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CLUSTERS
AND THE RAPIDITY INTERVAL METHOD

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ABSTRACT

An effective method of analyzing the experimental jets with the aim of discovering the clusterization phenomenon of particles produced in high energy inelastic processes is proposed. The method exploits the set of all possible rapidity intervals i.e. rapidity differences of two particles $y_{i+k+1} - y_i$ with some k particles having rapidities in between (usual gap corresponds to $k=0$). It provides the information which is in principle as complete as that obtained from the correlation functions. It is simple in application and physically clear..

The method is applied to various models and the rapidity interval distributions thus obtained are compared to experimental data for $\pi^- p$ 40 GeV and pp 70 and 200 GeV collisions. The important role of clusterization of secondary particles follows from this comparison. It is predicted that the two-bump structure should develop for $k \geq 4$ and large multiplicities in pp-interactions at 200 GeV if two baryonic clusters are produced.

1. INTRODUCTION

The particles produced in high energy inelastic collisions are seemingly created in a two-step process - with an intermediate stage of the production of the correlated groups-clusters. However, up to now the problem of the clusters' nature can not be considered as solved.

Unlike the widely spread opinion that clusters could be reduced to the set of prominent resonances we believe that besides such trivial two-or three-particle correlations there exist many-particle correlations which can be understood as a result of the production of fireball-type clusters (see, for example, [1,2]).

To discriminate between these two cases we propose the method of analyzing jets by the investigation of all available distributions of rapidity intervals.

The rapidity interval ${}^{n}r_k$ is defined as a difference of the rapidities of two particles in an n-charged-particle event x) with some k particles falling between these two particles' rapidities i.e.

$$ {}^{n}r_k = y_{i+k+1} - y_i \qquad (0 \leq k \leq n-2) \tag{1} $$

where $1 \leq i \leq n-k-1$. The distributions of such invervals are obtained by summing up, first all possible choices of

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x) All particles are ordered according to the increasing rapidity i.e. $y_{i+1} > y_i$.

pairs of particles (the sum over i) in each event with a given n and then all events together.

These distributions appear to be very sensitive to the clusterization of particles and could be compared with theoretical ones obtained within different assumptions about the clusters.

Most of the previously proposed methods of the clusterization analysis need careful treatment and classification of an individual event or assume the prevailing role of a definite mechanism of particle production [2]. The two-particle correlation (see, for example, [3]) and rapididy gap [4] methods, as well as partly the fluctuation analysis [5], do not suffer these shortcomings.

Our method is more general and provides more information than the two-particle corellation or rapidity gap x) methods. To exploit it one should know the rapidities of the particles produced.

The rapidity interval distributions have been obtained for different theoretical models which, in particular, take into account possible rapidity overlapping of clusters.

We have applied this method to $\pi^- p$-events at 40 GeV and to pp-events at 70 and 200 Gev and compared the experi-

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x) The rapidity gap is defined as an empty interval (i.e. k=0 in formula (1)). Thus our method generalizes the rapidity gap method.

mental and theoretical distributions. We have come to the conclusion that the non-resonant clusters should be taken into account to get the agreement with experiment. To check the extreme cluster model it is proposed to get the experimental distribution of intervals ${}^{10}\Gamma_4$ or ${}^{10}\Gamma_5$ at 200 GeV. We claim that it should possess two bumps if two heavy πN-clusters are produced at that energy.

Some other (but less distinctive) features of cluster production models are also seen.

First ,we derive the analytic expressions for the rapidity interval distributions in two simplified models of independent particle production and of independent cluster production. Then we get the similar distributions in two versions of the multiperipheral model both of which were previously succesfully applied to the desctiption of other experimental data. And, finally, by comparing the whole theoretical information with the experimental results we come to the conclusions stated above.

2. Independent particle production

The independent particle production model is the simplest although unrealistic one [6,7] .However for completeness we shall briefly discuss it. It is assumed that each particle in an n-charged-particle event is produced independently, the rapidities being randomly distributed within the physical rapidity region $0 \le y \le Y = \ln \frac{2E}{M}$ (E and M being the lab energy and mass of a colliding particle) with a constant density x).

In terms of relative rapidities $\Delta = y/Y$ the problem is reduced to the distribution of intervals between n points appearing at random inside the unit interval. As is known from the probability theory (see, for example, [8]) the simple geometrical arguments show that the probability for any two particles to lie at a distance Δ within the unit interval is proportional to $1-\Delta$. The probability for k particles to lie within Δ and for $n-k-2$ particles to lie outside it is equal to $\Delta^k (1-\Delta)^{n-k-2}$. Consequently, one gets [6] the polynomial distribution of intervals Δ :

$$\frac{dw_k^{(n)}}{d\Delta} = n C_{n-1}^{k} \Delta^k (1-\Delta)^{n-k-1} \qquad (2)$$

where $n C_{n-1}^{k}$ is the combinatorical coefficient.

The positions of the maxima of the distributions grow linearly with k and decrease like an inverse power of n :

$$\Delta_{max} = \frac{k}{n-1} \qquad (3)$$

x) The rapidity plateau is predicted by the multiperipheral theory and roughly agrees with experiment (see, for example, [2]).

We shall show below (see ch.5) that the independent particle production model results in too large rapidity intervals as compared to the experimental ones.

3. Independent cluster production.

The main shortcoming of the independent particle production model, the large rapidity intervals, could be overcome if the particles are produced via correlated groups-clusters. Accordingly, the problem is reduced to the distribution of the rapidity intervals if the "combs" are thrown at random over the unit interval. Each comb represents the cluster and the teeth of a comb correspond to particles in the cluster. The combs may overlap. The rapidity intervals are the intervals between the teeth ordered according to the rapidity.

To avoid complications due to the finite width of a comb we consider the infinite interval of rapidities ($Y \to \infty$) in which clusters are randomly distributed with a constant density. Let each cluster decay into m particles and the number of clusters follow the Poisson law [x]. Thus the model can be considered as a simplified version of the multiperipheral cluster theory at an infinite energy.

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x) It is easy to generalize the situation.

Now, the probability for all m particles of the cluster centered at $\hat{y}$ to escape the interval (y' , y'') is given by

$$w = \left(1 - \int_{y'}^{y''} D(y-\hat{y})\, dy\right)^{m} \tag{4}$$

where $D(y-\hat{y})$ is the decay distribution of particles generated within the cluster. For the isotropic decay it can be written as

$$D(y-\hat{y}) = \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[-(y-\hat{y})^2/2\delta^2\right] \tag{5}$$

with $\delta \approx 0.8 \div 0.9$.

The probability for both the particles to lie at the edges of the interval (y' , y'') is given by the derivative of (4) $-\frac{\partial^2 w}{\partial y' \partial y''}$, each differentiation producing the factor $D(y^{(i)}-\hat{y})$ and reducing m by 1.

The probability for k particles to lie within that interval could be obtained by replacing -1 in front of integral in (4) by the constant α , differentiating with respect to α k times and replacing it again by -1.

Each time the factor $\int_{y'}^{y''} D(y-\hat{y})\, dy$ describing the probability for a particle to lie within the interval (y' , y'') appears in front of (4) and the power in formula (4) is reduced and corresponds to the number of the remaining particles. The whole procedure could be described by the generating function x)

$$G(\alpha,r) = \exp\left\{\rho\int_{-\infty}^{\infty}d\hat{y}\left[\left(1+\alpha\int_0^r D(y-\hat{y})dy\right)^n - 1\right]\right\} \quad (6)$$

where we have summed up over all clusters and replaced y' and y'' by 0 and r because the whole expression depends on the difference of $y''-y'=r$ only.

The inclusive distribution of rapidity intervals r_k is given by the formula [9]:

$$w_k(r) = \frac{\partial^k}{\partial\alpha^k}\left[\frac{1}{\rho m\alpha^2}\frac{\partial^2}{\partial r^2}G(\alpha,r)\right]\Bigg|_{\alpha=-1} . \quad (7)$$

From formulae (5)-(7) it is easy to show that the rapidity gaps $(k=0)$ are distributed [9] according to

$$w_0(r) \sim \exp[-\rho m r] \quad (8)$$

at small r, and according to [4]

$$w_0(r) \sim \exp[-\rho r] \quad (9)$$

at large values of r.

The intervals containing k particles possess the distribution with a maximum whose position linearly increases with k and decreases with n but it is noticeably shifted

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x) Therefore, this method is as complete as the correlation function method where the whole set of many-particle correlations is used.

to smaller values of Γ as compared to the independent particle production model.

The models considered above provide an analytical guide to the cluster problem but suffer many shortcomings. The most important ones are the incorrect treatment of conservation laws, the ad hoc chosen distributions of clusters and of particles within clusters, the improper multiplicity distribution etc. It was shown in [10] that the numerical values of ρ and m are strongly changed if one takes into account the multiplicity distribution properly. That is why one cannot have any confidence in the numerical values thus obtained and their proper interpretation even though the general trends are demonstrated clearly and elegantly.

4. <u>Multiperipheral models</u>

Next we consider two versions of the multiperipheral model which were previously applied to inclusive distributions and other experimental results and appeared to be successful.

They were described in detail elsewhere (see the first version in [2,11], the second version in [12-15]) and here we just review them briefly. Besides the resonances clusters could be produced peripherally in both of them but the second version emphasizes this possibility stronger than the first one. The cluster's decay is described by statistical laws (in particular, it is isotropic; see (5)).

The first version emphasizes the production of the nonperipheral cluster decaying according to the hydrodynamical theory. The production of clusters in both the versions is necessary to get the agreement with experimental multiplicity distributions. The resonances considered alone give rise to extremely narrow multiplicity distribution and low average multiplicity.

The computer calculations provide the samples of "theoretical events" for both the versions equivalent to (or even more complete than) the experimental samples of jets in bubble chambers. The exclusive information about any event (up to 16-prong events) is available and any distribution can be calculated and printed by the computer. The multiperipheral dynamics and the phase space volume are correctly taken into account. Thus no shortcomings of the previous models appear but an analytical treatment is impossible.

5. Theory and experiment

To get the qualitative guide we shall compare first the results of analytical models with the experimental distributions of the rapidity gaps' length at the highest available energy. It was shown in [4] that the tail of the distribution of empty rapidity intervals $(k = 0)$ suggests the value of the parameter ρ close to 1 (see formula (9)). Using this value of ρ one can estimate [9] the parameter m by comparing the experimental curves at small r (see [4]) with formula (8). One gets the values for the number of the charged particles emitted in average per cluster equal to

2,3, 2,6 and 3,0 at the energy 102, 200 and 400 GeV respectively. As is shown in [10], these estimates could be considered just as the lowest limits of actual values if the multiplicity distribution is properly taken into account. Therefore we conclude that even the lowest limits indicate that the agreement with experiment could be restored only if the proper clusters-fireballs are created. The slight increase of the quoted numbers with energy could correspond to the slight increase of the average mass of clusters in that energy region.

However the rapidity gaps appear to be not the most sensitive indication of the clusterization phenomenon. The distributions of intervals with some particles inside them ($k \neq 0$) strongly contradict the independent particle production hypothesis [7]. We demonstrate this statement in Fig.1, where the experimental distributions of $^{8}\Delta_{k}$ strongly differ from the curves calculated according to formula (2). The inclusive distributions in independent cluster production model were calculated with the values of parameters $\rho = 1$ and $m = 2$. They may be compared with semiinclusive data of Fig.1 because n=8 is close to the average charged particles multiplicity at 200 GeV. The agreement is in general much better here than for the independent particle model. The shift of maxima to the left is clearly demonstrated. However we would like to stress once more that the actual values of parameters ρ and m could still differ from the adopted ones. We are using these models just to show the main qualitative features of the role of clusters. Nevertheless we believe that the necessity of clusterization is seen already from Fig.1.

Next we turn to the multiperipheral models. Both the versions were extensively compared [11-15] with traditional distributions for π^-p-processes at 40 GeV and for pp-interactions at 70 and 200 GeV and showed no strong discrepancy in any of them. More than twenty inclusive and semi-inclusive characteristics were compared with experimental data at each energy. No clear distinction between the distributions given by the two models was visible until the rapidity interval distributions were used [15]. According to this criterium the second version (with clusters) is preferable at 40 and 70 GeV as is illustrated in Figs. 2 and 3.

The similar distributions were obtained for pp-events at 200 GeV for the second version (see Fig. 4). The maxima of the theoretical distributions appeared to be shifted slightly to the right.This shows that the clusterization is still insufficient even for the second version. The most intefesting feature is the appearence of shoulders in theoretical curves at that energy. These shoulders are due to the precess of production of two rather heavy clusters with the baryon number B=1 which move rather fast and therefore can be separated. This effect is very similar to the original proposal of fireball production [1]. However, there is a remarkable difference. The cosmic ray people claim [1] that the boson-type fireballs are created but the colliding baryons are not strongly excited forming just the resonance states. In the developed scheme (second version) two baryonic clusters can be produced. This process is just of a Nova-type [16]. It should result in two-bump structure in some distributions. We have analysed the theoretical distributions for different mechanisms of particle production (two clusters; one cluster and resonances etc). At present we believe that the most effective way to separate the two-cluster process at 200 GeV is to choose the distributions ${}^{10}\Gamma_{4;5}$ for the events with protons having $|x| < 0.4$ x). They should possess the pronounced two-bumps' structure as it is shown in Fig.5 if the Nova-type process exists.

In general, the similar structure should be noticeable for k close to n/2 for any n close to or larger than the average multiplicity.

At 40 and 70 GeV this substructure is still undeveloped due to the small relative velocities of clusters.

The other way to separate clusters at 200 GeV is to choose the events with negative pions having $|x| < 0.1$. At larger values of x most pions are due to the resonance decay. Thus pions with $|x| < 0.1$ should be correlated with protons with $|x| < 0.4$.

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x) This is necessary to diminish the role of the process with resonances.

6. Conclusions

1) The rapidity interval method (RIM) can be used for the analyses of the clusterization phenomenon.

2) The simplest models indicate qualitatively that the clusters heavier than resonances should be produced.

3) This method provides the best test to discriminate the models with different percentage of clusters if all the rest of their predictions almost coincide.

4) The bump-like structure should develop in some distributions of rapidity intervals if nucleonic clusters are created.

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Figure captions

Fig. 1. The comparison of experimental rapidity intervals in 8-prong pp-events at 200 GeV (points) with the independent particle production model (curves I drawn according to formula (22)) and with the inclusive distribution in the independent cluster model (curves II drawn according to formula (7) at $\rho = 1$, $m = 2$, $\delta = 0.85$) suggests the important role of clustering of produced particles.

Fig. 2. The comparison of experimental rapidity intervals (points) in $\bar{\pi} p$-events at 40 GeV with two versions (I and II) of the multiperipheral model favours the cluster version (II).

Fig. 3. The comparison of experimental rapidity intervals in pp-events at 70 GeV with two versions of the multiperipheral model favours the cluster version (II) also.

Fig. 4. Experimental distributions (points) of rapidity intervals in pp-events at 200 GeV are shifted to the left as compared to the theory predictions according to the second (cluster) version of multiperiphera-lity that suggests some shortage of clusters in this version. The "shoulders" of theoretical distributions are pronounced.

Fig. 5. The theoretical distributions of rapidity intervals $,^{10}r_3$ for two-cluster graphs in the second (cluster) version of the multiperipheral model reveal two-bump structure (b).

The similar but less pronounced substructure is seen for other graphs (a), (c).

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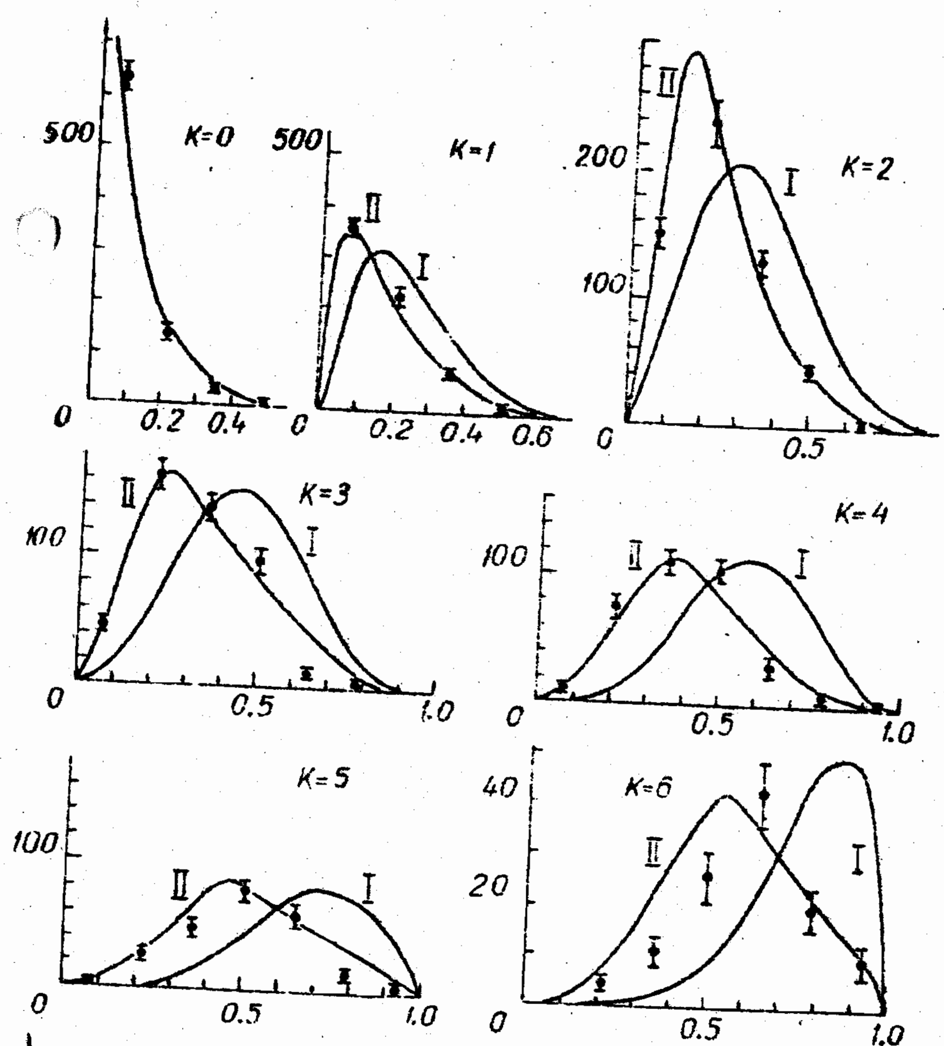


K=0
500
0
0.2
0.4
K=1
500
0.2
0.4
0.6
K=2
200
100
0.5
K=3
100
0.5
1.0
K=4
100
0.5
1.0
K=5
100
0.5
1.0
K=6
40
20
0.5
1.0

Fig. 1

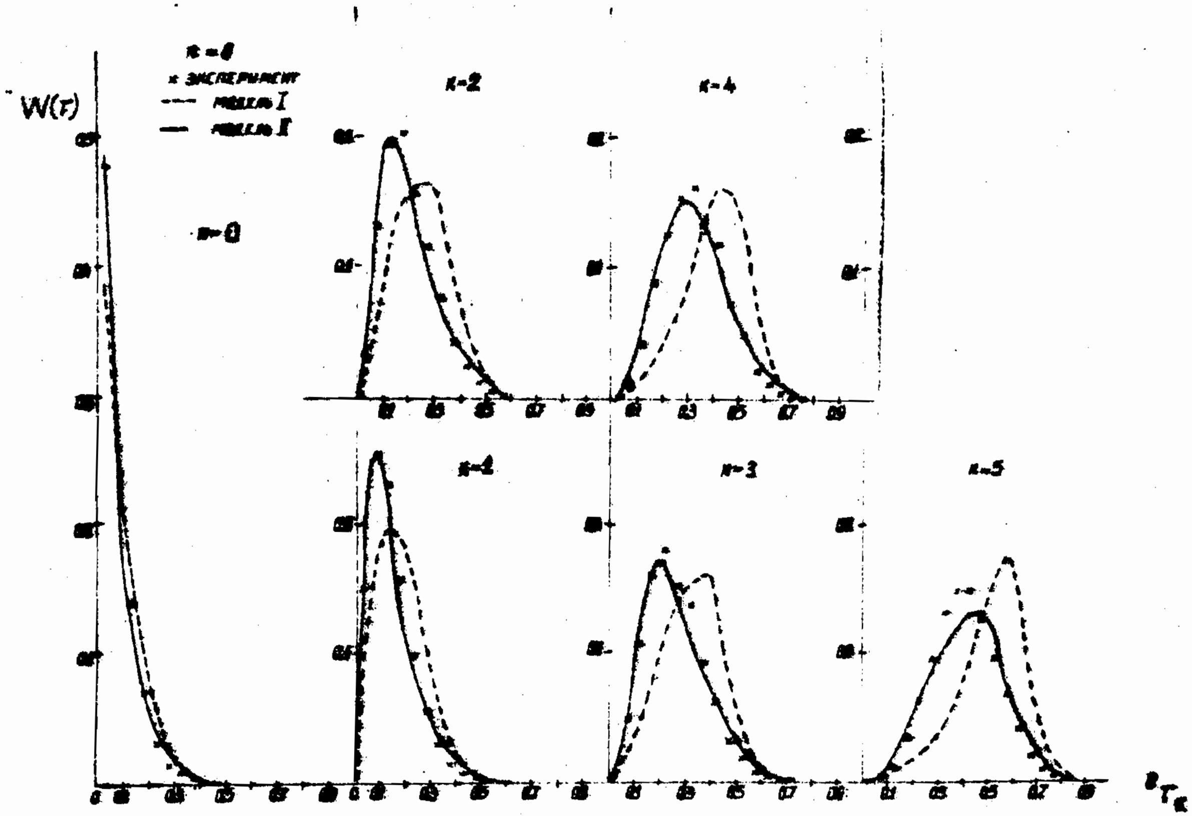


Fig. 2

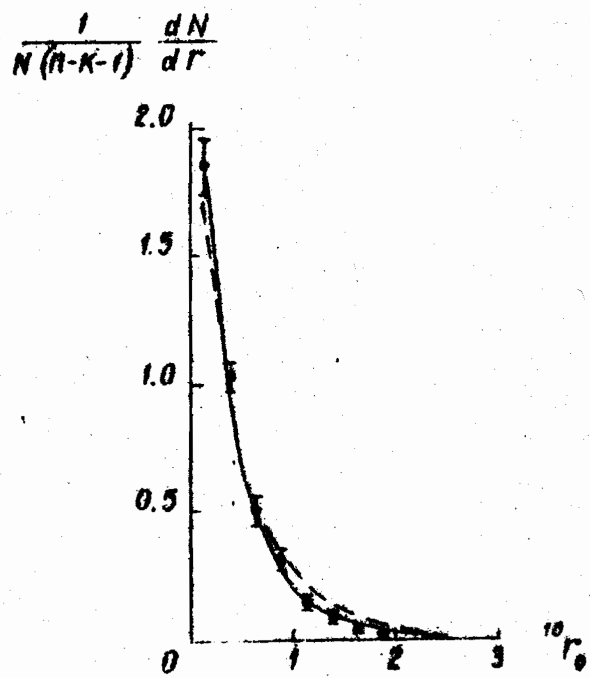


Fig. 3₂

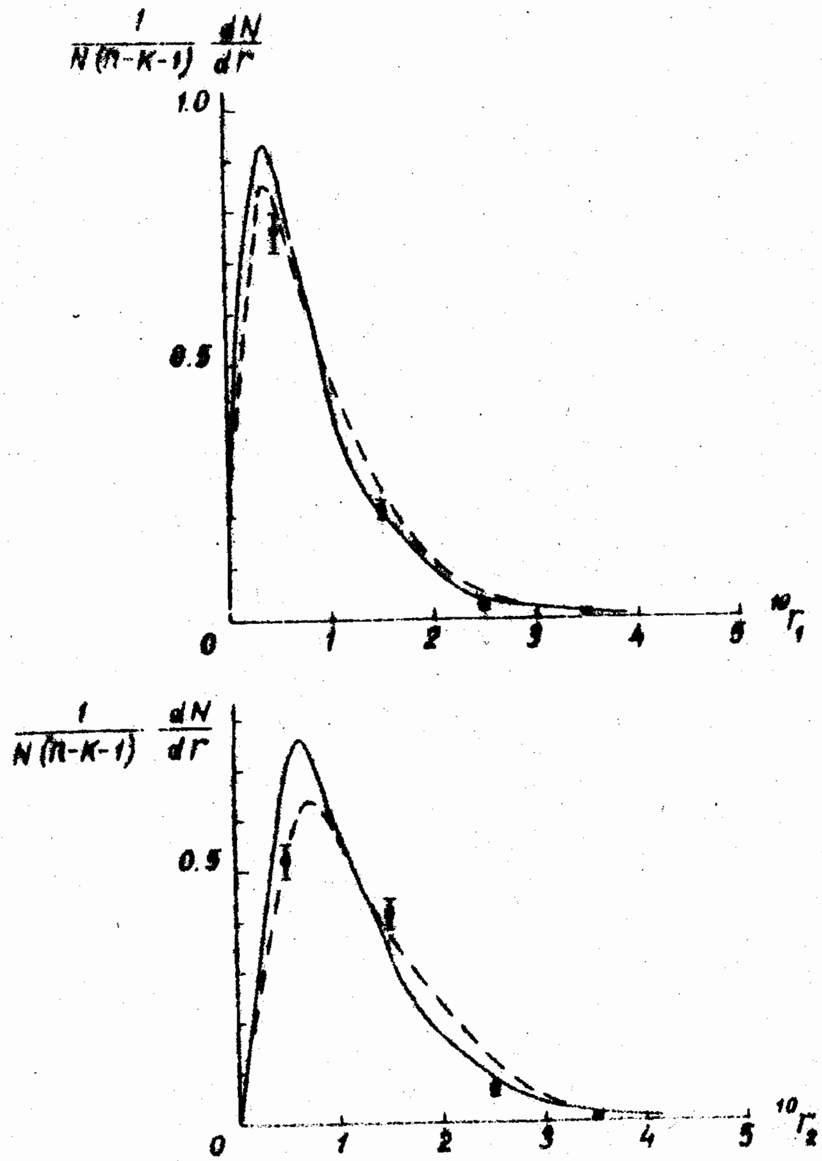


Fig. 3_2

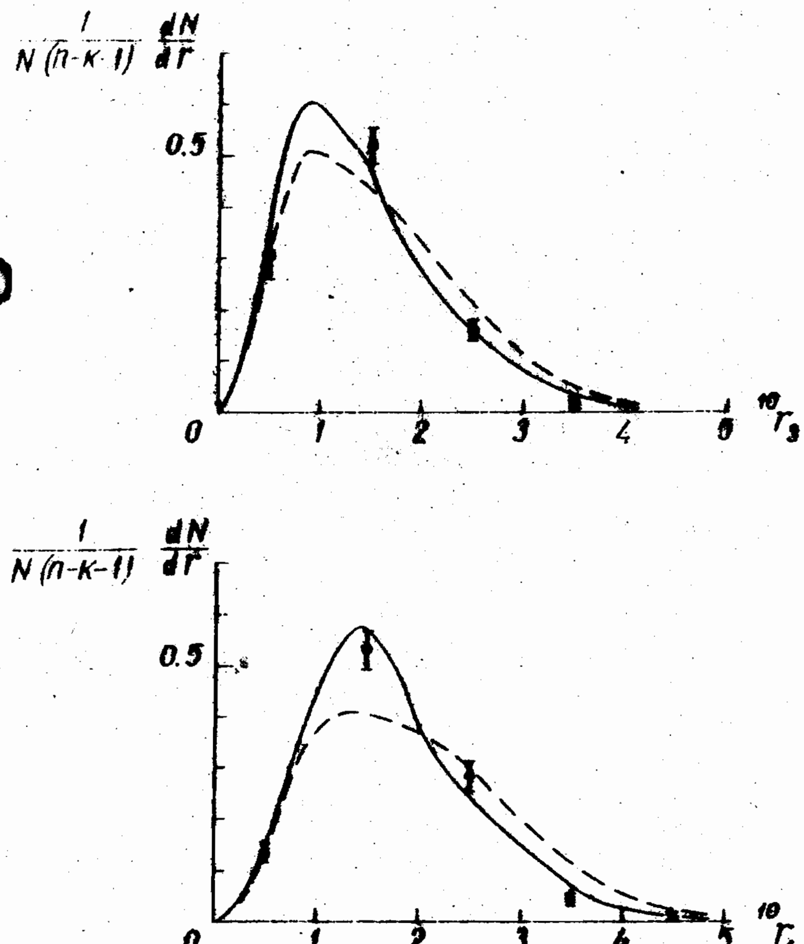


Fig. 3_3

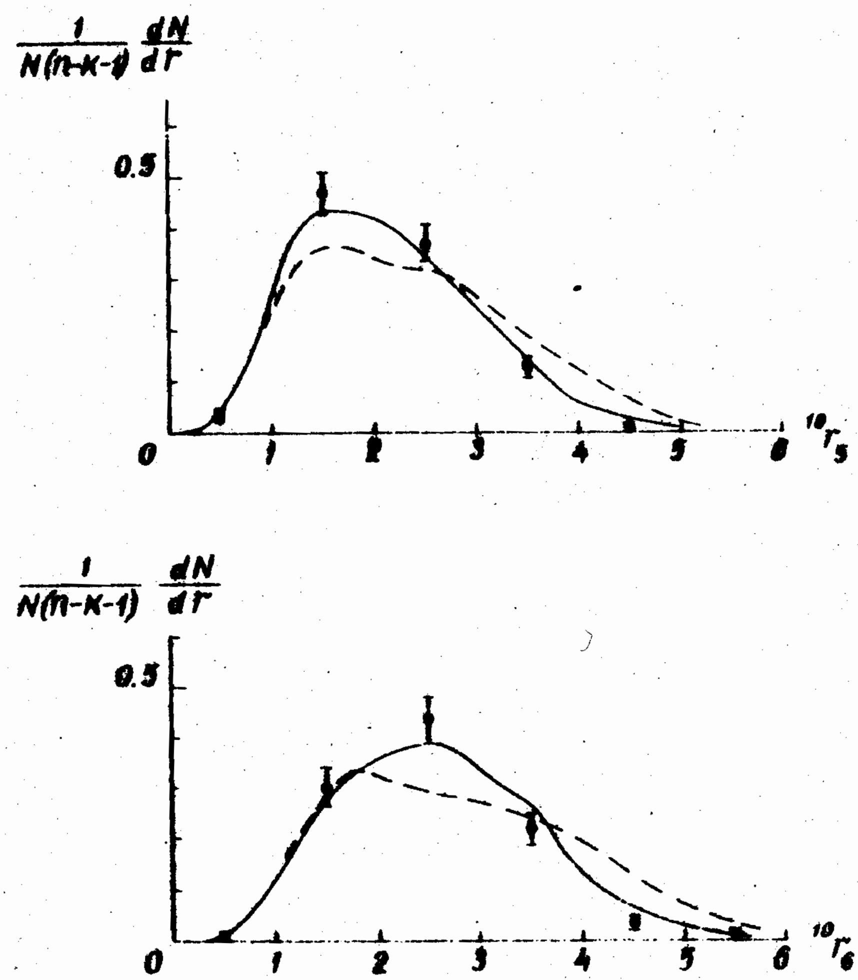
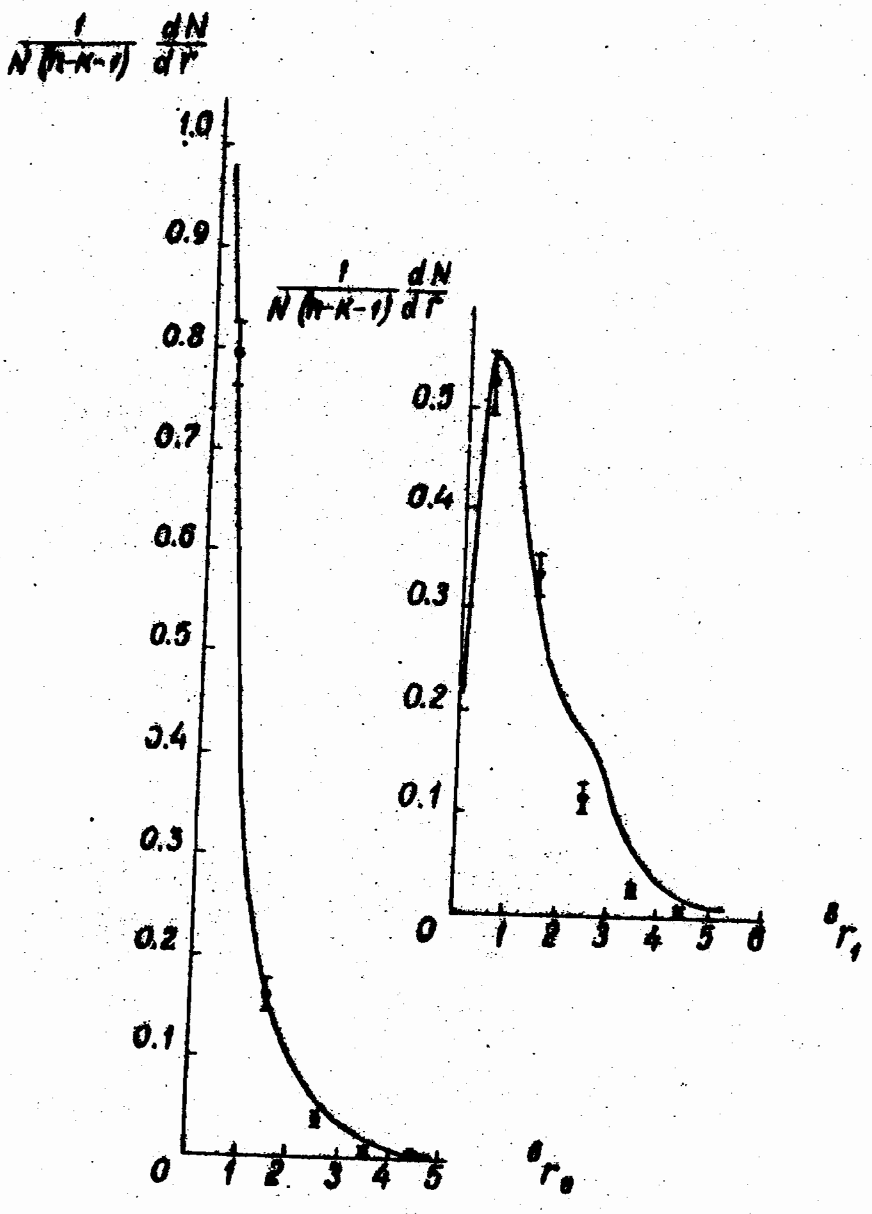
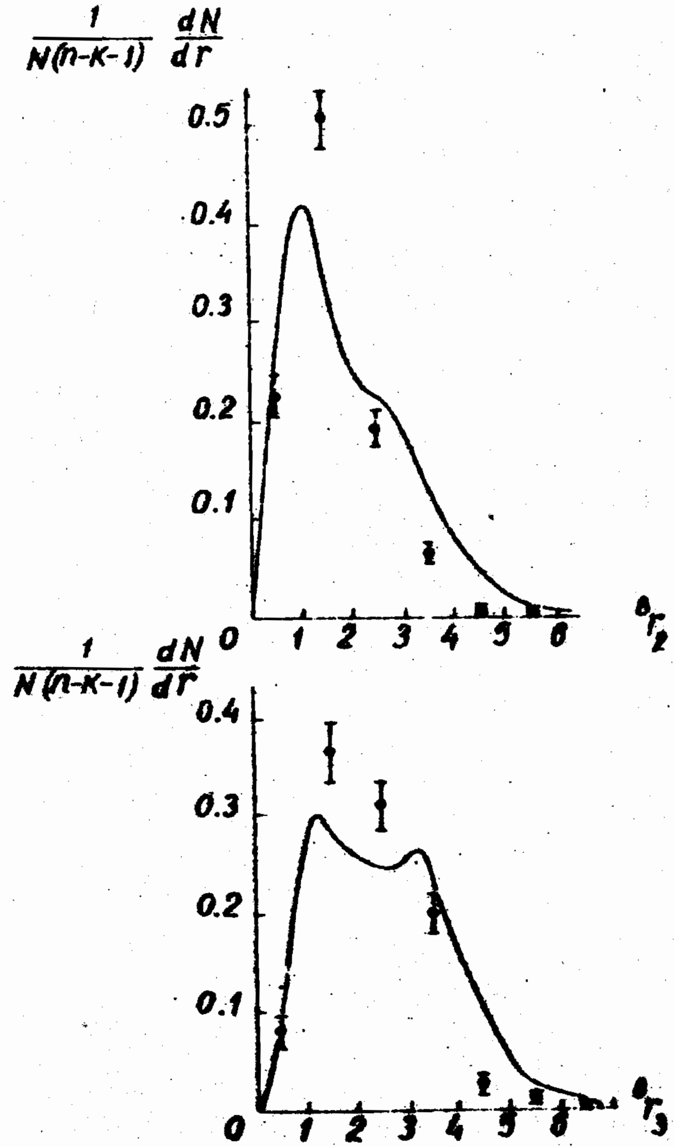


Fig. 3_4



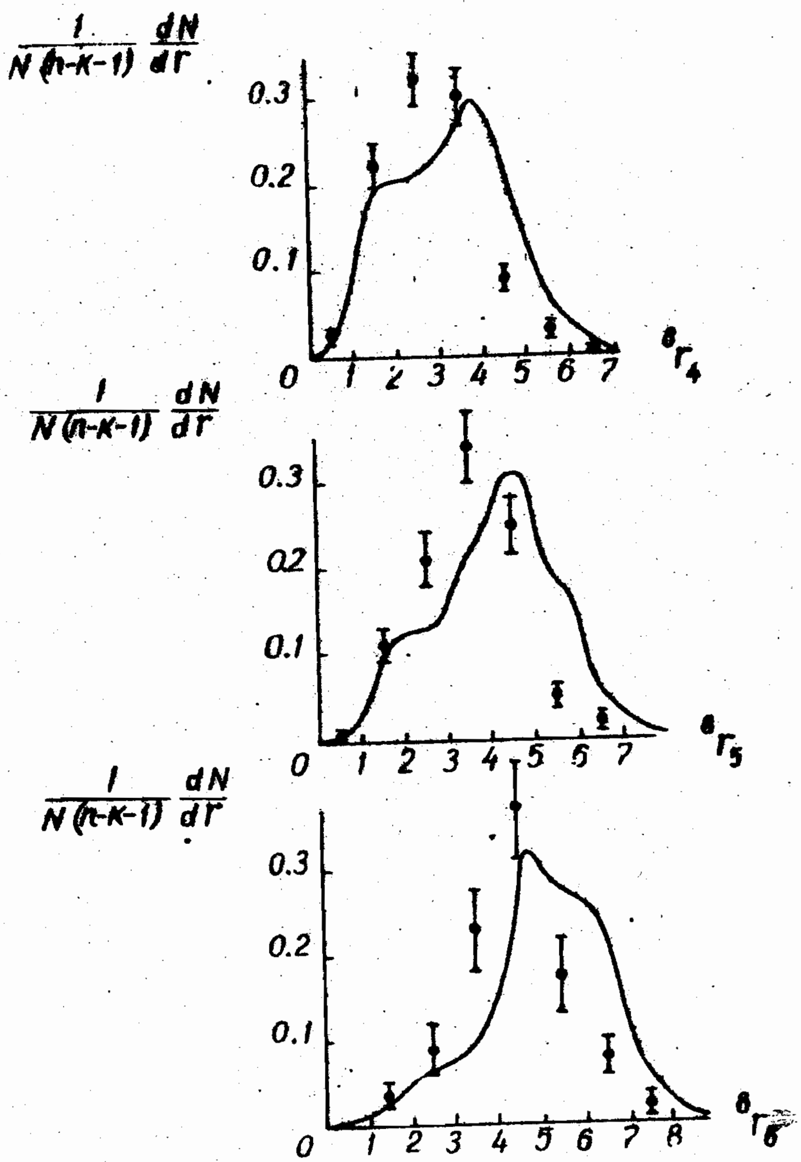
1/N(n-K-1) dN/dr
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
r₀
1/N(n-K-1) dN/dr
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
r₁

Fig. 4_1



$\frac{1}{N(n-K-1)}\frac{dN}{dr}$
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
r_2
$\frac{1}{N(n-K-1)}\frac{dN}{dr}$
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
r_3

Fig. 4₂



1/N(n-K-1) dN/dr
0.3
0.2
0.1
0
1 2 3 4 5 6 7
$^{8}r_4$
1/N(n-K-1) dN/dr
0.3
0.2
0.1
0
1 2 3 4 5 6 7
$^{8}r_5$
1/N(n-K-1) dN/dr
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8
$^{8}r_6$

Fig. 4$_3$

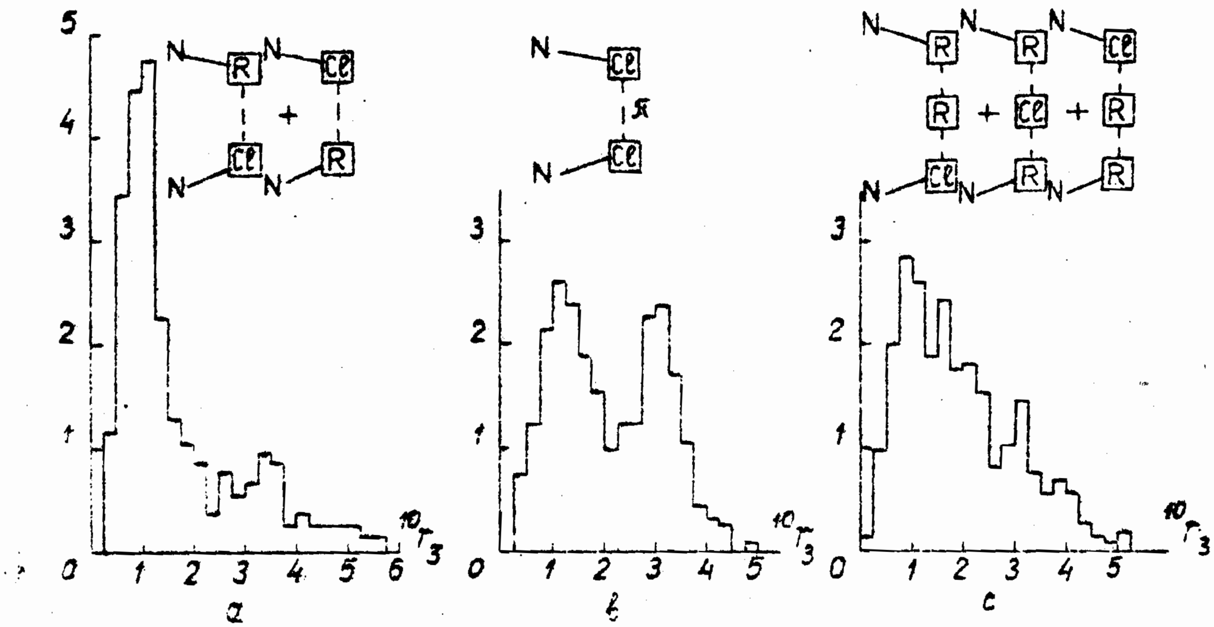


Fig. 5