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Neutrino Physics--Theoretical Considerations*

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NEUTRINO PHYSICS--THEORETICAL CONSIDERATIONS

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O. PROLOGUE

Neutrino physics is of interest not only for the study of intrinsic characteristics of the elusive neutrinos, but also as diagnostics of hadronic structures, and as a probe of weak interactions.

What follows is a more or less faithful reproduction of the transparencies used in the oral presentation. In preparing this talk, I relied heavily on the following excellent reviews:

S. Adler, "Accelerator Neutrino Physics, Present and Future," NAL-Conf-74/39-THY,
C.H. Llewellyn-Smith, Physics Reports 3C, No. 5 (1972),
E.A. Paschos, NAL-Conf-73/65-THY: Lectures delivered at the "Ettore Majorana" Summer School, 1973.

For comprehensive bibliographies, the above references should be consulted.

I have used a number of figures and illustrations from published papers available before this Conference. More up-to-date data have been presented at the Conference and the reader should consult other contributions in these Proceedings for these.

I. KINEMATICS AND STANDARD V-A THEORY

1. Kinematics.

$$\nu(k_1) + A(p) \rightarrow \ell(k_2) + B(p+q),$$

where $q = k_1 - k_2$ is the momentum transfer.

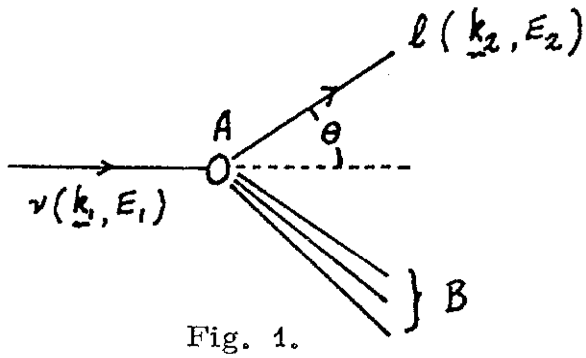

Fig. 1.
Kinematics of neutrino-induced reactions.

<u>Invariant variables:</u>

$$\nu = p\cdot q = m_A(E_1 - E_2),$$

$$q^2 = (k_1 - k_2)^2 = -4E_1E_2 \sin^2\frac{\theta}{2} \text{ (neglecting } \ell \text{ mass).}$$

2. V-A Theory.

Local Current × Current Interaction:

$$\mathscr{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \mathscr{J}_\mu^\dagger \mathscr{J}^\mu, \quad G_F \simeq 10^{-5}\, m_p^{-2}.$$

$$\mathscr{J}_\mu = J_\mu + \ell_\mu;$$

$$J_\mu = (V_\mu^{1+i2} - A_\mu^{1+i2}) \cos\theta_c + (V_\mu^{4+i5} - A_\mu^{4+i5}) \sin\theta_c,$$

$$\ell_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)e + \bar{\nu}'\gamma_\mu(1-\gamma_5)\mu,$$

and

$$\theta_c \cong 15° = \pi/12.$$

Current Algebra:

Normalization of V_μ^i and A_μ^i is fixed by Gell-Mann's SU(3) × SU(3) algebra:

$$[A_0^i(\vec{x},0), A_0^j(\vec{y},0)] = if^{ijk}\, V_0^k(\vec{x},0)\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}),$$

etc.

G-parity: $G = C e^{i\pi I_2}$.

$$G \begin{Bmatrix} V^{1+i2} \\ A^{1+i2} \end{Bmatrix} G^{-1} = \begin{Bmatrix} +V^{1+i2} \\ -A^{1+i2} \end{Bmatrix} \text{i.e., 1st class.}$$

Existence of the 2nd class currents not ruled out.

Lepton Number Conservation: See S. P. Rosen's discussion at this conference.

II. EXCLUSIVE REACTIONS

1. Quasi-elastic: A = N, B = N.

$$\nu_\mu + n \rightarrow \mu^- + p.$$

Hadronic matrix element:

$$\langle p(p_2)|J_\mu|n(p_1)\rangle = \cos\theta_c\,\bar{u}_p(p_2)\,\Gamma_\mu\,u_n(p_1),$$

where

$$\Gamma_\mu = \gamma_\mu F_V^1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_V^2(q^2) + \frac{q_\mu}{2m_N}F_V^3(q^2)$$

$$-\gamma_\mu\gamma_5 G_A(q^2) - \gamma_5 q_\mu H_A(q^2) + \frac{1}{m_N}\gamma_5(p_1+p_2)_\mu F_A^3(q^2)\,.$$

Absence of 2nd class current implies $F^3_{V,A} = 0$; CVC implies $F_V^3 = 0$, and $F_V^{1,2}$ are as in ep scattering. Since $q^\lambda\bar{\mu}\gamma_\lambda(1-\gamma_5)\nu \propto m_\mu$, H_A may be neglected. Parameterize $G_A = 1.24(1-q^2/M_A{}^2)^{-2}$.

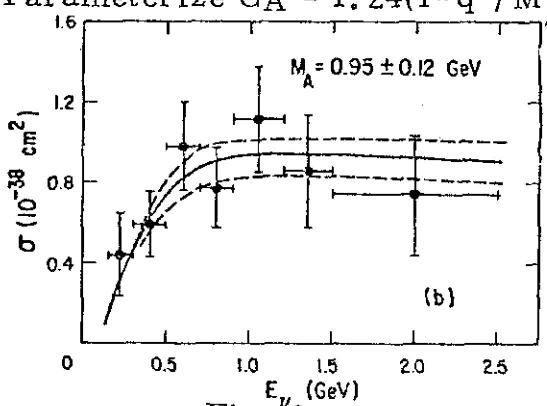

Fig. 2.

Cross Section for $\nu_\mu + n \to \mu^- + p$ and fit with $M_A = 0.95 \pm 0.12$ GeV. From W.A. Mann, et al., Phys. Rev. Letters 31, 844 (1973). See also P. Schreiner's contribution.

2. $\Delta(1236)$ Production: $A = N$, $B = \Delta\,(1236, P_{3/2}, 3/2)$.
$\quad\hookrightarrow N+\pi$.

Adler's theory [Ann. Phys., 50, 189 (1968)] : Relativistic version of the static model. Non-resonant amplitudes =

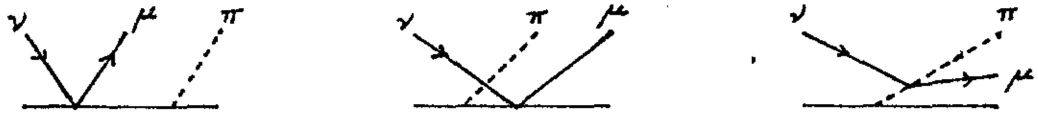

Fig. 3.
Born diagrams in Adler's theory.

Born approximations; Resonant multipoles = Born approximation + rescattering corrections via πN phase shifts. Complete and essentially unique predictions in terms of $\delta_{\ell\pm}$, $F_V^{1,2}$, G_A and H_A. Works o.k. for photoproduction (CGLN); for electroproduction ($q^2 \lesssim 1$ GeV) and also for ν production. For comparison with experiment, see J. Campbell et al., Phys. Rev. Letters 8, 335 (1973); P. Schreiner et al., ibid., 8, 339 (1973); P. Schreiner, in these Proceedings.

III. INCLUSIVE REACTIONS

1. Kinematics.

A=N, B= sum over all final states for fixed m_B. The cross section is

$$\sigma^{\nu,\bar{\nu}} = L^{\alpha\beta}(\nu,\bar{\nu})\, H^{\nu,\bar{\nu}}_{\alpha\beta}$$

where

$$H^{\nu}_{\alpha\beta} = \frac{1}{2} \sum_{B} \sum_{\substack{\text{spin}\\ A}} \langle N | J^{\dagger} | B\rangle \langle B | J_{\beta} | N\rangle$$

$$\times\ (2\pi)^3\, \delta^4(q+p-p_B)$$

$$(\nu \to \bar{\nu},\ J \longleftrightarrow J^{\dagger})$$

$$= -q_{\alpha\beta} W_1 + \frac{p_\alpha p_\beta}{m_N^2} W_2 - i\,\frac{1}{2m_N^2}\,\epsilon_{\alpha\beta\rho\sigma}\, p^{\rho} q^{\sigma} W_3$$

$$+ \frac{q_\alpha q_\beta}{m_N^2} W_4 + \frac{1}{2m_N^2}(p_\alpha q_\beta + p_\beta q_\alpha) W_5$$

$$+ i\,\frac{1}{2m_N^2}(p_\alpha q_\beta - p_\beta q_\alpha) W_6\ .$$

$$\frac{d^2\sigma}{d|q^2|d\nu} = \frac{G_F^2}{2\pi m_N^2}\left(\frac{E_\mu}{E_\nu}\right)\left[\cos^2\frac{\theta}{2} W_2 + 2\sin^2\frac{\theta}{2} W_1 \right.$$

$$\left. \mp \frac{E_\mu + E_\nu}{m_N}\sin^2\frac{\theta}{2} W_3 + \mathcal{O}(m_\mu^2)\right] \text{ for } \frac{\nu}{\bar{\nu}} .$$

Charge symmetry constraints ($\theta_c = 0$):

$$e^{i\pi I_2} J_\mu e^{-i\pi I_2} = -J_\mu^\dagger \Rightarrow W_i^{\nu n} = W_i^{\bar{\nu} p} ,$$

$$W_i^{\nu p} = W_i^{\bar{\nu} n} .$$

2. Current algebra prediction.

$$\left[\int d^3x\, J^0(\vec{x}, 0) e^{iq\cdot x}, \int d^3y\, J^{0\dagger}(\vec{y}, 0)\right]$$

$$= 4\, I_3 \cos^2\theta_c + (3y + 2 I_3)\sin^2\theta_c + \ldots$$

Take spin averaged matrix element between nucleons and go to infinite momentum frame $\Rightarrow$

Adler sum rule:

$$\frac{1}{m_N^2}\int_0^\infty d\nu\left[W_2^{\bar{\nu}}(\nu, q^2) - W_2^{\nu}(\nu, q^2)\right]$$

$$= \langle 4\cos^2\theta_c I_3 + (3y + 2 I_3)\sin^2\theta_c \rangle_N$$

$$\simeq \left.\begin{matrix} 2 & \text{for } N = p \\ -2 & \text{for } N = n \end{matrix}\right\} \text{ independent of } q^2!!$$

Alternatively

$$\lim_{E_\nu \to \infty} \left[\frac{d\sigma^{\nu p}}{d|q^2|} - \frac{d\sigma^{\bar{\nu} p}}{d|q^2|} \right] = - \frac{G_F}{\pi} (\cos^2 \theta_c + 2 \sin^2 \theta_c)$$

3. Scaling

Dimensional analysis gives

$$W_1(\nu, q^2) = G_1(x, |q|^2/m_N^2),$$

$$\frac{\nu}{m_N^2} W_2(\nu, q^2) = G_2(x, |q^2|/m_N^2),$$

$$\frac{\nu}{m_N^2} W_3(\nu, q^2) = G_3(x, |q^2|/m_N^2) .$$

Convenient scaling variables are

$$x = -g^2/2\nu \equiv \omega^{-1}, \quad y = \frac{\nu}{m_N E_\nu} = \left(1 - \frac{E_\mu}{E_\nu}\right) \cong \frac{E_B}{E_\nu} ,$$

$$0 \leq x, \ y \leq 1.$$

[Note: $z = xy = |q|^2/2m_N E_\nu = 2\left(\frac{E_\mu}{m_N}\right)\sin^2 \frac{\theta}{2}$ is scale-invariant.]

$$\frac{d^2\sigma}{dxdy} = \frac{G^2 m_N E_\nu}{\pi} \Big[(1 - y - \tfrac{1}{2}xy \frac{m_N}{E_\nu}) G_2$$

$$+ xy^2 G_1 \mp xy(1 - \tfrac{1}{2}y) G_3 \Big] .$$

<u>Bjorken scaling hypothesis</u>:

$$G_i(x, |q^2|/m_N^2) \xrightarrow[\substack{\nu, q^2 \to \infty \\ x \text{ fixed}}]{} F_i(x).$$

In Bj limit,

$$\left(\frac{\pi}{G_F^2 m_N E_\nu}\right)\frac{d^2\sigma}{dxdy} = xy^2 F_1(x) + (1-y)F_2 \mp xy(1-\frac{1}{2}y)F_3 .$$

Positivity considerations, $\epsilon^\alpha \epsilon^{\beta *} H_{\alpha\beta} \geq 0$, give

$$0 \leq \sigma_s \propto W_2\left(\frac{\nu}{-q^2 m_N^2} + 1\right) - W_1 \Rightarrow \frac{F_2}{2x} - F_1 \geq 0$$

$$0 \leq \sigma_{\substack{R\\L}} \propto W_1 \pm \frac{1}{2}\left(\frac{\nu^2}{m_N^4} - \frac{q^2}{m_N^2}\right)^{\frac{1}{2}} W_3 \Rightarrow F_1 + \frac{1}{2}F_3 \geq 0.$$

Thus

$$F_2 \geq 2xF_1, \quad F_1 \geq \frac{1}{2}|F_3| .$$

Integrate over y:

$$\frac{\pi}{G_F^2 m_N E_\nu}\frac{d\sigma}{dx}\begin{pmatrix}\nu\\ \bar\nu\end{pmatrix} = a_s^{\begin{pmatrix}\nu\\ \bar\nu\end{pmatrix}} + \begin{pmatrix}1\\ \\ \frac{1}{3}\end{pmatrix} xa_L^{\begin{pmatrix}\nu\\ \bar\nu\end{pmatrix}}$$

$$+ \begin{pmatrix}\frac{1}{3}\\ \\ 1\end{pmatrix} xa_R^{\begin{pmatrix}\nu\\ \bar\nu\end{pmatrix}}$$

where $a_s = \frac{1}{2}F_2 - xF_1$, $a_{\substack{L\\R}} = F_1 \mp \frac{1}{2}F_3$.

4. Simple consequences of scaling:

i) Adler sum rule

$$\int_1^\infty \frac{d\,\omega}{\omega}\,[F_2^{\bar\nu} - F_2^{\nu}] = \int_0^1 \frac{dx}{x}\,[F_2^{\bar\nu} - F_2^{\nu}]\,(x)$$

$$= \langle 4I_3 \cos^2\theta_c + (3y + 2I_3)\sin^2\theta_c \rangle_N .$$

In the scaling region, the left hand side is automatically q^2-independent.

ii) Total cross sections increase linearly with E_ν:

$$\sigma^{\nu,\bar{\nu}} = C_{\nu,\bar{\nu}} E_1$$

where

$$C_{\nu,\bar{\nu}} = \frac{G^2 m_N}{\pi} \int dx \left[a_3^{\nu,\bar{\nu}}(x) + \begin{pmatrix} 1 \\ \frac{1}{3} \end{pmatrix} x\, a_L^{\nu,\bar{\nu}}(x) + \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix} x\, a_R^{\nu,\bar{\nu}}(x) \right].$$

$$[\text{Also } \langle q^2 \rangle \propto \nu].$$

Experimentally

$$c_{\bar{\nu}}/c_\nu \simeq 1/3 \begin{pmatrix} 0.38 \pm 0.02 & \text{CERN} \\ 0.35 \pm 0.18 & \text{NAL} \end{pmatrix}.$$

To understand this, set $\theta_c = 0$ and consider an isosinglet target. From charge symmetry, get $F_i^{\nu p} = F_i^{\bar{\nu} n}$, $F_i^{\nu n} = F_i^{\bar{\nu} p}$, or

$$F_i^{\nu} = (F_i^{\nu p} + F_i^{\nu n})/2 = (F_i^{\bar{\nu} p} + F_i^{\bar{\nu} n})/2 = F_i^{\bar{\nu}} = F_i$$

$$\frac{\sigma^{\nu}}{\sigma^{\nu}} = \frac{\int a_s + \frac{1}{3}\int x a_L + \int x a_R}{\int a_s + \int x a_L + \frac{1}{3}\int x a_R}.$$

so

$$1/3 \leq \sigma^{\nu}/\sigma^{\nu} \leq 3,$$

and exp $\Rightarrow \int a_s = 0, \int x a_R = 0 \Rightarrow a_s \simeq 0,\ a_R \simeq 0.$

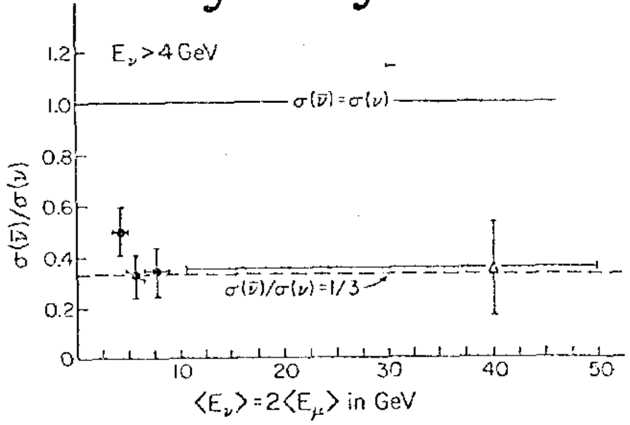

Fig. 4.
Plot of the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ as a function of $\langle E_\nu \rangle$. The value of 1/3 is expected in the scattering of neutrinos and antineutrinos by fundamental fermions such as electrons and muons. From Benvenuti et al., Phys. Rev. Letters 30, 1084 (1973).

$$a_S \simeq 0 \text{ means } F_2 \simeq 2x F_1.$$

(Callan-Gross relation: spin 1/2 constituents)

$$a_R \simeq 0 \text{ means } F_3 \simeq -2 F_1$$

(Maximal VA interference).

iii) y-distribution; above considerations predict y-distribution.

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 m_N E_1}{\pi} F_2(x) \qquad \text{for } \nu,$$

$$= \frac{G_F^2 m_N E_1}{\pi} F_2(x)(1-y)^2 \qquad \text{for } \bar{\nu}.$$

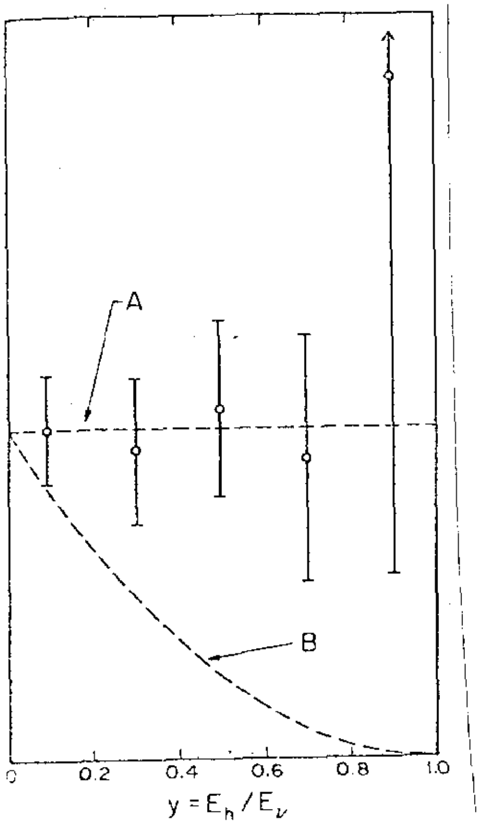

Fig. 5.
y-distribution in the inclusive ν process. From B. C. Barish et al., Phys. Rev. Letters 8, 565 (1973).

iv) Mean muon energy:

$$\left\langle \frac{E_2}{E_1} \right\rangle = \langle 1 - y \rangle = \begin{cases} 1/2 & \text{for } \nu \\ 3/4 & \text{for } \bar{\nu}. \end{cases}$$

This relation is flux independent.

v) Distribution in $z = xy$:

$$\frac{1}{N}\frac{dN}{dz} = \frac{d\sigma}{dz}\frac{1}{\sigma}$$

$$= \frac{\int_{v}^{1} \frac{dx}{x} F_2(x)}{\int_{0}^{1} dx\, F_2(x)}$$

This is flux, ν energy independent.

5. Regge Asymptotics.

From the Regge analysis of helicity amplitudes,

$$\left.\begin{aligned} W_1 &\xrightarrow[\nu\to\infty]{} \beta_1(q^2)\,\nu^{\alpha_1(0)} \\ W_2 &\longrightarrow \beta_2(q^2)\,\nu^{\alpha_2(0)-2} \end{aligned}\right\} \quad \text{Pomeron:}\ \alpha_1(0) \simeq \alpha_2(0) \simeq 1\,.$$

$$W_3 \longrightarrow \beta_3(q^2)\,\nu^{\alpha_3(0)-1} \qquad G = -1\ (\omega, \phi):\ \alpha_3(0) \simeq 1/2\,.$$

If Regge and Bjorken limits are simultaneously valid, β_i must be power behaved:

$$\beta_{1,3} \sim \left(\frac{1}{q^2}\right)^{\alpha_1,\,\alpha_3}, \quad \beta_2 \sim \left(\frac{1}{q^2}\right)^{\alpha_2 - 1},$$

and

$$F_1 \to \beta_1\,\omega^{\alpha_1(0)},$$

$$F_2 \to \beta_2\,\omega^{\alpha_1(0)-1} \ :\ \text{const as}\ \omega \to \infty\,,$$

$$F_3 \to \beta_3\,\omega^{\alpha_3(0)}.$$

Also

$$F_2^{\nu p} - F_2^{\bar{\nu} p} \sim \omega^{\alpha_\rho(0)-1} \sim \frac{1}{\sqrt{\omega}} \text{ as } \omega \to \infty, \text{ etc.}$$

IV. SCALING AND QUARK PARTON MODEL

1. Assumptions: Infinite momentum frame description of hadrons in terms of almost free constituents of light mass.

i) Leptons scatter incoherently from partons.

ii) Partons are almost free: They are near their mass shell before and after interaction.

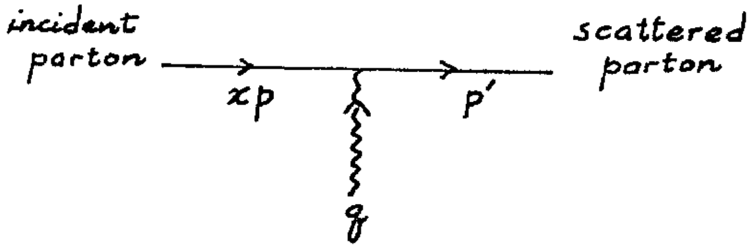

Fig. 6.
Parton kinematics. x is the fraction of longitudinal momentum carried by parton.

$$0 \simeq (m_{\text{parton}})^2 = p'^2 = (xp+q)^2 \simeq 2xp\cdot q + q^2,$$

or

$$x \simeq -q^2/2p\cdot q$$

i.e., for given q^2 and ν, deep inelastic scattering probes the parton distribution with longitudinal momentum fraction x. [References: R. P. Feynman, "Hadron-Photon Interactions", (Benjamin, NY., (1972); D. H. Perkins, in Proceedings of Chicago-Batavia Conference (1972), Vol. 4, p. 189.]

2. Structure function in the parton model.

$$H_{\alpha\beta}^{\nu,\bar{\nu}} = \sum_i u_i(x)\, h_{\alpha\beta}^{\nu,\bar{\nu}}$$

$u_i(x)$: probability that a parton (anti-parton) of type i be found with x.

$h_{\alpha\beta}$: structure function for a parton calculated in the Born approximation.

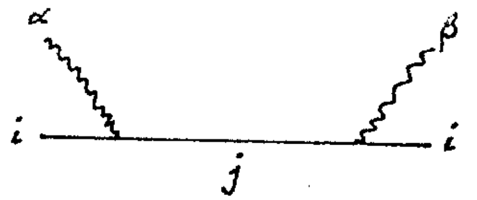

Fig. 7
Born diagram for the parton structure function $h_{\alpha\beta}$.

$$J_\mu = \cos\,\theta_c\, \bar{p}\, \underbrace{\gamma_\mu (1-\gamma_5) n}_{\text{pure V-A}}$$

$$h_{\alpha\beta} = -x\, g_{\alpha\beta} + 2x^2 \frac{p_\alpha p_\beta}{p\cdot q} \pm ix \frac{\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma}{p\cdot q} \text{ for } \begin{matrix}\text{parton}\\ \text{anti-parton}\end{matrix}$$

$$= -g_{\alpha\beta}\omega_1 + \frac{p_\alpha p_\beta}{m_N^2}\omega_2 - i\,\epsilon_{\alpha\beta\rho\sigma}\frac{p^\rho q^\sigma}{2m_N^2}\omega_3 .$$

So

$$\left.\begin{aligned} f_1 &= \omega_1 = x \\ f_2 &= p\cdot q\ \omega_2/m_N^2 = 2x^2 \\ f_3 &= p\cdot q\ \omega_3/m_N^2 = \mp 2x \end{aligned}\right\} \quad \begin{aligned} & f_2 = 2x f_1 \equiv F_2 - 2x F_1 = 0 \\ & \quad \text{(Callan-Gross)} \\ & f_3 = -2 f_1 \text{ (partons)} \\ & \quad = +2 f_1 \text{ (antipartons)} \end{aligned}$$

Experimentally $F_3 \simeq -2F_1 \Rightarrow u_{\bar{p}}, u_{\bar{n}}, u_{\bar{\lambda}} \simeq 0.$

With Callan-Gross relation we can write.

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 m_N E_1}{\pi} x \left\{ \begin{bmatrix} 1 \\ (1-y)^2 \end{bmatrix} F_L(x) + \begin{bmatrix} (1-y)^2 \\ 1 \end{bmatrix} F_R(x) \right\}$$

$$\text{for} \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix}$$

where

$$F_L^k = \sum_{i=p,n,\lambda} a_i^k u_i(x) ,$$

$$F_R^k = \sum_{i=\bar{p},\bar{n},\bar{\lambda}} b_i^k \bar{u}_i(x),$$

k denotes different processes, νp, $\bar{\nu} p$, $\bar{\nu} n$, νn, ep, and en.

3. Equality: From the above linear relations follow

$$12(F_1^{ep} - F_1^{en}) = F_3^{\nu p} - F_3^{\nu n} \text{(not tested)}$$

4. Sum Rules:

$$<S> = \int_0^1 dx\,[u_\lambda - u_{\bar{\lambda}}]$$

$$<I_3> = \int_0^1 dx\,\frac{1}{2}\,[u_p - u_{\bar{p}} - u_n + u_{\bar{n}}]$$

$$<B> = \int_0^1 dx\,\frac{1}{3}\,[u_p + u_n + u_\lambda - u_{\bar{p}} - u_{\bar{n}} - u_{\bar{\lambda}}]$$

From these

a. Adler sum rule $\Leftarrow I_3$.

b. Llewellyn Smith-Gross: $-\int_1^\infty \frac{d\omega}{\omega}(F_3^\nu + F_3^{\bar{\nu}}) =$

$$= <4B + Y(2 - 3\sin^2\theta_c) + 2I_3 \sin^2\theta_c> .$$

5. Inequalities: $u_i \geq 0$.

$$F_2^{ep} + F_2^{en} - \frac{5}{18}(F_2^{\nu p} + F_2^{\nu n}) \propto \text{pos. const } (u_\lambda + u_{\bar{\lambda}}) \geq 0.$$

Experimentally

$$\int_0^1 dx\,(F_2^{ep} + F_2^{en}) \simeq \frac{5}{18}\int_0^1 dx\,(F_2^{\nu p} + F_2^{\nu n})$$

implying $u_\lambda \approx u_{\bar{\lambda}} \approx 0$, and $\frac{18}{5}\,F_2^{ed}(x) \simeq F_2^{\nu d}(x)$.

This is a test for fractionally charged quark partons.

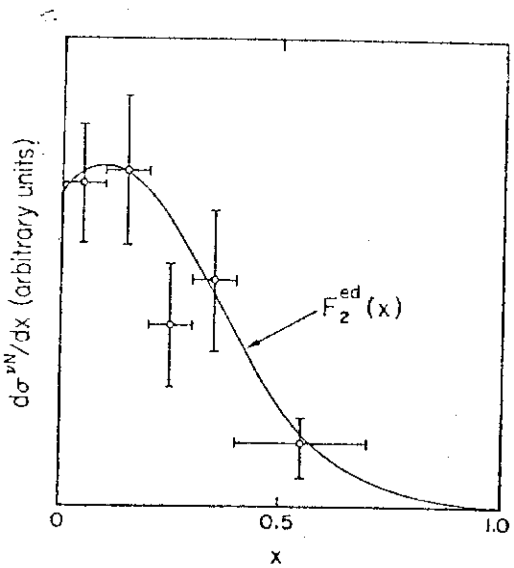

Fig. 8
The distribution $d\sigma^{\nu N}/dx$ (arbitrary units) versus $x = Q^2/2m\nu$ (the scaling variable). For comparison, the fit for $F_2^{ed}(x)$ is shown. From B. C. Barish et al., Phys. Rev. Letters 8, 565 (1973). See also F. Sciulli's contribution.

6. Overall picture

$$\frac{d\sigma}{dxdy} = \frac{G_F^2 m_N E_\nu}{\pi} 2x \begin{cases} u_n(x) & \nu p \\ (1-y)^2 u_p(x) & \bar{\nu} p \\ u_p(x) & \nu n \\ (1-y)^2 u_n(x) & \bar{\nu} n \end{cases}$$

V. SCALING AND LIGHT CONE

1. Light Cone Analysis

$$T_{\mu\nu}(p, q) = \int d^4x\, e^{iq\cdot x} \langle p|[\, j_\mu\left(\frac{x}{2}\right),\, j_\nu^\dagger\left(-\frac{x}{2}\right)]\,|p\rangle$$

$$= \int d^2x_\perp \int dt\, dz\, e^{i\frac{\nu}{m}(t-z) - imx(t+z)} \langle p|[\quad]|p\rangle$$

Dominant contribution comes from $x^2 \simeq 0$, because $t - z \simeq 0\left(\frac{m}{\nu}\right)$ and because of the commutativity of two currents at space-like separation.

Wilson's operator product expansion [K. Wilson, Phys. Rev. 179, 1499 (1969)].

$$\mathscr{J}_\mu(x)\,\mathscr{J}_\nu(0) = \sum_i \underbrace{\mathscr{C}_i(x^2)}_{\text{c number function of } x^2} N[\,\mathscr{O}^{\,i}_{\mu\nu}(0)]$$

Gell-Mann, Fritsche postulate that leading light cone singularity structure of $\mathscr{J}_\mu(x)\mathscr{J}^\dagger_\nu(0)$ same as that of the free quark model → reproduce predictions of the quark parton model, and scaling. [See, for example, Gross and Treiman, Phys. Rev. D4, 1059 (1971).]

More rigorously [see, for example, D. Gross, "Scaling in Quantum Field Theory," unpublished],

$$T[J_\alpha \tfrac{x}{2}\; J_\beta -\tfrac{x}{2}\,] = \sum_{n=0}^{\infty} \mathscr{C}^{(n)}(x^2)\, \mathscr{O}_{\alpha\beta\mu_1\ldots\mu_n}(0)X^{\mu_1}\ldots X^{\mu_n}$$

$$+ \text{ terms contributing to } W_1, W_3$$

$$+ \text{ operators of "twist"} > 2,$$

where

$$\mathscr{O}_{\alpha\beta\mu_1\ldots\mu_n} \simeq \bar\psi \underbrace{\gamma_\alpha\,\partial_\beta\,\gamma_{\mu_1}\ldots}_{\text{sym. and traceless in } \alpha\beta,\,\mu_1,\ldots\mu_n}\psi, \text{ etc.}$$

$$\begin{aligned} \text{twist} &= \text{dimensions} - \text{spin} \\ &= (3+n+1) - (n+2) = 2. \end{aligned}$$

Twist > 2 operators contribute to W's asymptotically down by full powers of $(q^2)^{-1}$.

Take the matrix element of $\mathscr{O}_{\alpha\beta\ldots}$ and spin average:

$$\bar\Sigma \langle p|\,\mathscr{O}_{\alpha\beta\mu_1\ldots\mu_n}(0)|p\rangle \sim p_\alpha p_\beta p_{\mu_1}\ldots p_{\mu_n}.$$

The n-th term contains n+2 factors of p. Compare this to the forward Compton amplitude:

$$p_\alpha p_\beta \int d\nu' \,\frac{W_2(\nu',q^2)}{\nu'-\nu} \cong p_\alpha p_\beta \sum_n (p\cdot q)^n \int dx\, x^n\cdot[\nu' W_2(\nu,q^2)]$$

Thus

$$\int_0^1 dx\ x^N \left[\frac{\nu W_2(\nu, q^2)}{m_N^2}\right] = C^{(N)}(q^2)$$

$$\tilde{C}^{(N)}(q^2) \simeq (q^2)^{n+1}\left(\frac{\partial}{\partial q^2}\right)^n \int d^4x\ e^{iq\cdot x}\mathscr{C}^n(x^2)$$

For free fields, $\tilde{C}^{(N)}(q^2)$ is independent of $q^2 \Rightarrow \nu W_2(\nu, q^2)$ a function only of x.

2. Interacting Fields and Renormalization Groups.

Coefficient functions $\tilde{C}^{(N)i}(q^2)$ satisfy the renormalization group equation .

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma^{(N)i}(g)\right]C_{AS}^{(N)i}(q^2/\mu^2, g) = 0.$$

The solution is of the form

$$C^{(N)i}(q^2/\mu^2, g) \xrightarrow[-q^2\to\infty]{} C^{(N)i}(-q^2)^{-\frac{1}{2}\gamma^{(N)i}(g_\infty)}$$

where g_∞ is a root of $\beta(g)$:

$$\beta(g_\infty) = 0 \text{ and } d\beta(g)/dg\Big|_{g=g_\infty} < 0$$

and $\gamma^{(N)i}$ is the anomalous dimension of $\mathscr{O}^{(N)i}$, $\gamma^{(N)i}(g=0) = 0$; increases with N (positivity of νW_2) and is convex downwards ($\to 2\gamma_\psi$ as $n\to\infty$).

Consider $\mathscr{O}^{(N)i} = \Theta_{\mu\nu}$: conserved energy momentum tensor (D=4, J=2, twist 2). $\Theta_{\mu\nu}$ has no anomalous dimensions, $\gamma^\Theta = 0$. Now moment $x^N \longleftrightarrow$ spin N + 2 operators.

$$\int_0^1 dx\ \nu W_2 = \underbrace{\text{const}}_{\text{contribution of } \Theta_{\mu\nu}} + \frac{C}{|q|^\nu}.$$

$\Theta_{\mu\nu}$ is isoscalar $\Rightarrow$

$$\int_0^1 dx\ \nu W_2^{\nu p}(\nu, q^2) = \int_0^1 \nu W_2^{\nu n}(\nu, q^2) \quad \text{as } -q^2 \to \infty$$

or, (with $\theta_c \to 0$)

$$\int_0^1 dx\ [\nu W_2^{\nu p} - \nu W_2^{\bar{\nu} p}] \to 0$$

We need consider two cases. [See, for example, S. Adler, NAL-Conf-74/39-THY] :

(1) $g_\infty \neq 0$. $\gamma^{(N)}(g_\infty) \neq 0$ in general $\Rightarrow$ moments go to zero as $(-q^2)^{-\frac{1}{2}\gamma^{(N)}}$.

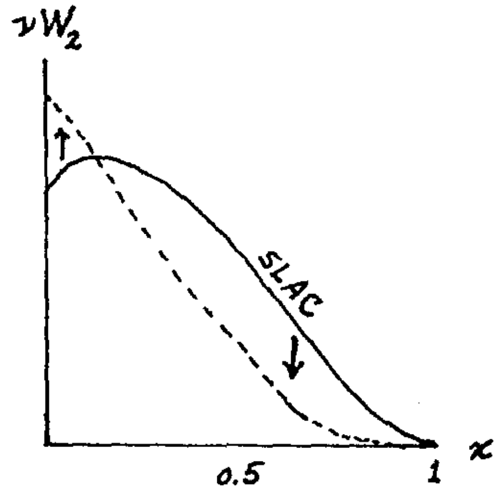

Fig. 9
Behavior of νW_2 as $q^2 \to \infty$ as predicted by field theories (schematic).

νW_2 will change as $-q^2 \to \infty$. (a) Near $x = 1$, decreases to make higher moments go to zero; (b) If $\gamma^{(2)i}$ all small, area remains $\simeq$ const; (c) near $x \simeq 0$ increases.

(2) $g_\infty = 0$ (Asymptotic freedom)

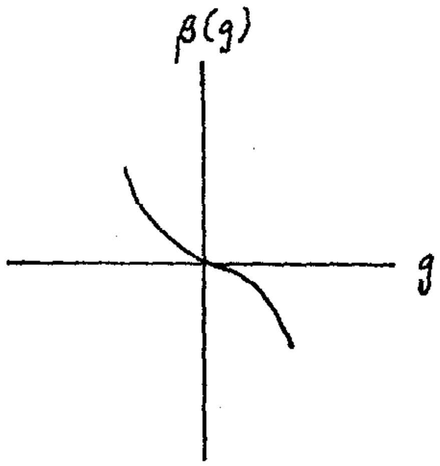

Fig. 10.
Behavior of $\beta(g)$ in asymptotically free gauge theory of strong interactions.

This happens in nonabelian gauge theories.

$$C^{(N)i}\ (q^2) \underset{-q^2\to\infty}{\rightarrow} (\ell n - q^2)^{-\frac{1}{2}\, a^{(N)i}}$$

where $a^{(N)i}$ are computable numbers, independent of g. Approach to asymptotic logarithmic; νW_2 behavior as $-q^2 \to \infty$ similar to above; since $a_{(n)i}$ known, possible to extrapolate W_2 from $(x, -q^2)$ to $(x, -q'^2)$. Combined with the Bloom-Gilman hypothesis, gives bounds on e.m. form factors. For implications of asymptotic freedom on neutrino physics, see the contribution of A. Zee.

3. Remarks

A. If exact scaling remains valid, all known field theory models of strong interactions in trouble.

B. Parton model works at SLAC and NAL ν energies, whether you believe in it or not--a case of Niels Bohr's horseshoe.

C. If scaling breaks down as field theory predicts, $\sigma_\nu - \sigma_\nu \to 0$. Adler sum rule OK, but not Llewellyn Smith-Gross.

VI. GAUGE THEORIES

1. Gauge Theories of Weak Interactions.

General References:

B. W. Lee, Proceedings of Chicago-Batavia Conference, (1972), Vol. 4, p. 249.

S. Weinberg, Proceedings of Aix-en-Provence Conference (1973); Rev. Modern Physics, to be published.

i) Unification of electromagnetic and weak interactions in a gauge theory (nonabelian).

ii) Spontaneously Broken Gauge Symmetry: masses of weak bosons arise thru this mechanism. The symmetry breaking is especially "soft" → The renormalizability of the unbroken gauge theory is preserved.

In a renormalizable theory, bad high energy behavior of a tree graph has to be cancelled by another. Example:

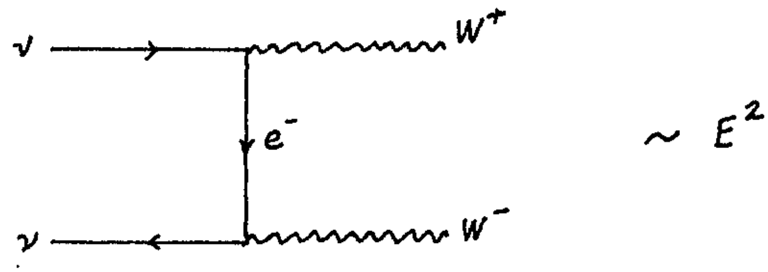

Fig. 11.
The process $\nu + \bar{\nu} \to W^+ + W^-$ in the conventional theory.

The E^2-, E-terms must be cancelled by one or the other of the following graphs:

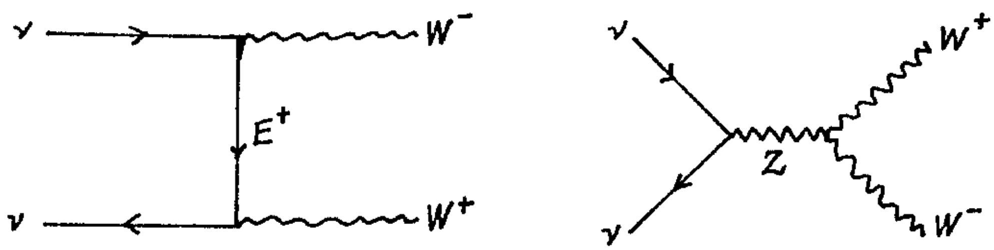

Fig. 12
Two tree diagrams for $\nu + \bar{\nu} \to W^+ + W^-$ that appear in a renormalizable theory.

Because gauge theories are renormalizable, they must contain either heavy leptons, neutral currents or both.

iii) Important Issues:

Heavy leptons
Neutral currents
Charms
Nuclear physics

2. Neutral Current in the Weinberg-Salam Theory.

$$Q = \underbrace{T_3}_{\text{weak isospin}} + \underbrace{Y/2}_{\text{weak hypercharge}}$$

Example: $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$, $T = 1/2$, $Y = -1$

e_R , $T = 0$, $Y = -2$.

Neutral current-gauge boson coupling:

$$\mathscr{L} = g\, j^3_\mu A^{3\mu} + g' \underbrace{(j^{em}_\mu - j^3_\mu)}_{\frac{1}{2} \times \text{"hypercharge" current}} B^\mu .$$

Let

$$\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W, & \sin\theta_W \\ -\sin\theta_W, & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \begin{matrix} \text{massive} \\ \text{massless } \gamma. \end{matrix}$$

Then

$$\mathscr{L} = e\, j^{em}_\mu A^\mu + \sqrt{g^2 + g'^2} \left[j^3_\mu - \sin^2\theta_W\, j^{em}_\mu \right] Z^\mu$$

with

$$e = g g' (g^2 + g'^2)^{-1/2}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad .$$

$$\mathscr{L}_Z = \sqrt{g^2 + g'^2}\, Z^\mu \left[J^3_\mu - \sin\theta_W\, J^{em}_\mu + \frac{1}{2} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu + \ldots \right]$$

Effective Local Coupling:

$$\frac{g^2 + g'^2}{8 m_Z^2} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \left[\underbrace{V^3_\mu - A^3_\mu}_{\substack{\text{isospin rotation} \\ \to \text{charged current}}} - 2 \sin^2\theta_W \underbrace{J^{em}_\mu}_{\text{e.m. current}} \right].$$

In the original construction of Weinberg and Salam, a particularly simple symmetry breaking mechanism was used, leading to the relation

$$\frac{m_Z^2}{m_W^2} = \frac{g^2 + g'^2}{g^2}, \text{ or } \frac{g^2 + g'^2}{m_Z^2} = \frac{g^2}{m_W^2}$$

In general, however, there need not be any relation between m_W and m_Z. Parameterize the strength of Z exchange by one parameter x:

$$\frac{g^2 + g'^2}{m_Z^2} = x \frac{g^2}{m_W^2}.$$

$$\Rightarrow \quad \frac{G_F}{\sqrt{2}} x \; \nu \gamma^\mu (1-\gamma_5) \nu \left[V_\mu^3 - A_\mu^3 - 2 \sin^2 \theta_W J_\mu^{em} \right].$$

3. Tests of the WS Theory in Hadronic Reactions.

While the properties of neutral current ought to be studied in the most general context, we will consider parameterizing neutral current effects a'la Weinberg-Salam theory.

i) Inclusive Reactions: Define

$$R_{\substack{\nu \\ \bar{\nu}}} = \sigma \begin{pmatrix} \nu_\mu + N \to \nu_\mu + B \\ \bar{\nu}_\mu \bar{\nu}_\mu \end{pmatrix} / \sigma \begin{pmatrix} \nu_\mu + N \to \mu^- + B \\ \bar{\nu}_\mu \mu^+ \end{pmatrix}.$$

(A) The hadronic neutral current is in the quark language (with $\theta_c = 0$):

$$\bar{p} \gamma_\mu [a(1-\gamma_5) + b(1+\gamma_5)] p$$
$$+ \bar{n} \gamma_\mu [c(1-\gamma_5) + d(1+\gamma_5)] n,$$

where

$$a = 1/2 - (2/3) \sin^2 \theta_W, \quad b = -(2/3) \sin^2 \theta_W$$

$$c = -1/2 + (1/3) \sin^2 \theta_W, \quad b = (1/3) \sin^2 \theta_W .$$

$$R_\nu = \frac{1}{2} - \sin^2 \theta_W + \frac{20}{27} \sin^4 \theta_W ,$$

$$R_{\bar{\nu}} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W .$$

[A. De Rujula et al., Revs. Mod. Phys., to be published.]

(B) Assuming scaling in electroproduction,

$$R_\nu \geq \frac{1}{2}\left\{1 - 2\sin^2\theta_W\sqrt{t}\right\}^2;$$

Assuming scaling both in eN and νN ($\bar{\nu}N$),

$$R_\nu \geq \frac{1}{2}\left\{\frac{2}{3} + \frac{1}{3}x - (1-x^2)t\right\}$$

where

$$t = \frac{G^2}{\pi}\frac{4}{3}m_N E_1 \int_0^1 dx\, F_2^{eN}(x)/\sigma(\nu_\mu + N \rightarrow \bar{\mu} + B).$$

Using

$$\int_0^1 dx\, F_2^{eN}(x) \simeq 0.14, \quad \sigma(\nu_\mu + N \rightarrow \bar{\mu} + B)$$

$$\frac{G^2}{\pi}m_N E_1 \;.\; 0.52,$$

one gets $t \cong 0.36$.

Take $t = 1/3$. One gets

$$R_\nu = \frac{1}{2}\cdot\frac{1}{3}(1 + x + x^2),$$

$$R_\nu = \frac{1}{2}(1 - x + x^2).$$

[Pais and Treiman, Phys. Rev. D6, 2700 (1972); Paschos and Wofenstein, Phys. Rev. D7, 91 (1973)].

ii) Exclusive Reactions

(A) Elastic: Theory predicts

$$0.15 \leq \sigma(\nu + p \rightarrow \nu + p)/\sigma(\nu + n \rightarrow \mu + p)$$

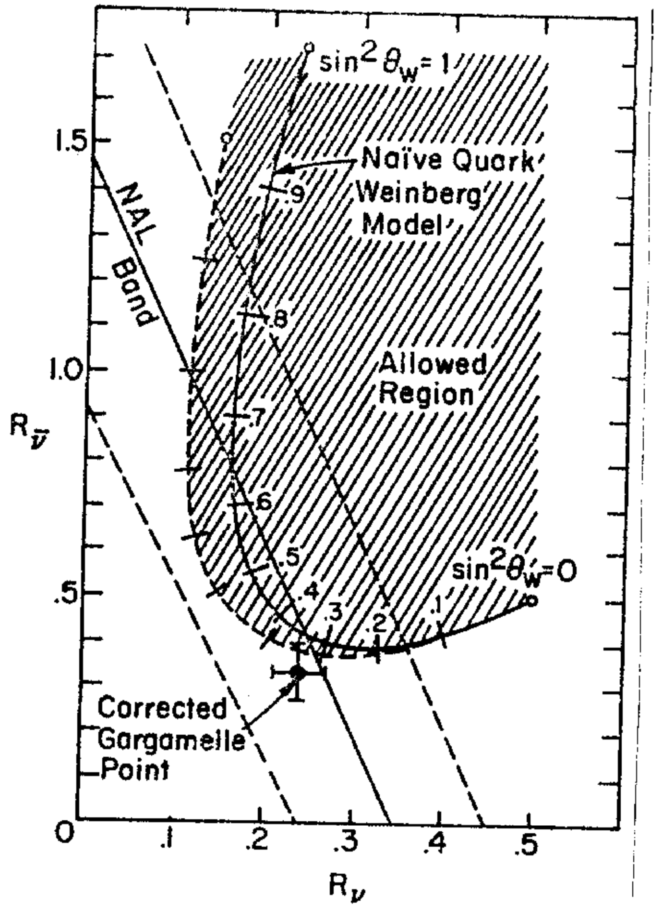

Fig. 13.
Comparison of theory and experiment for processes $(\nu, \bar{\nu}) + N \to (\nu + \bar{\nu}) +$ anything. From A. De Rujula et al., Revs. Mod. Phys., to be published.

$$\leq 0.25 \text{ for } \sin^2\theta \leq 1/2$$

Experiment (Gargamelle) gives 0.12 ± 0.06

(B) Δ Production. Define

$$R = \frac{\sigma(\nu_\mu + p \to \nu_\mu + p + \pi^\circ) + \sigma(\nu_\mu + n \to \nu_\mu + n + \pi^\circ)}{2\sigma(\nu_\mu + n \to \mu^- + p + \pi^\circ)} .$$

Δ dominance gives $R \geq 0.4 \sim 0.5$ for $\sin^2\theta_W < 1/3$.

Refined theory (Adler, Nussinov, and Paschos, Phys. Rev. D, to be published) gives

$\sin^2\theta_W$	Δ only	$+ I = 1/2$	+ charge exch. in Al
0.3	0.56	0.40	0.23
0.4	0.46	0.33	0.18

Experiment (W. Lee-Columbia) gives R < 0.14 (90% CL). [W. Lee, Phys. Letters 40B, 423 (1972)].

4. Tests of the WS Theory in Leptonic Processes.

$$H_W(\nu e \to e\nu) = \frac{G}{\sqrt{2}} \quad \bar{\nu}_\mu \gamma^\alpha(1-\gamma_5)\nu_\mu \bar{e}\gamma_\alpha(g_V - g_A\gamma_5)e$$

$$+ \bar{\nu}_e \gamma^\alpha(1-\gamma_5)\nu_e \bar{e}\gamma_\alpha(G_V - G_A\gamma_5)e$$

where

$$G_A \quad 1 + \quad g_A$$

$$G_V = 1 + \quad g_V$$

neutral current

charge current

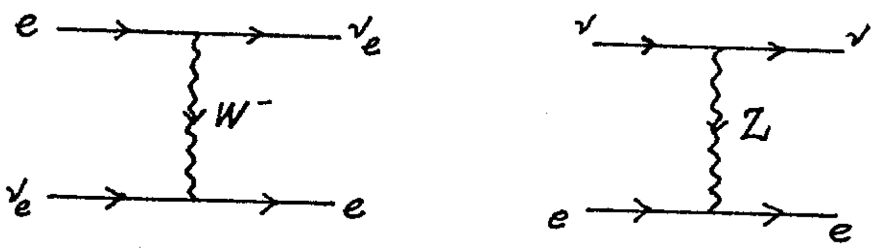

Fig. 14
Tree diagrams for $\nu + e \to \nu + e$.

$$g_V = (-\frac{1}{2} + 2\sin^2\theta_W)x$$

$$g_A = (-\frac{1}{2})x.$$

(A) $\sigma(\bar{\nu}_e + e \to \bar{\nu}_e + e) = C\, 10^{-41}\, cm^2\, (E_\nu/GeV).$

C = 0.54 in standard V-A

= 0.136 ~ 2.86 in WS

Gurr, Reines and Sobel, Phys. Rev. Letters 28, 1406 (1972) find

$$\sigma \leq 3\,\sigma_{V-A} \quad (90\%\ cL).$$

(B) Gargamelle had 370,000 pix each for ν_μ and $\bar{\nu}_\mu$.

	WS	background (est)	observed
ν_μ	$0.6 \sim 6.0$	0.3 ± 0.2	0
$\bar{\nu}_\mu$	$0.4 \sim 8.0$	0.03 ± 0.02	1

[F. J. Hassert et al., Phys. Letters 46B, 121 (1973)].

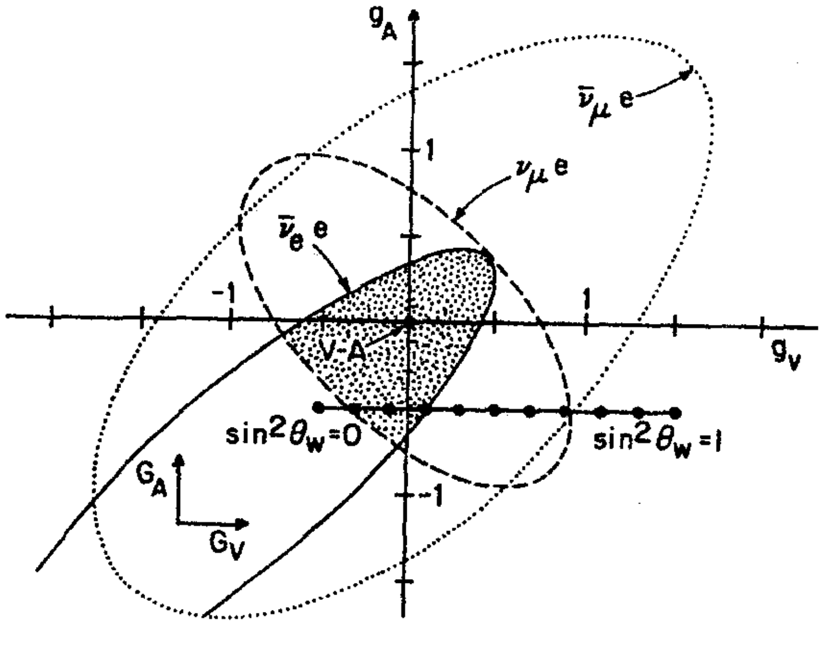

Fig. 15.

Various limits on g_A and g_V imposed on by $(\nu, \bar{\nu}) + e$ elastic scattering. From A. De Rujula et al., Revs. Mod. Phys. to be published.

5. Neutral Current in Nuclear Physics.

i) Nuclear coherent scattering of neutrinos. Measures the strength of vector current, i.e.,

$$<I_3 - 2\sin^2\theta_W Q> \equiv a \text{ in W-S.}$$

$$\sigma(100 \text{ MeV/c} < q_{\text{recoil}} < 300 \text{ MeV/c}) \text{ on } {}^{12}\text{C}$$

$$= a^2 \cdot 13.6 \times 10^{-39} \text{ cm}^2, \quad E_\nu > 1 \text{ GeV}$$

$$= a^2 \cdot 11.2 \times 10^{-39} \text{ cm}^2, \quad E_\nu \simeq 200 \text{ MeV}$$

[D. Z. Freedman, Phys. Rev. D, to be published.] Also astrophysical implications!

ii) G. T. transitions by $\vec{A}^3$. [T. W. Donnelly et al., SLAC-Stanford preprint (1973).]

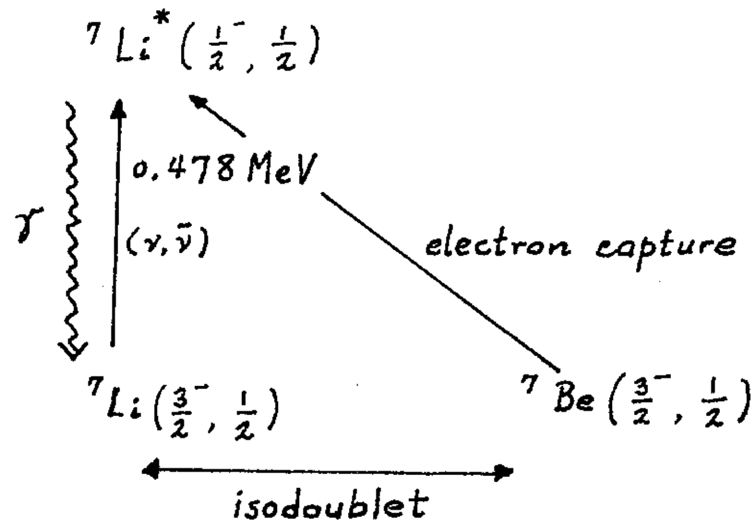

Fig. 16
Schematics of the reaction $(\nu, \bar{\nu}) + \mathrm{Li} \rightarrow (\nu, \bar{\nu}) + \mathrm{Li}^* \rightarrow \mathrm{Li} + \gamma$.

At Savannah River ($2 \times 10^{13}\ \bar{\nu}_e/\mathrm{cm}^2/\mathrm{sec}$), 4γ's/day/Kg ^{7}Li are expected.

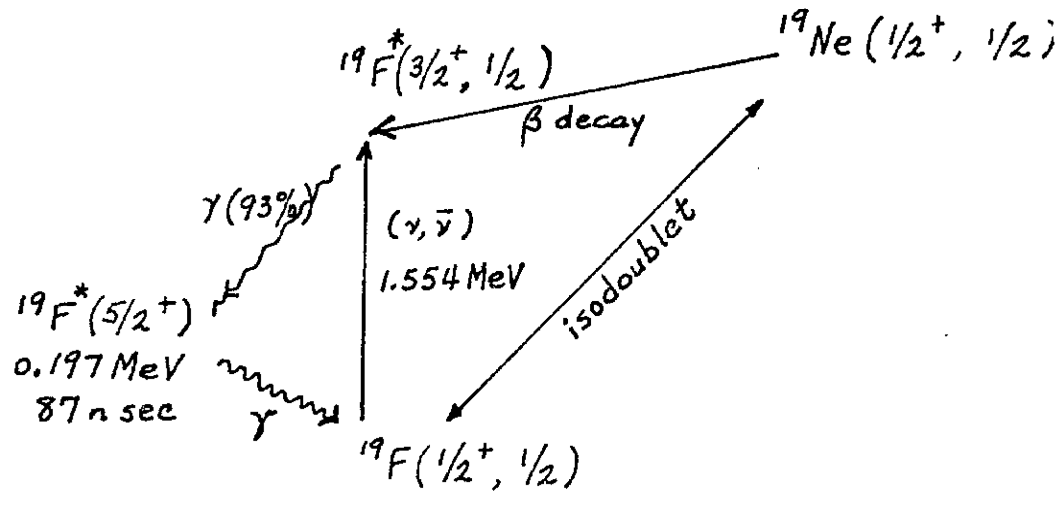

Fig. 17
Schematics of $(\nu, \bar{\nu}) + \mathrm{F} \rightarrow (\nu, \bar{\nu}) + \mathrm{F}^*$ and subsequent decays.

Delayed 2γ coincidence gives a unique signature!

iii) Giant dipole excitation. [Bilenky and Dadajan, Dubna preprint (1973).] (V_0^3 and $\vec{A}^3$)

$$\sigma_{\nu+\bar{\nu}} \geq (1 - 2\sin^2\theta_W)^2\,\sigma_0 \quad (V_0^3 \text{ contribution}).$$

where σ_0 can be estimated from photoabsorption cross section.

σ_0 on a typical nucleus.

30 MeV	50	100
10^{-42}	$10^{-(41\sim 40)}$	10^{-39} cm^2