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Chiral Symmetry and $\psi' \to \psi\pi\pi$ Decay

LOWELL S. BROWN
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and
Physics Department, University of Washington, Seattle, Washington 98195

AND

ROBERT N. CAHN
Physics Department, University of Washington, Seattle, Washington 98195

ABSTRACT

Unbroken chiral symmetry (with a vanishing "σ-term") relates the $\psi' \to \psi\pi\pi$ decay amplitude to three basic parameters. Two of these parameters put strong angular correlations in the amplitude which are, apparently, not observed. Taking these two parameters to vanish, we obtain an isotropic decay which is strongly peaked in the region where the invariant mass of the $\pi\pi$ system is large.

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Although the newly discovered narrow boson resonances[1-4] at $M = 3.095$ GeV $[\psi = J]$ and at $M' = 3.684$ GeV $[\psi']$ have yet to be fully explored, the data so far[5] indicate that these particles are of a hadronic nature with quantum numbers $J^P = 1^-$ and $I^{GC} = 0^{--}$. The ψ' has been observed[6] to decay into a ψ with a branching ratio of 0.57 ± 0.08. In these decays, the mode $\psi' \to \psi\pi\pi$ predominates with a charged $\pi^+\pi^-$ pair being produced with a probability[6] of 0.56 ± 0.10. The relative rates for $\psi' \to \psi\pi^+\pi^-$ and $\psi' \to \psi +$ neutrals indicates that the $\pi\pi$ system has $I=0$. Thus, including $\pi^0\pi^0$ pairs, the $\psi' \to \psi\pi\pi$ mode should account for some 84% of the $\psi' \to \psi$ decays.

The pions produced in the $\psi' \to \psi\pi\pi$ decay have modest energies, their squared invariant mass lying the interval $4m_\pi^2 \leq m_{\pi\pi}^2 \leq (M'-M)^2$ $\left[0.08 \text{ GeV}^2 \leq m_{\pi\pi}^2 \leq 0.35 \text{ GeV}^2\right]$. In this energy interval, pion-pion rescattering corrections are small.[7] Moreover, since the ψ' and ψ resonances are very narrow, their interaction with the pions is very small. Thus, the low energy constraints implied by chiral symmetry, which we assume is obeyed in the decay, should extrapolate into the whole physical momentum region without large corrections. We shall assume that the breaking of chiral symmetry is a small effect and neglect the "σ-term". We find that the decay amplitude then involves three parameters - coefficients of three different momentum - dependent terms. Two of these terms produce strong angular correlations in the decay. The data[6] suggest that the decay is isotropic and indicate that the two coefficients of these terms are small. Guided by the data, we take these two parameters to vanish and obtain an isotropic decay amplitude which is strongly peaked at large $m_{\pi\pi}$. Thus, chiral symmetry connects the absence of angular correlations with a strong dependence on the invariant mass of the produced $\pi\,\pi$ pair. We should emphasize that if the terms with the strong angular dependence were present (terms which fully respect the chiral symmetry), then the $m_{\pi\pi}$ mass distribution would no longer be expected to peak at large values of $m_{\pi\pi}$.

We turn now to our calculation. We denote the four momenta and polarization vectors of the ψ' and ψ by P'^{μ}, $\vec{\epsilon}\,'$ and P^{μ}, $\vec{\epsilon}$, respectively. In the laboratory frame, the produced ψ-particle is slowly moving, with a maximum velocity of 0.15 c. Hence, we can treat it as a non-relativistic particle with a purely spatial polarization vector ϵ_{ℓ}, and write the decay matrix element as a spatial sum,

$$M = \Sigma^{3}_{\ell,m=1}\,\epsilon_{\ell}\,M_{\ell m}\,\epsilon'_{m} \quad . \tag{1}$$

We denote the pion four-momenta and isospin indices by $q_1^{\ \mu}$, a and $q_2^{\ \nu}$, b, with the energy - momentum balance reading $q_1 + q_2 + P = P'$. Current algebra relates the decay amplitude to the matrix element of axial currents which have their pion poles removed, and an additional matrix element of the symmetry breaking σ-term. We have the exact identity[8]

$$M\delta_{ab} = F_{\pi}^{-2} q_1^{\ \mu} q_2^{\ \nu} \langle \psi, P, \epsilon \,|\, i\left(\bar{A}_{\mu,a}(q_1)\bar{A}_{\nu,b}(q_2)\right)_{+} |\psi', P', \epsilon'\rangle$$

(cont.)

$$-F_\pi^{-1}(q_1^{\,2}+q_2^{\,2}+m_\pi^{\,2})\langle\psi, P,\epsilon\,|\,\Sigma_{ab}\,|\,\psi', P',\epsilon'\rangle \quad . \tag{2}$$

On the pion mass shell $q_1^{\,2} = q_2^{\,2} = -m_\pi^{\,2}$, the second, σ-term above is on the order $m_\pi^{\,2}$, and we shall assume that it is negligibly small. The structure of the axial current matrix element measures properties of the $\psi' - \psi$ system. In the limit where this system is non-relativistic, there are three different terms, giving

$$M = F_\pi^{-2}\Big\{\vec{\epsilon}\cdot\vec{\epsilon}\,'\Big[-q_1^{\,\mu}q_{2\mu}A + q_1^{\,0}q_2^{\,0}B\Big] + (\vec{\epsilon}\cdot\vec{q}_1\,\vec{\epsilon}\,'\cdot\vec{q}_2 + \vec{\epsilon}\cdot\vec{q}_2\,\vec{\epsilon}\,'\cdot\vec{q}_1)C\Big\} \quad . \tag{3}$$

Here the momentum components $q_1^{\,\mu}$ and $q_2^{\,\nu}$ are measured in the laboratory frame. Except possibly for S-wave pion-pion rescattering corrections,[7] the momentum-dependent variation of the amplitudes A, B, C should be small, and we shall assume that these amplitudes may be taken to be (real) constant parameters. Relativistic corrections increase the number of independent amplitudes (for example they add a term involving $q_1^{\,\mu}q_{2\mu}\,\vec{\epsilon}\cdot\vec{P}\,\vec{\epsilon}\,'\cdot\vec{P}$), but these should also be small corrections.

The polarization of the ψ particle produced in the decay is analyzed by the leptonic decays $\psi\to\mu^+\mu^-$, $\psi\to e^+e^-$. Since these leptons are extremely relativistic, the leptonic decay modes involve the replacement $\Sigma_{pol}\,\epsilon_\ell\,\epsilon_m^{\,*} \to \delta_{\ell m} - \hat{k}_\ell\,\hat{k}_m$, where $\hat{k}$ is a unit vector in the direction of the relative lepton momentum $\vec{k}$ measured in the ψ rest frame, a frame which is well approximated by the laboratory frame.

Similarly, the original ψ' particle is produced by relativistic e^+e^- annihilation, and it is produced with an alignment $\langle \epsilon'_\ell \epsilon'^*_m \rangle = \frac{1}{2}(\delta_{\ell m} - \hat{z}_\ell \hat{z}_m)$, where $\hat{z}$ is a unit vector along the beam direction. Working out the three body phase space, we find that the distribution for the sequential decay $\psi' \to \psi\pi^+\pi^-$, $\psi \to \mu^+\mu^-$, e^+e^- is given by

$$\frac{d\Gamma}{dm_{\pi\pi}\, d\Omega_{q_{\pi\pi}}\, d\Omega_P\, d\Omega_k} = \frac{3}{2} B_\ell \, \frac{q_{\pi\pi} P}{(4\pi)^6 M'^2}$$

$$M^*_{\ell' m'} M_{\ell m}(\delta_{\ell'\ell} - \hat{p}_{\ell'}\hat{p}_\ell)(\delta_{m'm} - \hat{z}_{m'}\hat{z}_m) \quad . \tag{4}$$

Here $m_{\pi\pi}^2 = -(q_1+q_2)^2$ is the squared invariant mass of the pion - pion system, $q_{\pi\pi} = |\vec{q}_{\pi\pi}| = \frac{1}{2}(m_{\pi\pi}^2 - 4m_\pi^2)^{\frac{1}{2}}$ is the magnitude of the relative momentum of this system in its center of mass, and $P = \frac{1}{2M'}\left[(M'^2 - M^2)^2 - 2(M'^2 + M^2)m_{\pi\pi}^2 + m_{\pi\pi}^4\right]^{\frac{1}{2}}$ is the momentum of the ψ in the laboratory frame. The solid angle element $d\Omega_{q_{\pi\pi}}$ is that of the relative momentum $\vec{q}_{\pi\pi}$ measured in the π-π rest frame; $d\Omega_p$ and $d\Omega_k$ are the solid angle elements of the ψ momentum and relative lepton momentum measured in the laboratory frame. The branching ratio into the lepton pair is denoted by B_ℓ and the matrix elements $M_{\ell m}$ follow from comparing Eqs. (1) and (3). The total rate, without regard for the decay mode of the ψ particle, is obtained from Eq. (4) by deleting the factor B_ℓ and by integrating over the solid angle Ω_k .

If we write the laboratory momentum components q_1^μ and q_2^ν in terms of $q_{\pi\pi}$, P , Lorentz boost factors, and appropriate angular factors, we find that the terms associated with the parameters B and C involve strong angular correlations.[9] The data presently available[6] appear not to support much angular variation, and we shall tentatively assume that $B = C = 0$. Clearly, the actual values of these parameters need to be measured experimentally. With $B = C = 0$, the decay distribution (4) is independent of the directions of $\vec{q}_{\pi\pi}$ and $\vec{P}$ so that we may as well integrate over their solid angles to obtain

$$\frac{d\Gamma}{dm_{\pi\pi}d\Omega_k} = \frac{3}{2}B_\ell \frac{q_{\pi\pi}P}{(4\pi)^4 M'^2} \frac{1}{4}(m_{\pi\pi}^2 - 2m_\pi^2)^2 \left(\frac{A}{F_\pi^2}\right)^2 \cdot (1+\cos^2\theta_k) \quad , \tag{5}$$

where θ_k is the angle between the relative lepton momentum $\vec{k}$ and the beam direction $\hat{z}$. The total differential spectrum is given by

$$\frac{d\Gamma}{dm_{\pi\pi}} = \frac{1}{2} \frac{q_{\pi\pi}P}{(4\pi)^3 M'^2} (m_{\pi\pi}^2 - 2m_\pi^2)^2 \left(\frac{A}{F_\pi^2}\right)^2 . \tag{6}$$

This function is displayed in Fig. 1 together with the simple phase space curve (the factor $q_{\pi\pi}P$ normalized to the same area) and the modification brought about by the rescattering of the pions which are in a relative S state. We see that the factor $(m_{\pi\pi}^2 - 2m_\pi^2)^2$, introduced by the chiral symmetry coupled with the assumed vanishing of the parameters B and C, modifies the simple phase space curve substantially, giving a peak at large $m_{\pi\pi}$. The chiral symmetry breaking corrections which we have neglected give a constant term relative to the factor $(m_{\pi\pi}^2 - 2m_\pi^2)$ in the amplitude. They may alter somewhat the small $m_{\pi\pi}$ behavior of the decay spectrum (6).

Integrating (6) over the invariant π-π mass range gives a width

$$\Gamma(\psi' \to \psi\pi^+\pi^-) = 1.4 \frac{A^2}{4\pi} \text{ MeV} \quad . \tag{7}$$

Using [5,6] $\Gamma(\psi' \to \psi\pi^+\pi^-) = 0.32\ \Gamma(\psi' \text{Total}) \simeq 150$ keV , we find that the dimensionless parameter $\frac{A^2}{4\pi} \simeq \frac{1}{10}$.

We emphasize again that chiral symmetry does not determine uniquely the parameters of the decay $\psi' \to \psi\pi\pi$. It does predict a depletion of events with low $\pi\pi$ invariant mass if terms which give anisotropic distributions are small. Should this turn out to be an accurate description, there would still remain two important theoretical questions: Why are the anisotropic terms small and what is the precise role of chiral symmetry breaking (the σ-term)?

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[5]See F. J. Gilman, "Electron-Positron Annihilation and the Structure of Hadrons", in Invited Talk Presented at the Orbis Scientiae II, University of Miami, Coral Gables, Florida, January 1-24, 1975. SLAC-Pub-1537.

[6]G. S. Abrams, et al., LBL-3669, SLAC-Pub-1556 (unpublished).

[7]Pion - pion rescattering corrections are negligible for (even) angular momentum states higher than S-waves. We can estimate the effect for S-waves by the conventional $|D|^{-2}$ factor which accounts for the variation brought about by the elastic cut in the π-π scattering amplitude. A reasonable model of D appears in Eq. (7) of R. L. Goble and J. L. Rosner, Phys. Rev. D5, 2345 (1972), which is based on the earlier work of L. S. Brown and R. L. Goble, Phys. Rev. Letters 20, 346 (1968) and Phys. Rev. D4, 723 (1971). Here we use the value $M_0 = 1$ GeV in this model, as is suggested by recent data on π-π scattering, and find that $|D|^{-2}$ has a very broad peak in the center of the $m_{\pi\pi}$ range with the peak some 36% above the values of $|D|^{-2}$ at the endpoints of the $m_{\pi\pi}$ range. This variation is insignificant in comparison with the large

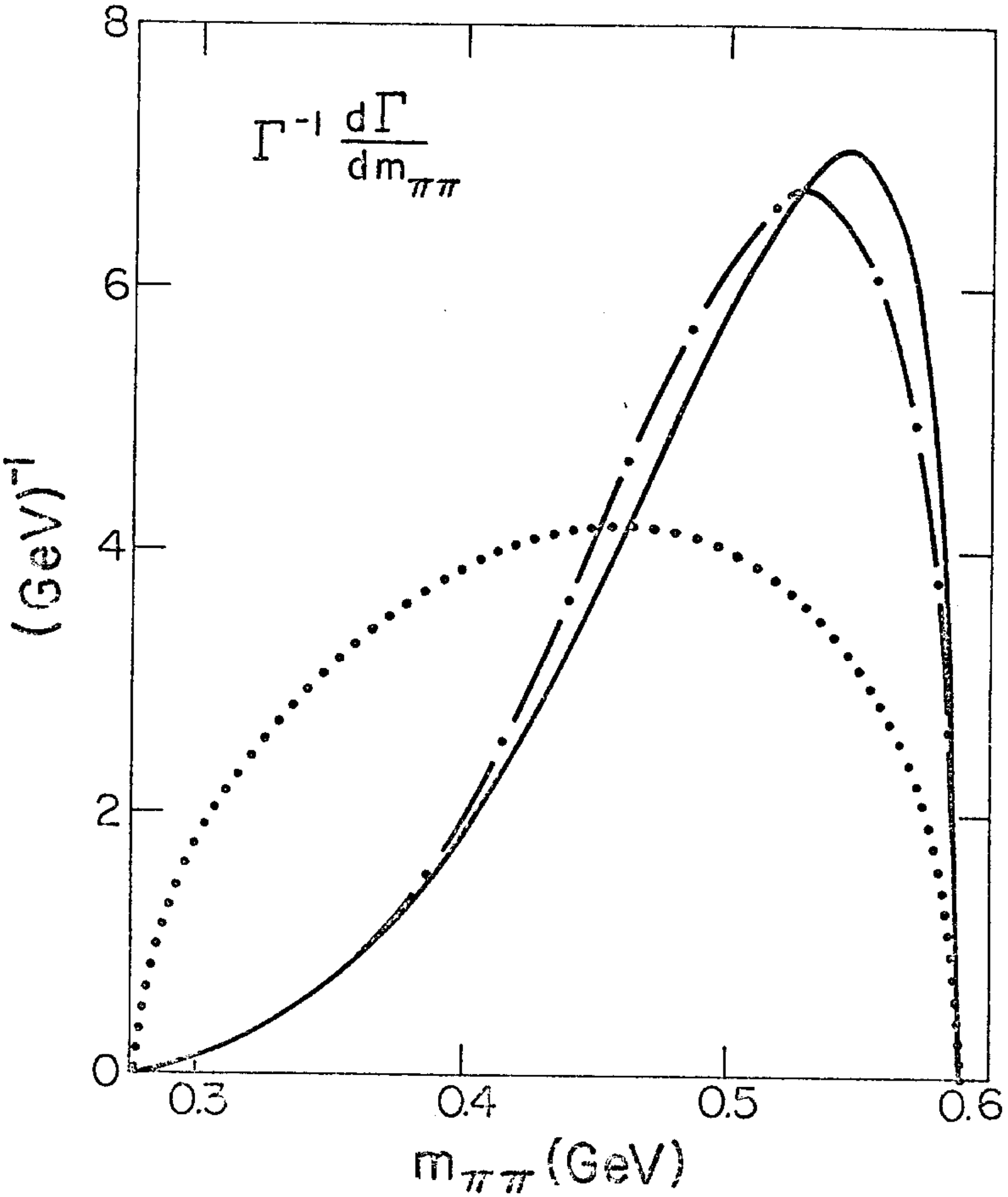
variation which we find are implied by chiral symmetry as displayed in Fig. 1.

[8]This identity is completely analogous to that for pion - nucleon scattering except that here there is no vector current, equal-time commutator contribution. See, for example, the discussion leading to Eq. (63) of L. S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D4, 2801 (1971).

[9]A full analysis of the decay correlations will be presented elsewhere.

FIGURE CAPTION

Fig. 1 The decay spectrum $\Gamma^{-1} \frac{d\Gamma}{dm_{\pi\pi}}$ as a function of $m_{\pi\pi}$ (a) given by Eq. (6) (solid line), (b) given by simple phase space (dotted line), (c) given by Eq. (6) modified by pion - pion rescattering (dot-dashed line).



$\Gamma^{-1} \frac{d\Gamma}{dm_{\pi\pi}}$
$(\mathrm{GeV})^{-1}$
8
6
4
2
0
0.3
0.4
0.5
0.6
$m_{\pi\pi}$ (GeV)