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Measurement of the Proton Structure Function from Muon Scattering*

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Results on the proton structure function, νW_2 are presented for $0.3 < q^2 < 50\ \text{GeV}^2$ and $5 < \nu < 130$ GeV. They are compared to earlier data and displayed to demonstrate violations of scaling. Values are reported for the energy-momentum sum rule and for $R \equiv \sigma_L/\sigma_T$ over a limited kinematic region.

In this paper we report values of the structure function νW_2 of the proton obtained by measuring the inelastic scattering of muons from hydrogen at the Fermi National Accelerator Laboratory. The data was taken with the muon scattering facility constructed by this group.

Details of the apparatus, trigger and analysis technique have been described in an earlier letter.[1] The data were obtained from three separate runs: at 147 GeV in 1974 and at 96 and 147 GeV in 1975. The results presented here are based on total fluxes of 1.0×10^{10} muons at 96 GeV and 3.5×10^{10} muons at 147 GeV. These runs yielded 2.8×10^4 useful events in the kinematic range $0.3 < q^2 < 50$ GeV^2 and $5 < \nu < 130$ GeV, where $-q^2$ is the squared momentum transfer of the muon and ν its laboratory energy loss. There were 1.5×10^4 events with $q^2 > 1.0$ GeV^2.

The cross-section for muon inclusive scattering in the one photon exchange approximation is given by

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{2\pi\alpha^2}{p^2 q^4} \frac{\nu W_2(q^2, \nu)}{\nu} \left[2EE' - \frac{q^2}{2} + \frac{(q^2 - 2m_\mu{}^2)(1 + \nu^2/q^2)}{1 + R(q^2, \nu)} \right]$$

where E, p, E', p' are the incident and scattered muon laboratory energies and momenta, $q^2 = 2(EE' - pp' \cos\theta - m_\mu{}^2)$, $\nu = E - E'$, θ the muon scattering angle, m_μ the muon mass, R the ratio of the total cross-sections on protons of longitudinal and transverse virtual photons and α the fine structure constant.

The 1975 data allow the determination of values of R by comparing cross-sections at the same q^2 and ν measured at different beam energies. The results are given in detail in Table 1. The average value of R in the range $1 < q^2 < 5$ GeV^2 and $64 < W^2 < 144$ GeV^2 is 0.05 ± 0.33 (W is the mass of the recoiling hadronic system). The precision is not high because the scattering has a weak dependence on R for our region of the kinematic variables. In view of this insensitivity the values of νW_2 were derived assuming the single constant value $R = 0.14$ in conformity with the bulk of present evidence.[2]

Figure 1 shows values of the proton structure function $\nu W_2(q^2, \omega)$ as a function of q^2 for various values of $\omega = 2M\nu/q^2$, where M is the proton mass. The values shown are weighted to give the correct value at the bin centre. The figures include data from earlier measurements at lower energies from MIT-SLAC.[2] Where the data sets overlap there is general agreement between our results and these earlier measurements once due account is taken of systematic uncertainties not indicated by the plotted points. These amount to 7% (12% for $\omega < 3$) in our case, and 3.4% overall normalization in the case of the MIT-SLAC points.

Our results considerably extend the range in q^2 over the MIT-SLAC measurement of νW_2, in particular by more than an order of magnitude at 15. They confirm a pattern of scaling violation that has been seen before[3] where νW_2 decreases with increasing q^2 for $\omega < 3$ and increases with q^2 for $\omega > 9$. One way of characterizing the observed scaling violations is to show a power law dependence in q^2 of νW_2 for various ω ranges, where

$$\nu W_2(q^2, \omega) = \nu W_2(q_o^2, \omega)(q^2/q_o^2)^b \qquad (1)$$

This is done in Fig. 2 where b is plotted as a function of x ($=1/\omega$). Also shown are the corresponding fits to the MIT-SLAC data. Note that $b = 0$ corresponds to Bjorken scaling.

In our case, the high beam energy allows comparatively small values of x where the increase in νW_2 with q^2 is not removed by the use of any scaling variables which have had some success at large x and lower energies.[4] As an overall measure of the scaling violation exhibited by our data for $q^2 > 2$ we fit with a form generalized from that used previously,[1, 3] where, in equation 1, $\nu W_2\ (q_o^2, \omega) = \sum_{i=3}^{5} C_i(1 - 1/\omega)^i$, a polynomial form that has been used before,[5] and for comparison, following Chang et al,[3] the scaling violation is expressed in terms of a single coefficient a by choosing $b = a \log \omega/6$. In the fits to our data the quantities C_i and a were allowed to vary keeping $q_o^2 = 3\ \mathrm{GeV}^2$. The fit to our data alone has $\chi^2 = 40.7$ for 39 degrees of freedom and gives $a = 0.145 \pm 0.024$. This compares with 0.072 $\pm$ 0.038 found for a deuterium target[1] and 0.099 $\pm$ 0.018 found for an iron target.[3] In our fit we found $C_3 = 2.799 \pm 0.493$, $C_4 = -4.048 \pm 1.134$, $C_5 = 1.615 \pm 0.649$. The errors in the C_i are strongly correlated among themselves but not with the error in a.

Fig. 3 shows our data and the MIT-SLAC data for $F_2(q^2, x)$ $(= \nu W_2)$ plotted as a function of x for various ranges of q^2. The change of the shape of this function with increasing q^2 is clear. The full curves are fits to both sets of data using the polynomial form $\sum_{i=3}^{5} C_i(1-x)^i$. The coeffients are given in Table 2.

We have calculated the integral of the structure function defined by

$$I_2^p\ (q^2) = \int_0^1\ F_2^p\ (x,\ q^2)\ dx$$

in different bands of q^2 using the following procedure. The fits shown in Fig. 3 are used to evaluate the integrals from x = 0.25 to 1. This region is dominated by the MIT-SLAC data. At low q^2, the requirement that $W > 2.0$ GeV severely limits the maximum value of x and the value of the integral is determined by the polynomial form of F_2. The results are shown in the second column of Table 3. Then summing over our data alone we evaluate the contribution from x_{min} to 0.25 where x_{min} is the minimum value of x observed in a given q^2 band (third and fourth columns, Table 3). We then estimate that the part from x = 0 to x_{min} is $x_{min}F_2(x_{min}) \pm 25\%$ (fifth column, Table 3). The uncertainties due to systematic effects are indicated within the brackets in the table. The integral shows little variation with q^2 although F_2 shows considerable variation.

Our averaged result $I_2^p = 0.171 \pm 0.006$ is to be compared with $1/2\ I_2^d = 0.154 \pm 0.005$ from a similar analysis of deuterium data[1] giving a proton neutron difference $I_2^p - I_2^n = 0.034 \pm 0.015$. According to the Callan-Gross sum rule I_2 gives the fraction of the total energy-momentum carried by the charged constituents, weighted by the square of their charges.[6,7,8] Values of I_2 have been calculated for various models. If the scattering were due to 3 valence quarks alone $I_2^p = 0.333$; agreement cannot be reached by simply adding $q\bar{q}$ pairs so that uncharged constituents are required to carry the balance of the momentum.[8] In the particular case of quantum chromodynamics, the 4 quark, 3 color version[7] in the asymptotic limit $I_2^p = I_2^n = 0.119$. The data show that we are some way from this limit and there is little sign of a movement towards it.

At values of q^2 below 1 GeV^2 parton models offer no guidance about the behavior of νW_2. We know, however, that in the limit $q^2 \to 0$, νW_2 must also go to zero and the approach to this limit may be understood within the framework of generalized vector dominance.[9] This decrease of νW_2 is clearly seen in the highest ω bins of Fig. 1.

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FIGURE CAPTIONS

Fig. 1 νW_2 for hydrogen as a function of q^2 for various ω bins. Open circles are MIT-SLAC data, ref. 5. Note the varying and suppressed zeros of the scales for the values of νW_2. The solid lines are the fits for equation 1 for this experiment alone.

Fig. 2 The scaling violation parameter b of Eq. 1 as a function of x. The closed circles are our data points and open circles MIT-SLAC data, Ref. 2.

Fig. 3 $F_2(x) \equiv \nu W_2 \; (q^2, \omega)$ for hydrogen as a function for x for various q^2 bins. The solid lines are fits to both this data (closed circles) and the MIT-SLAC data (open circles, Ref. 2) as described in the text. The top-right entry shows the fits superimposed to indicate the change with increasing q^2.

TABLE 1 Values of $R = \sigma_L/\sigma_T$. Errors are statistical and the systematic error is small.

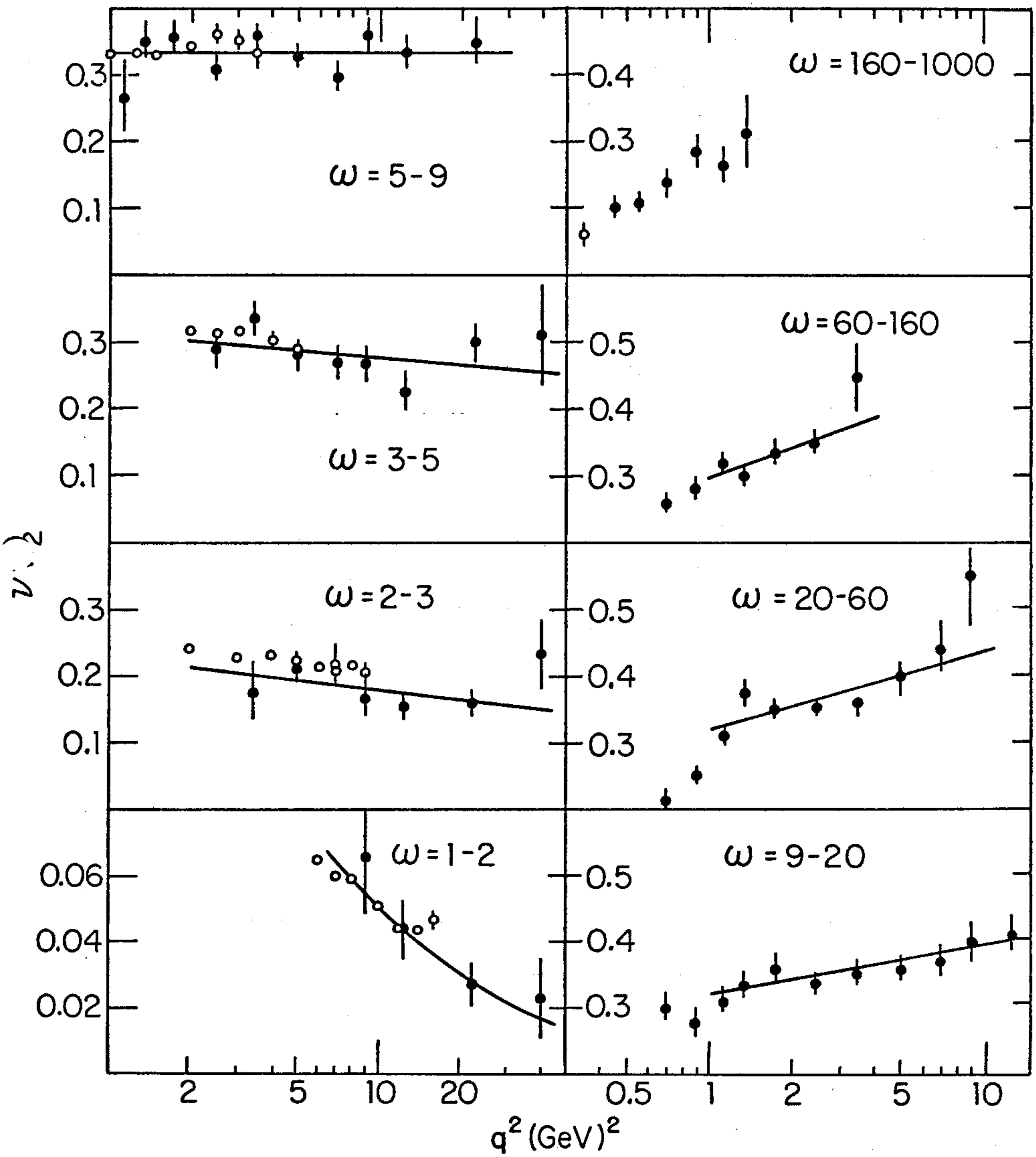
q^2 GeV^2	$\langle\omega\rangle$	$W^2 = M^2 + 2M\nu - q^2$ GeV^2	R
1 - 2	54	64 - 100	-0.35±0.50
	81	100 - 144	0.27±0.58
2 - 5	28	64 - 100	0.20±1.60
	41	100 - 144	0.59±0.79

TABLE 2. Values of the parameters C_i of Eq. 2 used to evaluate the sum rule of Eq. 3 between x = 0.25 and x = 1. See text for full explanation.

q^2 GeV^2	$1 < q^2 < 2$	$2 < q^2 < 4$	$4 < q^2 < 8$	$8 < q^2 < 15$	$15 < q^2 < 30$
C_3	3.919±0.246	3.212±0.108	3.072±0.067	2.302±0.062	0.848±0.370
C_4	-6.108±0.582	-4.552±0.280	-4.690±0.021	-3.331±0.242	-0.062±1.066
C_5	2.521±0.341	1.695±0.179	2.034±0.155	1.515±0.205	-0.253±0.800

TABLE 3. Contributions to the evaluation of the energy momentum sum rule. See text for explanation of the calculation of each contribution.

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6
q^2 GeV^2	$\int_{0.25}^{1} F_2^p\,dx$	x_{min}	$\int_{x_{min}}^{0.25} F_2^p\,dx$	$\int_{0}^{x_{min}} F_2^p\,dx$ (systematic error)	$\int_{0}^{1} F_2^p\,dx$ (systematic error)
1 - 2	0.0949 ± 0.0002	0.0042	0.0756 ± 0.0067	0.0018 (±0.0005)	0.1723 ± 0.0070 (±0.0060)
2 - 4	0.0883 ± 0.0007	0.0083	0.0800 ± 0.0018	0.0046 (±0.0012)	0.1729 ± 0.0019 (±0.0063)
4 - 8	0.0808 ± 0.0005	0.0167	0.0774 ± 0.0019	0.0098 (±0.0025)	0.1680 ± 0.0020 (±0.0064)
8 - 15	0.0689 ± 0.0012	0.0286	0.0812 ± 0.0031	0.0255 (±0.0064)	0.1756 ± 0.0033 (±0.0080)
15 - 30	0.0566 ± 0.0046	0.0909	0.0580 ± 0.0045	0.0531 (±0.0133)	0.1677 ± 0.0065 (±0.0140)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ω = 160-1000
ω = 3-5
ω = 60-160
ω = 2-3
ω = 20-60
ω = 1-2
ω = 9-20
0.3
0.2
0.1
0.4
0.5
0.06
0.04
0.02
2
5
10
20
0.5
1
q² (GeV)²

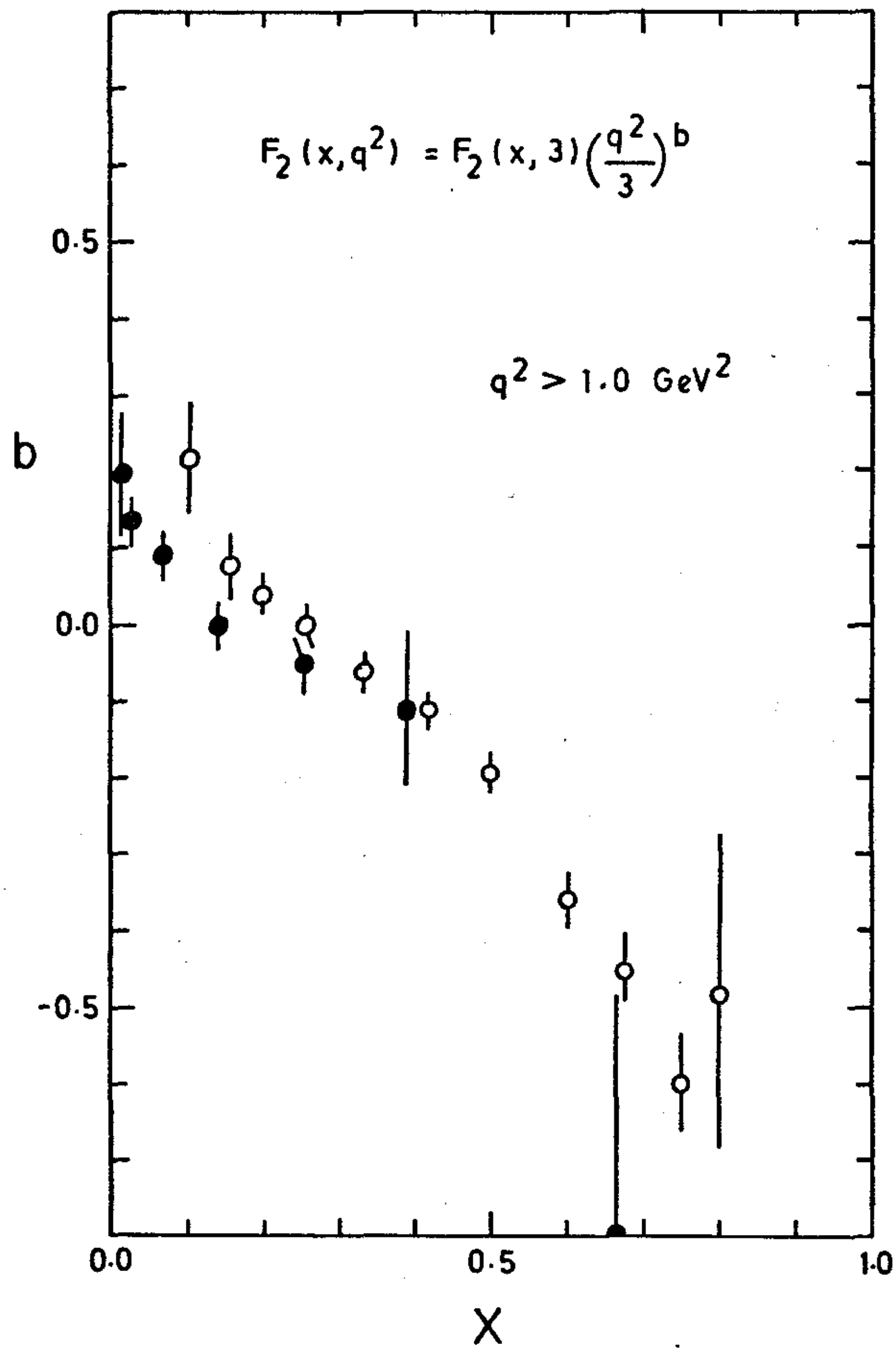
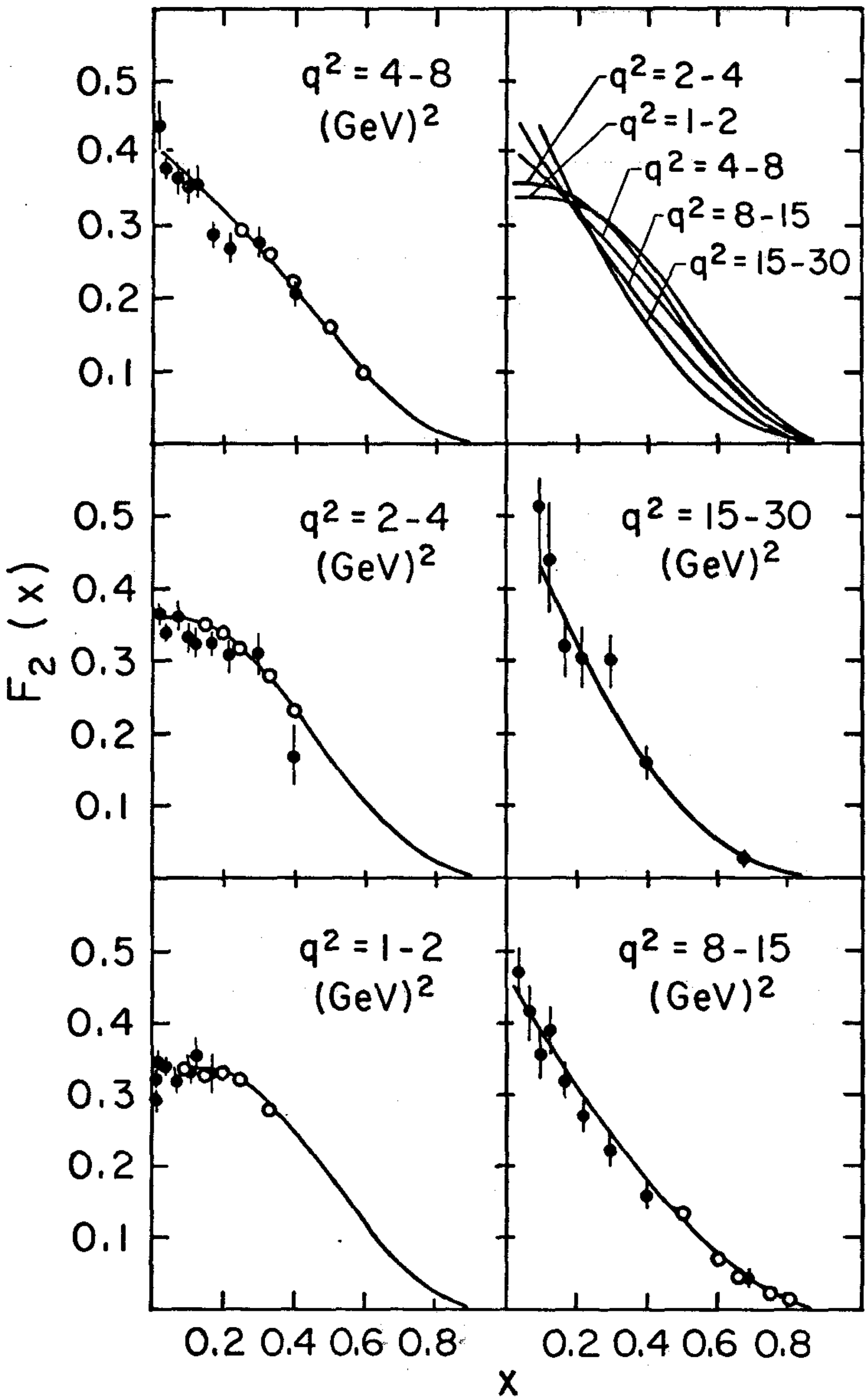


Fig 2



$q^2 = 4-8$ $(GeV)^2$
$q^2 = 2-4$
$q^2 = 1-2$
$q^2 = 4-8$
$q^2 = 8-15$
$q^2 = 15-30$
$q^2 = 2-4$ $(GeV)^2$
$q^2 = 15-30$ $(GeV)^2$
$q^2 = 1-2$ $(GeV)^2$
$q^2 = 8-15$ $(GeV)^2$
$F_2(x)$
0.5 0.4 0.3 0.2 0.1
0.2 0.4 0.6 0.8
x