Transport of Galactic Cosmic Rays in the Heliosphere: Stochastic Simulation Approach

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Abstract

In the present paper we consider a possibility of using stochastic simulation (Monte-Carlo) technique approach to the study of Galactic Cosmic Ray propagation in the Heliosphere. We developed a technique for calculation of the Cosmic Ray propagation in a spherically symmetric steady state approximation of the Heliosphere. In the frameworks of this approximation, we study the solar modulation of monoenergetic fluxes of Galactic Cosmic Rays entering the Heliosphere, in the particle's energy range 0.1 - 15 GeV. Besides, we present the first results of our simulation of 2D Heliosphere.

1 Introduction

During last decades, study of Galactic Cosmic Rays (GCR) transport in the Heliosphere has been improved and many models have been developed. Simple spherically symmetric steady state ones are good enough for a study of global modulation processes, while very sophisticated 2D and 3D time-dependent models are used for study of fine short-time scale processes. All the models developed so far use various kinds of finite differences numerical techniques. Since the equation of GCR transport in the Heliosphere takes a form of Fokker-Plank equation, one can apply a very flexible Monte-Carlo technique to solve it. An important advantage of Monte-Carlo techniques is that one can use a monoenergetic flux as the initial spectrum of GCR protons. This allows one to study modulation of monoenergetic fluxes of GCR, making it easy to obtain the modulated spectrum for any kind of assumed local interstellar spectrum (LIS). Besides, the use of monoenergetic fluxes allows us to study the details of modulation (such as time spent by a particle inside the Heliosphere or average energy loss) in dependence of the galactic proton's energy. In the present paper we show the first results of application of Monte-Carlo approach to the problem of GCR transport in the Heliosphere.

2 Simple Model

Transport of GCR in the Heliosphere is described by the Fokker-Plank equation which can be written in the spherically symmetric case as (Fisk, 1971):

$$\frac{\partial U}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa\frac{\partial U}{dr}) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 VU) + \frac{1}{3}\cdot\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V)\right)\cdot\left(\frac{\partial}{\partial T}(\alpha TU)\right) \tag{1}$$

where $U(r,T,t)$ is the cosmic ray number density per unit interval of kinetic energy T per nucleon, r - distance from the Sun, V - velocity of the radially directed solar wind, T - particle's kinetic energy per nucleon, κ - diffusion coefficient, $\alpha = (T + 2\cdot T_r)/(T + T_r)$, and T_r - proton's rest energy. We adopt for the diffusion coefficient the form (*e.g.* Perko, 1987) : $\kappa = \kappa_o\cdot\beta\cdot P$ ($P > P_c$) or $\kappa = \kappa_o\cdot\beta\cdot P_c$ ($P < P_c$), where P is particle rigidity, and P_c=1 GV.
The Heliosphere has the size of R_h=100 au; the solar wind velocity is taken to be a constant $V = 400$ km/s inside the Heliosphere. In our study, we make use of the stochastic simulation method based on the equivalence between Fokker-Plank equations and stochastic differential equations which can be solved numerically.

The realization of the numerical techniques we use here is similar to that applied recently for a study of solar particles' interplanetary transport (*e.g.* Kocharov et al., 1998). Note that the problem of GCR transport differs significantly from a problem of solar particle transport as the source of GCR particles is outside the Heliosphere. The details of the technique are given by Gervasi et al. (1999). We tested our technique by means of a comparison with the results obtained by other methods (Labrador & Mewaldt, 1997; Steenberg, 1998) and with a simple analytical approximation.

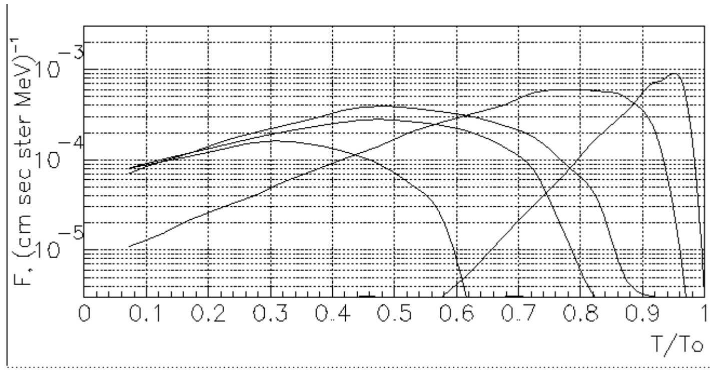

Figure 1: Modulated, at r=1 au, monoenergetic GCR fluxes for medium modulation. T_o=0.3, 0.7, 1, 3, 10 GeV.

The results of the monoenergetic fluxes modulation are shown in Fig.1. The figure shows the spread in energy of monoenergetic flux after modulation (at the Earth's orbit). The initial LIS is considered to be $\delta(T - T_o)$. The figure shows modulation of monoenergetic GCR fluxes for a set of initial energies T_o for medium (Φ=750 MV) modulation conditions, where $\Phi = V(R_h - 1au)/(3\kappa_o)$ - is the solar modulation strength (*e.g.* Gleeson & Axford, 1968).

Figure 2 shows the averaged energy losses of particles (due to adiabatic deceleration) before they reach the Earth's orbit in the dependence on the initial energy T_o for medium and weak (Φ=350 MV) modulation conditions. The energy loss is connected to the time spent by a particle diffusing in the Heliosphere before it reaches the Earth's orbit. This time is depending on the initial energy T_o for medium and weak modulation conditions. One can see that the time of diffusion varies from few days up to half an year. This is in agreement with the observed delays between the solar activity and long-time variations of cosmic ray flux detected by ground based neutron monitors (energy range: 100 MeV - few GeV) (Usoskin et al., 1998). The modulation depth is defined as a part of particle flux with the initial energy T_o which can reach the Earth's orbit. In other words, the modulation depth is an integral of curves in figure 1 over the energy. One can see that for the initial energy of few hundred MeV, the depressing of GCR flux varies from one (weak modulation) up to two orders of magnitude giving huge variations during a cycle of solar activity.

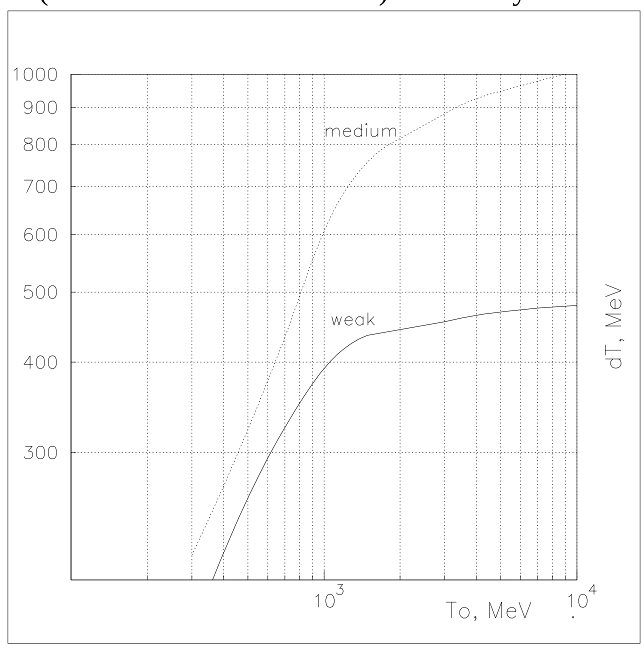

Figure 2: Averaged energy losses of GCR in the Heliosphere *vs.* T_o.

For the initial energy of about 10 GeV, the modulation depth is of the order of magnitude of 10 % though the variations of the GCR flux within a solar cycle are only few percent.

3 2D Model

The 2D equation of GCR transport in the Heliosphere (without drift terms) is (see *e.g.* Potgieter, et al., 1993):

$$\frac{\partial U}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 K_{rr}\frac{\partial U}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(K_{\theta\theta}\ sin\theta\frac{\partial U}{\partial\theta}) + \frac{1}{3r^2}\frac{\partial(r^2 V)}{\partial r}\frac{\partial}{\partial T}(\alpha T U) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V U). \quad (2)$$

The diffusion coefficients and magnetic field model are taken as in Burger & Potgieter (1989) and Potgieter (1993). For the simulation, we made use of the following model of the Heliosphere. The Heliosphere is considered to be a sphere of 100 au radius, which is symmetric with respect to the main axis as well as the ecliptics plane. Currently, no heliospheric neutral sheet is included into the model. The solar wind is considered to be radially directed with a constant velocity. We studied the latitudinal effect of GCR particle diffusion in the Heliosphere. Fig 3 shows lines of equal intensity (modulation depth in Section 2) in the Heliosphere (in XOY meridianal plane) for particles with initial energy $T_o = 1$ GeV. The lines correspond to the intensities of 0.05, 0.1, 0.15 and 0.2 (the intensity outside the Heliosphere is equal to 1.0). Note that these lines are similar to lines of equal intensity as used by *e.g.* Potgieter (1993). One can see that these particles can reach the Earth'sorbit mostly from the polar regions, meanwhile they can hardly come to the Earth along the ecliptics plane (note that the heliospheric neutral sheet is not currently included into the model). Fig.4 shows a couple of sample tracings of single ”particles” trajectories for $T_o = 1$ GeV and $T_o = 9.2$ GeV: time evolution of heliocentric distance of the ”particles”, their energy losses as well as ”particles” trajectories. One can see that ”particles” diffuse at middle heliocentric distances in the Heliosphere until they reached the polar region. After that, they fall rapidly to the distance of 1 au (or rapidly escape from the Heliosphere).

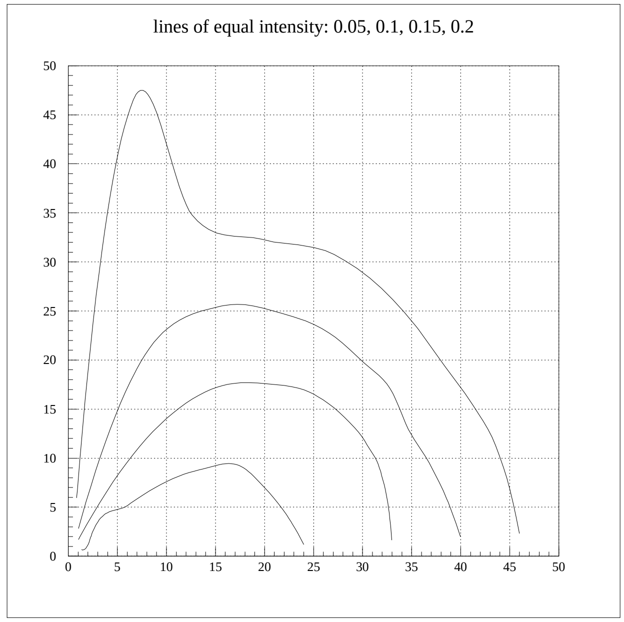

Figure 3: Lines of equal intensity (modulation depth) in the 2D model. The OY and OX axis correspond to the heliospheric polar and ecliptics directions, respectively. $T_o = 1$ GeV.

4 Concluding Remarks

We present the results of stochastic simulation approach to GCR propagation in the Heliosphere. The results for the spherically-symmetric case shows general behavior of solar modulation of GCR. Besides, we present the first results of our simulations for a simplified 2D model of the Heliosphere. Current version of the technique doesn't include drift, heliospheric neutral sheet, neither a latitudinal dependence of the solar wind velocity. Our next steps will be toward including the above effects into the model as well as towards studying time-dependent and spatially limited processes like modulation on interaction regions. As we have shown the stochastic simulation approach is a powerful tool for detailed study of processes of solar modulation of GCR.

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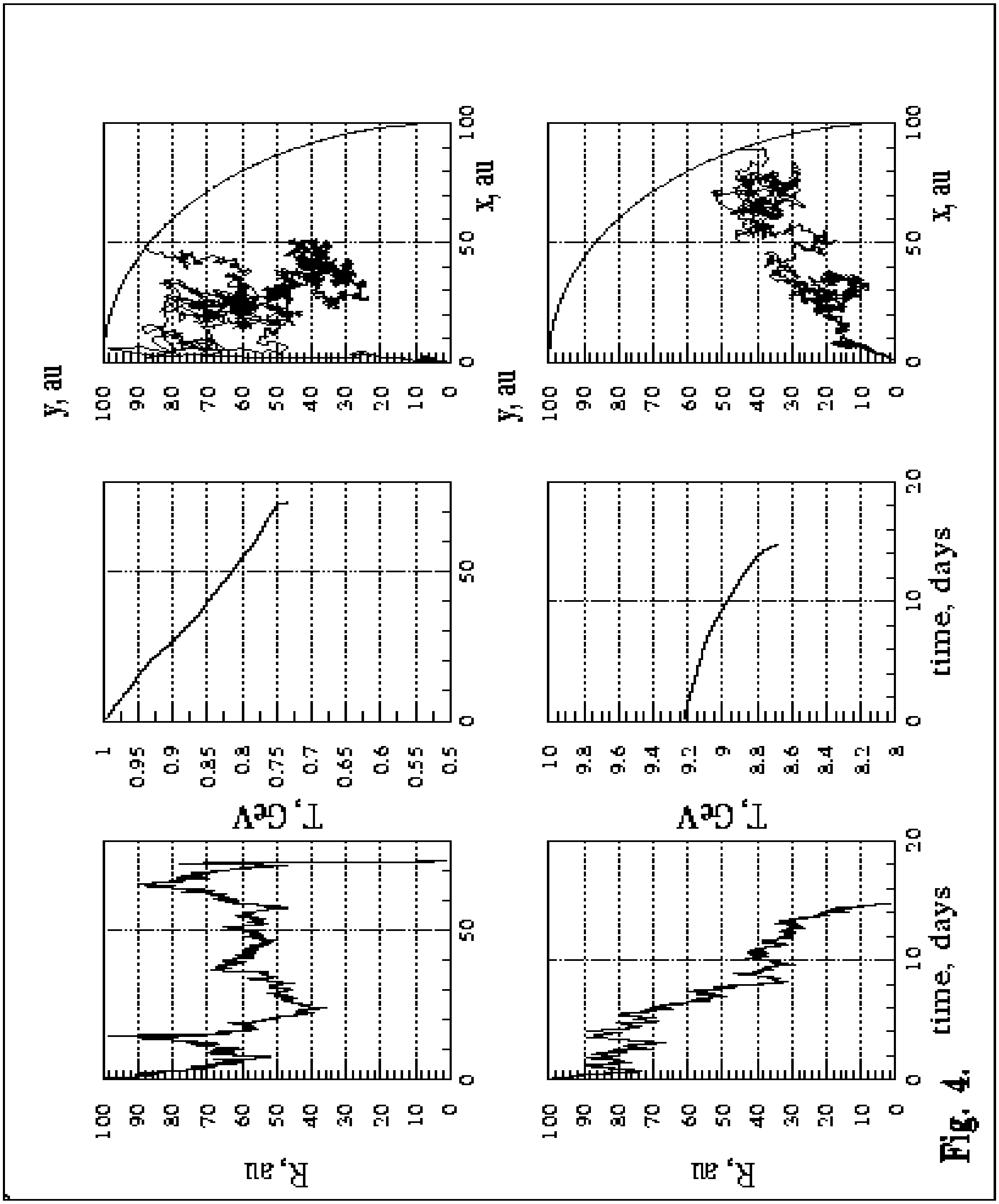

Figure 4. Samples of “particle” tracing inside the Heliosphere. Left panels: the initial energy of particle, T_o = 1 GeV. Right panels T_o = 9.2 GeV. Panels from the bottom to top are: radial distance vs time spent by “particle” inside the Heliosphere; energy losses; trajectory inside the Heliosphere.