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# Dynamics and Capture of Particles with Variable Electric Charge in Earth Magnetic Dipole Trap (Anomalous Cosmic Rays)

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**Abstract**

The main problem of the ACR dynamics analysis in the Earth's magnetic dipole trap is developing of the model of charged particle nonadiabatic motion and determination of the adiabatic motion boundary. The adiabaticity parameter $c$ is used for motion analysis. Here $c$ is the ratio of the particle gyration radius to the curvature radius of magnetic field line at the equator. We constructed a Poincare mapping for particle dynamics in a dipole magnetic field, based on the quasi-adiabatic motion model. The adiabatic motion boundary and energetic range for trapped particles at different *L*-shells were found by means of this mapping.

## 1 Introduction:

In the process of ACR ion stripping a jump-like change of magnetic rigidity $R$, and therefore, of the adiabaticity parameter $c = 5.0 * 10^{-2} RL^{-2}$ occurs, where $R$ is measured in GV. Hence, the particle motion changes qualitatively: from infinite CR motion ($\chi$>0.75) to finite motion of geomagnetically trapped particles ($\chi$<0.75) and, under certain conditions, to stable trapping of particles, moving adiabatically. These processes may be studied by means of numerical integration of the motion equation, however, along with an extensive amount of calculations a significant problem is the generalization of the calculated trajectory parameters and obtaining of the general regularities of particle motion.

We will study the processes of ACR stripping and trapping on the basis of the quasi-adiabatic model of particle motion, which we developed for the dipole magnetic field.

## 2 Quasi-Adiabatic Model of Particle Motion:

The main ideas of the quasi-adiabatic model of particle motion, based on the integrating of the motion equations, are formulated in the following way (Ilyina et al, 1993, Kuznetsov et al, 1993, Ilyin et al, 1997):

1. The character of particle motion is determined by the adiabaticity parameter $c$ .

2. The most suitable model of the particle guiding centre trajectory (GCT) is the central trajectory (CT) - the particle trajectory, crossing the dipole origin.

3. During particle motion from the equatorial plane towards the mirror point and back to the equator an analogue of the transverse adiabatic invariant (magnetic moment) $m^* = mv^2 \sin^2 a^* / 2B$ is conserved with high accuracy. Here $a^*$ is the angle between the particle velocity vector and the tangent to the CT.

4. During the equatorial plane crossing a jump-like change of $m^*$ occurs due to a distinction between the equatorial parameters of the CT, directed from the dipole centre (direct CT), and the

CT, directed towards the dipole centre (reversed CT). If for the direct CT the phase - the angle between the radius-vector and the projection of the tangent to the CT to the equatorial plane – is equal to $j_0$, then for the reverse CT it is equal to $p - j_0$. The other equatorial parameter – the angle $a_0$ between the magnetic field line and the tangent to the CT - is the same for both CT. The parameters $a_0$ and $j_0$ depend on $\chi$ only and are unambiguous functions.

5. It is necessary to use as the GCT the CT, corresponding to the effective value of the adiabaticity parameter $c_{eff} = c\,y\,(0) y^{-1}(a_0) \cong c \cos a_0$, where

$$y\,(a) = \frac{1}{\sqrt{2}\sin^2 a}\left(\frac{1+\sin^2 a}{\sin a}\ln\frac{1+\sin a}{\cos a} - 1\right)$$

6. The phase accumulation $\Delta f$ during half a bounce-period, measured in the coordinate system, associated with the CT, depends on the initial value of $f$ (Dmitriev et al, 1996).

# 3 Mapping:

We have constructed a Poincare mapping, corresponding to the quasi-adiabatic model. Three coordinate systems in two-dimensional (pitch-angle/ phase) phase space are employed in this mapping:

a) MFS, associated with the magnetic field line;

b) CTS1, associated with the reversed CT;

c) CTS2, associated with the direct CT.

Below we describe the mapping routine step by step.

1. The starting point of the particle is determined in MFS by coordinates $a_i$ and $j_i$.

2. The angle $a_i^*$ in CTS1 and $c_{eff}$ are found by means of numerical solution of the equation system, describing $a_0\left(c_{eff}(a_i^*)\right), j_0\left(c_{eff}(a_i^*)\right)$ and including the next one:

$$\cos a_i^* = \cos a_i \cos a_{0i} + \sin a_i \sin a_{0i} \cos(j_i - j_{0i}),$$

The phase in CTS1 is equal to $f_i = \arcsin\,[\sin a_{0i} \sin(j_i - j)/\sin a_i^*]$.

3.After mirroring the particle returns back to the equator, and its coordinates in CTS2 are

$$a_{i+1}^* = a_i^* + \Delta a_i^* \quad ; \qquad f_{i+1} = f_i + \Delta f_i$$

Here $\Delta f_i = 6 c^{-1}(\sin^{-1.348} a_i^* - 0.255)$,

$$\Delta a_i^* = 1.25 c^{0.753} \exp(-0.524/c) \sin^{1.258} a_i^* \,(1 - \sin^{1.09} a_i^*) \cos^{-1} a_i^* \sin 2 f_i$$

4. Coordinates of CTS2 origin in MFS are $a_0 = a_0\left(c_{eff}(a_{i+1}^*)\right);\ j_0 = j_0\left(c_{eff}(a_{i+1}^*)\right)$. Then $a_{i+1}$ and $j_{i+1}$ may be calculated, and it is possible to return to point 1 and continue this routine.

Thus, this mapping includes the main features of the quasi-adiabatic model: the quasi-moment $m^*$ conservation during half a bounce-period and the quasi-moment jump $\Delta m^*$ during equatorial plane crossing.

The studies of particle dynamics by means of this mapping showed that for a given $c$ there is a boundary value $a_{cr}^*$, separating two motion modes. Above $a_{cr}^*$ the particle motion is stable with conservation of $a^*$, i.e. adiabatic, below $a_{cr}^*$ the motion is stochastic. In Fig.1 dependencies of $a^*$ on $N$ - the number of half a bounce-period are shown for $c$ = 0.13 (initial $a^*$=15°.5 and 15°.6 ) and for $c$ =0.15 ($a^*$=21°.9 and 22°.0). The dependence of $a^*_{cr}$ on $c$ at 0.1 < $c$ <0.27 may be approximated as:

$$a^*_{cr} = \arcsin\,[0.486\,c^{-2}\exp(-0.615/\,c)] \qquad (1)$$

This dependence agrees well with previous estimates, based on the concept of motion stochastity due to resonant interaction between the Larmor gyration of the particle and its oscillations between mirror points (Ilyin et al, 1993).

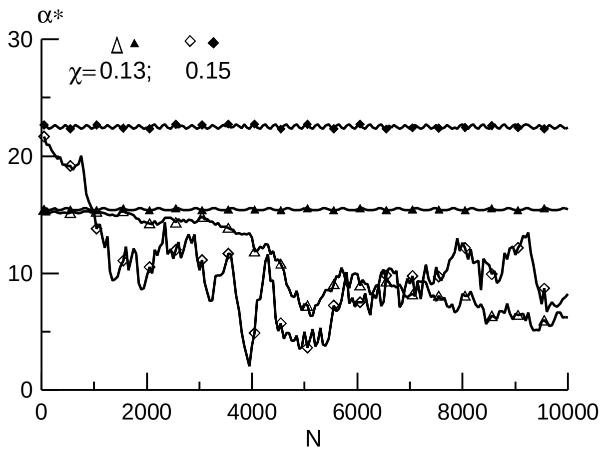


**Figure 1:** Long-term particle dynamics

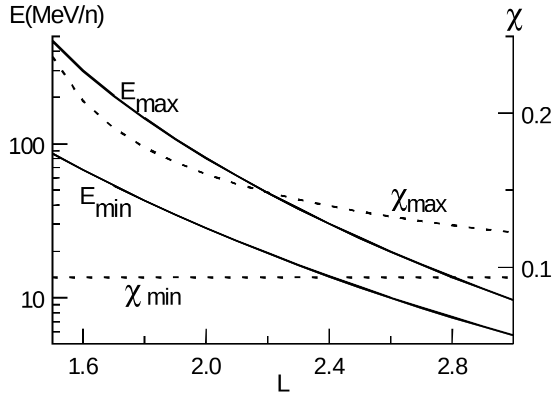


**Figure 2:** Trapping Boundaries

## 4 Trapping Boundaries for Full-Stripped Ions:

During analysis of ACR particle dynamics in the geomagnetical trap we assume, that particle motion occurs in a dipole field. At the same time the boundary conditions, determining the stripping and loss of particles in the upper atmosphere, are calculated according to IGRF90.

The lower boundary of trapping, determined from cut-off value $c$ =0.75 for the case of full stripping of single charged primary oxygen ions, is equal to $c_{\min}$ = 0.0933. It follows from (1) that at a given $a^*$ there is a maximum value of $c$, at which stable particle motion is possible. Taking as $a^*$ the value of the equatorial pitch-angle, $L$-dependent and corresponding to the stripping altitude $h_{str}$ in the South-Atlantic Anomaly, we obtain the upper limit $c_{\max}$ of trapping as a function of $L$. We assume that $h_{str}$ is equal to 250 km, which agrees with measured pitch-angle distribution of trapped ions (Selesnick et al, 1997). Determined in such way $c_{\max}$ is shown in Fig.2 as well as $c_{\min}$. In fact $c_{\max}$ is not the strict boundary since $h_{str}$ is not the exact value but extends from 220 to 350 km. Corresponding ambiguity of $c_{\max}$ is estimated to be about 1 %.

Kinetic energies, corresponding to $c_{\min}$ and $c_{\max}$ and calculated for full stripped oxygen ions, are also shown in Fig.2. It should be noted, that with decreasing $L$ both $E_{\min}$ and $E_{\max}$ increase, and the energetic range of trapped ions extends too. Similar changing of trapped ion energetic range was observed by Looper et al (1996).

According to Fig.2 primary ACR ions, having charge $Q$=2 (Mewaldt et al, 1996), after their stripping may be trapped at $L$<1.75.

Thus, the mapping permits to study single particle dynamics from the primary ACR particle stripping to the transition into the stable motion mode or entering into the loss cone. The primary ACR spectrum must be specified in the South-Atlantic Anomaly region, where the initial stripping is localized.

## 5 Conclusion:

1. The mapping discussed above is an effective tool for study of ACR ion stripping and trapping processes.
2. The adiabatic motion boundary is found as a function of adiabaticity parameter $c$ for the dipole magnetic field.
3. Upper and lower limits of trapped particle energies are defined for different $L$-shells.

## References


Dmitriev, A.V., Ilyin, V.D., Kuznetsov, S.N., & Yushkov, B.Yu. 1996, in "Radiation Belts:Models and Standards. Geophysical Monograph 97" p.43, Eds. J.F.Lemaire et al, AGU.

Ilyin, V.D., Ilyin, I.V., & Kuznetsov S.N. 1993, Cosmic Research, **31**, 687.

Ilyin, V.D., Ilyina, A.N., & Yushkov, B.Yu. 1997, Plasma Phys. Reports, **23**, 367.

Ilyina, A.N., Ilyin, V.D., Kuznetsov, S.N., Yushkov, B.Yu., Amirkhanov, I.V., & Ilyin I.V., 1993, JETP, **77**, 246.

Kuznetsov, S.N., Ilyin, V.D., & Yushkov, B.Yu. 1993, Proc.23 ICRC (Calgary), v.3, 830.

Looper, M.D., Blake, J.B., Klecker B., & Hovestadt, D., 1996, JGR, **101**,24,747.

Mewaldt, R.A., Selesnick, R.C., Cummings, J.R., Stone E.C., & von Rosenvinge, T.T., 1996, ApJ, **466**, L43.

Selesnick, R.C., Leske, R.A., Mewaldt, R.A., & Cummings, J.R., 1997, Proc.25 ICRC, (Durban), v.2, 305.