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Rigidity Dependence of Near-Earth Latitudinal Proton Density Gradients

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Abstract

The rigidity dependence of the latitudinal gradient of cosmic-ray protons observed by *Ulysses* poses a challenge to modulation models: it increases as function of rigidity up to about 1.4 GV and then decreases. Although models could reproduce the observed small positive gradient for an $A > 0$ solar polarity cycle, its maximum was at a rigidity well below 1 GV. After exploring various options, it turns out that changing the rigidity dependence of the perpendicular diffusion coefficient (DC) in the polar direction so that it differs from that of the parallel DC, is the most effective way to obtain good agreement with data. Specifically, we find that this DC must have a flatter rigidity dependence than the parallel DC in order to reproduce the observed rigidity dependence of the latitudinal gradient of protons during an $A > 0$ solar polarity cycle.

1 Introduction:

From September 1994 to July 1995, the *Ulysses* spacecraft executed a fast latitude scan (FLS) by moving from 80° South to 80° North at solar distances between 1.3 and 2.2 AU. During this first comprehensive exploration of the latitudinal dependence of modulation, a number of discoveries were made (see Simpson 1998 and McKibben 1998 for recent overviews). Those of relevance to this paper are the unexpected small latitudinal cosmic ray proton density gradients, and its rigidity dependence (Heber *et al*. 1996.) These authors also attempted to model the observed gradients. They found that the discrepancy between measurements and model results increased as rigidity is decreased. The magnitude problem was subsequently solved (Potgieter, Ferreira, & Heber 1997, Hattingh *et al*. 1997) by using anisotropic perpendicular diffusion (Jokipii & Kóta 1995). In this paper we show that it is the rigidity dependence of the perpendicular diffusion coefficient in the *polar* direction that controls that of the latitudinal gradient, and that this coefficient's rigidity dependence cannot be the same as that of parallel diffusion.

2 Modulation Model and Diffusion Tensor:

The modulation of galactic cosmic rays is described by Parker's transport equation (Parker 1965) for the omnidirectional distribution function $f_0(\mathbf{r}, P)$ for particles with rigidity P at position **r**, which can be written in the steady state as

$$\overbrace{\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\kappa_{rr}\right)\right]\frac{\partial f_0}{\partial r}+\left[\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\kappa_{\theta\theta}\right)\right]\frac{\partial f_0}{\partial\theta}+\kappa_{rr}\frac{\partial^2 f_0}{\partial r^2}+\frac{\kappa_{\theta\theta}}{r^2}\frac{\partial^2 f_0}{\partial\theta^2}}^{\textit{diffusion}}$$
$$\overbrace{-\left[\langle\mathbf{v}_d\rangle_r\right]\frac{\partial f_0}{\partial r}-\left[\frac{1}{r}\langle\mathbf{v}_d\rangle_\theta\right]\frac{\partial f_0}{\partial\theta}}^{\textit{drift}}\quad\overbrace{-V_w\frac{\partial f_0}{\partial r}}^{\textit{convection}}\quad\overbrace{+\left[\frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2V_w\right)\right]\frac{\partial f_0}{\partial\ln P}}^{\textit{adiabatic energy loss}}=0. \tag{1}$$

Here r and θ are heliocentric radial distance and colatitude (polar angle) respectively, V_w the solar wind speed, and $\mathbf{v}_d$ the drift velocity. The coefficient $\kappa_{\theta\theta}$ describes diffusion perpendicular to the mean

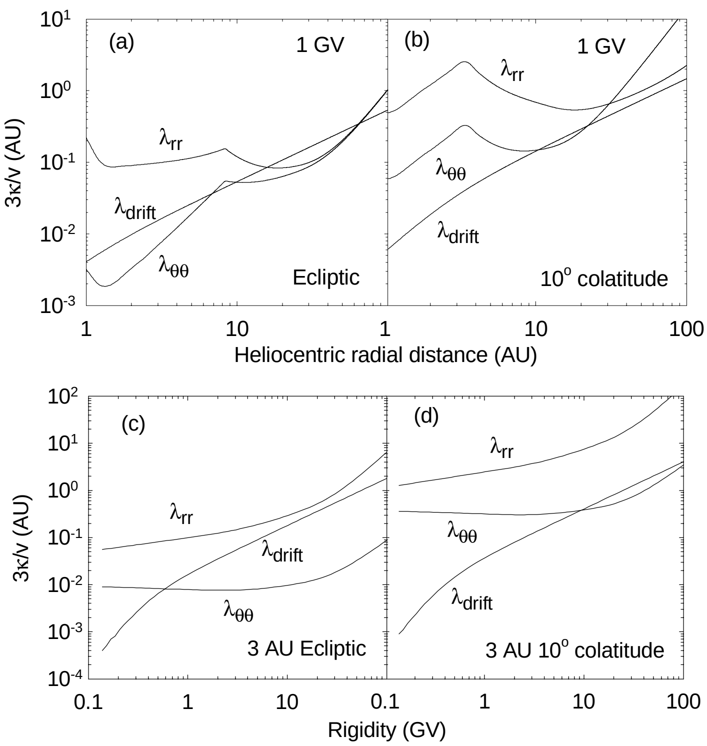

Figure 1: Radial dependence of the radial and polar mean free paths, and the drift scale for 1 GV protons in the ecliptic and at 10° colatitude (upper panels). The two lower panels show the rigidity dependence of the same variables at a radial distance of 3 AU. In all cases $\gamma = \eta = -0.4$ in Eq. (4).

magnetic field in the polar direction, while the radial coefficient is $\kappa_{rr} = \kappa_{\|} \cos^2 \Psi + \kappa_{\perp}^{r\phi} \sin^2 \Psi$, with $\kappa_{\|}$ the diffusion coefficient parallel to the mean magnetic field, $\kappa_{\perp}^{r\phi}$ the diffusion coefficient perpendicular to the field in the radial/azimuthal direction and Ψ the spiral angle. In our two-dimensional model this coefficient acts only in the radial direction. The various processes that play a role in our model are indicated in Equation (1).

We use a steady-state two-dimensional model that simulate the effect of a wavy current sheet (Burger & Hattingh 1995, Hattingh & Burger 1995) by using for the three-dimensional drift pattern in the region swept out by the wavy current sheet, an averaged field with only an r- and a θ-component. The heliospheric boundary is assumed at 100 AU while the solar wind speed is 400 km/s within ~30° of the ecliptic plane and increases within ~10° to 800 km/s in the polar regions. A modified heliospheric magnetic field (HMF) is used (Jokipii & Kóta 1989). The tilt angle of the wavy current sheet is 15°.

The diffusion tensor on which the current one is based, is described in detail in Burger & Hattingh (1998). (Our conclusions are valid for the present form of the diffusion tensor and various other options). For diffusion parallel to the magnetic field, we use

$$\kappa_{\|} = \frac{9 v B_0^{5/3} l_s^{2/3}}{28\pi^2 s C_s} \left[\frac{R}{c} \right]^{1/3} \tag{2}$$

if the quantity $D = (c/R) B_0 l_S$ is greater than 1, while if it is less than one

$$\kappa_{\|} = \begin{cases} \dfrac{v}{8\pi^2 s C_s l_s} \left(\dfrac{R}{c} \right)^2 \left[\dfrac{1}{4\delta} + \left(2 + \dfrac{\delta}{2} \right) D^2 - \dfrac{1}{12} \left(\dfrac{8}{7} + \delta^3 \right) D^4 \right] & \text{if} \quad \delta D \le 1 \\ \dfrac{v B_0}{4\pi^2 s C_s} \dfrac{R}{c} \left[\dfrac{1}{3} + D - \dfrac{1}{21} D^3 \right] & \text{if} \quad \delta D > 1 \end{cases} \tag{3}$$

In these expressions v is the particle speed, B_0 is the magnitude of the background magnetic field, l_S is the correlation length of the magnetic field, s is the fraction of slab turbulence, C_S is the level of the turbulence, c is the speed of light, and R is the particle rigidity. The quantity δ determines the transition from $\kappa_{\|}/v \propto R^1$ to $\kappa_{\|}/v \propto R^2$ for particles resonant with fluctuations in the energy range of the magnetic field power spectrum: if δ is equal to 1, only $\kappa_{\|}/v \propto R^2$ occurs, while if it is greater than one both occur. The terms in square brackets ensure a smooth transition from one rigidity dependence to the next. To describe the anisotropic diffusion perpendicular to the field, and drift, we use

$$\kappa_{\perp}^{r\phi} = 0.007\kappa_{\|}R^{\gamma} \text{ and } \quad \kappa_{\theta\theta} = \begin{cases} 0.007\kappa_{\|}R^{\eta} & \text{ecliptic region} \\ 0.1\kappa_{\|}R^{\eta} & \text{polar region} \end{cases}; \quad \kappa_T = \frac{vR}{3cB_0}\frac{10R^2}{1+10R^2}. \qquad (4)$$

The ecliptic region spans the solar equatorial plane with a half-angle of 35° and values for γ and η are given in the next section

Figure 1 (a) and (b) show the spatial dependence of the radial and polar mean free paths, and the drift scale, while (c) and (d) show their rigidity dependence [with $\gamma = \eta = -0.4$ in Eq. (4)]. The spatial and the rigidity dependence of the diffusion coefficients cannot be separated and this leads to the different behavior of these quantities in different regions in space. Note that in the ecliptic region [Fig. 1 (a)] , λ_{rr} approaches $\lambda_{\theta\theta}$ beyond 30 AU. Radial diffusion is dominated by $\kappa_{\perp}^{r\phi}$ at large radial distances where the field becomes azimuthal, and in the ecliptic region $\kappa_{\perp}^{r\phi} = \kappa_{\theta\theta}$. In the polar region [Fig. 1 (b)], $\lambda_{\theta\theta}$ exceeds λ_{rr} at large radial distances where $\kappa_{\perp}^{r\phi}$ again begins to dominate radial diffusion; but here $\kappa_{\perp}^{r\phi} < \kappa_{\theta\theta}$. Figure 1 (c) and (d) show that the polar mean free path has a flatter rigidity dependence than the radial mean free path. Drifts are slightly reduced (below about 1 GV) with respect to the weak scattering case which is proportional to R at all rigidities.

The parameters given in this Section [with $\gamma = \eta = -0.4$ in Eq. (4)] are chosen to fit solar minimum data at Earth for both solar polarity epochs, by changing *only* the sign of the magnetic field. Although optimized for protons, good fits to galactic helium and high energy electrons are also obtained.

3 Results:

The central result of this paper is Figure 2, which shows a comparison of the latitudinal gradient for cosmic ray protons calculated at 2 AU between the ecliptic and 10° colatitude, and *Ulysses* data obtained during the FLS. Comparing (a), (b) and (c) it is evident that changing the rigidity dependence of $\kappa_{\perp}^{r\phi}$ has little effect on the latitudinal gradient as γ changes from –0.4 to +0.4. It reduces the gradient somewhat at high rigidity, but is does not shift the maximum. In contrast, changing the rigidity dependence of $\kappa_{\theta\theta}$ changes both the magnitude and the position of the maximum as η changes from –0.8 to 0 in each panel. The best values are $\gamma = \eta = -0.4$.

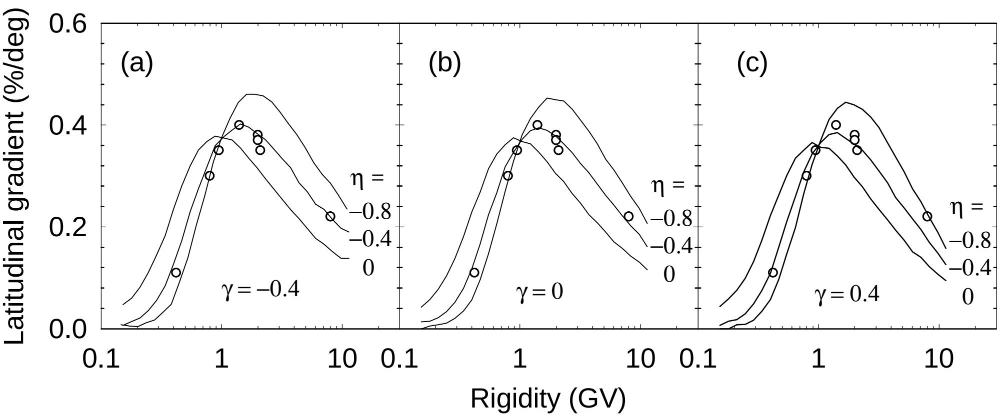

Figure 2: See text for description. The quantities γ and η are used in Eq. (4) for perpendicular diffusion. Data (open circles) are from Heber *et al*. 1996.

4 Discussion and Conclusions:

To obtain the correct magnitude of the observed near-Earth latitudinal gradient, enhanced latitudinal transport is required. In the present model, this is accomplished by increasing the cross-field diffusion in the polar direction with respect to that in the other direction perpendicular to the HMF. To obtain the observed rigidity dependence of this gradient, the cross-field diffusion in the polar direction must have a flatter rigidity dependence than parallel diffusion; at rigidities below about 10 GV $\kappa_{\perp}/v$ should be almost independent of rigidity. Before coming to this conclusion, numerous other options were tried. However, looking at the transport equation (1), it is obvious that $\kappa_{\theta\theta}$, which appears as a coefficient of $\partial f_0/\partial\theta$, should play a dominant role in governing latitudinal transport.

At this time, at least two other studies support our conclusions. Comparing *Ulysses* high-latitude data on the rate of change of integral cosmic ray intensities with *IMP8* data, Simpson (1998) concludes that if cross-field diffusion (as opposed to direct magnetic field "channeling"; see Fisk 1996) occurs, it should be independent of rigidity. In an independent study, Potgieter *et al.* (1999) came to a similar conclusion studying cosmic-ray electron modulation and using Ulysses electron data.

There are however other studies that at a first glance appear to contradict the above conclusion. A numerical simulation (Giacalone 1998) predicts $\kappa_\perp/v \propto R^{1/2}$ in the range 40 MV < R < 2 GV. A second different conclusion follows from the interpretation of *Voyager* anomalous nuclear component data (Cummings & Stone 1998 and references therein) which suggests that the perpendicular mean free path is proportional to R^2 below about 1 GV, in agreement with quasi-linear theory (e.g., Bieber, Burger, & Matthaeus 1995).

Can all these different results be reconciled? The answer is a guarded yes, but only if those that appear to have observational support are considered, i.e. if the numerical simulations reported by Giacalone (1998) is neglected for the moment. One possibility is that perpendicular transport in the ecliptic and in the polar region are different. In the ecliptic region, QLT may apply – this will explain the result reported in Cummings & Stone (1998). This can of course be tested, and such a study is in progress. The relative insensitivity of the rigidity dependence of G_θ on $\kappa_\perp^{r\phi}$ near Earth, is an indication (albeit not a strong one) that the results of the current study will not necessarily be invalidated if $\kappa_\perp^{r\phi}$ from QLT is used. More problematic is $\kappa_{\theta\theta}$. The enhanced latitudinal transport may also be due to direct magnetic field "channeling" in the HMF model of Fisk (1996). The question in this case is, if the transport is parallel to the field, should this process not have the same rigidity dependence as parallel diffusion? The first implementation of the Fisk field in a numerical modulation model (Kóta & Jokipii 1997) unfortunately did not address this issue.

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