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$I=\frac{1}{2}$ contributions to $\nu_\mu + N \to \nu_\mu + N + \pi^0$ in the Weinberg weak-interaction model

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We use a detailed dispersion-theoretic model for pion production in the (3, 3)-resonance region to calculate the ratio

$$R=\frac{\sigma(\nu_\mu+n\to\nu_\mu+n+\pi^0)+\sigma(\nu_\mu+p\to\nu_\mu+p+\pi^0)}{2\sigma(\nu_\mu+n\to\mu^-+p+\pi^0)}$$

in the Weinberg weak-interaction theory. We find that $I=\frac{1}{2}$ contributions do not substantially modify the earlier static model calculation of R given by B. W. Lee.

Neutral-pion production by neutrinos appears to be one of the best reactions for searching for the hadronic weak neutral current predicted by the Weinberg weak-interaction theory.[1] In fact, if one accepts the bound

$$\sin^2\theta_W \leq 0.35 \tag{1}$$

given by Gurr, Reines, and Sobel,[2] the static-model calculation by B. W. Lee[3] of

$$R=\frac{\sigma(\nu_\mu+n\to\nu_\mu+n+\pi^0)+\sigma(\nu_\mu+p\to\nu_\mu+p+\pi^0)}{2\sigma(\nu_\mu+n\to\mu^-+p+\pi^0)} \tag{2}$$

in the Weinberg theory is already in conflict with existing experiments[4] in complex nuclei. Two essential cautions are necessary, however, before concluding that the Weinberg theory is ruled out. First, charge-exchange effects are important in complex nuclei, and may result in an experimentally measured value of R which is smaller than the true single-nucleon-target value by a factor of up to 2.[5] Second, the static-model approximation, which neglects $I=\frac{1}{2}$ s-channel contributions to the reactions in Eq. (2), has the effect of overestimating R.[6] If the $I=\frac{1}{2}$ corrections are large enough, then, together with charge-exchange corrections, they may move experiment and theory back into agreement.

In this note we report the results of calculating the $I=\frac{1}{2}$ corrections to R using the detailed dispersion-theoretic model of weak pion production in the (3, 3)-resonance region which we developed some time ago.[7] The model is basically a generalization to weak pion production of the old CGLN model for pion photoproduction.[8] Nonresonant multipoles are treated in the Born approximation,[9] while the resonant (3, 3)-channel multipoles are obtained from the Born approximation by a unitarization procedure. The model is in excellent agreement with pion photoproduction data,[7] agrees well with pion electroproduction data up to a four-momentum transfer of $k^2 \approx 0.5$ (GeV/c)^2,[7] and is also in satisfactory accord with the recent Argonne measurements of weak pion production.[10] Because all terms contributing to the weak-production amplitude in the model are proportional to nucleon elastic form factors, the model fails badly in the region $k^2 \gg 0.5$ (GeV/c)^2, where scaling effects become visible and leptonic inelastic cross sections decrease more slowly with increasing k^2 than elastic form factors squared. Fortunately, this region of large k^2 makes a relatively small contribution to the individual cross sections in Eq. (2), and the errors will furthermore tend to cancel between numerator and denominator.

In the Weinberg model, the effective Lagrangian for the semileptonic strangeness-conserving weak interactions is

$$\mathcal{L}=\frac{G}{\sqrt{2}}\cos\theta_C\{\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu(J_\lambda^{V1}+iJ_\lambda^{V2}+J_\lambda^{A1}+iJ_\lambda^{A2}) +\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_\mu[J_\lambda^{V3}(1-2\sin^2\theta_W)+J_\lambda^{A3}-2\sin^2\theta_W J_\lambda^S]+\cdots\}, \tag{3}$$

where we have shown both the charged- and the neutral-current terms contributing to Eq. (2). In terms of the isospin matrix elements defined in Eqs. (2B.4) and (2B.5) of Ref. 7, the hadronic matrix element of the neutral current is

$$_{\text{out}}\langle\pi N|J_\lambda^{V3}(1-2\sin^2\theta_W)+J_\lambda^{A3}-2\sin^2\theta_W J_\lambda^S|N\rangle=(1-2\sin^2\theta_W)[a_E^{(3/2)}V_\lambda^{(3/2)}+a_E^{(1/2)}V_\lambda^{(1/2)}] -2\sin^2\theta_W a_E^{(0)}V_\lambda^{(0)}+a_E^{(3/2)}A_\lambda^{(3/2)}+a_E^{(1/2)}A_\lambda^{(1/2)}. \tag{4}$$

The amplitudes appearing in Eq. (4) are all ones which appear in either the pion-electroproduction or the weak-production calculations of Ref. 7, and so R can be evaluated by a simple adaptation of the computer routines used in the earlier work. The result of such a calculation is shown in Fig. 1, where we have assumed an incident lab neutrino energy $k_{10}^L = 1$ GeV and a nucleon axial-vector elastic form factor

$$g_A(k^2) = \frac{1.24}{[1 + k^2/(1\ \mathrm{GeV}/c)^2]^2}, \qquad (5)$$

and have integrated over the (3, 3)-resonance region up to a maximum isobar mass of $W = 1.47$ GeV. Curve a gives the result obtained from our model when both resonant and nonresonant multipoles are kept; curve b is the corresponding result obtained when only the resonant multipoles are kept, and hence when $I = \frac{1}{2}$ amplitudes are neglected. As expected, curve a lies below curve b, but the effect of the $I = \frac{1}{2}$ corrections is not dramatic. For comparison, we give in curve c the result obtained from Lee's static-model calculation.[11] If one assumes that θ_W is restricted as in Eq. (1), and includes a safety factor of 2 for charge-exchange effects, curve a is barely consistent with the present experimental upper bounds. Put conservatively, our calculations indicate that an experiment to measure R at the level of a few percent should be decisive.

Note added in proof. Recently, the possible observation of neutral current events has been reported in deep-inelastic inclusive neutrino reactions by the CERN Gargamelle group.[12] If confirmed, this experiment will establish the *existence* of neutral currents; however, more detailed questions, such as whether the phenomenological form of Eq. (3) is correct, will require the independent study of many different neutral-current induced reactions, among them the pion-production reaction considered in this note.

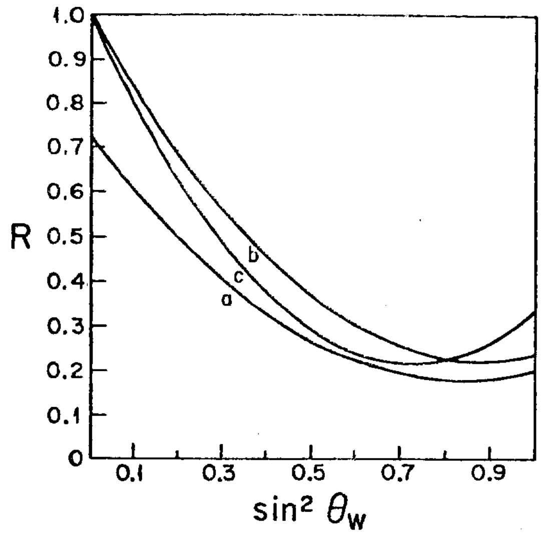

FIG. 1. Ratio R of Eq. (2) vs Weinberg angle θ_W. Curve a—resonant and nonresonant multipoles; curve b—resonant multipoles only; curve c—resonant multipoles in Lee's static-model calculation.

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967). By the "Weinberg theory" we will always mean Weinberg's original model. There are, of course, many variations on the gauge-theory idea in which weak neutral currents are not present.

[2]H. S. Gurr, F. Reines, and H. W. Sobel, Phys. Rev. Lett. 28, 1406 (1972).

[3]B. W. Lee, Phys. Lett. 40B, 420 (1972).

[4]W. Lee, Phys. Lett. 40B, 423 (1972); CERN Gargamelle experiment, as reported by P. Musset, Bull. Am. Phys. Soc. 18, 73 (1973).

[5]D. H. Perkins, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189.

[6]See, for example, footnote 18 of E. A. Paschos and L. Wolfenstein, Phys. Rev. D 7, 91 (1973).

[7]S. L. Adler, Ann. Phys. (N.Y.) 50, 189 (1968).

[8]G. F. Chew, F. E. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[9]In Ref. 7 we also evaluate dispersive corrections to the nonresonant partial waves arising from (3, 3)-resonance exchange in the u channel. We omit these corrections in the present note, because they are small but costly to evaluate in terms of computer time.

[10]J. Campbell *et al.*, Phys. Rev. Lett. 30, 335 (1973); P. Schreiner and F. von Hippel, *ibid.* 30, 339 (1973).

[11]Curve c is obtained by setting the parameter η in Ref. 3 equal to zero; this corresponds to the static limit of the model of Ref. 7.

[12]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).