SH.3.6.39

A 2D Simulation of the Proton Radiation Belt with Pellpack Code

A. Gusev[1], I. Martin[1], G. Pugacheva[1], A. Christy[2], W. Spjeldvik[3]
[1] *IFGW/DRCC, UNICAMP, Campinas, SP, 13083-970, Brazil*
[2] *Department of Computer Science, Purdue University, Indiana 47907-1398, USA*
[3] *Department of Physics, Weber State University, Ogden, Utah, USA*

Abstract

The numerical solution of diffusion equation for geomagnetically trapped protons taking into account deceleration of protons by Coulomb interactions with free and bounded electrons, the charge exchange process, the cosmic ray albedo neutron decay (CRAND) source and electric and magnetic radial diffusion was obtained using the PELLPACK code based on the finite element method. The advantage of the method in comparison with the traditional finite differences method is a greater speed of computation at the same precision. When boundary conditions at L=7 are given with the distribution function extracted from proton spectrum obtained on board of ATS 6 satellite, the code produces 2D unidirectional proton flux at the top of geomagnetic lines from L=1 up to L=7 which satisfactory agrees with the AP8 model proton flux for all proton energies in the range of 0.5 – 50 MeV. For less proton energies observational AP8 model trapped proton fluxes are several orders of magnitude greater than PELLPACK code fluxes at L < 3 that is difficult to explain by uncertainty of parameters of the equation.

1 Introduction:

Energetic magnetospheric charged particle fluxes are mainly concentrated in the inner region of the Earth's magnetosphere where the drift magnetic shells are permanently closed (L = 1.15 to 6). It has been generally assumed that protons and electrons constitute the predominant part of that population. The main sources and physical processes of formation of the charged particle population in the magnetosphere are known. We can infer for the sources nuclear interactions of cosmic rays with residual atmosphere, decay of the neutron albedo resulting these interactions, solar flares, and planetary magnetosphere origin particles. The principal physical processes are particle radial transport and pitch-angle diffusion due to perturbations in large scale electric and magnetic fields, injection, particle energy losses caused by interaction with the wave-particle interactions and Coulomb interactions. Diffusion theory intended to describe charged particle population in the magnetosphere accounts all these processes. The traditional steady radial diffusion version of the theory supposes that the quiet time structure of energetic particle population can be explained as equilibrium balance among adiabatic radial diffusive transport inward magnetosphere from a source located just within the first closed magnetic field lines in the outer region of the magnetosphere (L ~ 7) and the losses described above. Modern version of the theory also introduces inner source of particles presented by cosmic ray albedo neutron decay (*CRAND*). In the diffusion theory a description of the position and velocity of a trapped particle in the belt is equivalent to knowing its three adiabatic invariants: a mechanical moment M, second adiabatic invariant J and the third adiabatic invariant is a magnetic flux Φ through the drift L-shell :

$$M = P^2/2mB \qquad J = 2P\int_{s'}^{s''}\sqrt{(1-B/B_{mir})}\,ds \qquad \Phi = -\,1.953/L \;\; Gauss\; R_E^2$$

here P - is a particle moment; B, B_{mir} - a magnitude of geomagnetic field in a current point s of magnetic field line and in the mirroring points s', s''.

The diffusion equation may then be written in a form of a elliptic partial differential equation:

$$\frac{\partial f}{\partial t} = L^2\frac{\partial}{\partial L}\left[D_{LL}(L,M)\frac{1}{L^2}\frac{\partial f}{\partial L}\right] - \frac{\partial}{\partial M}\left[\left(\frac{\partial M}{\partial t}\right)_{fric} f\right] - \frac{\partial}{\partial J}\left[\left(\frac{\partial J}{\partial t}\right)_{fric} f\right] - \Lambda f + CRAND$$

Here D_{LL} represents the diffusion coefficient on the variable L; L is the Mac Ilwain parameter corresponding to a particle drift motion around the Earth; $(dM/dt)_{fric}$; dJ/dt_{fric} represent the loss terms, Λf is a term describing charge-exchange losses, i.e. neutralization of proton passing through residual atmosphere. The main feature of diffusion theory is a particle acceleration during cross-field transport: particles flowing inward to the Earth surface are accelerated due to conservation of M and J.

The adiabatic diffusion theory seems to be qualitatively adequate for description of practically all important (including not stationary) phenomena. But in spite of that a comprehensive **quantitative description of the trapped population is still not created**. The main reason of it is a problem of quantitative description of such complicated and multiparameter system as the geomagnetosphere. Analytical solutions of the diffusion equation only exists for a few simplified stationary cases and rather have a character of estimation and qualitative illustrations. In the same time many numerical solutions were obtained on the basis of the diffusion theory (Nakada and Mead, 1965; Cornwall, 1972; Lyons and Thorn, 1973; Spjeldvik, 1977; Riley and Wolf, 1992). Numerical methods give possibility to obtain more realistic and detailed description.
The recent example of exact numerical solution of 3D task for stationary case of protons was published by Beutier et al., (1995). There one may see also general agreement with the experimental values (Figure 1, (Beutier et al., 1995) but the difference of 1 order of magnitude between computed fluxes and observational fluxes of several MeV protons still remains. Thus, we may state that the parameters of diffusion theory still need to be adjusted to correspond observational data concerning even steady state of radiation belt particle fluxes. There exist stationary empirical models based on averages of satellite data from sixties, particularly those of NASA: AE8 - electron flux model and AP8 - proton flux model (Vette, 1991). The models are based on hundreds satellite experiments, made in the periods of minimum and maximum Solar activity. In this paper we will get numerical solution of diffusion equation for proton fluxes, will vary the parameters of the equation and will compare the results of numerical solution with the AP-8 model proton fluxes.

2 The Proton Continuity Equation:

In the phase space the stably state diffusion equation describing the time evolution of the phase space distribution function f ($f = dN/dE/P^2$, here P is a particle moment and dN/dE is a particle differential spectrum) at the top of geomagnetic field line (2D task, J=0), including Coulomb, charge-exchange losses and CRAND source is given as

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left[D_{LL}(L,M) L^{-2} \frac{\partial f}{\partial L} \right] + \frac{dM}{dt}\frac{\partial f}{\partial M} + f\frac{d^2 M}{dMdt} - \Lambda f + CRAND \quad = 0$$

Diffusion parameters. Particle transport mechanism across the geomagnetic field lines is driven by fluctuations in geomagnetic field (magnetic radial diffusion with a coefficient D_{LL}^{m}) and in large scale convection electric field (electric radial diffusion with a coefficient D_{LL}^{e}). Generally, it is assumed that $D_{LL}^{m}=D_0^{m}L^{10}$ and $D_{LL}^{e}=K_eL^{10}/(L^4+M^2)$ where D_0^{m} and K_e are parameters which depend on magnetic and electrostatic field fluctuations. In the publications made in 60ies - 70ies (Tverskoy, 1968; Cornwall 1972; Spjeldvik, 1977 and others) D_0^{m} and K_e were accepted as (2 - 5)10^{-15} 1/s; (2 - 5)10^{-10} 1/s correspondingly. In the recent publications $D_0^{m} = 1.1\ 10^{-13}$ 1/s, that is the 2 orders of magnitude greater than in the previous works. Below we show how particle space L-distributions change with a change of diffusion coefficients.
Coulomb energy losses are caused by proton collisions with free electrons of plasmasphere and with bound electrons of neutral atoms with their following ionization. The Coulomb losses are given by (Schulz and

$$\frac{dE}{dt} = -\frac{4\pi e^4}{m_e v}\left[N_i Z_i\left(\beta^2 - Log A_i\right) + N_e\left(\beta^2 - Log C_e\right)\right]$$

Lanzerotti, 1974) where $A_i = 2m_ec^2(\gamma^2-1)/I_i$; $C_e = 4\pi^2\Lambda_D m_e\beta c/h$; Λ_D - Debye length; h is Plank constant; N_e is the averaged density of free electrons in plasmasphere, N_i is the number density of the exospheric gas molecules, each one containing Z_i bound electrons, and $I_i=13Z_i$ eV is the mean excitation energy for the bound electrons. The function for free electron densities is (Cornwall, 1972): $N_e(L)=250(L/4.1)^{-4.64}$ $1/cm^3$ *for* $L < 4.1$ $N_e(L)=13(L/4.1)^{-4.64} 1/cm^3$ *for* $L > 4$. T = 5000K in plasmasphere temperature model (Cornwall, 1972) used for determination of Debye radius in term describing proton interactions with free electrons.
Charge Exchange process. Energetic protons when collide with neutral geocoronal atomic hydrogen strip of its bound electron and become fast non-thermal neutral hydrogen atoms, that are not affected or confined by the geomagnetic field and escape from radiation belt region. The change in phase space distribution function caused by the neutralization process is given by term $df/dt = -\Lambda f$ where $\Lambda = \sigma v N_h$; here v - is a proton velocity; σ is a charge exchange cross section for energetic protons in an atomic hydrogen (Alisson,

1958; Orsini et al,1994); N_h is the neutral hydrogen concentration. This last value depends on L and exospheric temperature and the model of N_h was been taken from Spjeldvik (1977) at the temperature 950K.

Boundary conditions. For boundary conditions we accepted L-shell L=7 and a proton differential spectrum at L = 7 in the energy range of 0.1 - 1000 MeV derived from AP- 8. We used also for boundary spectrum the spectrum suggested by Spjeldvik (1977) that was modeled from quiet time ion observations on board the geostationary satellite ATS 6 (Fritz et al, 1977). The solution intervals of L and E are the followings: $1 < L < 7$; 0.05 keV < E_{kin} < 1000 MeV. The low energy edge used is very low and has no physical sense. It was made to avoid the influence of the edge conditions on the distribution function of the physically important low energy protons (of about 50 keV) at low L-shells because particles are accelerated due to cross transport from L = 7 to L = 1 with an energy increase of about L^3, i.e. 350 times, and the 50 keV proton at L ~ 1 before penetration to this L-shell has energy of 0.15 keV at L = 7.

Numerical technique used is based on the method of finite elements. In the traditional method of finite differences, a rectangular grid is placed over the domain in order to determine approximations to the solution at each grid point. An algebraic equation is written for each such point that approximates the differential equation locally. The finite element method uses a set of basis functions (Bj(x, y), j=1,..N), and then determines coefficients Cj. The domain is approximated by elements (rectangular, triangular, etc in 2D; tetrahedral, etc in 3D) to define the basis functions. A grid point resolution in L space is 0.05 R_E, and 15 grid points per decade were assigned in M space and consequently in the energy spectra. In the whole *L, M* space there were used 6252 grid points to get estimations and 138100 grid points to get main results. To get estimations it is necessary several minutes.

3 Results and Discussions:

In Figure 1 we compare the unidirectional differential proton fluxes for 8 energies (0.1; 0.5; 1; 2.5; 5; 10; 50; 100 MeV) resulting of PELLPACK code and of the AP-8 model in the geomagnetic equatorial plane with the boundary proton spectrum from Spjeldvik (1977). Computed L-distributions with Spjeldvik-1977 boundary spectrum agree very well with the observational AP-8 proton flux model at all L-shells (excluding L ~ 7) in the energy range of 0.5 – 50 MeV. At the energies less 500 keV the computed and observational fluxes do not agree. We increased on several orders of magnitude a low energy part of boundary spectrum, but it did not help to decrease a difference between observational and computed results. In the Figure 2 we present results of computed proton L-distributions (with the Spjeldvik-1977 boundary spectrum) for magnetic radial diffusion coefficients D_0^m=2.3 10^{-15} 1/s and with various electric diffusion coefficients Ke=2.3$10^{-10}$ and 10^{-9} 1/s and L-distributions of AP8 model proton fluxes. One can see that the better agreement with experimental results is obtained when magnetic and electric diffusion coefficients are D_0^m= 2.3$10^{-15}$ and Ke=2.3$10^{-10}$; with other K_e coefficients the inner part of inner radiation belt does not agree with the AP8 model data. In Figure 3 the AP-8 model proton fluxes and the fluxes resulting from PELLPACK code but with the boundary proton flux extracted from AP-8 model at L = 7 are presented. One can see that at the energies of several MeV AP-8 model is not self-consistent from the point of view of theory: computed L-distributions of proton flux with boundary conditions of AP-8 at L=7 satisfactory agree with observational proton L-distributions from AP-8 model only at energies of about 1 MeV. The difference in the flux values reaches several order of the flux magnitude at several MeV energies.

4 Conclusion:

The undertaken attempt to describe stable magnetospheric proton fluxes with numerical theory shows: 1) that observational proton fluxes significantly exceed computational ones in the low energy range and a reason of it is still not found; 2) the observational AP-8 model, apparently, is not self-consistent because AP-8 proton spectrum at L=7 being used as a boundary spectrum in numerical theory do not reproduce AP-8 proton fluxes at lower L-shells, but Spjeldvik-1977 boundary spectrum makes it well.

References

Allison, S. K. 1958, Rev. Mod. Phys., 30, 1137.

Beutier, T., D. Boscher, and M. France. 1995, JGR, 100, 17181.
Cornwall, J.M. 1972, J. Geophys. Res., 77, A10, 1756.
Fritz, T.A., et al, 1977, Significant initial results from ATS-6, NASA, Tech Rep.
Lyons, L.R., and R.M. Thorne. 1973, J. Geophys. Res., 78, 2142.
Nakada, M.P., and G.D. Mead. 1965, J. Geophys. Res., 70, 4777.
Orsini, S., I. A. Daglis, M. Candidi, et al. 1994, J. Geophys. Res., 99, 13489.
Riley, P., and R.A. Wolf. 1992, J. Geophys. Res., 97, 16865 - 16876.
Schulz, M., & L.J. Lanzerotti. 1974, Particle Diffusion in the Radiation Belts, Springer-Verlag.
Tverskoy B. A. 1968, The Earth's radiation belts, Science, Moscow, (in Russian), 223.
Vette, J.I. 1991, The AE-8 trapped electron model environment, Rep. NSSDC 91-24, NASA.

Acknowledgments. The work was supported by CNPq (52211/96-3), Fapesp(93/4978-0), Faep of UNICAMP.

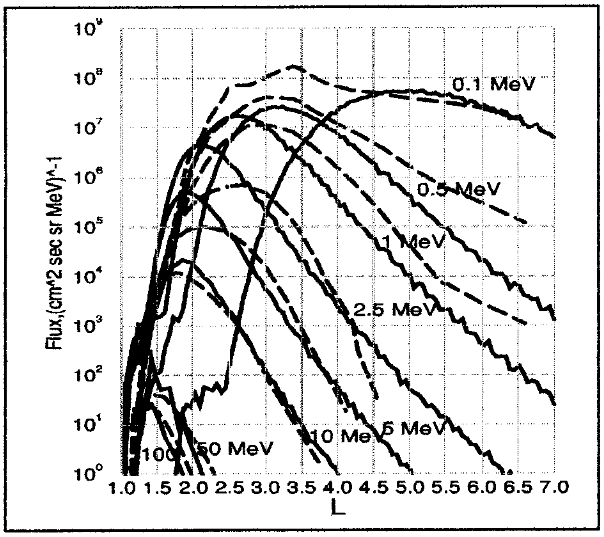

Figure 1. L-distributions of computed (solid) and AP-8 model (dashed) proton fluxes with boundary proton flux Spieldvik-1977.

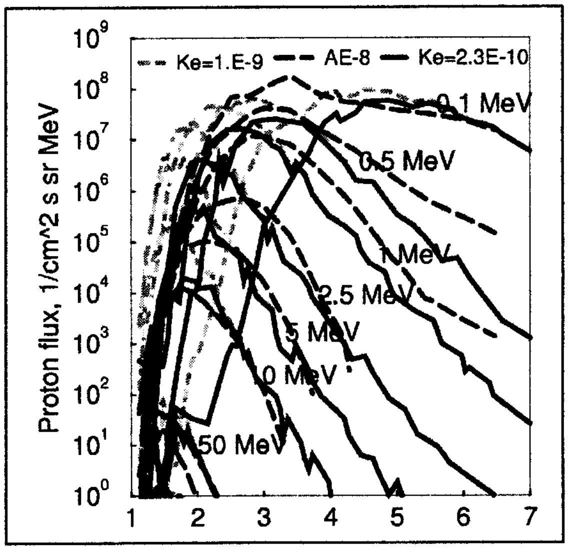

Figure 2. L-distributions of computed proton fluxes with electrostatic radial diffusion coefficients K_e= 10^{-9} ; $K_e = 2.310^{-10}$1/s and AP8 proton fluxes.

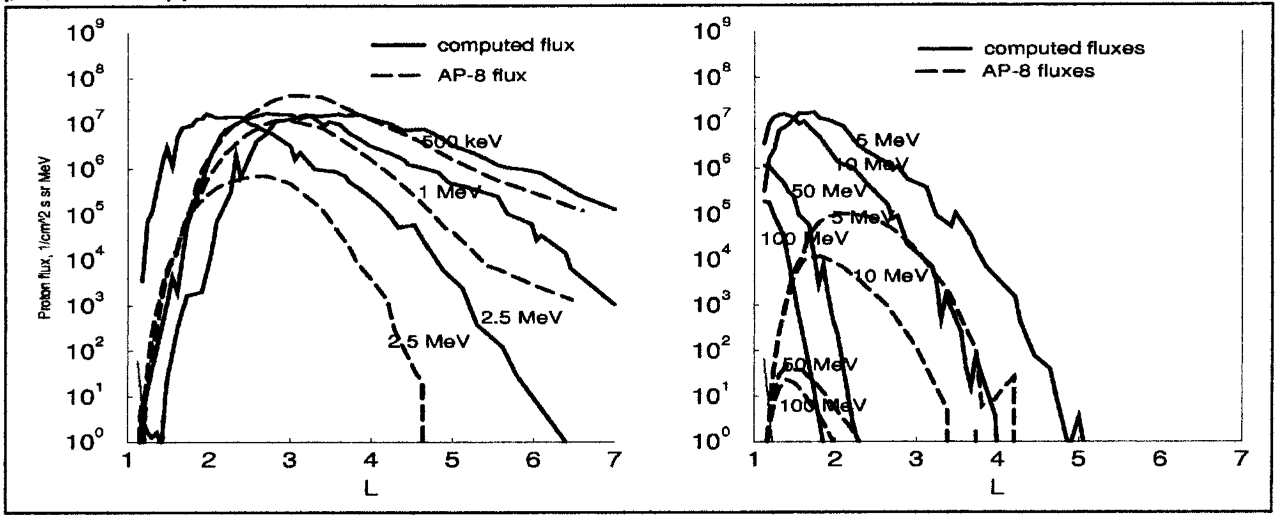

Figure 3. L-distributions of AP-8 proton fluxes and computed fluxes with the AP-8 boundary proton flux .