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Hadronic Decays of η_C

C. QUIGG[*] [†]
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and
JONATHAN L. ROSNER[‡]
Institute for Advanced Study, Princeton, New Jersey 08540

ABSTRACT

Hadronic branching fractions for the pseudoscalar psions are predicted on the basis of a simple model for $\psi(3095)$ decays. Results are given for hypothetical states at 2830, 3095, and 3455 MeV/c^2. Particular emphasis is laid on modes which may be observed as 1-c final states in e^+e^- annihilation into hadrons and on channels in which backgrounds are expected to be low.

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[‡] On sabbatical leave from School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455. Supported in part by the United States Energy Research and Development Administration under Contracts E-(11-1)-1764 and E-(11-1)-2220.

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I. INTRODUCTION

During the past two and one-half years, experiments have uncovered a rich spectrum of isoscalar hidden charm ($c\bar{c}$) states.[1] Two odd charge conjugation states $\psi(3095)$[2] and $\psi'(3684)$,[3] both with $J^{PC} = 1^{--}$, are identified as the 1^3S_1 and 2^3S_1 levels of the charmonium system.[4] Evidence has been presented as well for five states of even charge conjugation: X(2830),[5] $\chi(3414)$,[6] $\chi(3455)$,[7] $\chi(3508)$,[8] and $\chi(3552)$.[9] Three of these, $\chi(3414)$, $\chi(3508)$, and $\chi(3552)$ are well established and have tentative quantum number assignments[1, 10] as the 1^3P_0, 1^3P_1, and 1^3P_2 charmonium levels. It is appealing to associate the remaining candidates with the predicted pseudoscalar levels 1^1S_0 and 2^1S_0 (η_C and η_C'), but this assignment runs afoul of several theoretical expectations.[11] The rates for $\psi' \to \gamma + \chi(3455)$ and $\psi \to \gamma + X(2830)$ and the hadronic widths of $\chi(3455)$ and X(2830) seem far smaller than the straightforward predictions of the charmonium picture.[12, 4] In addition, the pseudoscalar-vector mass splittings of about 250 MeV/c^2 are much larger than the expected[4, 13] 30 - 100 MeV/c^2. Experimentally, no hadronic decay modes of either $\chi(3455)$ or X(2830) have been seen, and both states require confirmation.[14]

It is therefore of interest to ask at what level and in what modes the hadronic decays of the $^1S_0$ levels are to be expected. In advance of any detailed considerations we may expect, on the basis of known properties of the other psions[15, 1] and of charmed particles,[16] a very large number of multihadron decay channels each with a branching ratio on the order of a few percent. The object of this brief paper is to make more specific these general expectations. We shall present predictions for the branching ratios of the $^1S_0$ levels into hadronic final states in order to assess the meaning of the nonobservation of hadronic modes and to indicate fruitful modes for future searches. Our motivation and goals have much in common with earlier work by Koller and Walsh[17] but, in part because of the extensive data now available on hadronic decays of ψ, our results are sharper and more extensive.

The paper is organized as follows. In Section II we discuss two models for the multiplicity distribution of psion decays, and fit them to data on $\psi(3095)$ decay. In Section III we apply the models to the decays of (hypothetical) pseudoscalar psions with masses 2830, 3095, and 3455 MeV/c^2. The first and third are motivated by the experimental indications for X(2830) and $\chi(3455)$; the 3095 case is examined in deference to the simplest charmonium model expectation of relatively small splittings between ψ and η_C. Predictions are given for specific charge states within the general classes $\eta_C \rightarrow$ pions, $K\overline{K}$ + pions, $K\overline{K}K\overline{K}$ + pions, η + pions, η' + pions, and $N\overline{N}$ + pions. The predictions are discussed in detail in Section IV. We call attention to channels for which the expected branching ratios are relatively large, or for which backgrounds may be expected to be small.

II. MODELS FOR DECAYS OF MASSIVE HADRONS

A number of authors[17-22] have studied the hadron multiplicity distributions to be expected in the decays of massive particles or massive virtual photons. The models proposed have been characterized by a minimum of detailed dynamical assumptions, so only general features are expected to emerge. Let us review some of these approaches.

Lee, Quigg, and Rosner[20] advanced a version of the Fermi statistical model[23] appropriate to particle decay according to which the mean multiplicity of decay products is

$$\langle n \rangle = n_0 + 0.528(E/E_0)^{3/4} \quad . \tag{2.1}$$

Here E is the energy available in excess of the rest masses of the lowest multiplicity (n_0) decay channel. The scale E_0 is set by the hadronic radius R_0:

$$E_0 \equiv \hbar c/R_0 \quad . \tag{2.2}$$

For a radius of 1 fm (typical of bag models[24] of hadrons), $E_0 \simeq 0.2$ GeV. The statistical description is expected to apply in the "three-dimensional" regime in which decays are spherically symmetric, rather than jet-like.[25] The model has been applied to several situations of experimental interest.

The predicted mean charged multiplicity in the reaction $e^+e^- \rightarrow$ hadrons, given by

$$\langle n_{ch}\rangle = \frac{\langle n_{ch}(s)\rangle}{\langle n(s)\rangle}\left[2 + 0.528\left(\frac{\sqrt{s} - 2m_\pi}{E_0}\right)^{3/4}\right] , \qquad (2.3)$$

is compared with the data[26] in Fig. 1. The straight line represents a fit by Feldman and Perl.[26] For $\langle n_{ch}(s)\rangle/\langle n(s)\rangle$ we have used $\langle E_{ch}(s)\rangle/\sqrt{s}$ as quoted by Schwitters and Strauch,[26] fitting the experimental values to $\langle E_{ch}(s)\rangle/\sqrt{s} = 0.63 - 0.02(\sqrt{s}/1\ \text{GeV})$ for $2\ \text{GeV} \le \sqrt{s} \le 8\ \text{GeV}$. To make this empirical correction for neutrals may be to overlook the influence of "new physics" upon $\langle n_{ch}(s)\rangle$. It is arguably more faithful to the spirit of the model to fix $\langle n_{ch}(s)\rangle/\langle n(s)\rangle = 2/3$, which would lead to a somewhat larger "best" value of E_0.

For $E_0 = 0.17$ GeV, the expression (2.3) fits the data over a surprisingly large energy range. Consequently, we shall adopt this value in conjunction with Eq. (2.1) as one means of estimating $\langle n\rangle$ in Sec. III. We find it suggestive that the prediction (2.3) begins to diverge from the data at energies corresponding to the onset of hadron jets,[25] thus signalling the beginning of the "one-dimensional" regime. The separation is more pronounced if the choice $\langle n_{ch}(s)\rangle/\langle n(s)\rangle = 2/3$ is made.

The statistical model makes no unambiguous prediction for the distribution of multiplicities. Therefore several simple probability distributions have been tried. Koller and Walsh[17] assumed a Poisson distribution in n, suitably truncated. This probably overestimates high-multiplicity decay rates. The consequences of assuming a Poisson distribution in the variable $n - n_0$ have been elaborated for nonleptonic decays of charmed particles[20,21] and for the decays of psions.[22] In the latter work the predictions of the statistical approach were confronted with observations of visible (all charged particle) modes, without corrections for the unseen decays. We shall incorporate the neutral decays in our discussion of the data by means of statistical isospin weights.[27-31,17]

A second model with minimal dynamics was discussed by Gaillard, Lee, and Rosner[19] (GLR) in connection with the nonleptonic decays of charmed mesons. They assumed that the Lorentz-invariant matrix element for a decay into n + 1 particles is a constant times the matrix element for n-particle decay. On this picture, the ratio of partial widths for (n + 1)-particle and n-particle decays of a massive hadron α is

$$\frac{\Gamma\big(\alpha \to 1 + 2 + \ldots + (n+1)\big)}{\Gamma(\alpha \to 1 + 2 + \ldots + n)} = \frac{1}{(2\pi)^3 f^2} \frac{w_{\alpha:n+1}}{w_{\alpha:n}} \quad , \tag{2.4}$$

where f has dimensions of energy and the phase space weights are

$$w_{\alpha:n} = \int d^4p_1 \delta^{(+)}(p_1{}^2 - m_1{}^2) \ldots \int d^4p_n \delta^{(+)}(p_n{}^2 - m_n{}^2) \delta^4(p_\alpha - \sum_{i=1}^{n} p_i) \ . \tag{2.5}$$

In their original application, GLR used zero-mass phase space ($m_i \equiv 0$) and took the scale f of the matrix element to be equal to the current-algebra parameter for soft-pion emission, $f_\pi = 135$ MeV. (This value appears to be an overestimate, in part because of the neglect of emission of multipion resonances like ρ and ω.) Some minor improvements of estimates based upon the constant matrix element model for charmed meson decay have been made by Jackson,[32] who retained the kaon mass in decays $D \rightarrow K$ + pions, and in a preliminary report[21] of work which parallels the present study, in which f was chosen to reproduce the statistical model prediction of $\langle n \rangle$.

Here we rely less on a priori estimates and approximations. We have evaluated the multibody phase space weights numerically,[33] and we shall exploit the data on hadronic decays of $\psi(3095)$ to fix the value of f. Vanucci, et al.[15] have reported branching fractions for the direct decays into pions $\psi \rightarrow 3\pi$, 5π, 7π, 9π; for the decays $\psi \rightarrow KK\pi$, $KK2\pi$, $K\overline{K}3\pi$, $K\overline{K}4\pi$; and for the second-order electromagnetic decays $\psi \rightarrow 2\pi$, 4π, 6π. Corrections for unseen modes have been made according to the prescription of the statistical isospin model.[34] Separate fits of Eq. (4) to the three classes of decays yield a best value of $f^{-1} \approx 24\ \mathrm{GeV}^{-1}$ which slightly underestimates the branching ratios for high multiplicities. This value is also consistent with that obtained from preliminary data[35] on the branching fractions for the nonleptonic weak decays $D^0 \rightarrow \overline{K}\pi$, $\overline{K}2\pi$, $\overline{K}3\pi$.

We are able to perform one check on our assumption that the scale of the matrix element determined from $\psi(3095)$ decays can be applied to other psions. Preliminary data have been reported[36] on the relative branching ratios for $\chi(3410) \rightarrow 2\pi$, 4π, 6π; and $K\overline{K}$, $K\overline{K}2\pi$. These are well-fitted by the value $f^{-1} \approx 14\ \mathrm{GeV}^{-1}$, quite different from the value of $24\ \mathrm{GeV}^{-1}$ we found adequate for decays of $\psi(3095)$ and D(1867). We find the relative prominence of low multiplicities somewhat surprising, and we have no ready explanation for this apparent failure of our technique.

We shall now apply the constant matrix-element model and the statistical model with an assumed Poisson distribution to the hadronic decays of the pseudoscalar psions.

III. RESULTS: PREDICTED BRANCHING FRACTIONS FOR η_C

Because the psions are so massive, many different categories of decay channels are energetically open to them. We will consider here the decays $\eta_C \to$ pions, $K\overline{K}$ + pions, $K\overline{K}K\overline{K}$ + pions, η + pions, η' + pions, and $N\overline{N}$ + pions in the framework of both the constant matrix element model and the statistical model with a Poisson multiplicity distribution. For each category the apportioning of decays among various exclusive final states is prescribed, within either model, by the assumptions set forth in Section II. The distribution among charge states is computed using statistical isospin weights. In the case of the mesonic decays, simple assumptions of SU(3) invariance permit a prediction of the relative importance of different decay categories as well. Specifically, we assume that the reduced partial widths[37] for $\eta\pi\pi$, $\eta'\pi\pi$, and $K\overline{K}\pi$ occur in the ratio

$$\tilde{\Gamma}(\eta\pi\pi) : \tilde{\Gamma}(\eta'\pi\pi) : \tilde{\Gamma}(K\overline{K}\pi) : : 1 : 2 : 3 \qquad , \qquad (3.1)$$

and those for 4π, $K\overline{K}\pi\pi$, and $K\overline{K}K\overline{K}$ occur in the ratio

$$\tilde{\Gamma}(\rho\rho) : \tilde{\Gamma}(K^{*}\overline{K^{*}}) : \tilde{\Gamma}(\phi\phi) : : 3 : 4 : 1 \quad . \tag{3.2}$$

Relations of a similar character are in reasonable agreement with the data on decays of $\psi(3095)$.[15]

We give in Table I the predictions of the statistical model for the direct mesonic decays of η_C. The first of each pair of entries is for $\eta_C(2830)$; the second is for $\eta_C(3455)$. Predictions for a hypothetical $\eta_C(3095)$ may be obtained by interpolation. They are also shown graphically in Fig. 2. For our present purposes we have computed

$$\langle n\rangle = 2 + 0.528\left(\frac{M_{\eta_C} - M_A - M_B}{0.17\ \mathrm{GeV}/c^2}\right)^{3/4} \tag{3.3}$$

where M_A and M_B are the masses of the heaviest decay products within a category. We have then assumed[38] a Poisson multiplicity distribution in the variable $\langle n\rangle - 2$. In similar fashion, we give the predictions of the constant matrix element (phase space) model in Table II and Fig. 3, for $f^{-1} = 24\ \mathrm{GeV}^{-1}$. The predictions of our two models for decays $\eta_C \rightarrow N\overline{N}$ + pions are given in Tables III and IV, in which the entries represent the fractions of decays of this category alone.

IV. DISCUSSION

As was to be anticipated from the analogy with $\psi(3095)$ decays, both the recipes we have adopted suggest a vast number of competing decay channels. In this section we examine the results summarized in Figs. 2 and 3 and in Tables I - IV.

We notice first that the dominant channel among mesonic decays appears to be $\eta_C \to 2\pi^+ 2\pi^- 2\pi^0$, with a branching fraction of approximately 7 - 15%. Unfortunately, none of the experimental searches carried out so far has been able to reconstruct multineutral final states. The charged modes $\eta_C \to 2\pi^+ 2\pi^-$ and $\eta_C \to 3\pi^+ 3\pi^-$ occur with branching fractions of 2 - 5%. These are observable as 1-constraint fits in the reaction

$$e^+e^- \to \psi \text{ or } \psi' \to \gamma\, \eta_C \tag{4.1}$$
$$\hookrightarrow \text{charged particles},$$

or as 4-constraint fits in the Primakoff production reaction

$$\gamma A \to \eta_C A \tag{4.2}$$
$$\hookrightarrow \text{charged particles} \quad .$$

In the former case, one is at the mercy of a possibly minute branching ratio for the electromagnetic cascade.[11, 14] No signal has been observed for the X(2830)[14] or the $\chi(3455)$[39] in either mode. The disadvantage of

Primakoff production is the tiny cross section to be expected. For 200 GeV photons,

$$\sigma(\gamma A \to \eta_C A) \approx (0.1,\ 3,\ 16)\text{nb.} \times \frac{\Gamma(\eta_C \to \gamma\gamma)}{1\ \text{keV}} \tag{4.3}$$

for beryllium, copper, and lead targets.[40]

If (nearly) all branching fractions are on the order of a few percent, some advantage may be gained by searching channels in which low backgrounds are expected. From Tables III and IV, we notice that the $p\bar{p}$ mode accounts for about 5% of all decays $\eta_C(2830) \to N\bar{N}$ + pions. The 90% confidence level upper limit[41] on the product of branching ratios for the cascade

$$\begin{array}{l} \psi \to \gamma X(2830) \\ \qquad\quad \llcorner\!\!\to p\bar{p} \end{array}$$

of 4×10^{-5} leads to an upper limit of about 8×10^{-4} for the chain

$$\begin{array}{l} \psi \to \gamma X(2830) \\ \qquad\quad \llcorner\!\!\to N\bar{N} + \text{pions} \end{array} ,$$

which is uninviting. The relative importance of various baryonic modes can be estimated by an elementary SU(3) calculation, corrected for phase space effects. The rates for decays of $\eta_C(2830)$ into the two-body modes $\bar{p}p$ or $\bar{n}n$, $\bar{\Lambda}\Lambda$, $\bar{\Sigma}^-\Sigma^+$ or $\bar{\Sigma}^0\Sigma^0$ or $\bar{\Sigma}^+\Sigma^-$, $\bar{\Xi}^0\Xi^0$ or $\bar{\Xi}^+\Xi^-$ should be in the proportions 2.1, 1.8, 1.5, 1. The statistical model described in Sec. II then leads to the following semi-inclusive rates as fractions of the total baryonic decay rate of $\eta_C(2830)$:

$\Gamma(N\overline{N}$ + pions)	44%
$\Gamma(\Lambda\overline{\Lambda}$ + pions)	7%
$\Gamma(\Lambda\overline{\Sigma}$ or $\overline{\Lambda}\Sigma$ + pions)	24%
$\Gamma(\Sigma\overline{\Sigma}$ + pions)	20%
$\Gamma(\Xi\overline{\Xi}$ + pions)	6%

On this basis the $\overline{p}p$ mode is approximately 2% of all the baryonic decays. For $\psi(3095)$, the $\overline{p}p$ mode makes up about 0.3% of the direct hadronic decays.[42] This suggests that the semi-inclusive baryonic decays of $\eta_C(2830)$ make up about 15% of its hadronic width.

Another channel in which signal-to-noise may be favorable is $\eta_C \rightarrow K\overline{K}K\overline{K}$, a special case of which ($\phi\phi$) has been noticed by Lipkin (private communication). Finally, the decays $\eta_C \rightarrow \pi^+\pi^-(\eta$ or $\eta')$ have distinctive signatures. If η_C is not, as we have assumed, a pure SU(3) singlet but mixes appreciably[43] with η and η', the rates may be significantly larger than our estimates.

An additional complication may occur in the case of $\chi(3455)$, the candidate for η_C'. Just as the decay $\psi' \rightarrow (\pi\pi)_{s\text{-wave}}\,\psi$ is much more important than any direct decay $\psi' \rightarrow$ hadrons, we may anticipate a large rate for the decay $\eta_C' \rightarrow (\pi\pi)_{s\text{-wave}}\,\eta_C$.[4] [The analog of the other important cascade, $\psi' \rightarrow \eta\psi$ is forbidden by parity for the $\eta_C - \eta_C'$ system.] This will lead to a reduction of the semi-inclusive branching fraction for direct decays $\eta_C' \rightarrow$ hadrons, making the reconstruction of η_C' even more difficult. One may contemplate, however, the use of such a cascade decay for missing-mass detection of η_C in the chain

$$\begin{tikzcd} e^+e^- \arrow[r] & \psi' \arrow[r] & \gamma\eta_C' \arrow[d] & \\ & & {} \arrow[r] & (\pi\pi)^0 + \text{missing mass} \quad . \end{tikzcd} \tag{4.4}$$

As our results for η_C are not a large extrapolation from known decays of massive particles like ψ and D, we expect that the predictions recorded here will reliably anticipate the properties of hadronic decays of the pseudoscalar hidden charm states. Significant deviations from these expectations would point to the presence of important dynamical effects or (much more speculatively) might hint that the states in question are not levels of the $c\bar{c}$ spectrum. Hadronic branching ratios of the pseudoscalar psions are expected to be small for individual exclusive channels. It is important to carry out sensitive searches for these decays. In the absence of unambiguous observations, stringent upper limits combined with our tables and knowledge of the inclusive γ-spectra for ψ and ψ' decays will provide important new evidence on the nature of the states at 2830 and 3455 MeV/c^2.

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[37] By reduced partial width we mean the partial width divided by the appropriate phase space weight. For the vector meson final states we have used p-wave phase space.

[38] This prescription is not particularly compelling. Its chief virtue is that it gives mean multiplicities in reasonable agreement with the constant

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Table I. Decays of $\eta_C(2830)$ and $\eta_C(3455)$ into mesons, according to the statistical model $\left[\langle n_{ch}\rangle = 3.9 - 4.4\right]$.

	Decay Mode	All charge states	Specific charge states			
			π^0 0	2	4	6
Pions 31 - 29 % $\langle n\rangle = 6.0 - 6.7$	4π	9 - 6 %	4 - 2	5 - 3	1 - 0	
	6π	12 - 11	2 - 2	8 - 7	2 - 2	---
	8π	7 - 8	1 - 1	4 - 4	2 - 3	---
	$\geq 10\pi$	2 - 4				
			π^0 0	1	2	3
KK + pions[a)] 44 - 44% $\langle n\rangle = 5.2 - 5.9$	$K\bar{K}\pi$	6 - 4 %	4 - 2	2 - 1		
	$K\bar{K}2\pi$	10 - 7	5 - 3	3 - 2	2 - 1	
	$K\bar{K}3\pi$	10 - 9	3 - 3	5 - 4	2 - 2	1 - 0
	$K\bar{K}4\pi$	8 - 9	2 - 2	3 - 3	3 - 3	1 - 1
	$K\bar{K}5\pi$	5 - 7	1 - 1	2 - 2	2 - 2	1 - 2
	$K\bar{K} \geq 6\pi$	5 - 9				
			$2(K^+K^-)$	$K^+K^-K^0\bar{K}^0$	$2(K^0\bar{K}^0)$	
$2(K\bar{K})$ 1.7 - 1.4%	$K\bar{K}K\bar{K}$	1.7 - 1.4%	0.3-0.2	1.1-0.9	0.3-0.2	
			π^0 0	2	4	
η + pions 13 - 13% $\langle n\rangle = 5.5 - 6.3$	$\eta 2\pi$	3 - 1 %	2 - 1	1 - 0		
	$\eta 4\pi$	5 - 4	2 - 2	3 - 2	---	
	$\eta 6\pi$	3 - 4	1 - 1	2 - 2	1 - 1	
	$\eta \geq 8\pi$	2 - 3				
			π^0 0	2	4	
η' + pions 11 - 12% $\langle n\rangle = 5.0 - 5.8$	$\eta' 2\pi$	3 - 2 %	2 - 1	1 - 1		
	$\eta' 4\pi$	5 - 5	2 - 2	3 - 3	---	
	$\eta' 6\pi$	2 - 4	0 - 1	1 - 2	0 - 1	
	$\eta' \geq 8\pi$	1 - 2				

a) Each allowed $K\bar{K}$ charge state occurs with half the probability tabulated under "specific charge states."

Table II. Decays of $\eta_C(2830)$ and $\eta_C(3455)$ into mesons, according to the constant matrix element (phase space) model $\left[\langle n_{ch}\rangle = 3.5 - 4.1\right]$.

	Decay Mode	All charge states	Specific charge states			
			π^0 0	2	4	6
Pions 37 - 42% $\langle n\rangle$ = 5.4 - 6.3	4π	14 - 6 %	5 - 2	7 - 3	1 - 0	
	6π	20 - 24	4 - 5	13 - 15	3 - 4	---
	8π	3 - 11	0 - 1	2 - 6	1 - 4	---
	10π	0 - 1				
			π^0 0	1	2	3
$K\bar{K}$ + pions [a)] 39 - 36% $\langle n\rangle$ = 4.5 - 5.4	$K\bar{K}\pi$	6 - 2 %	4 - 1	2 - 1		
	$K\bar{K}2\pi$	14 - 7	7 - 3	5 - 2	2 - 1	
	$K\bar{K}3\pi$	13 - 11	4 - 3	6 - 5	2 - 2	1 - 1
	$K\bar{K}4\pi$	5 - 10	1 - 2	2 - 4	2 - 3	0 - 1
	$K\bar{K}5\pi$	1 - 5	0 - 1	0 - 2	0 - 1	0 - 1
	$K\bar{K}6\pi$	0 - 1				
			$2(K^+K^-)$	$K^+K^-K^0\bar{K}^0$		$2(K^0\bar{K}^0)$
$2(K\bar{K})$ + pions 3.2 - 3.0% $\langle n\rangle$ = 4.2 - 4.7	$K\bar{K}K\bar{K}$	2.6 - 1.4%	0.4-0.2	1.7-0.9		0.4-0.2
	$K\bar{K}K\bar{K}\pi$	0.6 - 1.2				
	$K\bar{K}K\bar{K}2\pi$	0 - 0.4				
			π^0 0	2	4	6
η + pions 13 - 12% $\langle n\rangle$ = 4.8 - 5.8	$\eta 2\pi$	3 - 1 %	2 - 0	1 - 0		
	$\eta 4\pi$	9 - 7	4 - 3	5 - 4	1 - 0	
	$\eta 6\pi$	2 - 5	0 - 1	1 - 3	0 - 1	---
	$\eta 8\pi$	---				
			π^0 0	2	4	6
η' + pions 8 - 7% $\langle n\rangle$ = 4.2 - 5.2	$\eta' 2\pi$	3 - 1 %	2 - 1	1 - 0		
	$\eta' 4\pi$	4 - 5	2 - 2	2 - 3	---	
	$\eta' 6\pi$	0 - 1	---	0 - 1	---	
	$\eta' 8\pi$	---				

a) Each allowed $K\bar{K}$ charge state occurs with half the probability tabulated under "specific charge states."

Table III. Decays of $\eta_C(2830)$ and $\eta_C(3455)$ into $N\bar{N}$ + pions, according to the statistical model[a)]

Mean Multiplicity, All Modes	Decay Mode	All charge states	Specific charge states: neutral pions				
			0	1	2	3	4
3.9 - 4.8	$N\bar{N}$	15 - 6%	15- 6				
	$N\bar{N}\pi$	28 - 17	19-11	9- 6			
	$N\bar{N}2\pi$	27 - 24	14-12	9- 8	4-4		
	$N\bar{N}3\pi$	17 - 22	5- 7	8-10	3-4	0-1	
	$N\bar{N}4\pi$	8 - 16	2- 3	3- 6	3-5	0-1	---
	$N\bar{N}5\pi$	3 - 9	0- 1	1- 3	1-3	1-1	---
	$N\bar{N}6\pi$	1 - 4	---	0- 1	0-1	0-1	---
	$N\bar{N} \geq 7\pi$	0 - 2					

a) Each allowed $N\bar{N}$ charge state occurs with half the probability tabulated under "specific charge states."

Table IV. Decays of $\eta_C(2830)$ and $\eta_C(3455)$ into $N\bar{N}$ + pions, according to the constant matrix element (phase space) model.$^{a)}$

Mean Multiplicity, All Modes	Decay Mode	All charge states	Specific charge states: neutral pions				
			0	1	2	3	4
3.1 - 4.1	$N\bar{N}$	20 - 3%	20- 3				
	$N\bar{N}\pi$	52 - 24	35-16	17- 8			
	$N\bar{N}2\pi$	25 - 42	12-21	8-14	4-7		
	$N\bar{N}3\pi$	3 - 24	1- 7	1-11	1-5	0-1	
	$N\bar{N}4\pi$	0 - 6	0- 1	0- 2	0-2	0-1	---
	$N\bar{N}5\pi$	0 - 1					

a) Each allowed $N\bar{N}$ charge state occurs with half the probability tabulated under "specific charge states."

FIGURE CAPTIONS

Fig. 1: Average number $\langle n_{ch} \rangle$ of charged tracks detected in $e^+e^- \rightarrow$ hadrons, from Schwitters and Strauch, Ref. 26, p. 138. Solid line: $\langle n_{ch} \rangle = 1.93 + 1.5 \log(\sqrt{s}/1\text{GeV})$ (Feldman and Perl, Ref. 26). Dash-dotted line: model described in text [Eq. (2.3)], $E_0 = 0.17$ GeV. Dashed line: same for $E_0 = 0.2$ GeV.

Fig. 2: Predictions of the statistical model for mesonic decays of a hypothetical $\eta_C(3095)$. The mean multiplicity $\langle n \rangle$ is estimated via Eq. (3.3), and a Poisson distribution in $n - 2$ is assumed.

Fig. 3: Predictions of the constant matrix element (phase space) model for mesonic decays of a hypothetical $\eta_C(3095)$. Scale of matrix element in Eq. (2.4) is taken as $f^{-1} = 24$ GeV^{-1} on the basis of fits to ψ decays described in text.

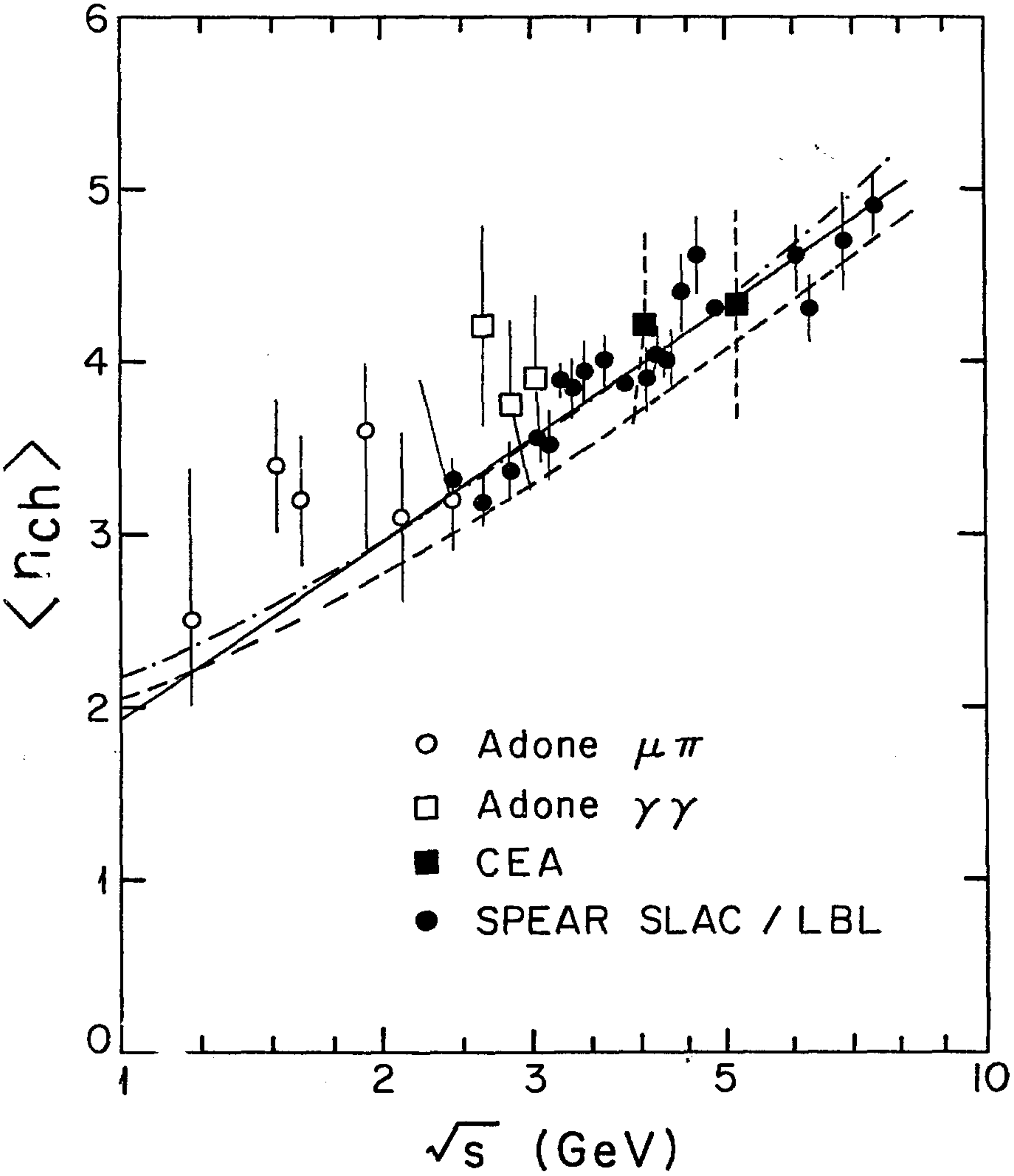
6
5
4
3
2
1
0
⟨ n_{ch} ⟩
1
2
3
5
10
$\sqrt{s}$ (GeV)
○ Adone $\mu\pi$
□ Adone $\gamma\gamma$
■ CEA
● SPEAR SLAC / LBL

Fig. 1

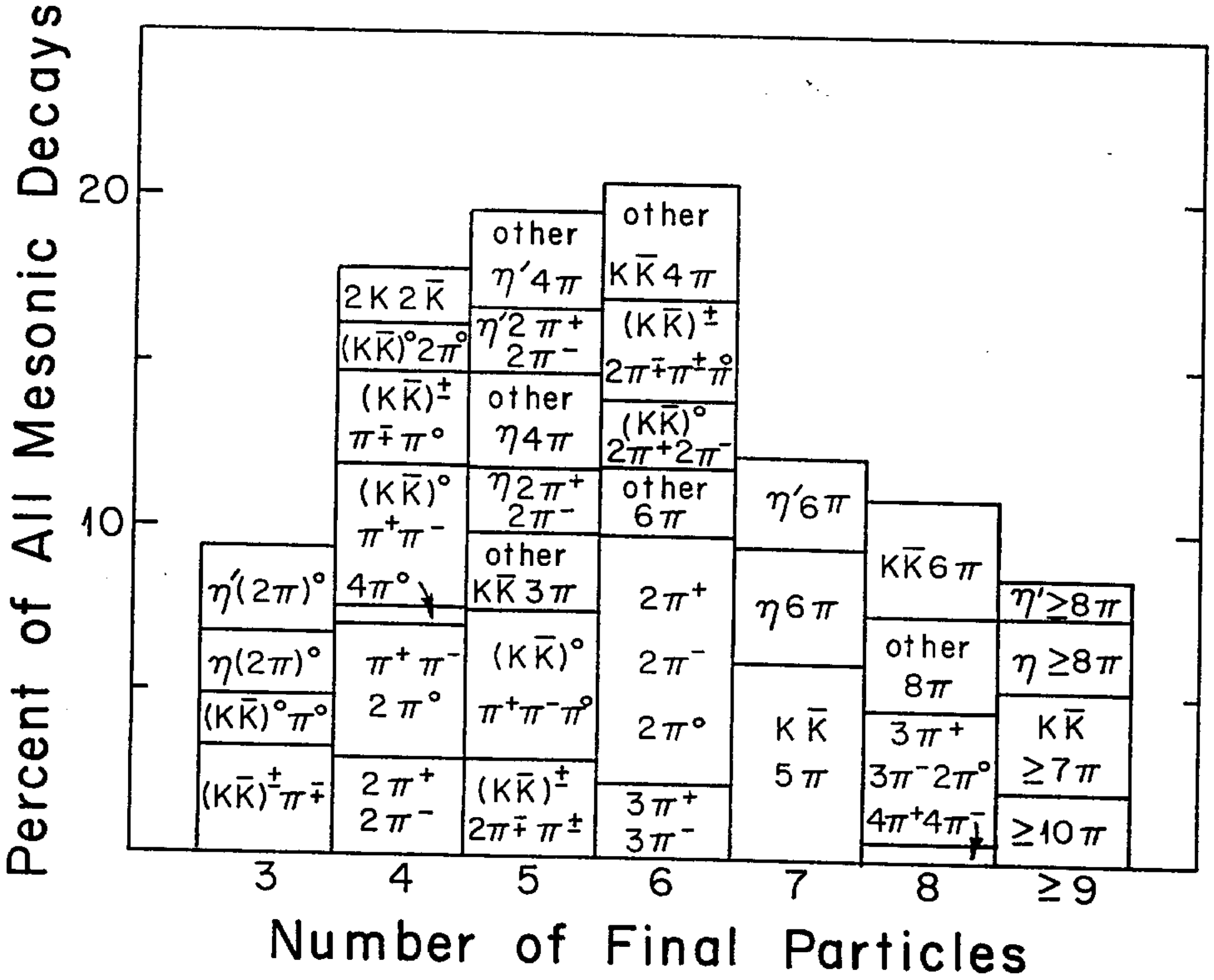
Percent of All Mesonic Decays
20
10
$\eta'(2\pi)^\circ$
$\eta(2\pi)^\circ$
$(K\bar{K})^\circ\pi^\circ$
$(K\bar{K})^\pm\pi^\mp$
$2K2\bar{K}$
$(K\bar{K})^\circ 2\pi^\circ$
$(K\bar{K})^\pm$ $\pi^\mp\pi^\circ$
$(K\bar{K})^\circ$ $\pi^+\pi^-$
$4\pi^\circ$
$\pi^+\pi^-$ $2\pi^\circ$
$2\pi^+$ $2\pi^-$
other $\eta'4\pi$
$\eta'2\pi^+$ $2\pi^-$
other $\eta 4\pi$
$\eta 2\pi^+$ $2\pi^-$
other $K\bar{K}3\pi$
$(K\bar{K})^\circ$ $\pi^+\pi^-\pi^\circ$
$(K\bar{K})^\pm$ $2\pi^\mp\pi^\pm$
other $K\bar{K}4\pi$
$(K\bar{K})^\pm$ $2\pi^\mp\pi^\pm\pi^\circ$
$(K\bar{K})^\circ$ $2\pi^+2\pi^-$
other 6π
$2\pi^+$ $2\pi^-$ $2\pi^\circ$
$3\pi^+$ $3\pi^-$
$\eta'6\pi$
$\eta 6\pi$
$K\bar{K}$ 5π
$K\bar{K}6\pi$
other 8π
$3\pi^+$ $3\pi^-2\pi^\circ$
$4\pi^+4\pi^-$
$\eta'\geq 8\pi$
$\eta\geq 8\pi$
$K\bar{K}$ $\geq 7\pi$
$\geq 10\pi$
3
4
5
6
7
8
≥ 9
Number of Final Particles

Fig. 2

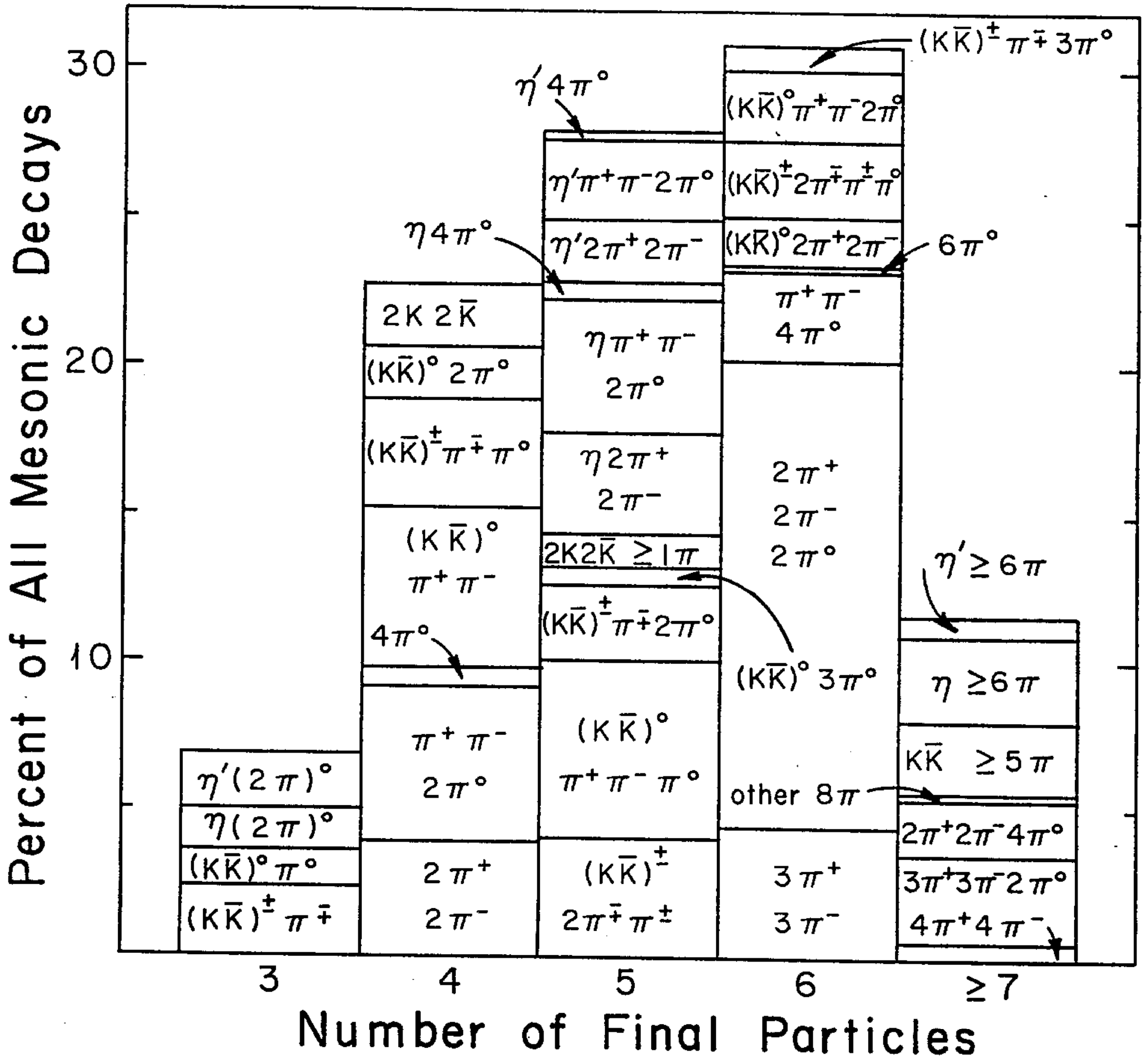
Percent of All Mesonic Decays
Number of Final Particles
30
20
10
3
4
5
6
≥7
$\eta'(2\pi)^0$
$\eta(2\pi)^0$
$(K\bar{K})^0\pi^0$
$(K\bar{K})^\pm\pi^\mp$
$2K2\bar{K}$
$(K\bar{K})^0 2\pi^0$
$(K\bar{K})^\pm\pi^\mp\pi^0$
$(K\bar{K})^0 \pi^+\pi^-$
$4\pi^0$
$\pi^+\pi^- 2\pi^0$
$2\pi^+ 2\pi^-$
$\eta' 4\pi^0$
$\eta'\pi^+\pi^- 2\pi^0$
$\eta' 2\pi^+ 2\pi^-$
$\eta 4\pi^0$
$\eta\pi^+\pi^- 2\pi^0$
$\eta 2\pi^+ 2\pi^-$
$2K2\bar{K} \geq 1\pi$
$(K\bar{K})^\pm\pi^\mp 2\pi^0$
$(K\bar{K})^0 \pi^+\pi^-\pi^0$
$(K\bar{K})^\pm 2\pi^\mp\pi^\pm$
$(K\bar{K})^\pm\pi^\mp 3\pi^0$
$(K\bar{K})^0\pi^+\pi^- 2\pi^0$
$(K\bar{K})^\pm 2\pi^\mp\pi^\pm\pi^0$
$(K\bar{K})^0 2\pi^+ 2\pi^-$
$6\pi^0$
$\pi^+\pi^- 4\pi^0$
$2\pi^+ 2\pi^- 2\pi^0$
$(K\bar{K})^0 3\pi^0$
other 8π
$3\pi^+ 3\pi^-$
$\eta' \geq 6\pi$
$\eta \geq 6\pi$
$K\bar{K} \geq 5\pi$
$2\pi^+ 2\pi^- 4\pi^0$
$3\pi^+ 3\pi^- 2\pi^0$
$4\pi^+ 4\pi^-$

Fig. 3