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Hadronic Decays of Charmed Mesons

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ABSTRACT

Hadronic branching fractions for the charmed mesons D^0, D^+, and the expected F^+ are predicted on the basis of models embodying a minimum of dynamical assumptions. These models entail average decay multiplicities between 3 and somewhat over 4. Suggestions are made for observing the F^+, for which the wide variety of available final states makes detection in any one state challenging. The branching ratio $\Gamma(F^+ \to K^+K^-\pi^+)/\Gamma(F^+ \to \text{hadrons})$ is probably $\lesssim 3$-6%; $\Gamma(F^+ \to K^+\bar{K}^0)/\Gamma(F^+ \to \text{hadrons}) \lesssim 4$-$6\%$; $\Gamma(F^+ \to \pi^+\eta)/\Gamma(F^+ \to \text{hadrons}) \lesssim 3$-$4\%$.

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Now that the charmed[1] mesons D^0 [2] and D^+ [3] have been discovered, can the predicted F^+ [4] be far behind? The present work is an estimate of branching ratios for hadronic decays of the F, as well as for those of the D mesons, on the basis of what is already known about the decays of heavy mesons. Our purpose is to provide a rough guide for experimental searches with a minimum of untried dynamical assumptions.[5] We shall indicate some likely prospects for detecting the F.
Similar methods have been applied already to charmed baryons,[6] to the η_c,[7,8] and to the ψ;[8] a preliminary account of some of our results was given in Ref. 9 (see also Ref. 10).

We shall recall the models briefly in Sec. II. Results are contained in Sec. III, and Sec. IV is devoted to a discussion.

II. MODELS

Two models for multiplicity distributions will be used: the "statistical model" (subsection A) and the "constant matrix element (phase space) model" (subsection B). Ref. 7 contains more details. Isospins are treated in a statistical manner (subsection C). Different kinds of decay modes are related via SU(3) where possible (subsection D).

A. Statistical Model

The nonleptonic decay of a charmed meson involves a hadronic state of definite quantum numbers which evolves into two or more pseudoscalar particles. (We shall neglect final states containing baryon-antibaryon pairs.) This state may be imagined to be confined within a radius R_0 at some temperature T. Both the total energy and the number of degrees of freedom (essentially the average number of pions) are functions of T. Eliminating T, one finds a relation[11,6,7] between the average decay multiplicity <n> and the energy available to populate these degrees of freedom. If a particle of mass M decays to one of M_A, one of M_B, and any number of pions, the relation is

$$\langle n\rangle = 2 + 0.528\left[\frac{M - M_A - M_B}{E_0/c^2}\right]^{3/4} \tag{2.1}$$

where

$$E_0 \equiv \hbar c/R_0 \quad . \tag{2.2}$$

One reasonable procedure[6,9] is to fix E_0 to reflect a typical hadronic radius: $R_0 \approx 1$ fm, or $E_0 \approx 0.2$ GeV, which gives an acceptable description of hadronic multiplicities in e^+e^- annihilations, over a wide range of energies.[7] A related alternative is to choose E_0 for a best fit of Eq. (2.1) to mean multiplicities in $e^+e^- \rightarrow$ hadrons. The resulting value,[7]

$$E_0 = 0.17 \text{ GeV} \quad , \tag{2.3}$$

will be adopted here.

The shape of the distribution in n will be specified[6,7,9] by a Poisson distribution in the variable n-2, suitably truncated and renormalized if the minimum allowed multiplicity exceeds two.[7]

B. Constant matrix element (c.m.e.) model

We may assume, as in Ref. 4, that the Lorentz-invariant matrix element for (n+1)-pion emission is that for n-pion emission divided by a constant scale factor f. We have found (see Ref. 7 for normalizations) that $f^{-1} = 24\ \mathrm{GeV}^{-1}$ provides an acceptable fit of this model to the relative branching ratios for $\psi \to$ pions, $\psi \to K\bar{K}$ + pions,[12] and to preliminary data on $\bar{D}^0 \to K\pi$, $K2\pi$, $K3\pi$.[13] The quality of this fit is shown in Fig. 1 for ψ decays; we shall see in Sec. III that it is satisfactory for $\bar{D}^0$ decays. The distributions predicted by this model are narrower than Poisson distributions. However, an increase in f^{-1} with multiplicity is not excluded (see Fig. 1a). This would tend to give distributions which are closer to Poisson distributions. We regard the statistical model as an example of a "broad" distribution, and the c.m.e. model as a "narrow" distribution. We expect the two to indicate reasonable variations in theoretical predictions.

C. Isospins

The charged D and the F decay nonleptonically to states of definite I and I_3 in the limit $\theta_C = 0$.[4,14,15] To estimate the charge distributions in their decays, we use a statistical isospin model.[16-18,14] The nonleptonic decays of D^0 can be expected to give rise to states with both $I = 1/2$ and $I = 3/2$. We shall use isospin weights appropriate to a statistical admixture of $I = 1/2$ and $I = 3/2$ final states.[14] The statistical isospin factors are quoted for convenience in Table I.

D. SU(3) relations

Decays to different kinds of final states may be related with the help of unitary symmetry at the two-body level. Since both the statistical model and the c.m.e. model specify decays with additional pions in terms of two-body decays, this method specifies the relative probabilities of a wide variety of multimeson decays.

The relative values of $\tilde{\Gamma}$(partial widths with phase space weights factored out) for two-meson decays of charmed mesons are shown in Table II.[19-23] The η and η' are assumed to be an unmixed octet state and singlet state, respectively.

The nonet scheme of the first column[22] ("EQ") of Table II treats $(\bar{D}^0, F^+)$ as decaying through a $(d\bar{s}, u\bar{d})$ intermediate state. That of the second column ("$[\underline{6}^*]$ enhancement")[9] involves $\bar{D}^0$ final states of the form $du\bar{s}\bar{u}$ and F^+ final states of the form $su\bar{d}\bar{s}$. Both are predicated on the assumption that the dominant nonleptonic $|\Delta C| = 1$ decays proceed through the action of the piece of the weak Hamiltonian which transforms as an SU(3) sextet.

Several <u>caveats</u> apply to Table II.

1. We are particularly uncertain about the branching ratios involving η'. The figures we shall present are based on maximum estimates of these branching ratios (EQ scheme) in order to highlight the possible dilution of remaining signals. They should <u>not</u> be taken as a source of optimism regarding detection of signals involving η'.

2. Our use of unitary symmetry for $\overline{D}^0$ decays is not consistent with the statistical ansatz for isospins. The EQ scheme involves a final state of pure I = 1/2, while the $[\underline{6}^*]$-enhancement scheme involves pure I = 1/2 for $K\pi$. For final states involving more than one pion, the results of Table I are very similar to those[14] for a pure I = 1/2 final state, so that this inconsistency should be unimportant at the level of the present illustrative calculations. For two-body decays, separate discussions are possible,[4,9,15,20-23] and the statistical model should be regarded as particularly crude.

3. We cannot reliably estimate the importance of multi-pion decays of F^+ relative to other modes. It is conceivable that decays $F^+ \rightarrow$ (pions) could be the dominant decay modes if the F^+ decays through a $u\bar{d}$ intermediate state. However, this state may be suppressed by helicity factors[23,24] if the u and $\bar{d}$ act as very light quarks. We shall present separate results for $F^+ \rightarrow$ (pions) and for $F^+ \rightarrow$ ($K\overline{K}$ + pions, η + pions, η' + pions).

III. RESULTS

A. Charged D decays

The I = 3/2 final states populated by $D^{\pm}$ nonleptonic decays will be assumed to consist entirely of a single kaon and pions. Decays such as $D^{+} \rightarrow \overline{K}K\overline{K}$ and $D^{+} \rightarrow \overline{K}\eta\pi$ (plus possible pions) will be ignored. By analogy with the neutral D decays to be discussed below, we expect the ignored modes to account for no more than 10-15% of the nonleptonic $D^{\pm}$ decays.

The branching fractions for D^{-} nonleptonic decays to various states of K + pions are shown in Table III and Fig. 2. Results are given for the statistical model of §IIA and for the constant matrix element (c.m.e.) model of §IIB.

B. Neutral D decays

The calculated branching ratios for $\overline{D}^{0} \rightarrow$ (K + pions, $K\eta$ + pions, $K\eta'$ + pions) are given in Table IV and Fig. 3. The EQ nonet scheme for η' production is assumed. The less restrictive model based only on $[\underline{6}^{*}]$ -dominance entails η' rates 1/4 of those shown, with a consequent change in overall normalization.

C. F decays

Separate calculations have been made for F → pions and for F → other hadrons, as noted in § IID. Results for the two classes are presented separately, in Tables V and VI and in Figs. 4 and 5. The results are based on the EQ nonet scheme for η' production.

IV. DISCUSSION

A. Charged D decays

The decay mode $D^- \to K^+ 2\pi^-$ is expected to be prominent, and has been seen.[3] The cross section σ in e^+e^- annihilations times branching ratio B is[13]

$$2\sigma(D^-)B(D^- \to K^+2\pi^-) = 0.38 \pm 0.09 \text{ nb} \quad (E_{c.m.} = 4.028 \text{ GeV}) \tag{4.1}$$

$$= 0.33 \pm 0.12 \text{ nb} \quad (E_{c.m.} = 4.41 \text{ GeV}) \quad . \tag{4.2}$$

When combined with results of Table III, these values imply that

$$[\sigma(D^+) + \sigma(D^-)]B(D^\pm \to K + \text{pions}) = 1.7 \text{ to } 5.2 \text{ nb} \quad (E_{c.m.} = 4.028 \text{ GeV}) \tag{4.3}$$

$$= 1.2 \text{ to } 5.0 \text{ nb} \quad (E_{c.m.} = 4.41 \text{ GeV}) \quad . \tag{4.4}$$

The decay $D^- \to K^0\pi^-$ may have been seen,[25] but at a level below that implied by Table III and Eqs. (4.1) and (4.2):[13]

$$[\sigma(D^+) + \sigma(D^-)]\, B(D^- \to K^0\pi^-) < 0.20 \text{ nb} \quad (E_{c.m.} = 4.028 \text{ GeV}) \tag{4.5}$$

$$< 0.18 \text{ nb} \quad (E_{c.m.} = 4.41 \text{ GeV}) \quad . \tag{4.6}$$

The suppression of $\Gamma(D^- \to K^0\pi^-)/\Gamma(D^- \to K^+2\pi^-)$ below statistical expectations of ≈ 1 could indicate that the nonleptonic $\Delta S = \Delta C$ weak Hamiltonian obeys an approximate $\Delta V = 0$ rule.[20-23, 9] The limits (4.5) and (4.6) are not sufficient to decide this point, however.[26]

One interesting mode still not observed is $D^- \to K^0 2\pi^-\pi^+$, predicted in Table III to be 11-14% of the decays $D^- \to K$ + pions.

B. Neutral D decays

We may fit measured values of σB[13] to the models of Table IV by treating σ as a free parameter. The results are shown in Table VII. A clear distinction between the two models is not possible at present. At $E_{c.m.} = 4.028$ GeV, the cross sections for neutral D production are larger than those of Eq. (4.2) for charged D production. This behavior is understandable if a large number of D's come from D^*s.[27] The total cross sections for charged and neutral D production are

$$[\sigma(D^0) + \sigma(\bar{D}^0)]\, B(D^0 \to \text{hadrons}) + [\sigma(D^+) + \sigma(D^-)]\, B(D^\pm \to \text{hadrons}) = 8 - 20 \text{ nb} \quad (E_{c.m.} = 4.028 \text{ GeV}) \tag{4.7}$$

$$= 6 - 15 \text{ nb} \quad (E_{c.m.} = 4.41 \text{ GeV}) \quad . \tag{4.8}$$

If one notes that B(D →hadrons) could be as low as 60%,[23] with the remainder taken up by semileptonic decays,[28] Eqs. (4.7) and (4.8) are not unreasonable. The behavior of R at 4.028 and 4.41 GeV[25] implies that the total charmed pair production cross section is probably 10-15 nb at the former energy and slightly smaller at the latter. The higher numbers in (4.7) and (4.8), based on the predictions of the statistical model, thus are probably in better agreement with present data.[26]

We have ignored decays involving more than one kaon. These can be estimated to account for no more than a few percent of nonleptonic D decays.[9]

C. F decays

Our results illustrate the wide variety of final states to which the F can decay. However, there are a few easily identified decay modes for which the branching ratios could exceed several percent. The $K^+K^-\pi^+$ state should comprise 3-6% of the nonpionic channels noted in Table V. If the multi-pion states dilute this signal appreciably, they should be visible themselves at the level of several percent. (The $K^+\bar{K}^0$ state is expected to be 4-6% of the decays listed in Table V, but could be suppressed by a cancellation of two terms in the weak nonleptonic Hamiltonian.[23]) The $\pi^+\eta$ mode is by no means negligible, but requires good neutral detection.

It has already been possible to observe D decays to final states which, according to the present models, have branching ratios of only several percent. The branching ratios of F into observable final states such as $K^+K^-\pi^\pm$ should not lie much below this figure. What if the F steadfastly refuses to show itself well below present levels in e^+e^- annihilations or in photoproduction? What possibilities remain?

1. One can blame the strong interactions. In e^+e^- annihiliations, the virtual photon presumably produces a $c\bar{c}$ pair, which then materializes into charmed mesons by a "dressing" procedure as yet poorly understood. It is possible that strange quarks (needed to form F's) are not readily produced in this process. However, up to now the failure of strange quarks to materialize readily in hadronic processes can be ascribed to a large extent to the inequality $m_K >> m_\pi$ and to the effect of barrier factors, as in the comparison of SU(3) predictions for resonance decays with experiment.[29] By contrast, one expects $m_F \approx m_D + 140$ MeV,[30] a fractional difference too small to imply any appreciable kinematic suppression of F's. The ratio $\sigma(F)/\sigma(D)$ in e^+e^- annihilations thus can have important bearing on strong interaction dynamics.

One mechanism for producing F's which does not compel the strong interactions to give rise to strange quarks is the diffractive process[31]

$$(\nu, \bar{\nu}) + N \rightarrow \mu^\mp + F^\pm + \text{anything} \quad . \tag{4.9}$$

However, if this process accounts for less than a percent of all neutrino interactions its effects may be difficult to observe even in large-statistics bubble chamber experiments.

2. The QCD calculations[30] of the F mass could be so grievously wrong that the wrong mass range is searched, or worse, the F^* could lie lower than the F and could decay dominantly to lepton pairs.[32] We regard these possibilities as remote in the light of the successes of the QCD calculations[30] for the properties of D mesons.

3. The whole charm picture could be wrong or seriously incomplete. In the former case, it is hard to imagine the D mesons appearing with properties so close to those expected. In the latter, supposing that there are still more quarks with masses ≈ 2 GeV, it is hard to imagine that they should have a significant effect on the properties of the F.

To conclude, we expect that the F will be seen within the year. Once the level of its production has been established, the F will prove to be a useful tool for setting lower bounds on certain neutral lepton masses.[33] It will also provide a number of useful confirmations, both of the charm hypothesis and of the general picture of the decays of massive particles.

Note added: After this paper was written, we received an experimental report from R. Brandelik, et al. (DASP Collaboration), DESY preprint 77/44, in which evidence is given for the $\pi^+\eta$ decay of $F^+(2030 \pm 60)$.

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Table I. Fractions for individual charge states in decays of $\overline{D}^0 \rightarrow K +$ pions, according to the statistical isospin postulate of Ref. 16. The final state is taken as a statistical average of I = 1/2 and I = 3/2.

Final state \ n_{π^0}	0	1	2	3	4	5	6
$K\pi$	1/2	1/2					
$K2\pi$	9/20	8/20	3/20				
$K3\pi$	12/45	21/45	9/45	3/45			
$K4\pi$	50/245	88/245	78/245	24/245	5/245		
$K5\pi$	45/357	120/357	108/357	66/357	15/357	3/357	
$K6\pi$	245/2667	660/2667	900/2667	552/2667	255/2667	48/2667	7/2667

Table II. Relative reduced partial widths $\tilde{\Gamma}$ for decays of charmed mesons into two pseudoscalars.

Decay mode	EQ nonet scheme [Ref. 11]	$[\underline{6}^*]$ -enhancement [Ref. 12]
$\overline{D}^0 \rightarrow K\pi$	9	9
$K\eta$	1	1
$K\eta'$	8	2
$F \rightarrow K\overline{K}$	3	3
$\eta\pi$	2	2
$\eta'\pi$	4	1

Table III. Branching ratios for decays $D^- \to K + \text{pions}$ as fractions of all $K + \text{pions}$ modes.[a]

a) Statistical Model: $\langle n \rangle \simeq 4.3$, $\langle n_{ch} \rangle \simeq 3.1$.

Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3	4
$K\pi$	9.7%	10	--			
$K2\pi$	22.7	9	14			
$K3\pi$	26.4	11	8	7		
$K4\pi$	20.5	4	10	4	3	
$K5\pi$	11.9	2	3	5	1	1
$K \geq 6\pi$	8.7					

b) Constant matrix element model: $\langle n \rangle \simeq 3.4$, $\langle n_{ch} \rangle \simeq 2.5$.

Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3
$K\pi$	12.1%	12	--		
$K2\pi$	42.7	17	26		
$K3\pi$	35.4	14	11	10	
$K4\pi$	9.1	2	4	2	1
$K \geq 5\pi$	0.8				

[a]For the mean multiplicity, π^0, K^0, η, and η' are treated as stable particles; for the mean charged multiplicity K^0, η, and η' are treated as 0.67, 0.58, and 1.90 charged particles, respectively.

Table IV. Branching ratios for decays $\bar{D}^0 \to K + \text{pions},\ K\eta + \text{pions},\ K\eta' + \text{pions}.$[a]

a) Statistical model: $\langle n \rangle \approx 4.1$, $\langle n_{ch} \rangle \simeq 3.0$.

Mean multiplicity	Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3	4	5
$\langle n \rangle \simeq 4.3$, $\langle n_{ch} \rangle \simeq 3.0$	$K\pi$	8.0%	4	4				
	$K2\pi$	18.7	8	7	3			
	$K3\pi$	21.8	6	10	4	1		
	$K4\pi$	16.9	3	6	5	2	-	
	$K5\pi$	9.8	1	3	3	2	-	-
	$K \geq 6\pi$	7.2						
$\langle n \rangle \simeq 3.7$, $\langle n_{ch} \rangle \simeq 2.4$	$K\eta$	0.8	1	-				
	$K\eta\pi$	1.4	1	1				
	$K\eta 2\pi$	1.2						
	$K\eta \geq 3\pi$	1.1						
$\langle n \rangle \simeq 3.0$, $\langle n_{ch} \rangle \simeq 3.3$	$K\eta'$	4.7	5					
	$K\eta'\pi$	4.8	2	2				
	$K\eta' 2\pi$	2.5	1	1	-			
	$K'\eta \geq 3\pi$	1.1						

Table IV, cont'd.

b) Constant matrix element model: $\langle n\rangle \simeq 3.3$, $\langle n_{ch}\rangle \simeq 2.4$.

Mean Multiplicity	Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3	4
$\langle n\rangle \simeq 3.4$	$K\pi$	10.6%	5	5			
$\langle n_{ch}\rangle \simeq 2.4$	$K2\pi$	37.6	17	15	6		
	$K3\pi$	31.1	8	15	6	2	
	$K4\pi$	8.0	2	3	3	1	-
	$K \geq 5\pi$	0.7					
$\langle n\rangle \simeq 2.9$	$K\eta$	1.1	1				
$\langle n_{ch}\rangle \simeq 1.8$	$K\eta\pi$	1.9	1	1			
	$K\eta \geq 2\pi$	0.6					
$\langle n\rangle \simeq 2.3$	$K\eta'$	6.2	6				
$\langle n_{ch}\rangle \simeq 2.8$	$K\eta'\pi$	2.3	1	1			
	$K\eta' \geq 2\pi$	0.0					

[a] See footnote to Table III.

Table V. Branching ratios for decays $F^+ \to$ pions, as fractions of all purely pionic decays. The F^+ mass is chosen as 2.01 GeV/c^2.[a]

a) Statistical model: $\langle n\rangle \simeq 5.2$, $\langle n_{ch}\rangle \simeq 3.4$.

Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3	4	5
3π	15.6%	9	--	6			
4π	23.5	--	19	--	5		
5π	23.6	7	--	15	-	2	
6π	17.7	--	9	--	8	-	1
7π	10.7						
≥ 8π	8.9						

b) Constant matrix element model: $\langle n\rangle \simeq 4.2$, $\langle n_{ch}\rangle \simeq 2.8$.

Decay Mode	All charge states	Specific charge states: neutral pions 0	1	2	3	4	5
3π	23.6%	14	--	9			
4π	40.5	--	32	--	8		
5π	27.0	8	--	17	-	2	
6π	7.8	--	4	--	3	-	0
≥ 7π	1.1						

[a]See footnote to Table III.

Table VI. Branching ratios for decays $F^+ \to \bar{K}K$ + pions, η + pions, η' + pions, as fractions of all such modes. The F^+ mass is chosen at 2.01 GeV/c^2.[a]

a) Statistical model: $\langle n \rangle \simeq 4.1$, $\langle n_{ch} \rangle \simeq 3.2$.

Mean Multiplicity	Decay Mode	All charge states	Specific charge modes: neutral pions 0	1	2	3	4	5
			$\bar{K}^0$ K^-	$\bar{K}^0$ K^-	$\bar{K}^0$ K^-	$\bar{K}^0$ K^-	$\bar{K}^0$ K^-	$\bar{K}^0$ K^-
	$K\bar{K}$	4.3%	4 -					
	$K\bar{K}\pi$	8.7	3 3	2 -				
$\langle n \rangle \simeq 4.0$	$K\bar{K}2\pi$	8.8	3 1	2 2	1 -			
$\langle n_{ch} \rangle \simeq 3.0$	$K\bar{K}3\pi$	5.9	1 1	2 1	1 1	0 -		
	$K\bar{K}4\pi$	3.0						
	$K\bar{K} \geq 5\pi$	1.8						
	$\eta\pi$	3.0	3					
	$\eta 2\pi$	7.4	-	7				
$\langle n \rangle \simeq 4.5$	$\eta 3\pi$	9.1	5	-	4			
$\langle n_{ch} \rangle \simeq 2.9$	$\eta 4\pi$	7.5	-	6	-	1		
	$\eta 5\pi$	4.6	1	-	3	-	0	
	$\eta \geq 6\pi$	3.6	-	1	-	1	-	0
	$\eta'\pi$	5.0	5					
	$\eta' 2\pi$	9.4	-	9				
$\langle n \rangle \simeq 3.9$	$\eta' 3\pi$	8.7	5	-	3			
$\langle n_{ch} \rangle \simeq 3.8$	$\eta' 4\pi$	5.4	-	4	-	1		
	$\eta' 5\pi$	2.5	1	-	2	-	0	
	$\eta' \geq 6\pi$	1.3						

Table VI, cont'd.

b) Constant matrix element model: $\langle n \rangle \simeq 3.3$, $\langle n_{ch} \rangle \simeq 2.6$.

Mean Multiplicity	Decay Mode	All charge states	Specific charge states: neutral pions 0		1		2		3	
			$\bar{K}^0$	K^-	$\bar{K}^0$	K^-	$\bar{K}^0$	K^-	$\bar{K}^0$	K^-
	$K\bar{K}$	5.7%	6	-						
$\langle n \rangle \simeq 3.2$	$K\bar{K}\pi$	15.6	6	6	4	-				
$\langle n_{ch} \rangle \simeq 2.5$	$K\bar{K}2\pi$	8.6	3	1	2	2	1	-		
	$K\bar{K}3\pi$	1.2								
	$\eta\pi$	4.0	4							
	$\eta 2\pi$	16.6	-		17					
$\langle n \rangle \simeq 3.6$	$\eta 3\pi$	16.5	10		-		7			
$\langle n_{ch} \rangle \simeq 2.3$	$\eta 4\pi$	5.3	-		4		-		1	
	$\eta 5\pi$	0.6								
	$\eta'\pi$	6.7	7							
$\langle n \rangle \simeq 3.0$	$\eta' 2\pi$	13.4	-		13					
$\langle n_{ch} \rangle \simeq 3.2$	$\eta' 3\pi$	5.1	3		-		2			
	$\eta' 4\pi$	0.4								

[a]See footnote to Table III.

Table VII. Fits to cross section × branching ratio for the production and decay of $(D^0, \bar{D}^0)$ in e^+e^- annihilations.

$E_{c.m.}$	$[\sigma(D^0) + \sigma(\bar{D}^0)] \times B$	experiment (Ref. 13)	statistical model	c.m.e. model
4.028 GeV	$K^{\mp}\pi^{\pm}$	0.51±0.08 nb.	0.51 nb.	0.44 nb.
	$K^0\pi^+\pi^-$	1.07±0.30	1.08	1.41
	$K^{\mp}2\pi^{\pm}\pi^{\mp}$	0.75±0.24	0.74	0.69
	K+π's+(≤1 η or η')	---	12.8 ± 1.6	8.4 ± 1.7
	χ^2/d.o.f.		~0/2	2.0/2
4.41 GeV	$K^{\mp}\pi^{\pm}$	0.28±0.08 nb.	0.34 nb.	0.30 nb.
	$K^0\pi^+\pi^-$	0.92±0.30	0.71	0.96
	$K^{\mp}2\pi^{\pm}\pi^{\mp}$	0.91±0.39	0.49	0.47
	K+π's+(≤1 η or η')	---	8.4 ± 1.1	5.6 ± 1.1
	χ^2/d.o.f.		2.2/2	1.4/2

FIGURE CAPTIONS

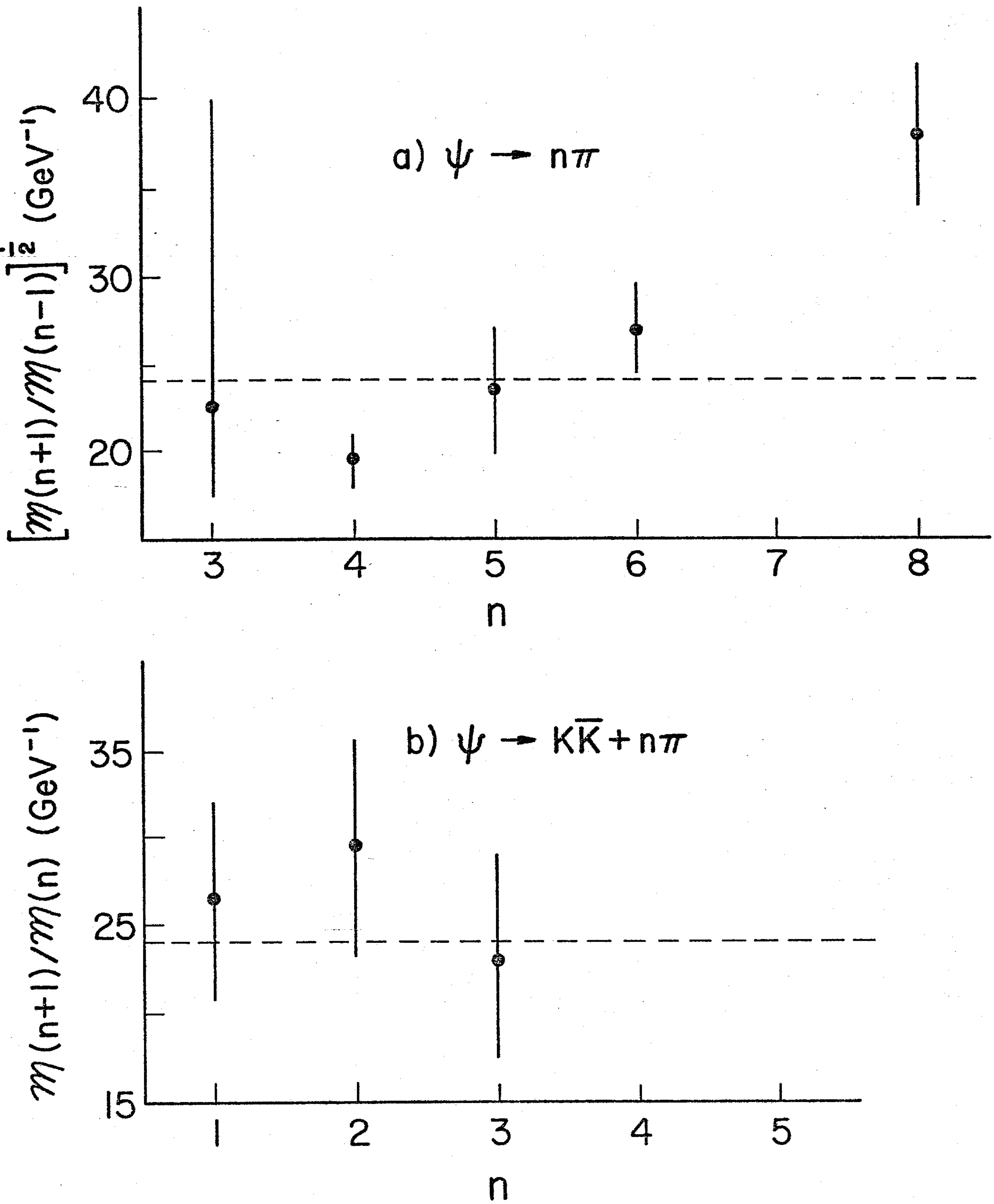
Fig. 1: Constant matrix element model fits to decays (a) $\psi \to$ pions and (b) $\psi \to K\bar{K}$ + pions. Decays to an even number of pions are presumed to occur via a one-photon intermediate state. The horizontal dashed lines indicate the matrix element ratio $f^{-1} = 24\ \mathrm{GeV}^{-1}$.

Fig. 2: Predicted branching ratios for specific states of $D^- \to K$ + pions, relative to all such decays. (a) statistical model; (b) constant matrix element (c.m.e.) model with $f^{-1} = 24\ \mathrm{GeV}^{-1}$.

Fig. 3: Predicted branching ratios for specific states of $\bar{D}^0 \to$ (K + pions, $K\eta$ + pions, $K\eta'$ + pions), relative to all such decays. EQ nonet scheme is assumed (see text). (a) statistical model; (b) c.m.e. model with $f^{-1} = 24\ \mathrm{GeV}^{-1}$.

Fig. 4: Predicted branching ratios for specific states of $F \to$ (pions), relative to all such decays. (a) statistical model; (b) c.m.e. model with $f^{-1} = 24\ \mathrm{GeV}^{-1}$.

Fig. 5: Predicted branching ratios for specific states of $F \to$ ($K\bar{K}$ + pions, η + pions, η' + pions) relative to all such decays. EQ nonet scheme is assumed. (a) statistical model; (b) c.m.e. model with $f^{-1} = 24\ \mathrm{GeV}^{-1}$.



$[\mathcal{M}(n+1)/\mathcal{M}(n-1)]^{-\frac{1}{2}}$ (GeV^{-1})
40
30
20
a) $\psi \rightarrow n\pi$
3
4
5
6
7
8
n
$\mathcal{M}(n+1)/\mathcal{M}(n)$ (GeV^{-1})
35
25
15
b) $\psi \rightarrow K\bar{K} + n\pi$
1
2
3
4
5
n

Fig. 1

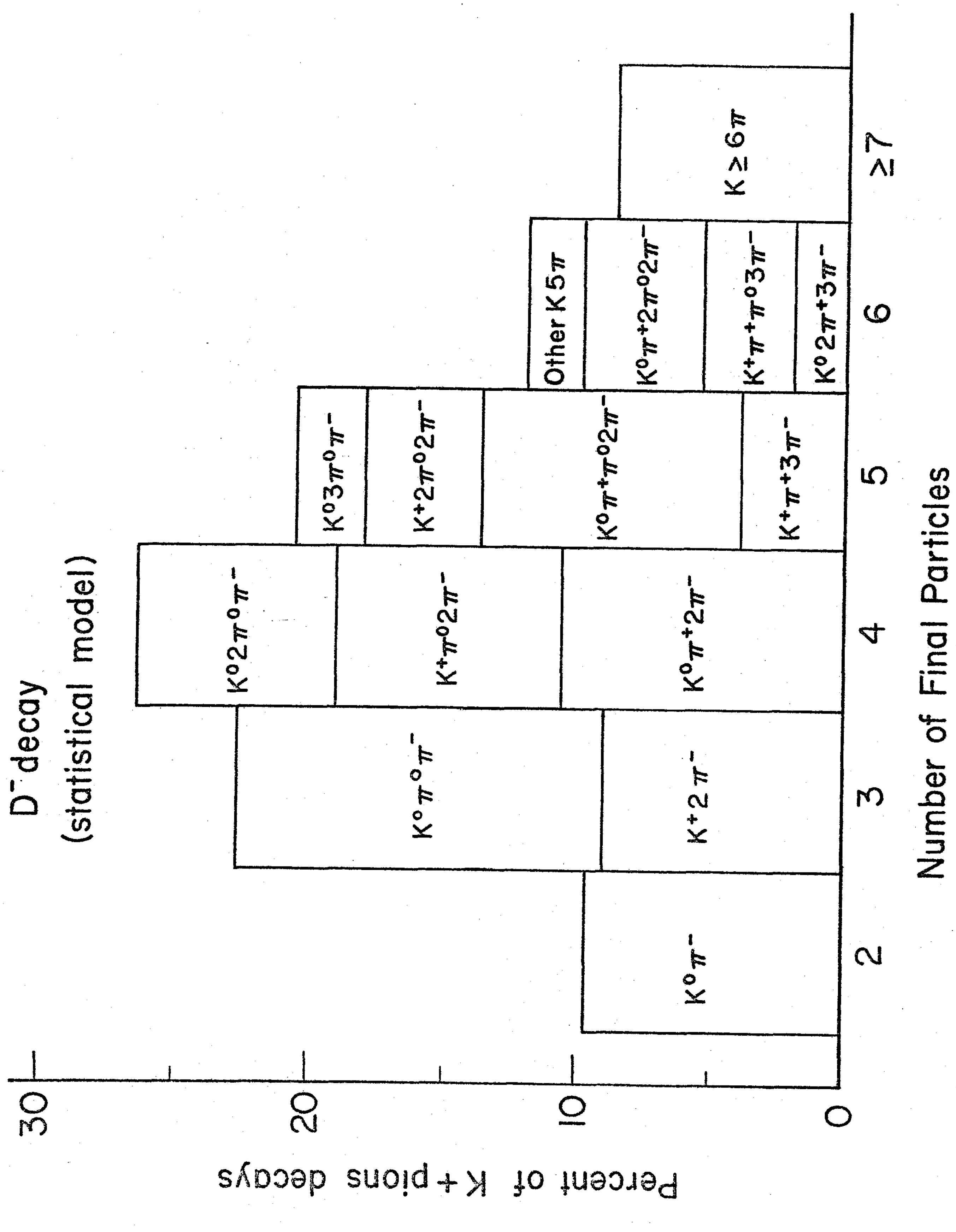
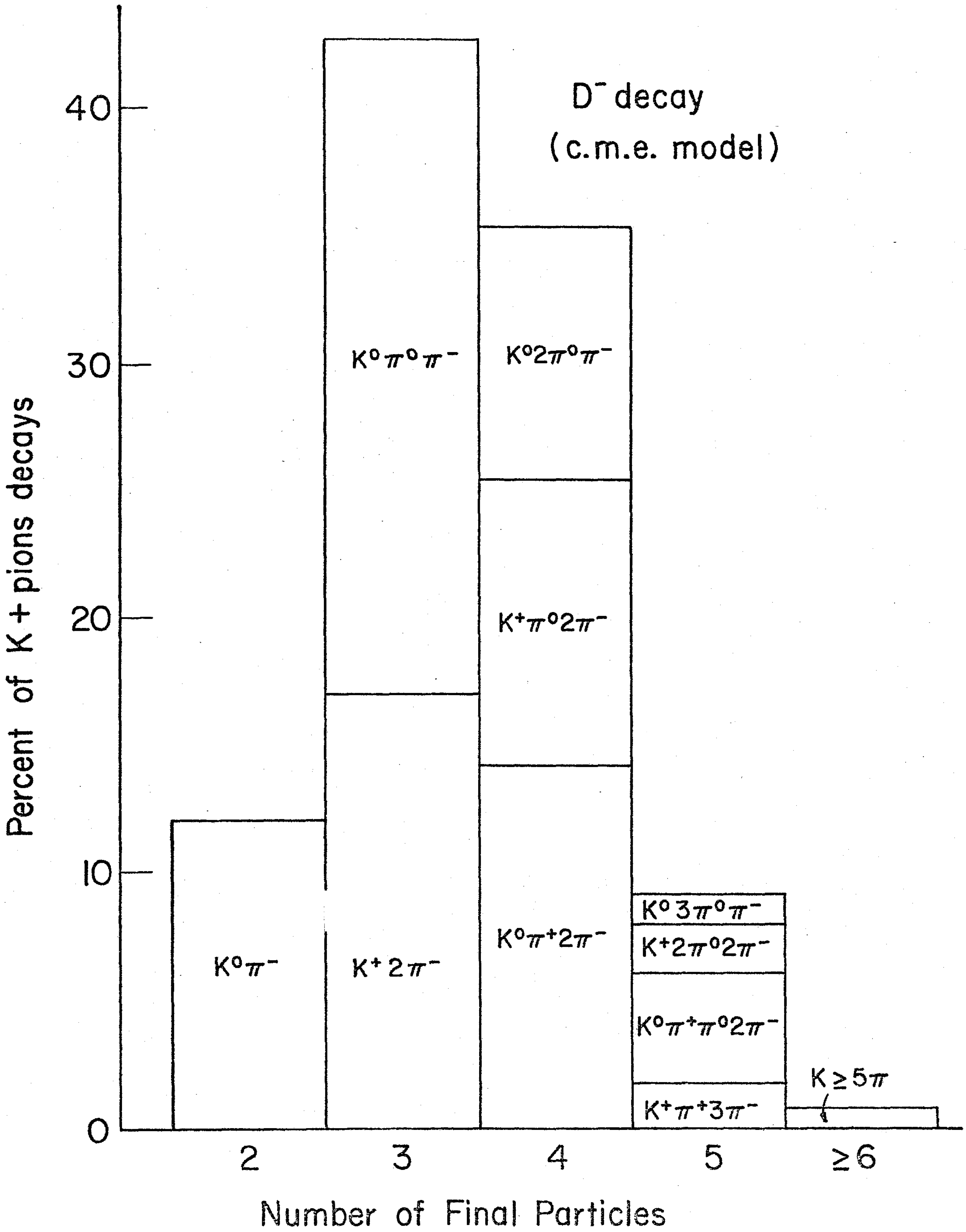
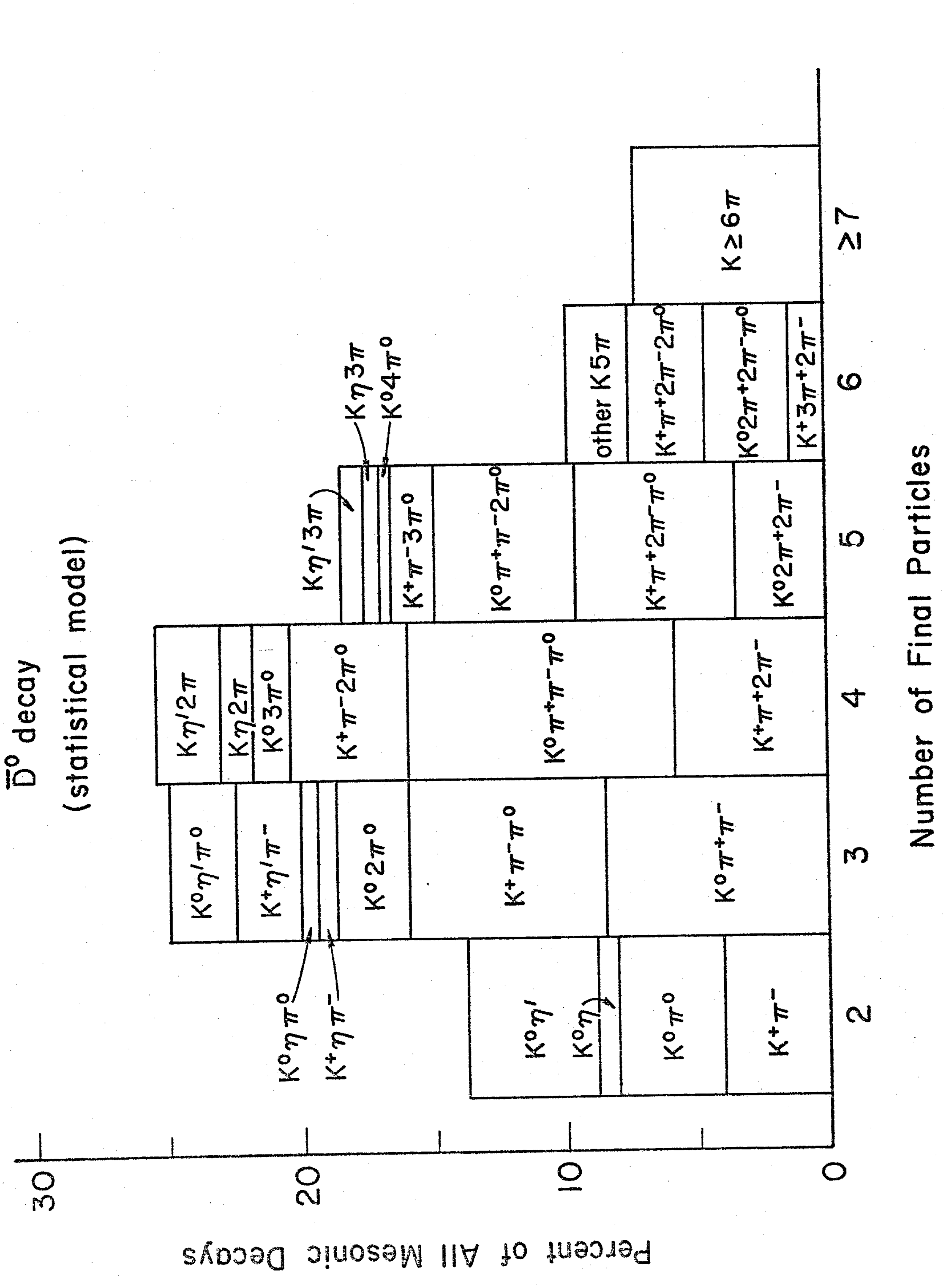

D^- decay
(statistical model)
Percent of K + pions decays
0
10
20
30
Number of Final Particles
2
3
4
5
6
≥7
$K^0\pi^-$
$K^0\pi^0\pi^-$
$K^+2\pi^-$
$K^02\pi^0\pi^-$
$K^+\pi^02\pi^-$
$K^0\pi^+2\pi^-$
$K^03\pi^0\pi^-$
$K^+2\pi^02\pi^-$
$K^0\pi^+\pi^02\pi^-$
$K^+\pi^+3\pi^-$
Other K 5π
$K^0\pi^+2\pi^02\pi^-$
$K^+\pi^+\pi^03\pi^-$
$K^02\pi^+3\pi^-$
$K \geq 6\pi$

Fig. 2a



D⁻ decay
(c.m.e. model)
Percent of K + pions decays
0
10
20
30
40
K°π⁻
K⁺2π⁻
K°π°π⁻
K°π⁺2π⁻
K⁺π°2π⁻
K°2π°π⁻
K°3π°π⁻
K⁺2π°2π⁻
K°π⁺π°2π⁻
K⁺π⁺3π⁻
K≥5π
2
3
4
5
≥6
Number of Final Particles

Fig. 2b



D̄⁰ decay
(statistical model)
Percent of All Mesonic Decays
0
10
20
30
Number of Final Particles
2
3
4
5
6
≥7
K⁰η′
K⁰η
K⁰π⁰
K⁺π⁻
K⁰ηπ⁰
K⁺ηπ⁻
K⁰η′π⁰
K⁺η′π⁻
K⁰2π⁰
K⁺π⁻π⁰
K⁰π⁺π⁻
Kη′2π
Kη2π
K⁰3π⁰
K⁺π⁻2π⁰
K⁰π⁺π⁻π⁰
K⁺π⁺2π⁻
Kη′3π
Kη3π
K⁰4π⁰
K⁺π⁻3π⁰
K⁰π⁺π⁻2π⁰
K⁺π⁺2π⁻π⁰
K⁰2π⁺2π⁻
other K5π
K⁺π⁺2π⁻2π⁰
K⁰2π⁺2π⁻π⁰
K⁺3π⁺2π⁻
K ≥ 6π

Fig. 3a

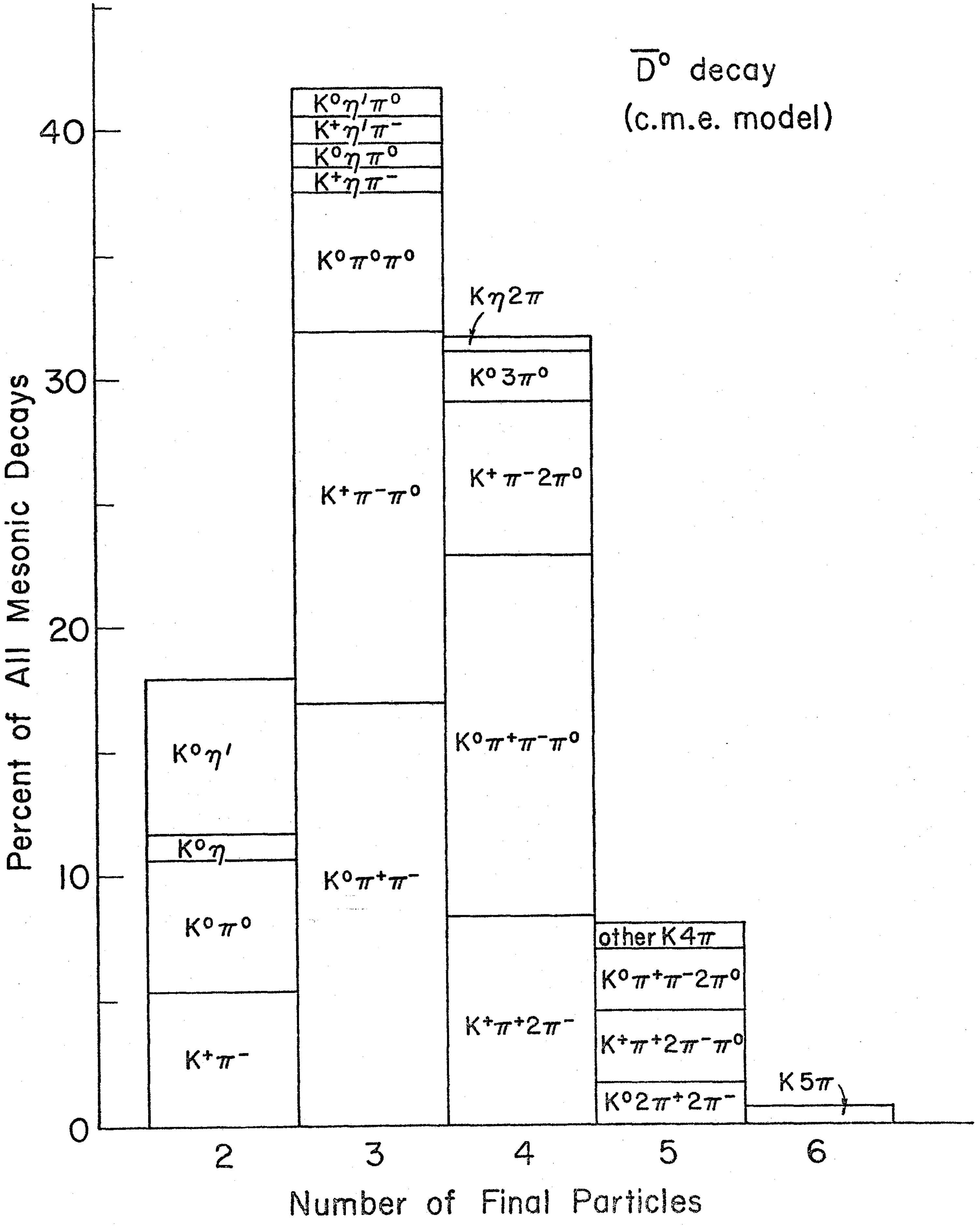
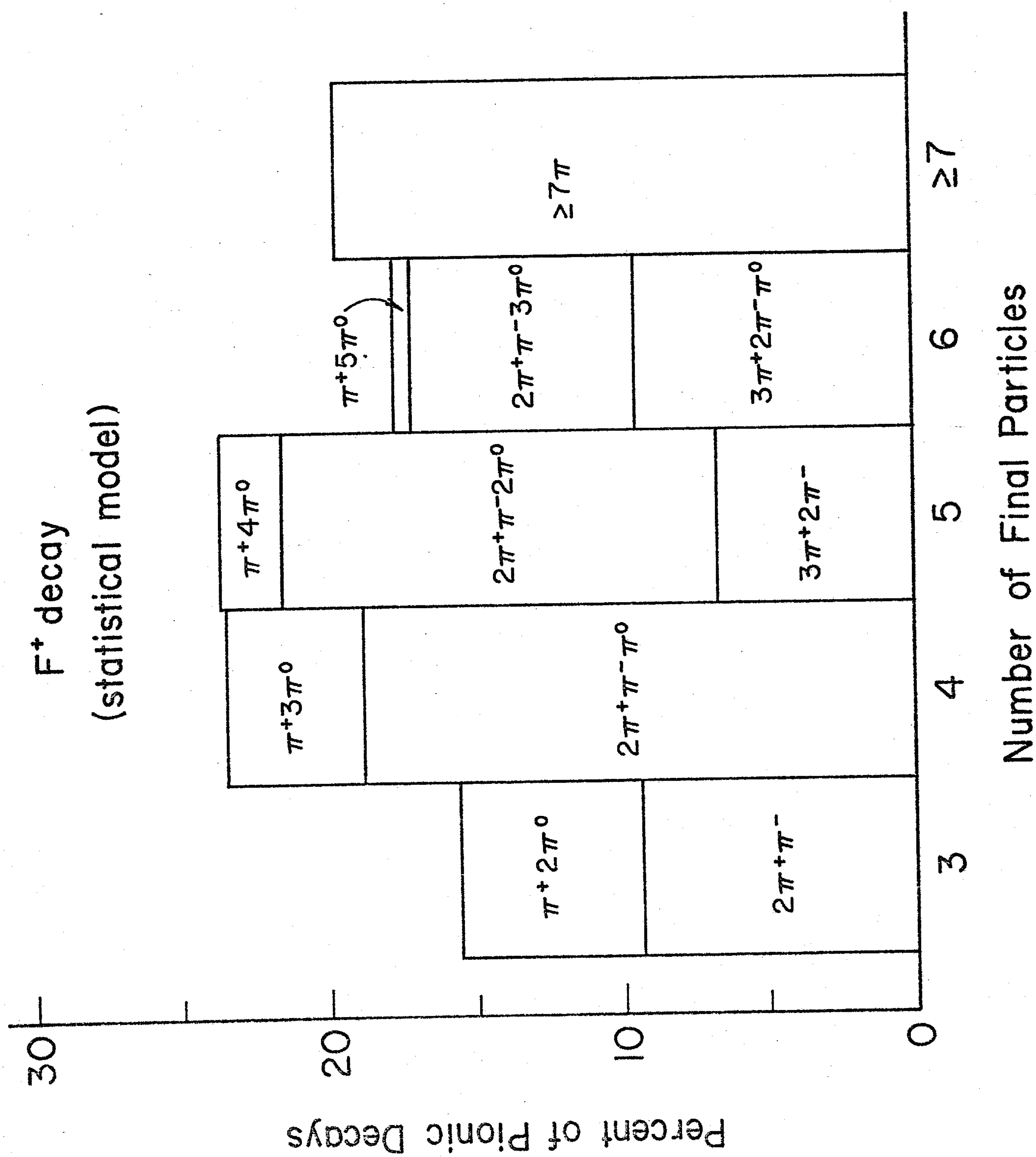
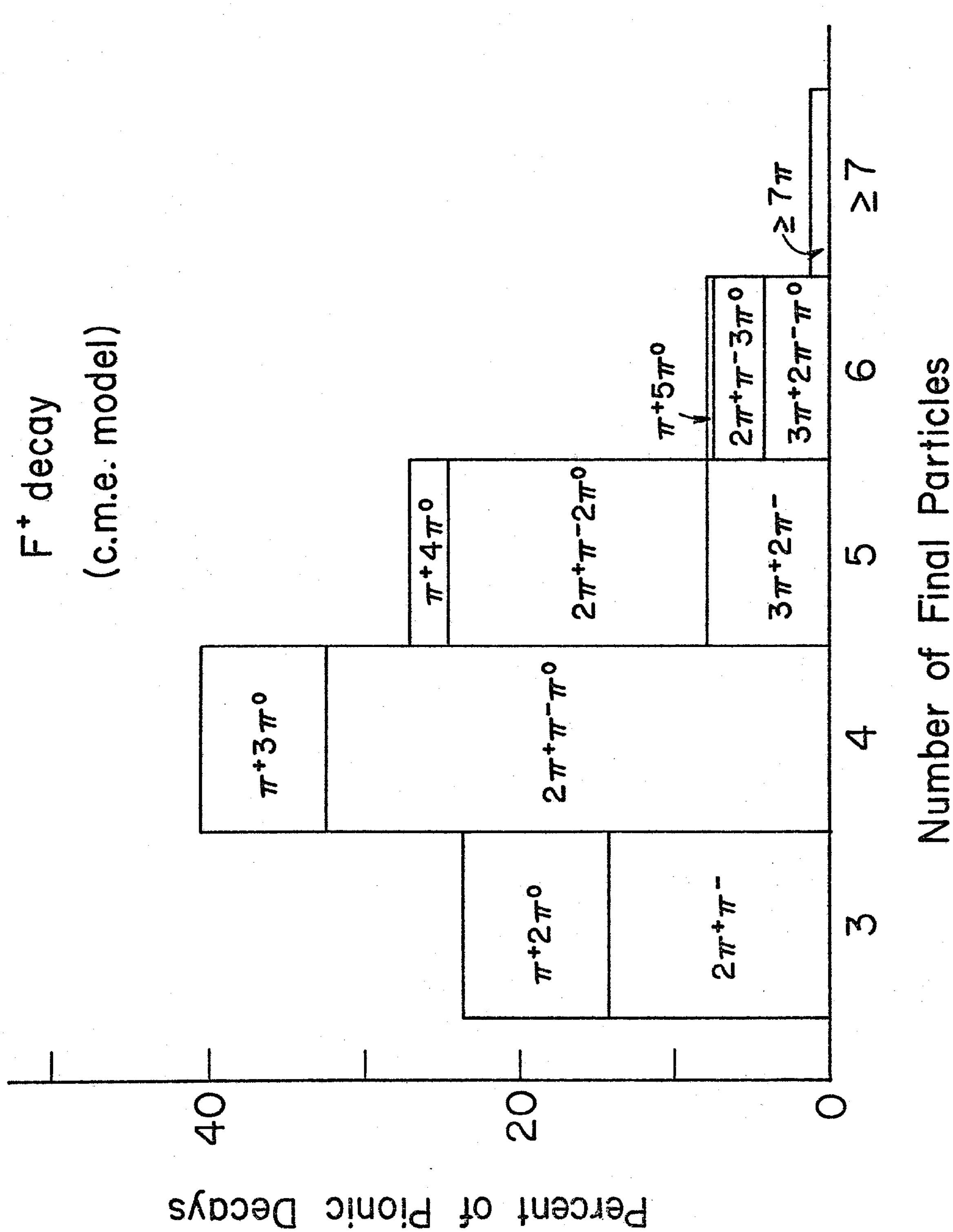

$\overline{D}^0$ decay
(c.m.e. model)
Percent of All Mesonic Decays
0
10
20
30
40
$K^+\pi^-$
$K^0\pi^0$
$K^0\eta$
$K^0\eta'$
$K^0\pi^+\pi^-$
$K^+\pi^-\pi^0$
$K^0\pi^0\pi^0$
$K^+\eta\pi^-$
$K^0\eta\pi^0$
$K^+\eta'\pi^-$
$K^0\eta'\pi^0$
$K^+\pi^+2\pi^-$
$K^0\pi^+\pi^-\pi^0$
$K^+\pi^-2\pi^0$
$K^0 3\pi^0$
$K\eta 2\pi$
$K^0 2\pi^+ 2\pi^-$
$K^+\pi^+2\pi^-\pi^0$
$K^0\pi^+\pi^-2\pi^0$
other $K4\pi$
$K5\pi$
2
3
4
5
6
Number of Final Particles

Fig. 3b



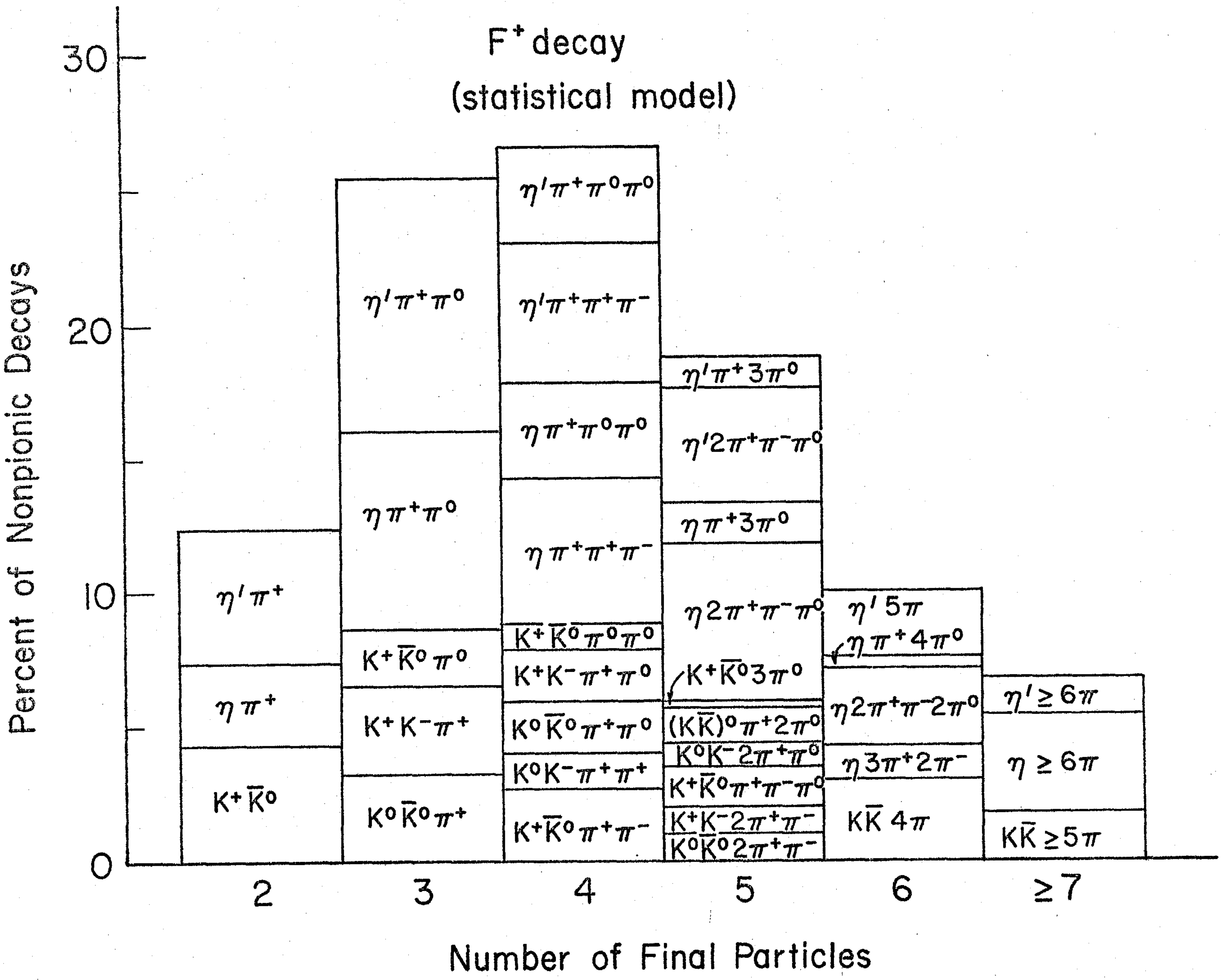
F⁺ decay
(statistical model)
Percent of Pionic Decays
0
10
20
30
Number of Final Particles
3
4
5
6
≥7
π⁺2π⁰
2π⁺π⁻
π⁺3π⁰
2π⁺π⁻π⁰
π⁺4π⁰
2π⁺π⁻2π⁰
3π⁺2π⁻
π⁺5π⁰
2π⁺π⁻3π⁰
3π⁺2π⁻π⁰
≥7π

Fig. 4a



F^+ decay
(c.m.e. model)
$\pi^+2\pi^0$
$2\pi^+\pi^-$
$\pi^+3\pi^0$
$2\pi^+\pi^-\pi^0$
$\pi^+4\pi^0$
$2\pi^+\pi^-2\pi^0$
$3\pi^+2\pi^-$
$\pi^+5\pi^0$
$2\pi^+\pi^-3\pi^0$
$3\pi^+2\pi^-\pi^0$
$\geq 7\pi$
0
20
40
3
4
5
6
≥ 7
Number of Final Particles
Percent of Pionic Decays

Fig. 4b



F^+ decay
(statistical model)
Percent of Nonpionic Decays
Number of Final Particles
0
10
20
30
2
3
4
5
6
≥7
$K^+\bar{K}^0$
$\eta\pi^+$
$\eta'\pi^+$
$K^0\bar{K}^0\pi^+$
$K^+K^-\pi^+$
$K^+\bar{K}^0\pi^0$
$\eta\pi^+\pi^0$
$\eta'\pi^+\pi^0$
$K^+\bar{K}^0\pi^+\pi^-$
$K^0K^-\pi^+\pi^+$
$K^0\bar{K}^0\pi^+\pi^0$
$K^+K^-\pi^+\pi^0$
$K^+\bar{K}^0\pi^0\pi^0$
$\eta\pi^+\pi^+\pi^-$
$\eta\pi^+\pi^0\pi^0$
$\eta'\pi^+\pi^+\pi^-$
$\eta'\pi^+\pi^0\pi^0$
$K^0\bar{K}^0 2\pi^+\pi^-$
$K^+K^- 2\pi^+\pi^-$
$K^+\bar{K}^0\pi^+\pi^-\pi^0$
$K^0K^- 2\pi^+\pi^0$
$(K\bar{K})^0\pi^+ 2\pi^0$
$K^+\bar{K}^0 3\pi^0$
$\eta 2\pi^+\pi^-\pi^0$
$\eta\pi^+ 3\pi^0$
$\eta' 2\pi^+\pi^-\pi^0$
$\eta'\pi^+ 3\pi^0$
$K\bar{K}\ 4\pi$
$\eta 3\pi^+ 2\pi^-$
$\eta 2\pi^+\pi^- 2\pi^0$
$\eta\pi^+ 4\pi^0$
$\eta' 5\pi$
$K\bar{K} \geq 5\pi$
$\eta \geq 6\pi$
$\eta' \geq 6\pi$

Fig. 5a

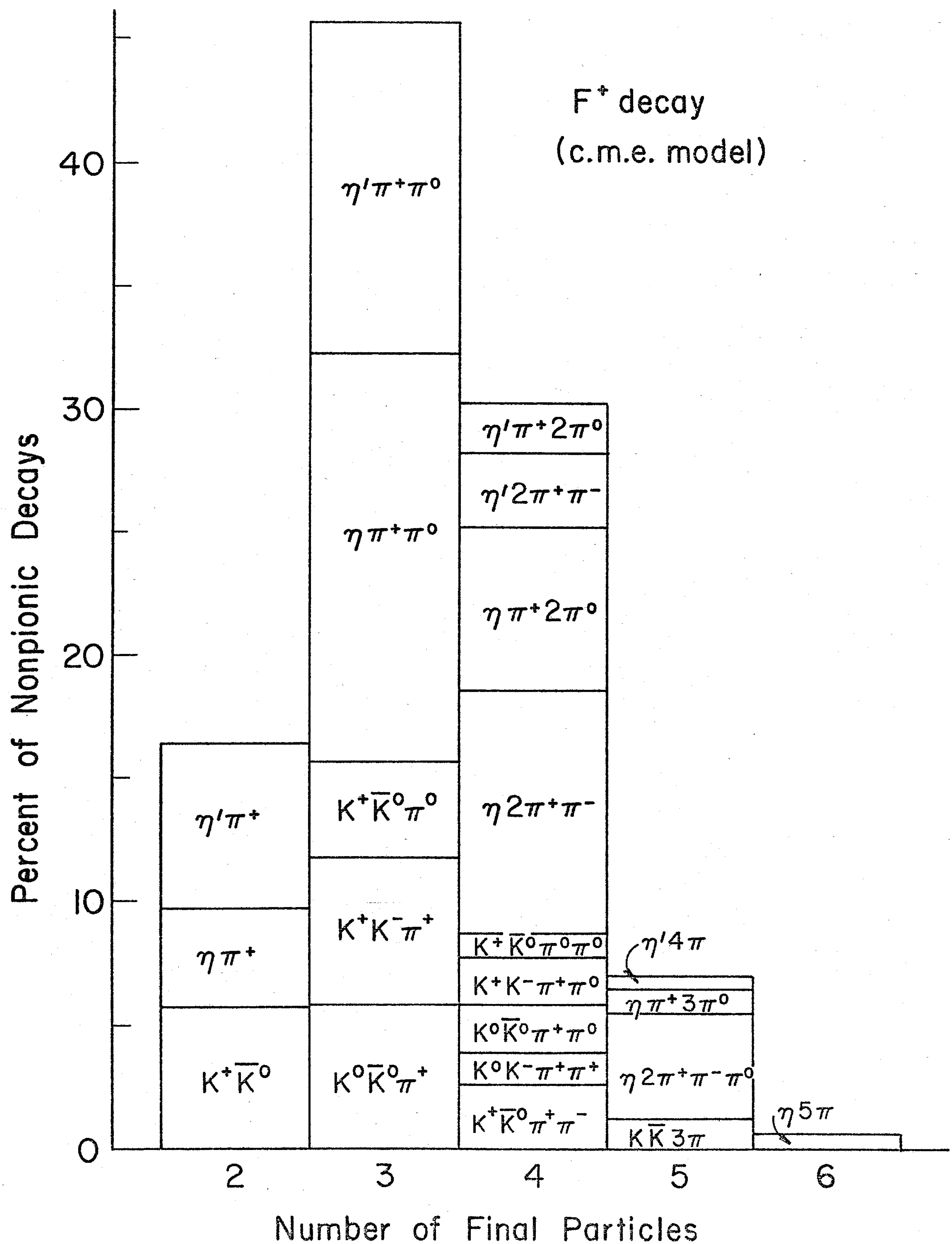

F⁺ decay
(c.m.e. model)
Percent of Nonpionic Decays
Number of Final Particles
0
10
20
30
40
2
3
4
5
6
$\eta'\pi^+$
$\eta\pi^+$
$K^+\bar{K}^0$
$\eta'\pi^+\pi^0$
$\eta\pi^+\pi^0$
$K^+\bar{K}^0\pi^0$
$K^+K^-\pi^+$
$K^0\bar{K}^0\pi^+$
$\eta'\pi^+2\pi^0$
$\eta'2\pi^+\pi^-$
$\eta\pi^+2\pi^0$
$\eta 2\pi^+\pi^-$
$K^+\bar{K}^0\pi^0\pi^0$
$K^+K^-\pi^+\pi^0$
$K^0\bar{K}^0\pi^+\pi^0$
$K^0K^-\pi^+\pi^+$
$K^+\bar{K}^0\pi^+\pi^-$
$\eta'4\pi$
$\eta\pi^+3\pi^0$
$\eta 2\pi^+\pi^-\pi^0$
$K\bar{K}3\pi$
$\eta 5\pi$

Fig. 5b