Calculation of energetic heliospheric ion spectra in the KeV-to-MeV range based on simultaneous diffusion processes in phase space

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Abstract

Based on solutions of the general kinetic transport equation we describe how heliospheric pick-up ions by means of non-linear wave-particle interactions while being convected outwards with the solar wind are systematically processed to higher energies. As we can show these ions of initial energies at 1 KeV/nuc by simultaneous operations of Fermi-1 and Fermi-2 scattering processes in the region close to the termination shock eventually appear as Anomalous Cosmic Ray (ACR) particles in the 10 MeV/nuc energy regime. We present as a function of solar distance ion energy spectra covering the range from 1 KeV/nuc to 100 MeV/nuc which result from solutions of the kinetic transport equation including simultaneous operations of convective and diffusive transport processes in heliospheric phase space. The solutions are obtained after transcription of the transport equation into an equivalent system of Itô-stochastic differential equations and integrating the latter for mega-particle samples. We can convincingly prove with these calculations that the kick-up from pick-up's to ACR's completely works on known physical grounds with no need for a specific injection of ACR's at the shock. We also prove that the resulting spectra near the termination shock are far from being a pure power-law as expected by classical shock acceleration theories.

1 Introductory briefing

Origin and kinetic properties of anomalous cosmic rays (ACRs) in the heliosphere are unsolved problems till today, even though the general production mechanism most probably is correctly identified already since quite some time (Fisk et al. 1974): It is accepted that neutral interstellar atoms penetrate into the inner heliosphere and become ionized there. As newly created ions they are picked up by the frozen-in solar wind magnetic field and are convected outwards. Some fraction of these ions is accelerated to ACR energies near or at the solar wind termination shock by means of diffusive shock acceleration processes or by shock drift processes (see Axford et al. 1977, Pesses et al. 1981, Scholer 1985, Potgieter & Moraal 1988, Jokipii 1992, Lee et al. 1996, Zank et al. 1996, Giacalone et al. 1994).

As raised by Jokipii (1992), only particle acceleration at quasi-perpendicular portions of the termination shock can explain the observed ACR spectra. But even in this limited framework there are still some unsolved problems manifest. Amongst these of prime importance is the question how primary pick-up ions are eventually injected into the ACR energy regime. After some tracegiving studies by Chalov et al. (1995) and Chalov & Fahr (1996) one may be inclined to believe that the necessary energization of pick-up ions occurs as a two-step process: During their propagation to the outer parts of the heliosphere pick-up ions not only suffer pitch-angle scattering but also are preaccelerated by Alfvénic and magnetosonic turbulence (or transit-time damping), as well as by interplanetary shock waves (CIRs) (see Fisk 1976a/b/c, Klecker 1977, Lee 1983, Fisk 1986, Isenberg 1987, Bogdan et al. 1991, Petuhov & Nikolaev 1993, Chalov et al. 1995, Fichtner et al. 1996, le Roux & Fichtner 1997). Upstream of the termination shock the energy distribution of pick-up ions has already developed a high-energy tail such that upon arrival at the shock selected particles from this tail experience a first reflection at the shock running again upstream till by pitch angle scattering being convected back to the shock (Leroy 1983, Liewer et al. 1993, Kucharek & Scholer 1995, Chalov & Fahr 1996). These ions then are subject by reasonable probabilities to further acceleration processes of the Fermi-1 type till eventually they reach ACR energies.

It was soon recognized that the phase space transport of energetic particles allows for a split into two distinct forms: i) Convection, adiabatic cooling and momentum diffusion of low energy particles, and ii) spatial diffusion and adiabatic acceleration of high energy particles. The relevant kinetic transport equation allows to derive two different solutions for these different cases both of which were treated separately in the literature up to now. As bridge between these two cases an inconsistent injection of ACR particles at the shock was introduced describing the percentage of preaccelerated pick-up's injected into the diffusive ACR regime.

In this paper we present a new numerical model to study the simultaneous operation of all relevant phase-space transport processes like preacceleration of convected pick-up ions, partial reflection at the termination shock, and diffusive acceleration by the shock-induced solar wind velocity profile without any need to specify an injection process.

2 Theoretical approach and solution

We aim at a solution of the complete kinetic transport equation describing all relevant convective and diffusive transport processes in heliospheric phase space for particle energies between 1 KeV/nuc and 100 MeV/nuc without prescription of ACR injection processes. Assuming pitch-angle isotropy for the distribution function $f(r,p,t)$ (r: radial distance, p: particle momentum, t: time) we can formulate the relevant transport equation in its conservation law form valid for the flux of the differential particle density $N = 4\pi r^2 p^2 f(r,p,t)$ in the form:

$$\begin{aligned}\frac{\partial N}{\partial t} = &- \frac{\partial}{\partial r}\left[\left(\frac{\partial \kappa}{\partial r} + \frac{2\kappa}{r} + u\right) N\right] - \frac{\partial}{\partial p}\left[\left(\frac{\partial D}{\partial p} + \frac{2D}{p} - \frac{p}{3r^2}\frac{\partial\left(r^2 u\right)}{\partial r}\right) N\right] \\ &+ \frac{1}{2}\frac{\partial^2}{\partial r^2}\left(2\kappa N\right) + \frac{1}{2}\frac{\partial^2}{\partial p^2}\left(2DN\right) + 4\pi r^2 p^2 S\end{aligned} \qquad (1)$$

Here $\kappa(r,p)$ and $D(r,p)$ denote the spatial and the momentum diffusion coefficients, $u(r)$ is the solar wind velocity, and $S(r,p,t)$ is the local pick-up ion production rate. The above equation is a nonlinear partial differential equation of second order which we solve after transcribing it into an equivalent system of Itô-stochastic differential equations (see Mac Kinnon & Craig 1991, Achterberg & Krülls 1992, Krülls & Achterberg 1994, Chalov & Fahr 1997, 1998). For this purpose we introduce the advection vector $\mathcal{A}$ and the diffusion matrix $\boldsymbol{\mathcal{B}}$ to describe the stochastic phase space transports by (for details see Dworsky 1999):

$$\mathcal{A} = \left(\frac{\partial \kappa}{\partial r} + \frac{2\kappa}{r} + u\,,\ \frac{\partial D}{\partial p} + \frac{2D}{p} - \frac{p}{3r^2}\frac{\partial\left(r^2 u\right)}{\partial r}\right) \quad \text{and} \quad \boldsymbol{\mathcal{B}} = \begin{pmatrix} 2\kappa & 0 \\ 0 & 2D \end{pmatrix} \qquad (2)$$

By use of a two-dimensional Wiener variable $W = W_r, W_p$ to describe the stochastics in the two-dimensional phase space we arrive at the following system of Itô-stochastic differential equations:

$$dr = \left(\frac{\partial \kappa}{\partial r} + \frac{2\kappa}{r} + u\right) dt + \sqrt{2\kappa}\, dW_r \quad \text{and} \quad dp = \left(\frac{\partial D}{\partial p} + \frac{2D}{p} - \frac{p}{3r^2}\frac{\partial\left(r^2 u\right)}{\partial r}\right) dt + \sqrt{2D}\, dW_p \qquad (3)$$

This system can now be integrated with Runge-Kutta methods for a representative sample of about 10^6 pseudo particles and at a significant phase-space coverage permits to synthesize the local distribution function $f(r,p,t)$ all over in the heliosphere. The energy-dependent momentum diffusion coefficient $D(r,p)$ is adopted here in the form also used by Chalov and Fahr (1995) and Fichtner et al. (1996). On the other hand, the spatial diffusion coefficient $\kappa(r,p)$ which is applied here is taken in the same parametrized form like that used by le Roux and Fichtner (1997). The pick-up ion production rate is set equal to: $S(r,p) = S(r)\delta(p - p_0)$, where $S(r)$ is taken from Rucinski et al. (1993), and where p_0 is defined by: $p_0^2/2m = 1$ KeV. The transport equation (1) is not dynamically coupled to the solar wind equations, but is solved here in a test particle approximation. We use inconsistent solutions for the solar wind velocity profile $u = u(r)$ taken in our case from le Roux and Fichtner (1997) with a shock at 79.9 AU and a shock compression factor of $s = 3.4$.

3 Summary of the main results

In Figure 1 we show a comparison of two spectra a) and b) both obtained at the shock (79.9 AU). The spectrum a) (i.e. the complete spectrum called: standard spectrum) is obtained with all processes operating in the way formulated in the preceding section of this paper. In contrast, spectrum b) is generated if diffusive acceleration (Fermi-1) is suppressed by setting the spatial diffusion coefficient $\kappa(r,p)$ to 0. In contrast Figure 2 demonstrates how much various spectra b) deviate from the standard spectrum a), if the momentum diffusion coefficient $D(r,p) = d \cdot D_0(r,p)$ compared to its standard representation ($d = 1$) is changed by various factors d between 0.01 and 10.0. With an increased momentum diffusion the peak spectral intensity will be systematically shifted to higher energies and the spectrum becomes smoothed. The effect of changing the spatial diffusion coefficient (n. b.: not shown in this paper) is less univocal: While the peak intensity nearly stays unshifted, the high energy spectral slope is stronger (i. e. steeper spectral decrease occurs) both for larger and smaller than standard values of $\kappa(r,p)$.

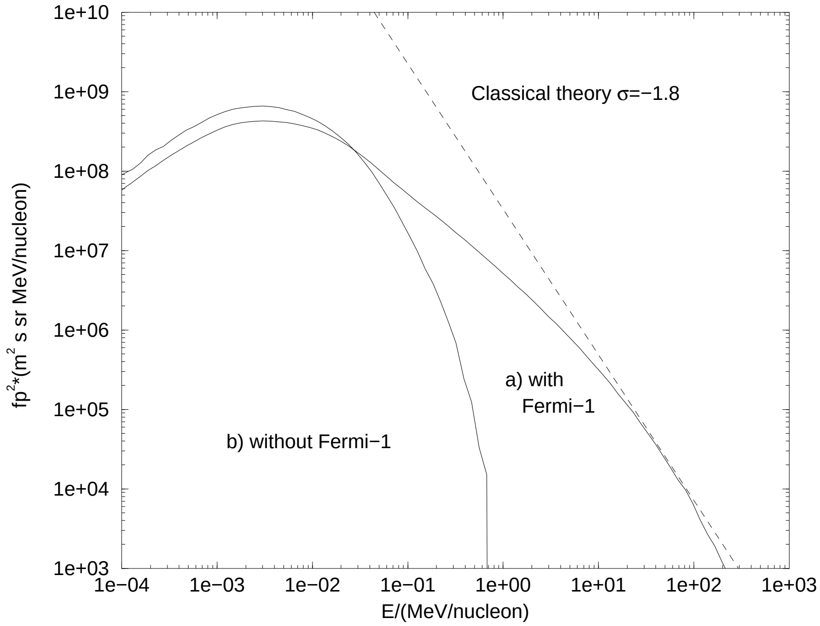

Figure 1: Shown are two pick-up ion/ACR spectra a) and b) calculated at the position of the heliospheric termination shock (at 79.9 AU): a) Standard spectrum with all relevant processes operating as taken into account in this paper; b) Spectrum obtained for identical conditions, but the diffusive shock acceleration (Fermi-1) is suppressed by setting $\kappa(r,p) = 0$. The dashed line shows the spectrum expected from classical shock acceleration theory.

An interesting new insight is also gained when one compares the standard spectrum a) shown in Figure 1 with the spectrum that would be expected by classical shock acceleration theory (see Drury 1983, or Potgieter & Moraal 1988). According to the latter theory applicable to the case of an unmodulated planar Rankine-Hugoniot shock a pure power law $fp^2 \propto E^\sigma$ can be expected for the spectrum at the shock with a power spectral index of σ connected with the compression ratio s by $\sigma = (s+1)/(1-s)$.

For the case treated by us in this paper here a compression factor of $s = 3.4$ applies at the shock yielding a spectral index of $\sigma = -1.8$. In Figure 1 we have shown this power law spectrum in comparison with the generated standard spectrum. As one can clearly see, even at the shock position the resulting real spectrum is very much different from a simple power law putting attempts of Stone et al. (1996) into severe problems to derive the actual compression ratio at the solar wind termination shock from demodulated VOYAGER spectra. From the ACR spectra which are presently obtained by VOYAGER-1/2 (i. e. at about 70 AU) which already look similar to our standard spectrum a) of Figure 1 it thus cannot be excluded that these NASA spaceprobes are already very close to the termination shock, since no modulation of an original power law might be required.

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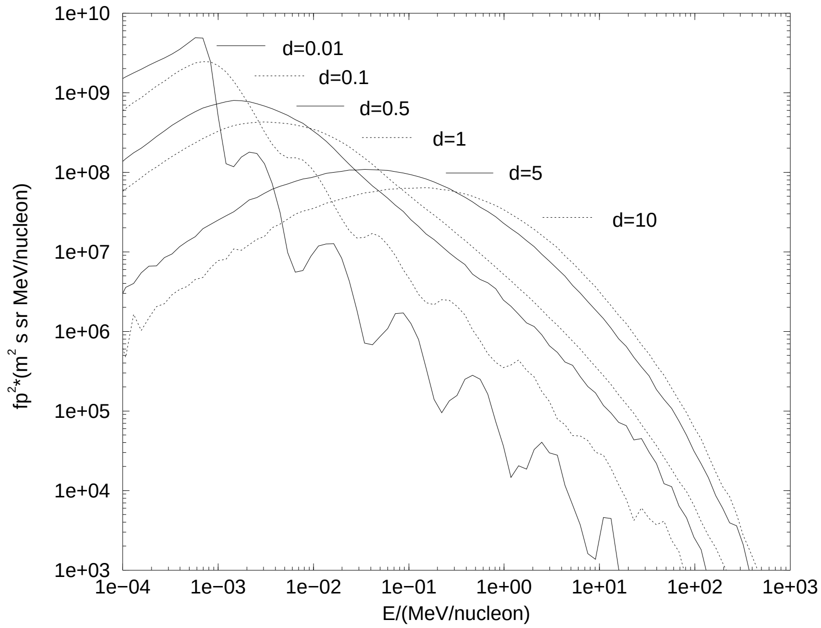

Figure 2: Shown are again spectra obtained at the termination shock calculated for conditions identical to those adopted in Figure 1. With the factor d the momentum diffusion coefficient $D(r,p) = d \cdot D_0(r,p)$ is modified with respect to the standard one, i. e. $D_0(r,p)$. The standard spectrum a) from Figure 1 is thus appearing here as the curve for $d = 1$.

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