# Calculated Vertical Cutoff Rigidities for the International Space Station During Magnetically Quiet Times

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#### Abstract

We have calculated a world grid of cosmic ray cutoff rigidities each 5 degrees in latitude and 15 degrees in longitude at 450 km altitude. The geomagnetic cutoff rigidity values have been calculated employing the Tsyganenko magnetic field model combined with the International Geomagnetic Reference Field for 1995.0 The cutoff values were calculated by the trajectory-tracing method for particles arriving from the vertical direction during magnetically quiet conditions. These cutoff rigidity values are intended to be a basic reference for charged particle access to the International Space Station.

### **1 Introduction:**

We have derived a world grid of cosmic ray cutoff rigidities appropriate for a spacecraft orbiting at 450 km altitude employing the Tsyganenko (1989) magnetospheric field model combined with the International Geomagnetic Reference Field for epoch 1995.0 (Sabaka et al., 1997) in the manner described by Flückiger and Kobel (1990). The orbit of a particle of specified rigidity was traced backwards by numerical methods, through a model of the magnetosphere, to ascertain if the particle access from space is allowed or forbidden at a given location. The results presented in this paper are for magnetically quiet conditions represented by Kp activity levels of 0 and 2. Geomagnetic cutoff rigidities were calculated each  $5^{\circ}$  in latitude and  $15^{\circ}$  in longitude in the vertical direction. (See Smart and Shea, 1994, for a discussion of the application of cutoff rigidities to spacecraft measurements.)

## 2 Method:

Cosmic ray trajectory calculations were initiated in the vertical direction from a distance of 6821.2 km from the geocenter, (450 km altitude above the average earth radius of 6371.2 km). The "sensible" atmosphere of the earth was considered to extend 20 km above the international reference ellipsoid, and any trajectory path that came lower than this distance was considered to be re-entrant and hence forbidden. In this work, "vertical" is the direction radial from the earth center. The trajectory-tracing technique employed was developed by Kobel (1990) and utilizes the Bulirsch-Stoer numerical integration technique (Stoer and Bulirsch, 1980; Press et al., 1989) to minimize the number of steps required in a charged particle trajectory computation. Each step length was about 1% of a gyro-distance (Smart and Shea, 1981), the distance the particle of the specified rigidity would travel during one gyration in a uniform magnetic field of the same intensity.

The cutoff rigidities are determined by calculating charged particle trajectories at discrete rigidity intervals starting with a rigidity value high above the highest possible cutoff and decreasing the rigidity to a value that satisfied our criteria that the lowest allowed trajectory had been calculated. As these calculations progress down through the rigidity spectrum, the results change from the easily allowed orbits to a complex structure of allowed, forbidden, and quasi-trapped orbits (loosely called penumbra) and finally to a set of rigidities where all trajectories intersect the solid earth. As a result of these trajectory calculations we determined the calculated upper cutoff rigidity ( $R_U$ ) which is the rigidity value of the highest allowed/forbidden pair of adjacent cosmic ray trajectories, the calculated lowest cutoff rigidity ( $R_L$ ) which is the rigidity value of the lowest allowed/forbidden pair of adjacent cosmic ray trajectories, the penumbra. (See Cooke et al., 1991, "effective cutoff rigidity" ( $R_C$ ) that allows for the transparency of the penumbra.

for definitions of cosmic ray cutoffs.) Rigidity intervals of 0.01 GV were used for trajectories between  $R_U$  and  $R_L$  to provide a reasonable sample of the cosmic ray penumbra. The effective cutoff rigidity  $R_C$  was found by summing the allowed orbits through the penumbra as described by Shea et al. (1965).

## **3** Results and Discussion:

These results are intended to be a reference for estimating the cosmic ray geomagnetic cutoffs at the International Space Station during quiet geomagnetic conditions. Iso-rigidity contours of the calculated effective vertical geomagnetic cutoff rigidities at 450 km for very quiet magnetic conditions (Kp = 0) are presented in Figure 1. Figure 2 illustrates the iso-rigidity contours of the calculated effective vertical geomagnetic cutoff rigidities during "quiet" conditions (Kp = 2). A reduced (for the lack of space) tabulation of effective vertical cutoff rigidities (R<sub>C</sub>) are given in the tables. These results, for an internal magnetic field epoch of 1 January 1995, are the average of cutoff calculations for 00, 06, 12 and 18 UT.

A comparison of the figures and tables shows an equatorward movement of the cutoff rigidity contours as the magnetic activity increases. When these results are compared with cutoff rigidities calculated for the same positions utilizing only the internal geomagnetic field (Smart and Shea, 1997) we find that including the external magnetospheric current systems as represented by the Tsyganenko (1989) model, results in a systematic reduction of the geomagnetic cutoff.

We have found that Störmer theory, in a coordinate system using a "proper" magnetic vertical direction (a radial direction from the offset dipole position), and "proper" magnetic north direction, is extremely effective in ordering the trajectory-derived cutoffs computed at various azimuth and zenith angles. Störmer theory can be used to extrapolate these calculated vertical cutoff rigidities to other directions. We have also found that the cutoff change with radial distance is proportional to  $L^{-2}$  (Smart and Shea, 1967), and that the McIlwain (1961) L parameter is very useful in interpolating cutoff rigidities to other altitudes.

# 4 Conclusions:

These cutoff rigidity values derived from the Tsyganenko magnetic field model for magnetically quiet conditions combined with the International Geomagnetic Reference Field for 1995 are intended to be a basic reference for charged particle access to the International Space Station.

# Acknowledgments:

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VERTICAL CUTOFF RIGIDITIES AT 450 KM Tsyganenko model Kp = 0

Figure 1. Cutoff rigidity contours at 450 km altitude for very quiet (Kp = 0) magnetic conditions.

Table 1.	Effective	Vertical	Cutoff Rigidit	ies in G	GV at 450	km (Epo	ch 1995;	Kp = 0	))
			U			· · ·		1	

					0 1				0			
Lat	0	30	60	90	120	150	180	210	240	270	300	330
Lat	0.00											
90	0.00	0 01	0 01	0 01	0 01	0 01	0 01	0 00	0 00	0 00	0 00	0 00
00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
75	0.01	0.02	0.03	0.04	0.04	0.03	0.02	0.01	0.00	0.00	0.00	0.00
70	0.05	0.00	0.00	0.05	0.10	0.09	0.00	0.05	0.01	0.00	0.00	0.01
65	0.08	0.14	0.20	0.25	0.28	0.29	0.17	0.08	0.02	0.00	0.01	0.03
60	0.31	1 12	1 28	1 40	1 57	1 66	1 34	0.22	0.00	0.02	0.02	0.07
55	1 67	1 98	2 16	2 34	2 56	2 75	2 32	1 37	0.13	0.05	0.00	0.35
50	2 74	3 14	3 40	3 62	3 96	4 13	3 50	2 28	1 12	0.15	0.30	1 81
45	4 17	4 49	4 75	5 12	5 53	5 72	4 70	3 46	1 94	1 11	1 46	3 00
40	5 91	6 30	6 78	7 48	7 85	8 09	6 59	4 67	3 00	1 84	2 35	4 46
35	8.13	8.28	8.97	9.32	9.95	9.51	7.87	6.50	4.35	2.78	3.49	6.54
30	9.59	10.07	10.87	11.50	11.91	11.22	9.61	8.14	5.77	3.74	4.71	8.67
25	11.01	11.69	12.48	13,13	13.25	12.34	11.17	9.82	7.99	5.13	6.62	10.03
20	11.99	12.78	13.57	14.32	14.14	13.15	12.05	11.08	9.04	6.50	7.90	11.02
15	12.55	13.29	14.15	14.87	14.63	13.58	12.65	11.80	10.07	7.16	9.71	11.61
10	12.58	13.40	14.37	15.05	14.77	13.91	12.83	12.18	11.02	9.28	10.68	11.87
5	12.37	13.18	14.17	15.01	14.79	14.00	13.29	12.54	11.66	10.48	11.08	11.82
0	11.85	12.68	13.82	14.60	14.40	13.73	13.31	12.79	12.01	10.93	11.23	11.56
-5	11.13	11.91	13.13	13.85	13.58	13.24	13.15	12.80	12.20	11.25	11.20	11.10
-10	10.25	10.97	12.14	12.77	12.41	12.30	12.52	12.54	11.97	11.43	11.02	10.47
-15	9.06	9.73	10.65	11.14	10.72	10.93	11.67	11.83	11.91	11.29	10.65	9.74
-20	7.69	8.13	8.75	9.09	8.96	8.81	10.29	10.92	11.52	11.02	10.20	8.78
-25	6.32	6.80	7.14	6.45	6.33	7.10	8.37	9.42	10.96	10.58	9.54	7.71
-30	5.31	5.27	4.96	4.54	4.39	4.97	6.71	7.59	9.92	10.01	8.74	6.60
-35	4.33	3.90	3.60	2.98	2.85	3.51	4.79	6.79	8.22	9.32	7.91	5.63
-40	3.47	3.01	2.44	1.81	1.71	2.17	3.46	4.74	6.94	8.44	7.03	5.13
-45	2.80	2.19	1.66	1.02	0.85	1.21	2.24	3.63	5.16	7.37	6.37	4.18
-50	2.20	1.59	1.01	0.43	0.28	0.54	1.33	2.50	3.93	5.59	5.18	3.35
-55	1.69	1.11	0.51	0.12	0.07	0.12	0.65	1.58	2.81	3.99	3.83	2.67
-60	1.25	0.70	0.20	0.05	0.02	0.05	0.20	0.93	1.86	2.87	2.95	2.05
-65	0.86	0.37	0.09	0.02	0.00	0.01	0.07	0.43	1.15	1.84	1.99	1.44
-70	0.51	0.17	0.05	0.01	0.00	0.00	0.03	0.12	0.60	1.10	1.23	0.94
-75	0.24	0.09	0.03	0.00	0.00	0.00	0.01	0.07	0.23	0.51	0.63	0.49
-80	0.11	0.05	0.02	0.00	0.00	0.00	0.01	0.04	0.09	0.15	0.20	0.16
-85	0.05	0.03	0.02	0.01	0.01	0.01	0.01	0.03	0.05	0.07	0.08	0.07
-90	0.02											

Geographic East Longitude	Geo	graphic	East	Longitude
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Figure 2. Cutoff rigidity contours at 450 km altitude for quiet (Kp = 2) magnetic conditions.

Table 2.	Effective	Vertical	Cutoff F	Rigidities	in GV	/ at 450	km ()	Epoch	1995:	Kp = 2	)
							(-				/

				00	ogra		Lasi	LUI	igitu	ue		
Lat	0	30	60	90	120	150	180	210	240	270	300	330
20 90	0.00											
85	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
80	0.01	0.02	0.02	0.03	0.03	0.03	0.02	0.01	0.00	0.00	0.00	0.00
75	0.02	0.05	0.06	0.07	0.08	0.07	0.05	0.02	0.00	0.00	0.00	0.01
70	0.06	0.10	0.14	0.18	0.21	0.21	0.11	0.06	0.02	0.00	0.00	0.02
65	0.22	0.40	0.53	0.59	0.70	0.75	0.53	0.15	0.04	0.01	0.02	0.06
60	0.76	1.03	1.17	1.30	1.50	1.55	1.24	0.58	0.11	0.04	0.04	0.30
55	1.60	1.93	2.09	2.28	2.52	2.67	2.26	1.29	0.43	0.12	0.23	0.87
50	2.70	3.07	3.35	3.58	3.91	4.04	3.47	2.22	1.04	0.44	0.68	1.78
45	4.11	4.47	4.77	5.10	5.53	5.68	4.68	3.39	1.88	1.03	1.40	2.95
40	5.85	6.26	6.77	7.41	7.84	8.06	6.57	4.63	2.95	1.80	2.31	4.42
35	8.13	8.27	8.91	9.33	9.90	9.48	7.83	6.48	4.31	2.73	3.42	6.51
30	9.54	10.09	10.84	11.48	11.88	11.17	9.61	8.10	5.75	3.71	4.62	8.66
25	10.98	11.68	12.43	13.13	13.22	12.33	11.13	9.79	7.94	5.08	6.60	10.02
20	11.97	12.74	13.57	14.29	14.12	13.15	12.09	11.06	9.02	6.47	7.85	11.00
15	12.53	13.27	14.13	14.84	14.60	13.58	12.63	11.78	10.06	7.10	9.68	11.59
10	12.59	13.38	14.35	15.04	14.77	13.90	12.82	12.16	11.00	9.27	10.63	11.83
5	12.35	13.16	14.16	15.00	14.76	13.98	13.27	12.54	11.67	10.44	11.07	11.80
0	11.84	12.65	13.81	14.59	14.41	13.73	13.31	12.77	12.02	10.89	11.21	11.54
-5	11.11	11.91	13.11	13.85	13.56	13.23	13.13	12.77	12.20	11.23	11.17	11.08
-10	10.23	10.96	12.13	12.75	12.39	12.28	12.49	12.52	11.94	11.41	11.00	10.48
-15	9.02	9.72	10.66	11.14	10.67	10.96	11.65	11.79	11.90	11.27	10.64	9.74
-20	7.63	8.10	8.78	9.06	8.93	8.79	10.29	10.87	11.52	11.00	10.16	8.74
-25	6.29	6.74	7.06	6.39	6.31	7.07	8.37	9.40	10.93	10.55	9.51	7.73
-30	5.25	5.22	4.92	4.46	4.36	4.96	6.66	7.55	9.89	9.98	8.75	6.59
-35	4.30	3.86	3.59	2.92	2.83	3.44	4.75	6.76	8.16	9.29	7.90	5.60
-40	3.40	2.98	2.43	1.76	1.63	2.11	3.40	4.71	6.94	8.40	6.99	5.11
-45	2.73	2.12	1.58	0.94	0.74	1.15	2.16	3.58	5.13	7.33	6.34	4.15
-50	2.15	1.49	0.91	0.36	0.21	0.47	1.22	2.41	3.86	5.52	5.15	3.35
-55	1.61	1.00	0.43	0.09	0.06	0.09	0.57	1.51	2.75	3.99	3.80	2.65
-60	1.18	0.60	0.15	0.04	0.02	0.04	0.15	0.82	1.81	2.78	2.92	2.00
-65	0.75	0.29	0.07	0.02	0.00	0.01	0.06	0.35	1.06	1.76	1.92	1.38
-70	0.42	0.11	0.04	0.01	0.00	0.00	0.02	0.10	0.52	1.00	1.14	0.86
-75	0.15	0.07	0.02	0.00	0.00	0.00	0.01	0.05	0.17	0.43	0.54	0.40
-80	0.08	0.04	0.01	0.00	0.00	0.00	0.01	0.03	0.07	0.12	0.14	0.12
-85	0.04	0.03	0.01	0.01	0.00	0.01	0.01	0.02	0.04	0.05	0.06	0.06
-90	0.02											

Geographic East Longitude