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Couplings of the Vacuum Trajectory

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ABSTRACT

We discuss in a unified and straightforward manner the various requirements for the vanishing of the couplings of a vacuum pole with $\alpha_V(0) = 1$ and factorizable residues. We show how the proof of these results goes through even in the presence of J-plane branch cuts which may collide with the pole at $J = 1$, if the residue at the pole remains factorized. Special attention is devoted to the question of the two particle-vacuum pole coupling which sets the scale of total cross sections. We indicate how the argument that this may also vanish can be incorrect.

I Introduction

Over the past several years rather severe restrictions have been given on the allowed couplings of the vacuum trajectory when it is supposed to have $\alpha_V(0)=1$, in order to account for the remarkable constancy of hadron total cross sections. Several of these restrictions are moderately old and familiar[1.] and require the two vacuum-one particle coupling to vanish when the Reggeon four momenta, q_i, have $q_i^2=0$. Some of these restrictions have been more recently derived, such as the vanishing of the three vacuum vertex appearing in the Reggeon calculus[2.] and in inclusive reactions[3.] and the coupling of the vacuum trajectory to any two states of unequal mass[4.,5.] or even to any Reggeon and other particles.[4.,5.,6.] The most striking result[4.,7.] is that at $q^2=0$ the vacuum trajectory appears at first sight to decouple from two particles; for example, the two pion-vacuum coupling would vanish. This, of course, immediately implies that the Pomeron does not contribute to total cross sections and the whole raison d'etre for discussing it would be lost.

In this note we would like to present a straightforward, and we believe instructive, derivation of these restrictions. It sets the issues out rather plainly and because of the importance of the problem being considered, it may be of some general value to have in hand a unified discussion. Beyond pedagogy we show how the decoupling phenomena transpire even in the presence of J-plane cuts which may collide with poles at t=0. Also we give a critical presentation of the result that the Pomeron-two particle coupling vanishes at $q^2=0$ pointing out that the argument could well be invalidated by additional structure in the vacuum coupling to many particle states at $q_\perp=0$. A detailed account of the source and consequences of such structure is reserved for a separate article.[8.]

II Derivation of the Restrictions on Vacuum Couplings

Our procedure here will be to follow the technique of Reference 5 and use the Schwartz inequality and the vanishing of the three Pomeron vertex measured in inclusive processes to provide restrictions on the coupling of the vacuum trajectory.

To establish our notation and to provide some connection among the works in the various cited papers let us begin with the amplitude for the exclusive reaction $a+b \to c + N$ particles with invariant mass M. This is shown in Figure 1. We choose the usual variables $s = (p_a + p_b)^2$, $q^2 = (p_a - p_c)^2$ and imagine that an appropriate choice of cluster variables for the N particle missing mass has been made. Denote these variables by v; besides invariant subenergies and momentum transfers these variables must specify the orientation of the cluster particles with respect to a, b, and c. We want to consider this exclusive amplitude for large incident energy s with q^2, M, and v fixed. The behaviour in this limit coming from the exchange of a factorizable Regge pole with trajectory $\alpha_R(q^2)$ will be

$$T_{ab\to cN}(s, q^2, M, v) \underset{s\to\infty}{\sim} s^{\alpha_R(q^2)} \beta_{acR}(q^2)\, \beta_{bNR}(q^2, v, M), \qquad (1)$$

where β_{acR} is the Reggeon-two particle coupling and β_{bNR} is the Reggeon-particle- N particle coupling. Any behaviour of $T_{ab\to cN}$ arising from other Regge poles or from J-plane branch cuts can be distinguished at this point by their distinct s dependence, and therefore, we may quite generally, even when poles and cuts are colliding, speak separately about any given Regge pole exchange as in (1).

It is always possible to label the couplings β by some choice of invariants like q^2, but it is often advantageous[2.] to give explicitly the components of momenta, especially the Reggeon momentum q, perpendicular to the plane (in four space) formed by the incoming momenta p_a and p_b. To see something of the equivalence of the characterizations let us work in the rest frame of particle b and choose a to move along the 3-axis with large momentum p:

$$p_a = (E, 0, 0, p), \tag{2}$$

and
$$p_b = (m_b, 0, 0, 0). \tag{3}$$

In this frame we give q as

$$q = (q_0, \mathbf{q}_\perp, q_3). \tag{4}$$

As $p \to \infty$

$$s = m_a^2 + m_b^2 + 2m_b E \sim 2m_b p + O(1/p). \tag{5}$$

We require $p_a \cdot q$ to remain finite in the limit corresponding to (1), so

$$p_a \cdot q = p(q_0 - q_3) + O(1/p) = \text{finite} \tag{6}$$

and
$$q_0 - q_3 = O(1/p). \tag{7}$$

In addition since $M^2 = (q + p_b)^2$ is fixed, $p_b \cdot q = m_b q_0$ is finite. Thus the mass squared of the Reggeon is

$$q^2 = (q_0 - q_3)(q_0 + q_3) - q_\perp^2 = -q_\perp^2 + O(1/s) \tag{8}$$

where $q_\perp = |\mathbf{q}_\perp|$. So to order 1/s it suffices to give $q_\perp$ instead of q^2 as a label for the Reggeon. We remind the reader that all these statements are really familiar from the use of infinite momentum variables in field theory.

Now since the whole idea of the dominance of some J-plane singularity, pole or cut, is meaningful only to O(1/s), the two labelings are identical in physical content. The $q_\perp$ notation has the additional virtue of reminding us that statements we will make about Reggeons are in reference to exchanged objects in some process where a time, 3-axis plane is defined. We shall adopt this designation in our subsequent discussion.

After this small kinematic digression we are ready to turn our attention to the problem at hand. Choose particle c in Equation (1) to be the same as a, but with momentum p_a'. We may now pick out from the asymptotic behaviour of $T_{ab\to aN}$ the contribution of the vacuum trajectory $\alpha_V(q_\perp)$:

$$T^V_{ab\to aN} = s^{\alpha_V(q_\perp)} \beta_{aaV}(q_\perp)\,\beta_{bNV}(q_\perp, M, \nu). \qquad (9)$$

Form the quantity (shown in Figure 2)

$$\sum_N T^V_{ab\to aN}(s, q_\perp, M, \nu)\, T^*_{df\to N}(q_d^2, q_f^2, M)\,\delta^4(q+p_b-p_N), \qquad (10)$$

where d and f are any two hadron states, with invariant masses q_d^2 and q_f^2 respectively, which can communicate with the chosen N particle state. The sum on N includes the usual phase space integration consistent with $p_N^2 = M^2$. This sum in (10) is, up to some constants, just the imaginary part of the "five point" function a + b $\longrightarrow$ a + d + f taken in the variable M^2. The Schwartz inequality provides an upper bound to this sum in the form

$$|\mathrm{Sum}\ (10)|^2 \leq \left\{ \sum_N |T^V_{ab\to aN}|^2 \delta^4(q+p_b-p_N) \right\} \times$$

$$\times \left\{ \sum_N |T_{df\to N}|^2 \delta^4(q_d+q_f-p_N) \right\}, \qquad (11)$$

where we recognize the first term on the right as the vacuum pole contribution to the inclusive cross section for a + b $\longrightarrow$ a + anything in the region $s\to\infty$, q^2, M^2 fixed, while the second term is the forward absorptive part of the quasi- two body process d + f $\to$ d + f. (Again see Figure 2.)

Our next step is to choose the hadron state f such that the quantum numbers in the $\bar{b}f$ channel include the vacuum and then consider the limit of (11) as $M^2 \to \infty$. It is necessary to take this limit maintaining $s/M^2 \to \infty$, so the kinematic statements above are still correct.

On the right hand side of our inequality we encounter as the leading behaviour in M^2

$$\left\{ g_V(q_\perp)\,\beta_{bbV}(0)\,(\Delta/M^2)^{2\alpha_V(q_\perp)}(M^2)^{\alpha_V(0)} \right\} \times$$

$$\times \left\{ \beta_{ddV}(0)\,\beta_{ffV}(0)\,(M^2)^{\alpha_V(0)} \right\}, \qquad (12)$$

where $g_V(q_\perp)$ is the three vacuum pole coupling with two legs carrying $q_\perp$. On the left hand side we have a process with minimum momentum transfer $t_{fb} = (p_b - q_f)^2$ of order $1/M^4$ for large M^2, so we may extract out the leading contribution in M^2 even at $(q_{fb})_\perp = 0$. This is

$$\left[\beta_{VdV}(q_\perp, q_d^2, (q_{fb})_\perp = 0)\,\beta_{bfV}(q_f^2, (q_{fb})_\perp = 0)(\Delta/M^2)^{\alpha_V(q_\perp)}(M^2)^{\alpha_V(0)} \right]^2. \qquad (13)$$

It is important to note that <u>branch cuts in the J-plane may be present</u> and contribute both to (12) and to (13), but at $(q_{fb})_\perp = 0$ the pole term provides the leading behaviour by powers of $\log M^2$. We have written (12) and (13) in factorized form making quite explicit our assumption that the leading pole at $\alpha_V(0)$ factorizes even in the presence of colliding cuts.

At this point we call upon References 2 and 3 to remind us that if $\alpha_V(0) = 1$, $g_V(q_\perp = 0) = 0$. So for $q_\perp = 0$, the leading behaviour on the right hand side of the inequality vanishes, and so it must on the left. We learn then (see Figure 3)

$$\beta_{VdV}(q_\perp = 0, q_d^2, q'_\perp = 0) = 0 \qquad (14)$$

that is, two vacuum trajectories with $q_\perp = 0$ must decouple from any hadron state d identically for all choices of variables for d. This is the key result.[4.,5.] The various decouplings listed in the introduction follow from this restriction on the two vacuum coupling.

First, split d into two clusters d_1 and d_2 each of which can couple to vacuum quantum numbers. As (14) must vanish for all $q_d^2 = (q_{d_1} + q_{d2})^2$, the contribution of any particular exchange between the clusters must also vanish. Indeed the term coming from the factorizable vacuum pole (see Figure 4)

$$(q_d^2)^{\alpha_V(k_\perp)} \beta_{Vd_1V}(q_\perp = 0, q_{d_1}^2, k_\perp)\, \beta_{Vd_2V}(k_\perp, q_{d_2}^2, q'_\perp = 0) \qquad (15)$$

must be zero, where $k = (q - q_{d_1}) = (q_{d_2} - q')$. Thus,

$$\beta_{VdV}(q_\perp = 0, q_d^2, k_\perp) = 0, \qquad (16)$$

and one vacuum pole with $q_\perp = 0$ decouples.

Second, split d into two clusters again, this time choosing each cluster to carry the quantum numbers of some Regge trajectory R. The contribution to (14) of the exchange of this particular trajectory (Figure 5)

$$(q_d^2)^{\alpha_R(k_\perp)} \beta_{Vd_1R}(q_\perp = 0, q_{d_1}^2, k_\perp)\, \beta_{Rd_2V}(k_\perp, q_{d_2}^2, q'_\perp = 0), \qquad (17)$$

must be zero. So we learn

$$\beta_{VdR}(q_\perp = 0, q_d^2, k_\perp) = 0; \qquad (18)$$

that is, a vacuum pole with $q_\perp = 0$ decouples from any Regge trajectory and any hadron state with the appropriate quantum numbers.[4.,5.,6.]

This last result can be extrapolated with no ambiguity to a single particle state, call it b, at $k_\perp^2 = -m_b^2$ when $q_d^2 \neq m_b^2$. (We will discuss this important case and the argument of Brower and Weis[7.] in one moment.) This means that the coupling of a Pomeron at $q_\perp = 0$ to a particle b and any hadron state d vanishes. Precisely this coupling is measured in the diffraction dissociation cross section a + b → a + anything. Our result[4.,5.] says that the contribution of the vacuum pole to the differential cross section $d\sigma(a+b \to a' +$ Missing Mass$)/dq^2dM^2$ at $q^2=0$ will vanish for <u>any value of</u> the missing mass $M \neq m_b$. The depth of a dip at $q^2=0$ or more correctly the tendency toward a dip at $(q^2)_{min}=O(1/s^2)$ will be set by the size of secondary contributions, cuts and poles. Of course, we are unable to say anything quantitative about that. However, if one sees the dip at all, then he must see it become deeper as the initial energy is increased.

This covers all of the physically interesting decoupling results except the most intriguing one[4.,7.] that the vacuum pole at $q_\perp = 0$ must also decouple from two particles. The next section is devoted to that.

III The Vacuum Pole - Two Particle Coupling

In this section we shall discuss the question of whether the decoupling of a $q_\perp = 0$ Pomeron from any Reggeon and any hadron state indeed necessitates the vanishing of the Pomeron-two particle coupling which is supposed to give the scale of constant total cross sections. This matter has been treated in detail by Brower and Weis[7], and we shall comment on their work after our own treatment.

Let us begin with the statement of Equation (18) extrapolated to $k_\perp^2 = -m_b^2$

$$\beta_{bdV}(m_b^2, q_d^2, q_\perp = 0) = 0, \qquad (19)$$

and inquire how this can occur. To be concrete consider b a pion and d a state of three pions. From the point of view of singularities in variables connected with the state d, any contribution to (19) which involves a Pomeron at $q_\perp = 0$ connecting states of different mass must be zero. Definite contributions not suffering from this problem are pole terms such as the one shown in Figure 6. The residue at this pole and at each of the other poles is proportional to $\beta_{\pi\pi V}(0)/q_c$. If these are the only terms singular as $q_o \to 0$, which one would expect from the usual notions of analyticity, then (19) requires $\beta_{\pi\pi V}(0) = 0$. The same argument applied to other choices for the states b and d leads to the conclusion that $\beta_{bbV}(0) \propto \sqrt{\sigma^{bb}_{Total}} = 0$. It is clear from this very elementary argument that the key point is the persistance of a conventional pole structure in β_{bdV}. That additional singularities might well be playing a role may be seen by remembering that our coupling (19) has been extracted as the residue of a Regge pole in some larger process. Just at the point of interest, namely $q_o \to 0$, however, the energy across that Regge pole is no longer allowed to become large, and it is no longer appropriate to consider that pole exchange as an asymptotic representation of the overall amplitude. Indeed, the full amplitude certainly has a pole at $k_\perp^2 = -m_b^2$ whose residue is connected with β_{bbV}. It is not correct to assume that only some particular piece of the amplitude has that particle pole unless it is the appropriate representation of the full amplitude in a region of phase space. When, in (19), $q_d^2 \neq m_b^2$ (that is, $q_o \neq 0$), then the Regge pole exchange term is a valid representation and the pole must be in it. So our result on diffraction dissociation stands.

It is our opinion that the right way to look at the apparent vanishing of $\beta_{bbV}(0)$ is that there must be additional structure in multiparticle amplitudes at $q_o=0$ and one should find the physics behind it. Precisely such a suggestion will be explored in detail in Reference 8.

Although their argument is couched in rather different terms our observation applies to the work of Brower and Weis[7] as well. They proceeded as follows. In the decoupling result (18) they take $q_d^2=m_d^2$ of some single particle state d lying on the Regge trajectory R. Then they remark that this coupling appears in the five point function of Figure 7, whose asymptotic behaviour we have written in (13) for large s and $s_2=M^2$

$$\left(s/s_2\right)^{\alpha_V(q_\perp)} \left(s_2\right)^{\alpha_R(k_\perp)} \beta_{VdR}(q_\perp, k_\perp), \qquad (20)$$

which using the kinematic relation

$$\frac{s_1 s_2}{s} = \left(m_d^2 + k_\perp^2 + q_\perp^2\right), \qquad (21)$$

correct as usual to leading order in energy, they write as

$$(s_1)^{\alpha_V(q_\perp)} (s_2)^{\alpha_R(k_\perp)} R(q_\perp, k_\perp). \qquad (22)$$

The relation between R and β

$$R(q_\perp, \underset{\sim}{k}_\perp) = (k_\perp^2 + m_d^2 + q_\perp^2)^{-\alpha_V(q_\perp)} \beta(q_\perp, \underset{\sim}{k}_\perp) \qquad (23)$$

causes no problems when $q_\perp \neq 0$, but appears to introduce a spurious singularity at $k_\perp^2 = -m_d^2$ into R when $q_\perp = 0$. Brower and Weis argue in essence that no such behaviour can be present in the full five point function and use knowledge of the analytic properties of the vertex R to see how this singularity may be canceled in the asymptotic form (22) by itself. They conclude this requires $\beta_{ddV}(0) = 0$.

This is certainly an unassailable exercise as long as (22) is a proper representation of the $2 \rightarrow 3$ amplitude. We see from (21), however, that in the case $q_\perp = 0$, taking $k_\perp^2 = -m_d^2$ means $s_1 s_2/s$ is zero. Since we want there to still be a vacuum pole exchanged across the energy s_1, this means s_2 cannot be large. The five point function is therefore no longer given by (22) and one need not cancel the singularities just mentioned by the structure of the two Reggeon-particle vertex itself. It is quite possible forthere to appear additional structure for finite s_2 which can be put together with (22), say, to remove the unwanted singularities. Contact is made with our previous argument by noting that when $q_\perp = 0$, $k_\perp^2 \rightarrow -m_d^2$, then $q_0 \rightarrow 0$.

IV Discussion and Observations

We have enumerated several processes from which the vacuum trajectory with $\alpha_V(0)=1$ must decouple at $q_\perp=0$. The method of derivation makes it clear that, except for the vanishing of the elastic transition $\beta_{bbV}(0) \propto \sqrt{\sigma^{bb}_{Total}}$, the results are rather tight. From our point of view the most interesting result with experimental content is the decoupling of the Pomeron in the diffraction dissociation of any missing mass at zero momentum transfer. This will exhibit itself as a dip in the diffraction dissociation cross section as $q^2 \rightarrow (q^2)_{min}=O(1/s^2)$.

If this phenomenon is observed and total cross sections persist in behaving as constants for $s \rightarrow \infty$, then it would seem very compelling to search (theoretically) for structure in the coupling of a $q_\perp=0$ Pomeron to multiparticle systems in addition to the conventional structure. A beginning in direction will be presented in Reference 8. It seems appropriate to have some confidence that a consistent picture of diffraction scattering will emerge from such an investigation.

Acknowledgements

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References

1. I. Verdiev, O.V. Kanchelli, S. G. Matinyan, A. M. Popova, and K. A. Ter-Martiroysan, JETP 44, 341 (1963) (Sov. Phys. JETP 19, 1148 (1964)); O.V. Kanchelli, Yad. Fiz. 7, 1119 (1968); and J. Finkelstein and K. Kajantie, Phys. Letters 26B, 305 (1968).

2. V. N. Gribov and A. A. Migdal, Yad. Fiz. 8, 1002 (1968) (Sov. Journal Nuc. Phys. 8, 583 (1969).)

3. H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Lett. 26, 937 (1971). Also see C. E. deTar and J. H. Weis, Phys. Rev. D4, 3141 (1971).

4. V. N. Gribov and O. V. Kanchelli, unpublished.

5. H. D. I. Abarbanel, S. D. Ellis, M. B. Green, and A. Zee, NAL-THY preprint, April, 1972.

6. C. E. Jones, F. E. Low, S.-H. Tye, G. Veneziano, and J. E. Young, MIT preprint CTP-264, March, 1972.

7. R. C. Brower and J. H. Weis, MIT preprint CTP-280, May, 1972

8. V. N. Gribov and O. V. Kanchelli, to be published.

Figure Captions

Figure 1 The amplitude $T_{ab\to cN}$ to produce a distinguished particle c and N particles of invariant mass M in the collision of a and b.

Figure 2 The Schwartz inequality relating the absorptive part in $M^2=(p_a + p_b - p_a')^2 = (p_d + p_f)^2$ of $T_{ab\to adf}$ to the inclusive process $a + b \to a$ + anything and the forward absorptive part of "elastic" d,f scattering. d and f are any hadron states. The inequality is exhibited for large $s=(p_a + p_b)^2$.

Figure 3 Illustration of the vanishing of the coupling of two vacuum poles at $q_\perp=0$ to any hadron state d.

Figure 4 Illustration how the result in Figure 3 leads to the vanishing of the coupling of one vacuum pole with $q_\perp=0$ with another vacuum pole with $q_\perp'\neq 0$ and any hadron state d.

Figure 5 Illustration of how the decoupling of two vacuum poles with $q_\perp=0$, Figure 3, leads to the decoupling of one vacuum pole with $q_\perp=0$ to any Reggeon with any $k_\perp$ and any hadron state d. The last step shown extrapolates this result to a particle pole on the trajectory and illustrates the vanishing of the diffraction dissociation transition at $q_\perp=0$.

Figure 6 A pole contribution to the diffraction dissociation transition Pomeron + $\pi \to 3\pi$. The residue at the pole is proportional to $\beta_{\pi\pi V}$.

Figure 7 The kinematics of the $2\to 3$ amplitude in the double Regge regime where the Pomeron-Reggeon-particle vertex appears.

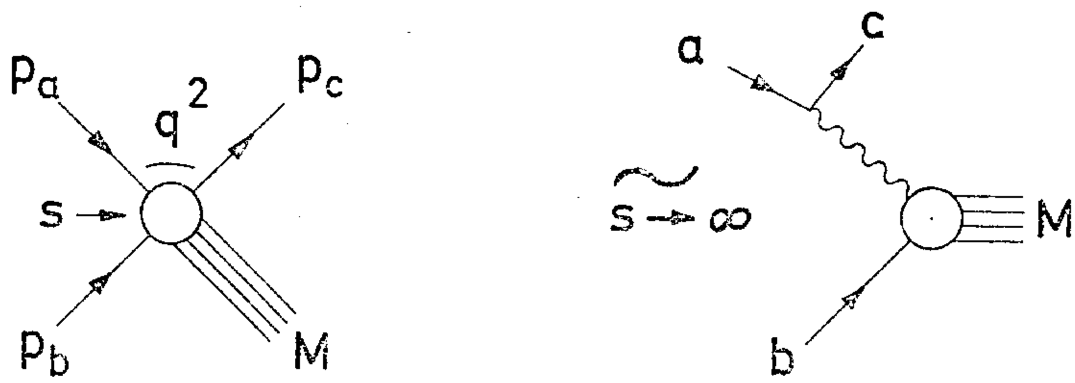


Figure 1

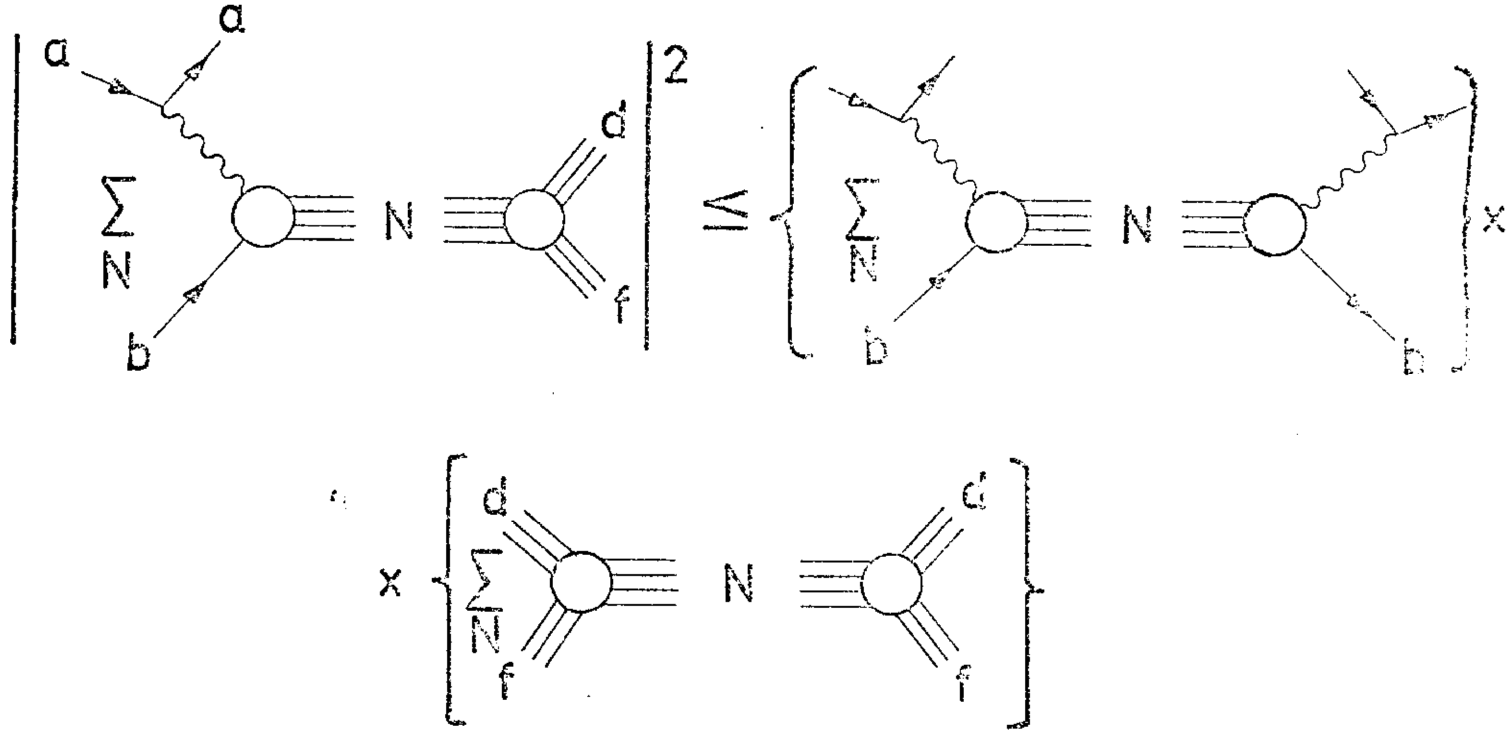


Figure 2

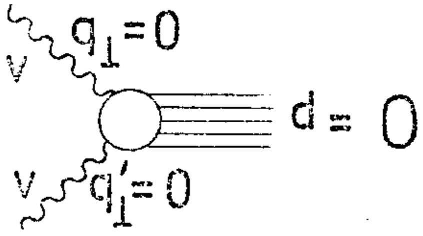


Figure 3

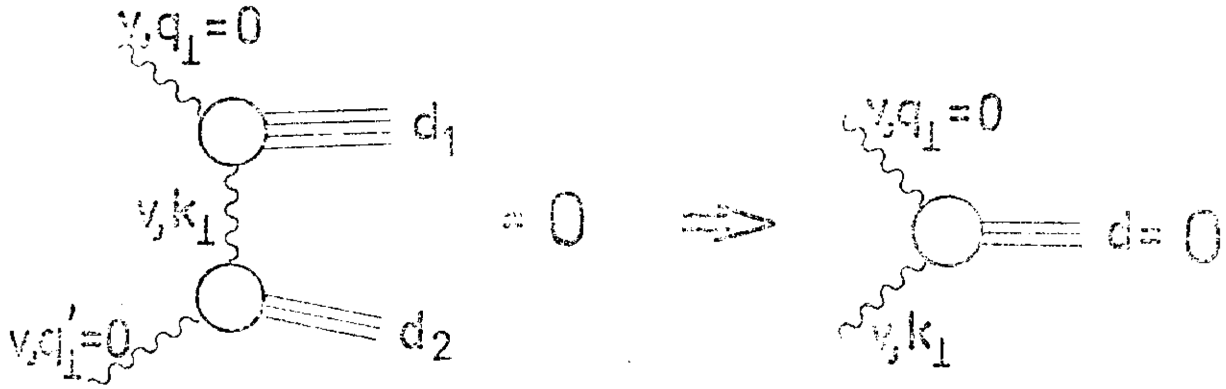
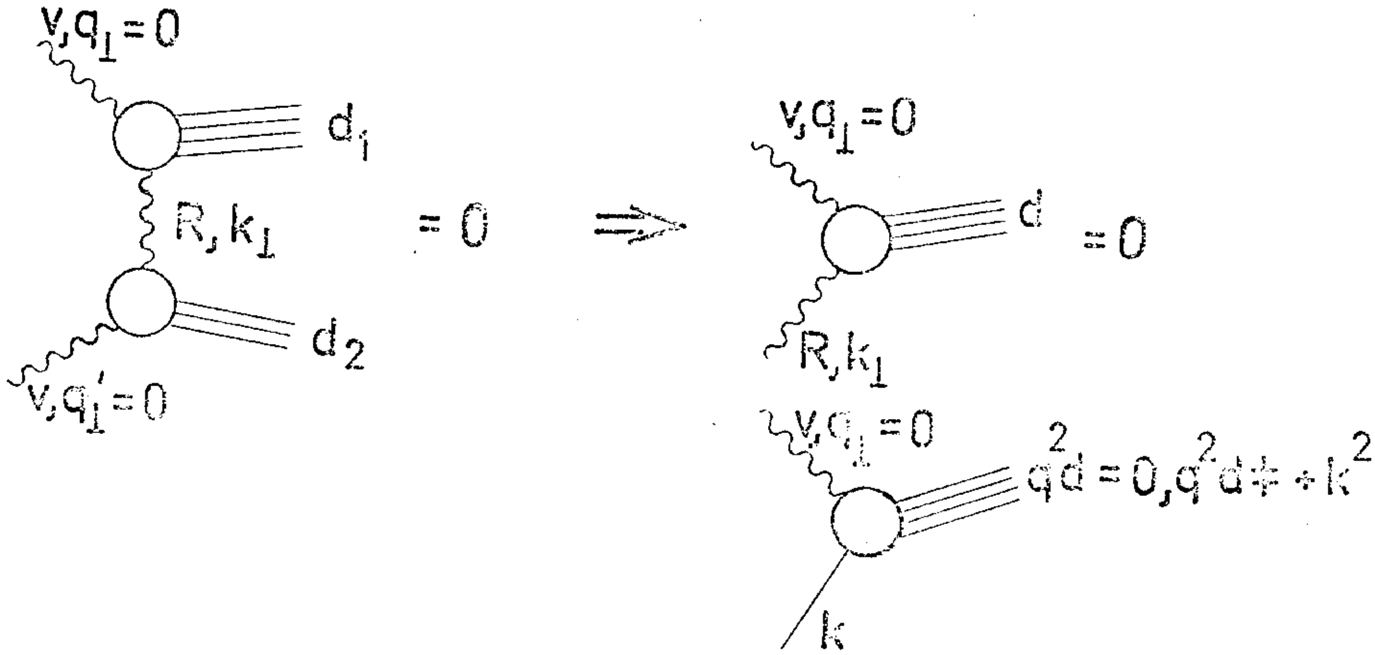
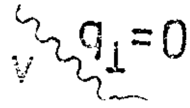


Figure 4



$v, q_\perp = 0$
d_1
$R, k_\perp$
d_2
$v, q'_\perp = 0$
$= 0$
$\Longrightarrow$
$v, q_\perp = 0$
d
$= 0$
$R, k_\perp$
$v, q_\perp = 0$
$q^2 d = 0, q^2 d \neq + k^2$
k

Figure 5



v
$q_\perp = 0$

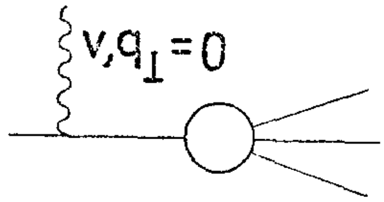


Figure 6

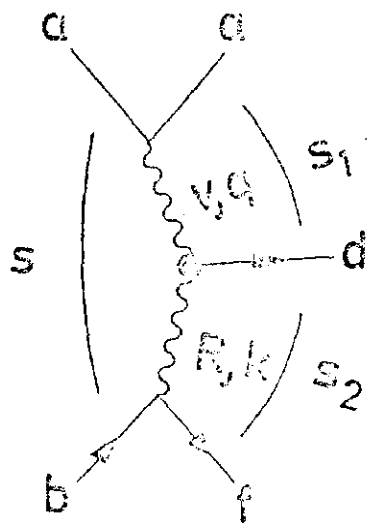


Figure 7