)$, $X(3600) \rightarrow \gamma + \psi(3095)$, with a combined probability of $(2.8\pm1.2)\times10^{-3}$, was presented in Singapore by H. Schopper and at the XIX International Conference on High Energy Physics, Tokyo, by G. Flügge. The data are from the DESY-Heidelberg NaI-Pb Glass detector. Apparently there is some inconsistency between this experiment and earlier reports of $\chi(3455)$. The average (Feldman and Perl, Ref. 9) of previous measurements for $\Gamma(\psi(3684) \rightarrow \gamma + \chi(3455))/\Gamma(\psi(3684) \rightarrow \text{all})$ is $(0.6\pm0.4)\times10^{-2}$, but the preliminary DESY-Heidelberg analysis gives an upper limit of about 0.2×10^{-2}.

[11]Thorough reviews have been given by K. Gottfried, in Proc. 1977 Int. Symposium on Lepton and Photon Inter-

actions at High Energies, August 25-31, Hamburg, ed. F. Gutbrod, (Hamburg: DESY), p. 667-710;

J.D. Jackson, "New Particle Spectroscopy," in Proc. 1977 European Conf. on Particle Physics, Budapest, Hungary, 4-9 July 1977, eds. L. Jenik and I. Montvay, (Budapest: Central Res. Inst. for Physics), Vol. I, p. 603-630;

V.A. Novikov, L.B. Okun, M.A. Shifman, A.I.Vainshtein, M.B. Voloshin, and V.I. Zakharov, Physics Reports 41C, 1 (1978);

T. Appelquist, R.M. Barnett, and K. Lane, Charm and Beyond, SLAC-PUB-2100 (March 1978), to appear in Vol. 28 of Annual Review of Nuclear and Particle Science, December 1978.

[12] A recent, but abbreviated, assessment appears in J.D. Jackson, C. Quigg, and J.L. Rosner, LBL-7977, summary of parallel session B8 at the XIX International Conference on High Energy Physics, Tokyo.

[13] An appealing (but perhaps rose-colored) response to the apparently large $^3S-^1S$ hyperfine splitting is to deny the existence (or at least the identity) of the pseudoscalar candidates. See the discussion in Ref. 12.

[14] A summary of the data appears in the Tokyo Conference Rapporteur talk by L.M. Lederman, "Dilepton Production in Hadron Collisions."

[15] T. Yamanouchi, parallel session A10, Tokyo Conference.

[16]These results are preliminary. The errors are those of the fitting program, do not include a systematic normalization uncertainty, and are to be taken with a grain of salt.

[17]P. Mockett, parallel session A10, Tokyo Conference.

[18]Unconventional color assignments have been considered by Yee Jack Ng and S.-H. H. Tye, Phys. Rev. Lett. 41, 6 (1978); P.G.O. Freund and C.T. Hill, University of Chicago Preprint EFI 78/21; H. Fritzsch, Wuppertal University Preprint WU-B 78/19. A critical assessment of nontriplet assignments for the b-quark appears in Ref. 12.

[19]C. Quigg and J.L. Rosner, Comments Nucl. Part. Phys. 8A, 11 (1978) contains a summary of scaling laws for the Schrödinger equation.

[20]C. Quigg and J.L. Rosner, Phys. Lett. 71B, 153 (1977).

[21]For simplicity, in discussing magnetic transitions we assume that only one mass scale is important, whether the two bound particles have equal masses ($=2\mu$), or because one (with mass μ) is much lighter than the other.

[22]R. Van Royen and V.F. Weisskopf, Nuovo Cimento 50, 617 (1967); 51, 583 (1967). A factor of three has been inserted because of quark color. Reservations about this connection are reviewed in Ref. 12.

[23]This is because $\Delta E/\mu$ does not grow as μ increases for $\varepsilon \geq -1$. We assume $M(\Upsilon) \simeq 2(2\mu)$ + small binding cor- when the mass of each constituent of Υ is 2μ.

[24] The region of space probed by the wavefunction at the origin is illustrated in Figs. 1 and 2 of H.B. Thacker, C. Quigg, and J.L. Rosner, Phys. Rev. D18, 287 (1978).

[25] A.M. Boyarski, et al., Phys. Rev. Lett. 34, 1357 (1975); V. Lüth, et al., Phys. Rev. Lett. 35, 1124 (1975).

[26] C.W. Darden, et al., Phys. Lett. 76B, 246 (1978), and Ch. Berger, et al., Phys. Lett. 76B, 243 (1978) reported the observation of Υ in $e^+e^- \to$ hadrons.

[27] Additional data on Υ were reported at the Tokyo Conference (Parallel Session B1) by H. Spitzer (PLUTO Collaboration).

[28] W. Schmidt-Parzefall (DASP2 Collaboration) and G. Heinzelmann (DESY-Heidelberg Collaboration) reported further results on Υ and the observation of Υ' at parallel session B1 of the Tokyo Conference. Somewhat refined results are published in J.K. Bienlein, et al., Phys. Lett. 78B, 360 (1978) and C.W. Darden, et al., Phys. Lett. 78B, 364 (1978).

[29] C. Quigg and J.L. Rosner, Phys. Lett. 72B, 462 (1978).

[30] E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977).

[31] J.L. Rosner, C. Quigg, and H.B. Thacker, Phys. Lett. 74B, 350 (1978). For a further discussion see T. Leung and J.L. Rosner, J. Math. Phys. (to be published).

[32] C. Quigg, J.L. Rosner, and H.B. Thacker, unpublished; H. Grosse and A. Martin, private communication.

[33] Thacker, Quigg, and Rosner, Ref. 24.

Table I. Combined analyses of all Columbia-Fermilab-Stony Brook Data (Preliminary)

	3 Peak Fit	2 Peak Fit	3 Peak Fit[a]
$m(\Upsilon)$ GeV/c^2	$9.46\pm0.001\pm0.10$	$9.45\pm0.006\pm0.10$	$\underline{9.460}$
$\dfrac{B\frac{d\sigma}{dy}(\Upsilon)}{\frac{d^2\sigma}{dm\cdot dy}(\text{cont.})}$	0.96 ± 0.03 GeV/c^2	1.02 ± 0.03 GeV/c^2	0.93 ± 0.03 GeV/c^2
$m(\Upsilon')-m(\Upsilon)$ GeV/c^2	0.590 ± 0.035 $m(\Upsilon')=10.05$	0.709 ± 0.012 10.18	$\underline{0.556}$ 10.02
$\dfrac{B\frac{d\sigma(\Upsilon')}{dy}}{B\frac{d\sigma}{dy}(\Upsilon)}$	0.31 ± 0.03	$0.35\ \pm0.015$	0.30 ± 0.03
$m(\Upsilon'')-m(\Upsilon)$ GeV/c^2	0.96 ± 0.06 $m(\Upsilon'') = 10.42$	-	0.92 ± 0.03 10.38
$\dfrac{B\frac{d\sigma(\Upsilon'')}{dy}}{B\frac{d\sigma(\Upsilon')}{dy}}$	0.16 ± 0.04	-	0.16 ± 0.015
χ^2/DF	213/225	247/227	195/209

a) Assuming DESY Values for masses of Υ and Υ'.

Table II. Measurements of the leptonic width of ϒ(9.46)

Experiment	$\Gamma(\Upsilon\to e^+e^-)$, keV	Reference
PLUTO	1.3±0.4	26
(DASP2	1.3±0.4)	26
DASP2	1.5±0.4	28
DESY-Heidelberg	1.1±0.3	28
Average	1.26±0.21	

Table III. Measurements of the leptonic branching ratio of ϒ(9.46)

Experiment	$\Gamma(\Upsilon\to\mu^+\mu^-)/\Gamma(\Upsilon\to\text{all})$	Reference
PLUTO	(2.7±2.0)%	27
DASP2	(2.5±2.1)%	28
Average	(2.6±1.4)%	

Table IV. Υ' parameters determined at DORIS (Ref. 28)

Experiment	Mass, GeV/c^2	$M(\Upsilon')-M(\Upsilon)$, GeV/c^2	$\Gamma(\Upsilon'\to e^+e^-)$, keV
DASP 2	10.012±0.010	0.555±0.003	0.5 ±0.2
DESY-Heidelberg	10.016±0.004±0.010	0.557±0.005	0.32±0.10
Average			0.36±0.09

Table V. Effective power-law potentials $V(r) \propto r^{\varepsilon}$ deduced from ψ and Υ leptonic widths.

n \ m_Q/m_c	3	4
1	ε = -0.56±0.13	-0.16±0.16
2	0.03±0.35	+0.56±0.45

FIGURE CAPTIONS

Fig. 1: A schematic representation of the spectrum of positronium (e^+e^-). Principal decay modes are indicated.

Fig. 2: The spectrum of charmonium ($c\bar{c}$). Branching fractions (in per cent) are shown for the important classes of decays.

Fig. 3: Spectroscopic notation ($n^{2s+1}L_J$) for the levels of charmonium. The identification of $^1S_0$ levels is speculative.

Fig. 4: High-intensity data on the dimuon invariant mass spectrum observed in 400 GeV/c pN collisions by the Columbia-Fermilab-Stony Brook Collaboration.

Fig. 5: High-intensity data of the CFS group, after continuum subtraction. The curves show the elements of a three-resonance fit to the data.

Fig. 6: High-resolution data of the CFS group, after continuum subtraction. The curves show a three-resonance fit to the data.

Fig. 7: Dimuon invariant mass spectrum observed in 400 GeV/c pN collisions by the MNWT Collaboration. An exponential representation of the continuum is shown.

Fig. 8: MNWT mass spectrum after continuum subtraction.

Fig. 9: The upsilon spectrum, according to the three-peak fit to the CFS data. One-standard-deviation errors in the fitted masses are indicated by the shaded areas.

Fig. 10: Visible cross sections for $e^+e^- \to$ hadrons in the region of Υ(9.46) observed in the two early experiments at DESY.

(a) DASP2 Collaboration;

(b) PLUTO Collaboration.

See Ref. 26.

Fig. 11: Visible cross sections for $e^+e^- \to$ hadrons in the region of Υ(9.46) observed in the second run at DORIS.

(a) Final data of the DASP2 Collaboration;

(b) DESY-Heidelberg Collaboration.

See Ref. 28.

Fig. 12: Visible cross sections for $e^+e^- \to$ hadrons in the region of Υ'(10.02) observed at DORIS.

(a) DASP2 Collaboration;

(b) DESY-Heidelberg Collaboration.

See Ref. 28.

Fig. 13: Quark-mass-dependence of the wavefunction at the origin for the n=1 and n=2 quarkonium levels. The data are from Ref. 25 for the

$\psi(3.095)$● and $\psi'(3.684)$■ and from Ref. 26 and 28 for the $\Upsilon(9.46)$○ and $\Upsilon'(10.02)$□. The mass dependence characteristic of several simple potentials is indicated by slopes of the straight lines.

Fig. 14: Square of the wavefunction at the origin deduced from leptonic widths of the psions. Possible mixing between the $s^3S_1(3684)$ and $3^3D_1(3772)$ levels has been neglected. The solid line is a best fit proportional to n^p, with $p=-0.98\pm0.20$, assuming the conventional 4S assignment for $\psi(4414)$. The dashed line, which refers to an alternative 5S assignment for $\psi(4414)$, corresponds to $p=-0.92\pm0.18$.

Fig. 15: Expectations for leptonic widths of Υ and Υ'. The lower bounds (4.40) are indicated for $e_Q=-1/3$ (solid lines) and $e_Q=+2/3$ (dashed lines). The shaded region shows the widths predicted for $e_Q=-1/3$ on the basis of twenty potentials from Ref. 33 which reproduce the ψ and ψ' positions and leptonic widths. The experimental point represents the average of measurements made at DORIS.

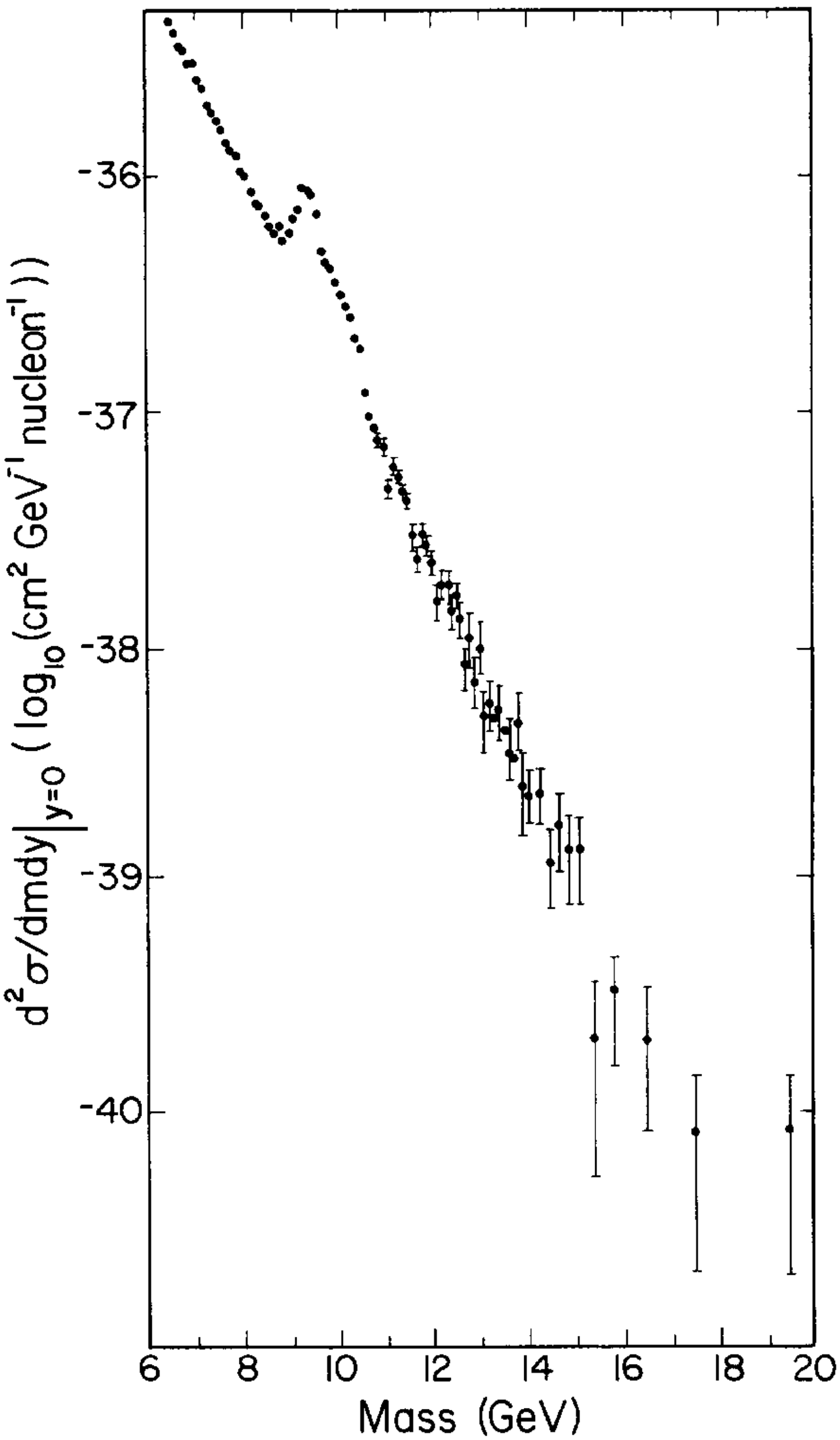
-36
-37
-38
-39
-40
$d^2\sigma/dmdy|_{y=0}$ ($\log_{10}$(cm^2 GeV^{-1} nucleon^{-1}))
6
8
10
12
14
16
18
20
Mass (GeV)

Fig. 4

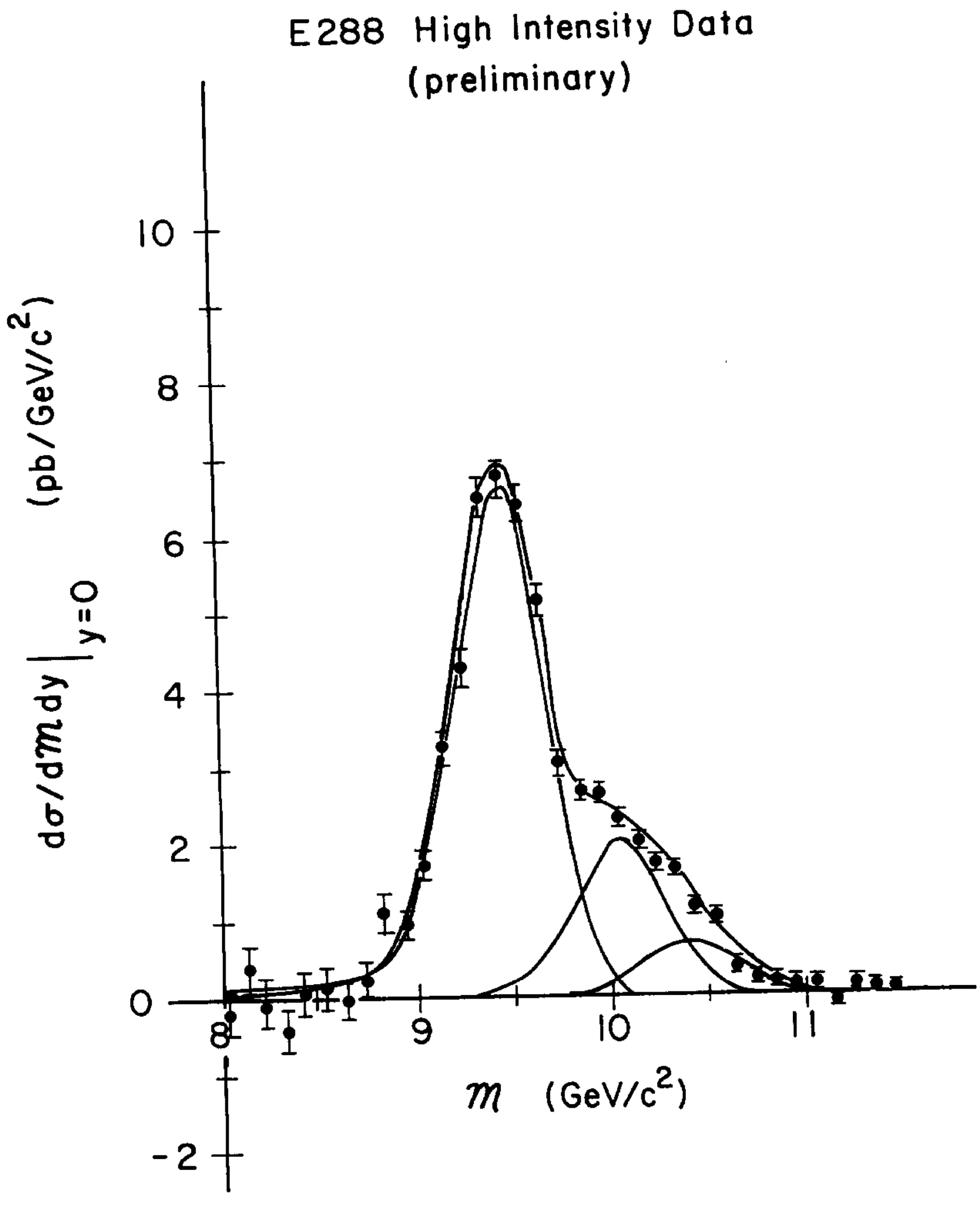
E288 High Intensity Data
(preliminary)
dσ/d𝓂dy |y=0 (pb/GeV/c2)
10
8
6
4
2
0
-2
8
9
10
11
𝓂 (GeV/c2)

Fig. 5

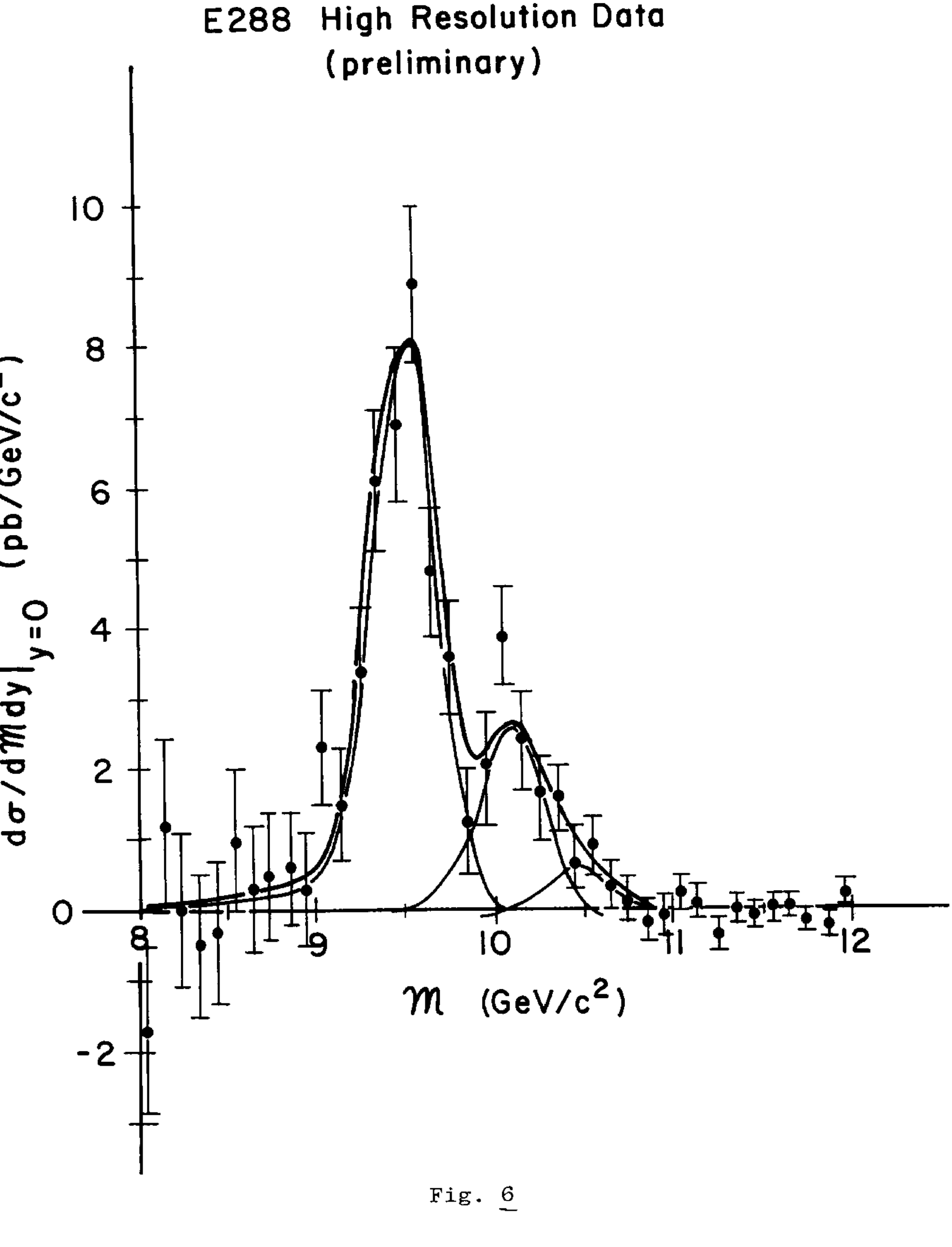
E288 High Resolution Data
(preliminary)
10
8
6
4
2
0
-2
8
9
10
11
12
$d\sigma/d\mathcal{M}dy|_{y=0}$ (pb/GeV/c²)
$\mathcal{M}$ (GeV/c²)

Fig. 6

RESULTS(8/13/78) MNWT COLLABORATION FERMILAB EXP. 439

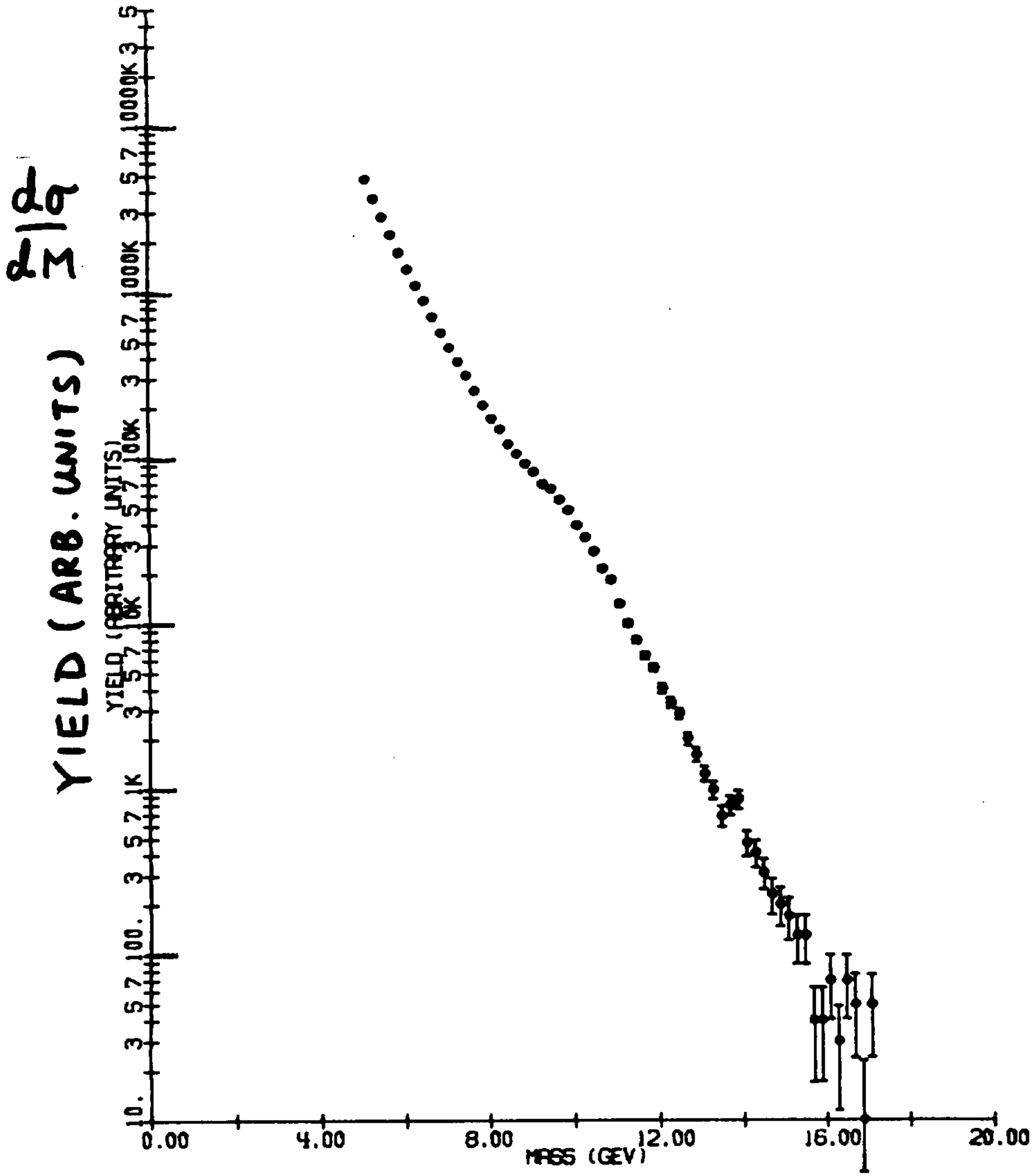

Fig. 7

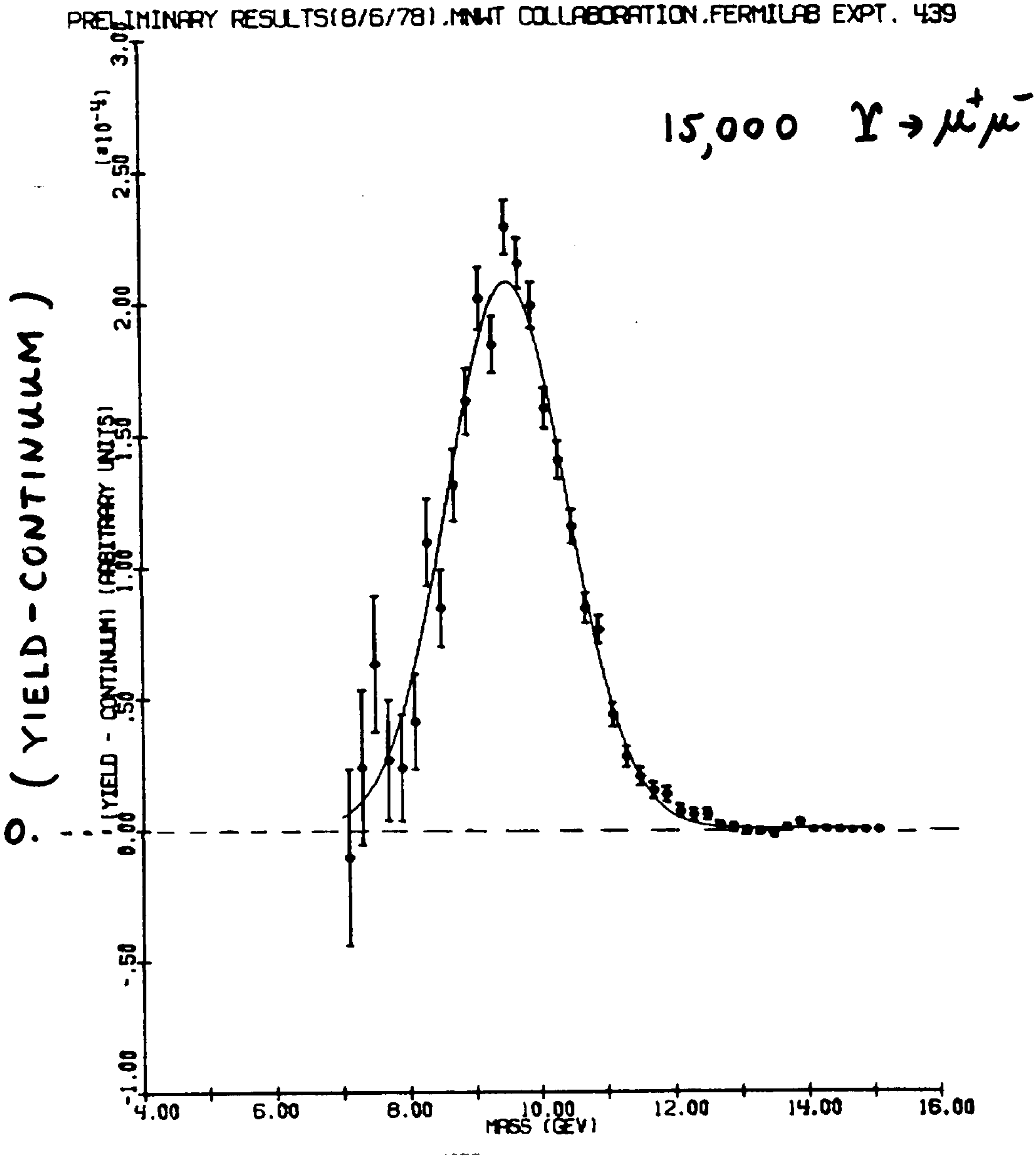
PRELIMINARY RESULTS(8/6/78).MNWT COLLABORATION.FERMILAB EXPT. 439
15,000 Υ → μ⁺μ⁻
(YIELD - CONTINUUM)
0.
(YIELD - CONTINUUM) (ARBITRARY UNITS)
(*10⁻⁴)
3.00
2.50
2.00
1.50
1.00
.50
0.00
-.50
-1.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
MASS (GEV)

Fig. 8

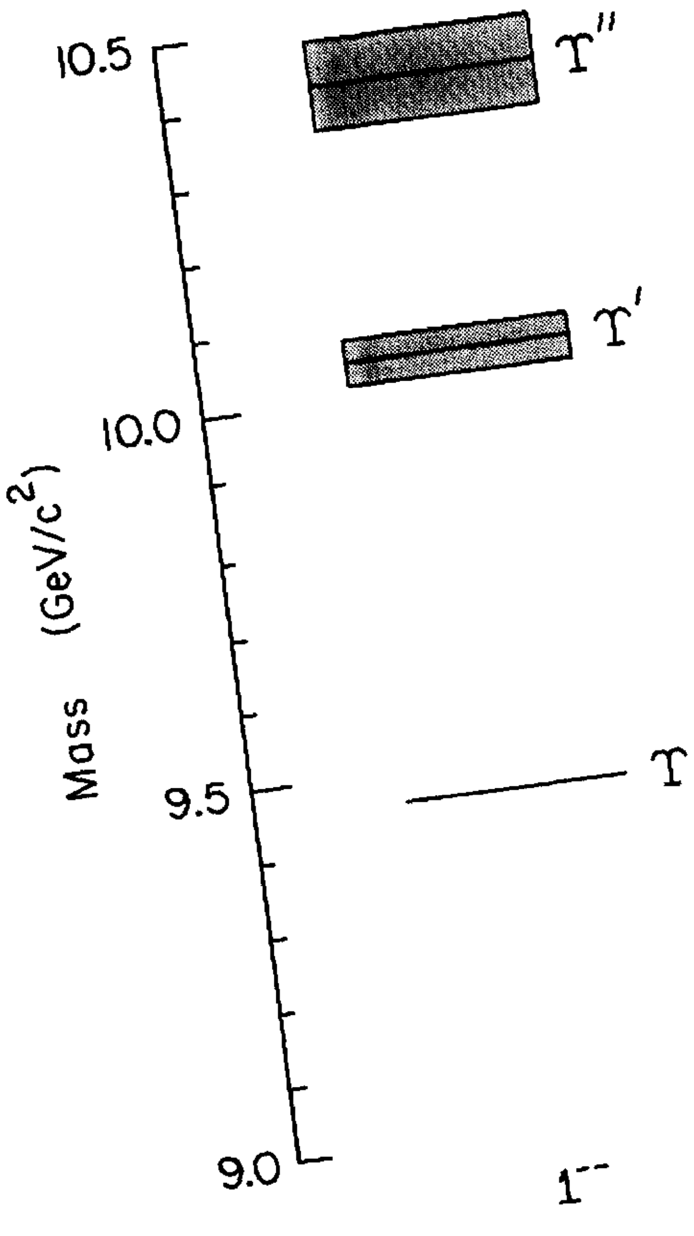

Fig. 9

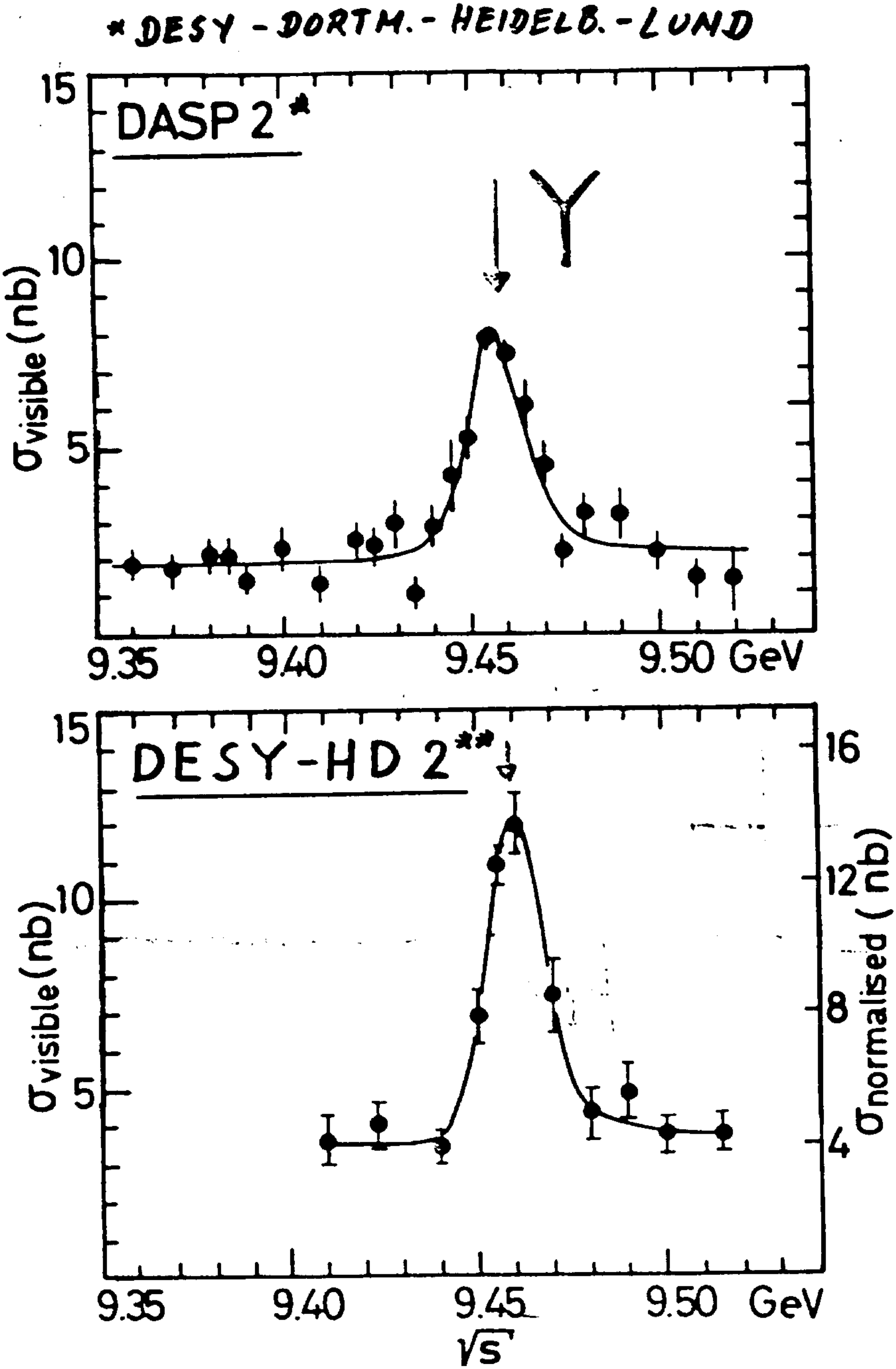
* DESY - DORTM. - HEIDELB. - LUND
DASP 2 *
ϒ
$\sigma_{visible}$ (nb)
15
10
5
9.35
9.40
9.45
9.50 GeV
DESY-HD 2 **
$\sigma_{normalised}$ (nb)
16
12
8
4
$\sqrt{s}$

Fig. 11

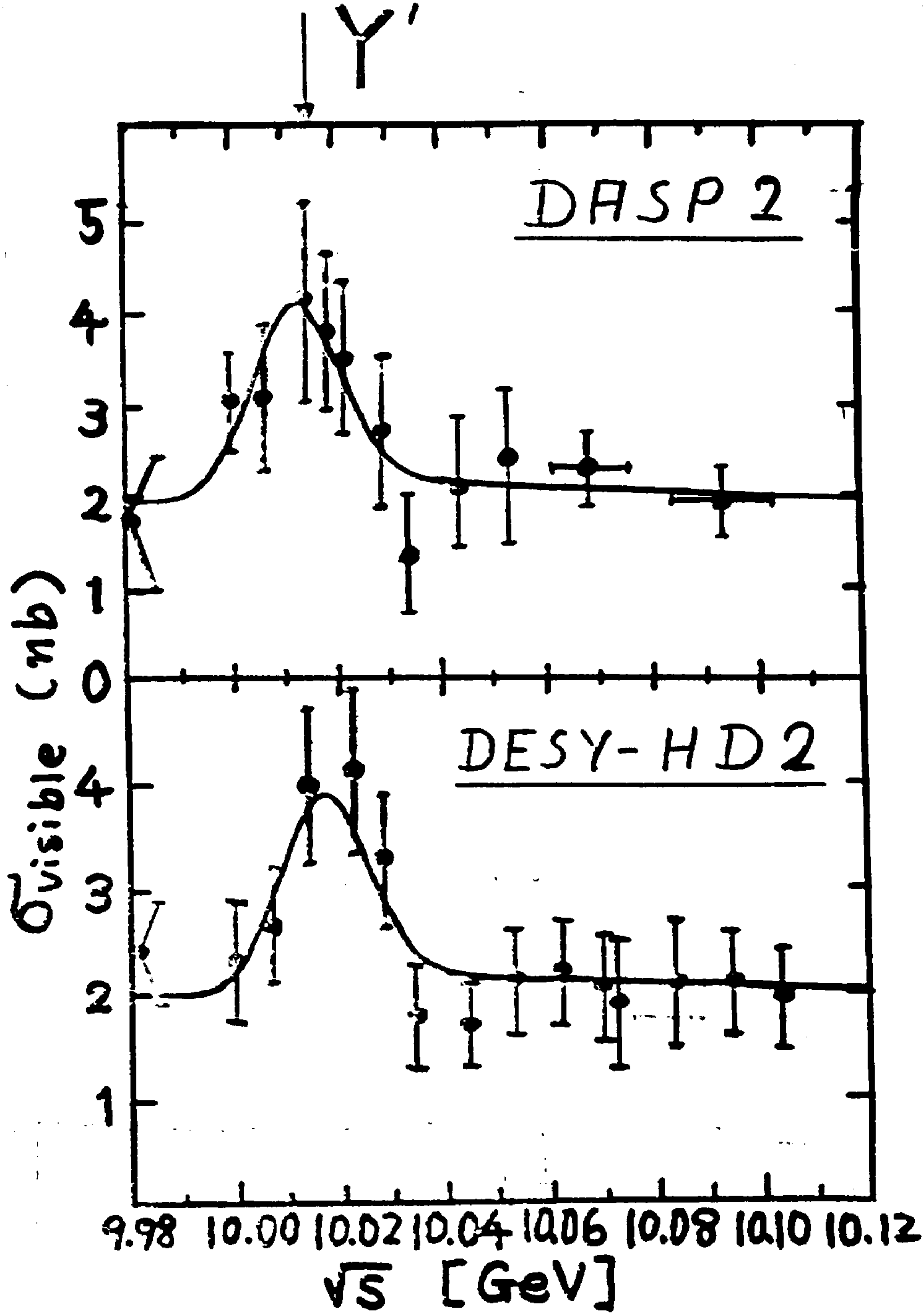
Υ'
DASP 2
DESY-HD2
σvisible (nb)
5
4
3
2
1
0
4
3
2
1
9.98
10.00
10.02
10.04
10.06
10.08
10.10
10.12
√s [GeV]

Fig. 12

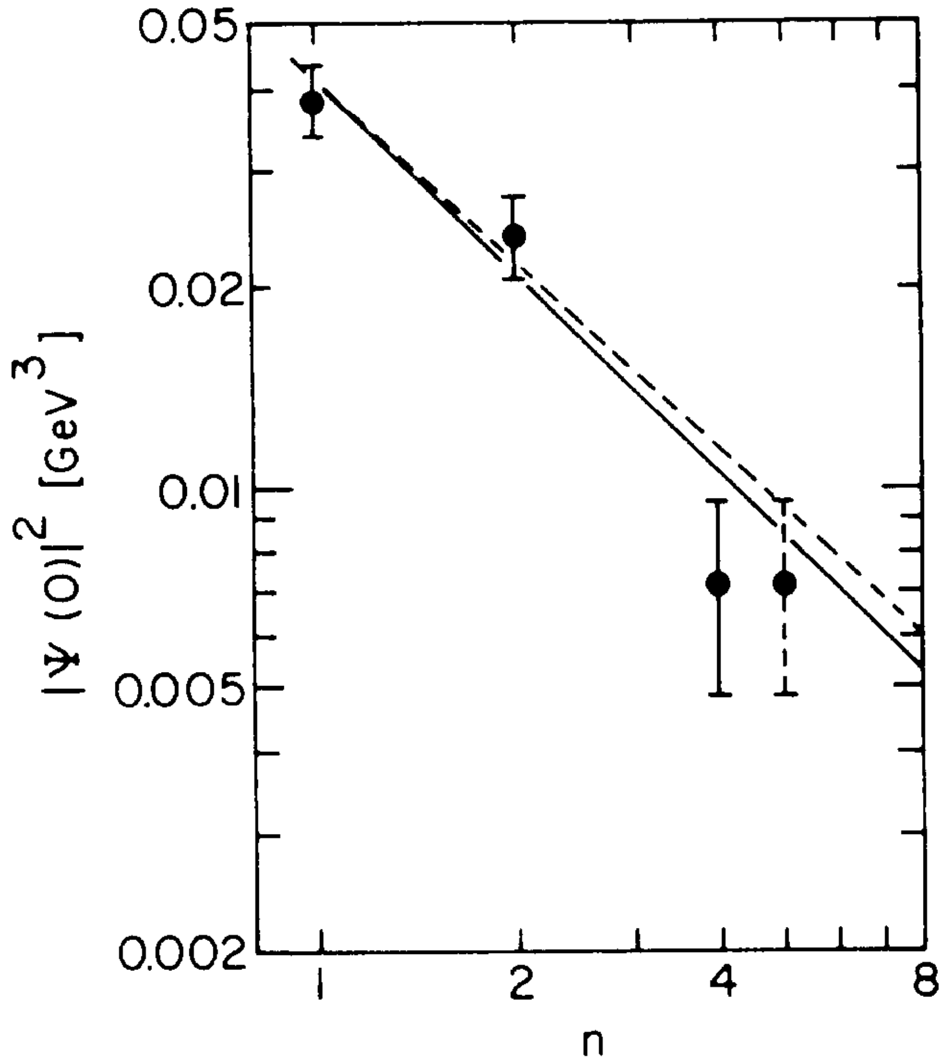

Fig. 14

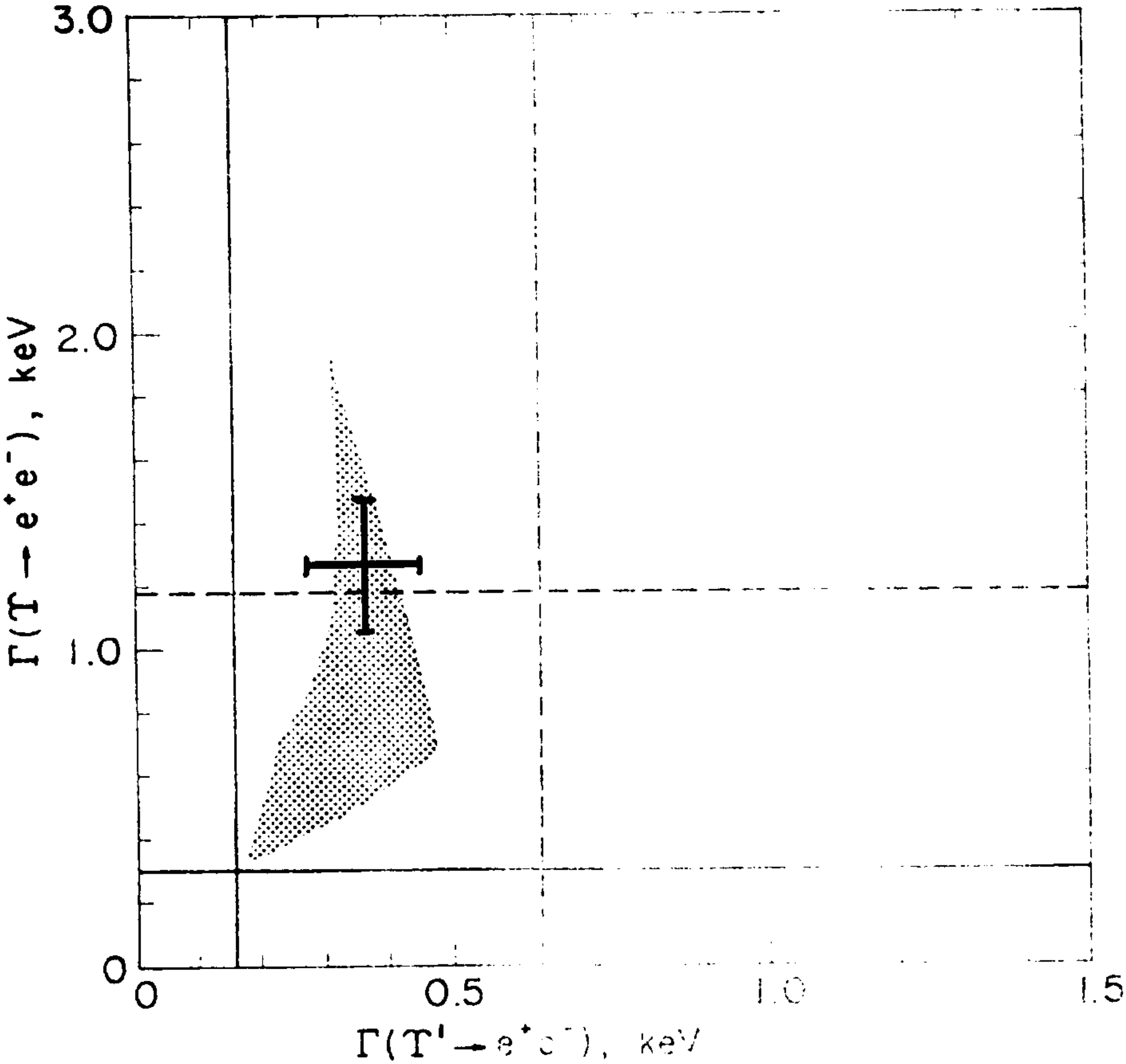

Fig. 15