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Rapidity Gap Distributions and Clustering in Multiparticle Production

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ABSTRACT

We obtain a general formula for the rapidity gap distribution in short-range order models and relate the exponential slope at large gaps to the behavior of topological cross sections.

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The distribution P(r) of the length r of rapidity gaps between charged secondaries produced in high-energy hadron collisions has been the subject of extensive experimental study and has also been analyzed using various theoretical models [1-3]. In $\pi^- p$ and pp collisions throughout the Fermilab energy range [4], and according to recent data on pp collisions at the CERN ISR [5], the observed fall of P(r) at large r is well described by an exponential function

$$P(r) \sim \exp(-r/R), \quad r \gtrsim 1.5, \qquad (1)$$

with slope parameter $R \approx 1$. We refer here to the so-called "nondiffractive" events. Furthermore, the distribution of rapidity gaps between negatively-charged secondaries has the same exponential form at large r, and the data are compatible with

$$R_- = R. \qquad (2)$$

This behavior of rapidity gap distributions can be simply accounted for [1] if one postulates that particle production proceeds by independent emission of clusters. Indeed, under the usual assumptions about cluster decay and for r large enough, the distribution of rapidity gaps between the decay products of clusters approaches the distribution of rapidity gaps between clusters. The assumed Poisson distribution in the number of produced clusters then implies that

$$P(r) \underset{r \to \infty}{\sim} \exp(-\rho_C r), \qquad (3)$$

where ρ_C is the mean density of clusters in rapidity. The equality (2) is predicted to hold as well.

In this Letter we derive, without reference to any specific model, a simple and general relation between two distinct pieces of experimental information, multiplicity distributions and rapidity gap distributions, and thereby estimate the slope parameter R. This relation shows directly that Eqs.(1) and (2) are consequences of short-range order in the structure of inclusive spectra. We also estimate R using two different models of particle production, namely the cluster emission model and the multiperipheral model. Although they are based upon different input data, all these estimates are close to each other, and close to the observed value of R. The overall consistency is remarkable.

Let us calculate the probability $G(y', y'')$ that no particle lies in the rapidity interval (y', y''). The inclusive densities will be denoted by $\rho_j(y_1, y_2, \ldots, y_j)$. Now assume for simplicity that the number of particles in (y', y'') cannot exceed N and let $y_j \in (y', y'')$ for $j = 1, 2, \ldots N$. Then $\rho_N(y_1, \ldots, y_N)$ is equal to the exclusive probability to find N particles at $y_1, \ldots, y_N$. Similarly, $\rho_{N-1}(y_1, \ldots, y_{N-1}) - \int dy_N \, \rho_N(y_1, \ldots, y_N)$ is the exclusive probability to find N-1 particles at $y_1, \ldots, y_{N-1}$, etc. The probability $G(y', y'')$ is obtained by subtracting from unity the probabilities to find N, N-1, ... 1 particles in the interval. Upon letting N tend to infinity we obtain

$$G(y',y'') = 1 + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!} \int_{y'}^{y''} dy_1 \ldots \int_{y'}^{y''} dy_j \, \rho_j(y_1,\ldots,y_j). \quad (4)$$

The point of this heuristic derivation was to make Eq. (4) more transparent, but a rigorous derivation can be carried through using functional techniques. Results of this sort have been rediscovered relatively recently in high-energy physics, but have been known for many years in the study of stochastic point processes [6].

Using the definition of the inclusive correlation functions $C_j(y_1,\ldots,y_j)$, we may rewrite Eq. (4) as

$$G(y',y'') = \exp\left[\sum_{j=1}^{\infty} \frac{(-1)^j}{j!} \int_{y'}^{y''} dy_1 \ldots \int_{y'}^{y''} dy_j \; C_j\,(y_1,\ldots,y_j)\right]. \quad (5)$$

We now assume that hadronic scaling holds and restrict our attention to the central region, in which the effect of phase-space boundaries can be ignored. In this regime the correlation functions depend only upon rapidity differences, so $G(y',y'') = G(y' - y'')$. As discussed in [1], the rapidity gap distribution is computed as

$$P(r) = (1/\rho_1)(d^2/dr^2)\, G(r). \quad (6)$$

Following this rather general discussion, let us make explicit the consequences of the short-range order (SRO) property of inclusive spectra. It is convenient to define the generating function

$$\Omega(z,Y) = \left(1/\sigma_{inel}(Y)\right) \sum_n z^n \,\sigma_n(Y)\,, \quad (7)$$

where Y denotes the available rapidity interval. Under the SRO hypothesis, we have

$$\log \Omega(z,Y) \underset{Y\to\infty}{\sim} Ya(z) + O(1) . \tag{8}$$

The correlation moments f_j are simply related to derivatives of a(z) by

$$\frac{1}{Y} f_j \underset{Y\to\infty}{\sim} \left.\frac{d^j}{dz^j} a(z)\right|_{z=1} + O(1/Y) . \tag{9}$$

If the rapidity gap r greatly exceeds the length λ over which correlations are appreciable, we obtain

$$\int_{y'}^{y''} dy_1 \cdots \int_{y'}^{y''} dy_j \, C_j(y_1,\ldots,y_j) = r \left.\frac{d^j}{dz^j} a(z)\right|_{z=1} + O(1), \tag{10}$$

so that

$$G(r) \underset{\substack{Y\to\infty \\ r>>\lambda}}{\sim} \exp\left[r \sum_{j=1}^{\infty} \frac{(-1)^j}{j!} \left.\frac{d^j}{dz^j} a(z)\right|_{z=1} + O(1)\right] . \tag{11}$$

The sum in the exponent is a formal expression for a(0) - a(1). Since a(1) = 0 by construction, Eqs. (6) and (11) lead to a rapidity gap distribution of the form (1), with

$$1/R = -a(0) . \tag{12}$$

In addition, because $n \equiv n_{charged} = 2n_- +$ incident charge, we have the relation $a_-(z^2) = a(z)$. In analogy to Eq. (12) we also have

$$1/R_- = -a_-(0) , \tag{13}$$

whereupon we observe that R and R_- are equal.

To evaluate R we exploit the connection between a(0) and the multiplicity distribution. Following the suggestion of [7] , we invert Eq. (7) to obtain an integral representation for $\sigma_n(Y)$. The integral is evaluated by saddle-point techniques which yield

$$\sigma_n(Y)/\sigma_{inel}(Y) \underset{\substack{Y \to \infty \\ \frac{n}{Y} \text{ fixed}}}{\sim} e^{Ya(0)} \frac{Y^n}{n!} e^{Yb(n/Y)} , \tag{14}$$

where b(n/Y) depends on dynamical details and satisfies b(0) = 0. To disentangle the several factors which contribute to the energy dependence of $\sigma_n(Y)$, we plot in Fig. 1(a)

$$L(n/Y) \equiv \frac{1}{Y} \left[\log\left(n!\,\sigma_n(Y)/\sigma_{inel}(Y)\right) - n \log Y\right] , \tag{15}$$

a form suggested by the SRO result (14). If the SRO hypothesis holds, the data at all energies should follow a scaling curve given by L(n/Y) = a(0) + b(n/Y). The data indeed scale as remarked in [7] , so we may estimate a(0) ≈ -1 from the intercept of the scaling curve.

In principle we should exclude from $\sigma_n(Y)$ the contributions of diffractive events. To explore how this would alter the estimate of a(0), we have subtracted 2, 3 and 1.4 mb from σ_2, σ_4, σ_6. [We regard these as generous, if not particularly precise, estimates of the diffractive contributions.*] The resulting plot of L(n/Y), shown in Fig. 1(b), again shows a universal behavior and yields a(0) ≈ - 1.2. We conclude that $R \approx 1$ with an uncertainty of roughly 20%, comparable to the uncertainty in the determination of R from rapidity gap distributions.

We turn now from a relation of data with data to estimates of R in specific SRO models. In multi-Regge exchange models, for example, if the $\sigma_n(Y)$ were exclusive cross sections we would extract from (14) the familiar result

$$a(0) = 2\alpha_R(0) - \alpha_P(0) - 1, \tag{16}$$

where $\alpha_R(0)$ and $\alpha_P(0)$ are the Reggeon and Pomeron intercepts. The summation over unobserved neutrals in the topological cross section $\sigma_n(Y)$ is expected to renormalize the value of a(0) given by (16). The correction to (16) depends on the strength of the correlation between the number of charged and neutral secondaries. If the number of neutrals n_0 were completely determined by n, the renormalization would be negligible. Conversely, the correction to (16) would be maximal if n and n_0 were totally uncorrelated; a tentative estimate with a Chew-Pignotti model gives $a(0) = -2/3$ instead of the value $a(0) = -1$ given by (16) with $\alpha_R(0) = 1/2$, $\alpha_P(0) = 1$. The behavior of $\langle n_0(n)\rangle$ at high energies [10] indicates a rather strong correlation between n_0 and n, so we consider $a(0) = -1$ a reliable estimate, with an uncertainty significantly smaller than 30%.

Consider next the independent emission of clusters. From (1) and (3) we obtain $1/R = \rho_C$. As emphasized in [1], the observation of the tail of P(r) yields an estimate of ρ_C which is independent of any assumption on the intracluster multiplicity distribution. Such assumptions are necessary to extract ρ_C from inclusive or semi-inclusive spectra. On the other hand, as pointed out by Stodolsky [11], if clusters are emitted independently, the inclusive distribution of the emitting source (the leading particle) should take the form

$$\frac{d\sigma}{dx}_{\text{leading}} = \rho_C (1 - |x|)^{\rho_C - 1} \quad , \qquad (17)$$

where x is the Feynman variable. The observed variation with x of the leading particle spectrum is weak, suggesting $\rho_C \approx 1$, so that again we estimate $R \approx 1$.

In conclusion, we have shown that inclusive short-range order, a hypothesis which embraces a wide variety of models for clustering, leads to the experimental results summarized in (1) and (2) and connects the exponential slope at large rapidity gaps with the energy-dependence of topological cross sections. The parameter R can be regarded as one measure of clustering: if particles were emitted independently one would have $R = 1/\rho_1 \approx 0.5$. Inclusive short-range order alone is, however, compatible with almost any value of R, so the consonance of the values of R obtained from the gap distribution (1) and the leading particle spectrum (17) can be taken as a strong argument in favor of the cluster model. Furthermore, $R\rho_1$ can be interpreted as the mean charged multiplicity of the cluster decay products. The fact that $R\rho_1 \approx 2$, a value suggestive of the decays of "ordinary" low mass boson resonances, leads to the conjecture that clustering can be interpreted in terms of resonance production. The compatibility of our two model estimates of R then illustrates an interesting sort of duality. The same amount of clustering is predicted assuming either that it results from resonance production or that it is due to the presence of exchange forces with range in rapidity specified by the Regge intercept $\alpha_R(0)$.

A consequence of the short-range order interpretation is the prediction that the quantity L(n/Y) will continue to show a scaling behavior at higher energies, and that the exponential behavior observed in gap distributions will persist at increased energies.

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FOOTNOTE

*That these are reasonable estimates may be seen by comparison with [9].

REFERENCES

[1] C. Quigg, P. Pirilä, and G.H. Thomas, Phys. Rev. Lett. 34 (1975) 290; P. Pirilä, G.H. Thomas and C. Quigg, Phys. Rev. D. (to be published).

[2] D.R. Snider, Phys. Rev. D11 (1975) 140.

[3] T. Ludlam and R. Slansky, Los Alamos preprints 75-481, 483 (to be published). These authors do not interpret the experimental distributions in terms of short-range order.

[4] Michigan-Rochester Collaboration, 102 and 405 GeV/c pp (to be published); Argonne-Fermilab-Stony Brook Collaboration, 205 GeV/c pp (to be published); Notre Dame-Duke-Particle Physics Institute of Canada Collaboration and Berkeley-LBL-Fermilab Collaboration, 205 GeV/c $\pi^- p$ (to be published).

[5] Aachen-CERN-Heidelberg-Munich Collaboration (to be published).

[6] R. L. Stratonovich, Topics in the Theory of Random Noise, Vol. I (Gordon and Breach, New York-London).

[7] R.C. Arnold and G.H. Thomas, Phys. Lett. 47B (1973) 371;

G. H. Thomas, Phys. Rev. D8 (1973) 3042.

[8]Topological cross sections in pp scattering:

28.4 GeV/c: E. L. Berger, B. Y. Oh and G. A. Smith, Phys. Rev. Lett. 29 (1972) 675.

50 and 69 GeV/c: Franco-Soviet Collaboration, Phys. Lett. 42B (1972) 519.

102 GeV/c: C. Bromberg, et al., Phys. Rev. Lett. 31 (1973) 1563.

205 GeV/c: S. Barish, et al., Phys. Rev. D9 (1974) 2689.

303 GeV/c: A. Firestone, et al., Phys. Rev. D10 (1974) 2080.

405 GeV/c: Michigan-Rochester Collaboration, to be published.

[9]F. T. Dao, et al., Phys. Lett. 45B (1973) 402.

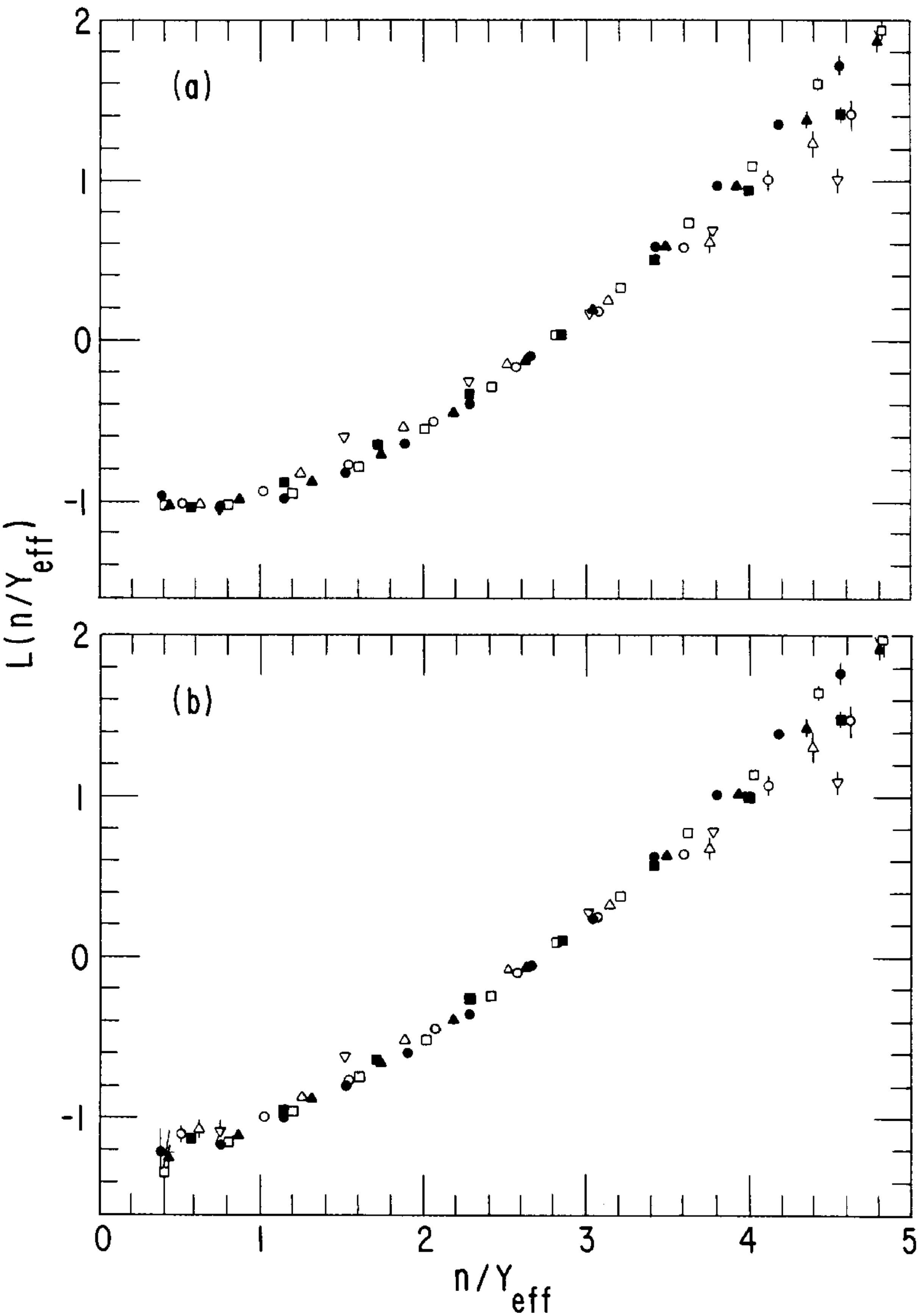
[10]F. T. Dao and J. Whitmore, Phys. Lett. 46B (1973) 252.

[11]L. Stodolsky, in Proceedings of the VIIth Rencontre de Moriond, ed. J. Tran Thanh Van (Orsay, 1972), Vol. 2; and quoted in S. Pokorski and L. Van Hove, Acta. Phys. Polon. B5 (1974) 229.

FIGURE CAPTION

Fig. 1 (a) $L(n/Y)$ for pp collisions at 28.4 ▽, 50 △, 69 ■, 102 ○, 205 ▲, 303 □, and 405 ● GeV/c [8]. We employ an effective rapidity interval $Y_{eff} \equiv \log(s/M_p^2)-1.5$, which corresponds to the effective length of the central region.

(b) The same plot after the attempted removal (described in the text) of diffractive events.



(a)
(b)
L(n/Y_eff)
n/Y_eff
2
1
0
-1
0
1
2
3
4
5