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Cosmic Ray Diffusion Coefficients determined for different Models of Perpendicular Diffusion on the basis of a MHD Transport Model for Solar Wind Turbulence

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Abstract

We present calculations of the radial cosmic ray diffusion coefficient in the ecliptic plane on the basis of 3 different theories for perpendicular diffusion assuming that large-scale field line random walk dominates resonant perpendicular diffusion. The radial dependence of the radial diffusion coefficient is determined completely theoretically using a recent model for the transport of MHD turbulence in the solar wind.

1 Introduction:

Quasi-linear theory (QLT) for the parallel diffusion ($\kappa_{\parallel}$) of cosmic rays (CRs) appears to be understood reasonably well, unlike perpendicular diffusion ($\kappa_{\perp}$). This hampers our understanding of CR modulation in the context of well-established CR transport theory. We present calculations of the radial CR diffusion coefficient (κ_{rr}) in the ecliptic plane on the basis of 3 different theories for $\kappa_{\perp}$ assuming that large-scale field line random walk dominates resonant perpendicular diffusion. The radial dependence of κ_{rr} is determined completely theoretically using a promising recent model for the combined transport of a predominantly 2D component (80%), and a minor slab component (20%) of MHD turbulence in the solar wind. The consequences of the different models for $\kappa_{\perp}$ for CR modulation are discussed in another contribution to this conference (le Roux et al., 1999a).

2 The Diffusion Coefficients:

On the basis of standard QLT for the cyclotron resonant interaction of CRs with random heliospheric magnetic field (HMF) slab fluctuations we derived the CR parallel mean free path ($\lambda_{\parallel}$) as

$$\lambda_{\parallel} = 2.433\frac{r_g^{1/3} l_b^{2/3}}{A^2}\left[0.0972\left(\frac{r_g}{l_b}\right)^{5/3} + 1\right] \tag{1}$$

where r_g is the particle gyroradius, l_b is the wavelength for slab turbulence at the break point in the power spectrum of HMF fluctuations, A is the normalized amplitude of the x-component of the slab fluctuations ($A = (\delta B_x/B)$ where B is the magnitude of the mean HMF; $(\delta B_x/B)^2 = 0.05$ and $B = 5$ nT at 1 AU).

The first model for $\kappa_{\perp}$ is given by

$$\kappa_{\perp} = \frac{1}{4} v l_c A^2 \tag{2}$$

where v is article speed, l_c ($l_c = 0.79256 l_b$ where $l_b = 0.03$ AU at 1 AU) is the correlation length of slab turbulence, and the amplitude A is the sum of slab and 2D turbulence amplitudes. Equation (2) corresponds to the QLT of Jokipii (1971) for slab turbulence and implies that CRs are tied to and moving along a large-scale random-walking field line without experiencing resonant spatial diffusion. The only modification is that A denotes the sum of slab and 2D turbulence instead of just the slab component. This theory is tied to the condition $\lambda_{\parallel} >> l_c$ indicating applicability for rigidities $R >> 2 \times 10^{-4}$ GV at 1 AU. Thereby, all R-values of relevance for CR modulation are covered. This model is referred to as the modified QLT (MQLT) model (see also Zank et al., 1998).

In the limit $\lambda_{\|} << l_c$ or $R << 2 \times 10^{-4}$ GV at 1 AU, $\kappa_{\perp}$ is given by $\kappa_{\perp} = 0.5 A^2 \kappa_{\|}$ where A is the sum of the amplitudes of slab and 2D turbulence. This expression is an outflow of the QLT by Chuvilgin and Ptuskin (1993) on anomalous perpendicular diffusion. It means that CRs are resonantly diffusing primarily along and weakly across large-scale random walking field lines, so allowing CRs to change field lines. It implies that the rate at which large-scale neighboring field lines separate then plays a major role in determining the effective $\kappa_{\perp}$ of CRs across B (see large ratio of $\kappa_{\perp}/\kappa_{\|}$ in above expression). Unfortunately, this model applies at R below that of interest for CR modulation. However, test particle simulations by Giacalone (1998) suggest that large-scale field line separation effects also occur at energies relevant for CR modulation, but at a reduced level. Using their work as a guide, we assume for the 2nd model of $\kappa_{\perp}$

$$\kappa_{\perp} = 0.02\left(\frac{A}{A_0}\right)^2 \kappa_{\|} \tag{4}$$

where A_0 is the amplitude of the turbulence at 1 AU. This model is referred to as the modified anomalous diffusion (MAD) model.

The 3rd model makes use of the well-known basic expression for $\kappa_{\perp}$ given by

$$\kappa_{\perp} = \frac{1}{3} v r_g \frac{\omega\tau}{1+\omega^2\tau^2} \tag{5}$$

where ω is the particle gyrofrequency, and τ is the scattering or relaxation time. Bieber and Matthaeus (1997) suggest that $\omega\tau$ can be expressed as

$$\omega\tau = \frac{2}{3}\frac{r_g}{D_{B\perp}} \tag{6}$$

where $D_{B\perp}$ describes large-scale field line wandering across B. The expression for $D_{B\perp}$ is given by

$$D_{B\perp} = \frac{1}{2} D_{sl} + \frac{1}{2}\sqrt{D_{sl}^2 + 4D_{2D}^2} \tag{7}$$

where

$$D_{sl} = \frac{1}{2} l_{sl} A_{sl}^2; \quad D_{2D} = l_{2D} A_{2D} \tag{8}$$

In equation (8), D_{sl} describes the magnetic field wandering for slab turbulence, A_{sl} is the amplitude of this turbulence, and l_{sl} is its correlation length along B, while D_{2D} describes the magnetic field wandering for 2D turbulence, A_{2D} is the amplitude of this turbulence, and l_{2D} is its correlation length across B (Matthaeus et al., 1995). The advantage of this approach is that there is a clear distinction between l_c along and across B tied to the 2 components of solar wind turbulence. The MQLT model allows just for l_c parallel to B. In addition, the approach is not limited to small amplitudes as QLT. Assuming $\omega\tau >> 1$, and $D_{2D} = 0$, $\kappa_{\perp}$ in equation (5) is the same as $\kappa_{\perp}$ in equation (2) for slab turbulence. Although not well known, the expectation is that $l_{2D} >> l_{sl}$ so that $\kappa_{\perp}$ is larger compared to the MQLT model in the limit $\omega\tau >> 1$ and smaller when $\omega\tau << 1$. We use $l_{2D} = 100 l_{sl}$ and for reference we call this model the nonperturbative (NP) model (see also Zank et al., 1998).

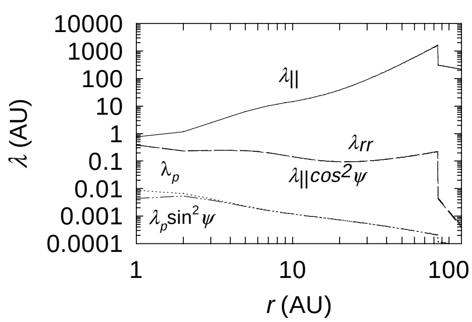

Figure 1: Mean free paths in the ecliptic plane for the modified quasi-linear (MQLT) model of $\kappa_{\perp}$. The curves denote 930 MV (28 MeV nuc.$^{-1}$) anomalous CR He^+. Note that $\lambda_{\perp} = \lambda_p$ in the figures.

The dependence of the diffusion coefficients on radial distance (r) is theoretically determined with the MHD model for HMF turbulence transport in the solar wind according to Zank et al. (1996). The model gives a good reproduction of the observed r-dependence in the energy density of HMF fluctuations and also specifies the r-dependence of l_c. Key elements in its success are the generation of turbulence by corotating interaction regions close to the Sun, and by isotropizing pickup ion (PI) ring distributions beyond the ionization cavity (r > 6 AU). In an extended version of the model (le Roux et al., 1999b), it is shown that near isotropic PI distributions can also damp turbulence for r > 30 AU, but that turbulence generation by PIs still dominates. The increase in the energy density of the turbulence and the decrease in l_c across the termination shock is estimated simply with the extended model.

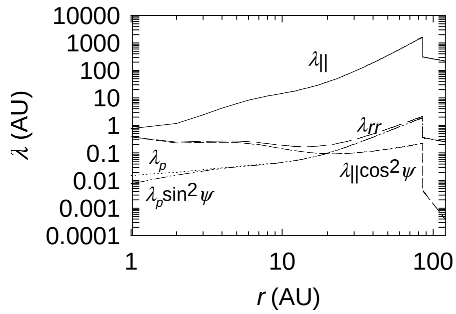

Figure 2: As Figure 1, but for the modified anomalous diffusion (MAD) model of $\kappa_\perp$.

3 The Results:

In Figure 1 we show theoretically calculated mean free paths for 930 MV CR He^+ with the MQLT model for $\kappa_\perp$ in the ecliptic plane as a function of increasing r from the Sun. The radial mean free path (λ_{rr}) is calculated according to $\lambda_{rr} = \lambda_{\|}\cos^2\psi + \lambda_\perp \sin^2\psi$ where ψ is the Parker field spiral angle, and $\lambda_\perp = 3\kappa_\perp/v$ ($\lambda_\perp = \lambda_p$ in the figures) is the perpendicular mean free path. The 3 important results to emerge from the MQLT model are as follows: (1) λ_{rr} is determined solely by $\lambda_{\|}$ without any contribution from $\lambda_\perp$ so that large negative radial gradients in λ_{rr} exist for R << 1 GV. (2) There is a big decrease in the magnitude of $\lambda_{\|}$ across the termination shock at 85 AU ($\cos^2\psi \propto u^2$ where u is the solar wind speed) implying that a strong modulation barrier to galactic CRs exists downstream. (3) Close to 1 AU $\lambda_{rr} \propto \lambda_{\|} \propto R^{1/3}$ because CRs interact resonantly with the inertial range of the power spectra. At larger distances $\lambda_{rr} \propto \lambda_{\|} \propto R^2$ because of resonant interaction of CRs with the energy range.

In Figure 2 we show calculations that consider the MAD model for $\kappa_\perp$. The main results are: (1) $\lambda_\perp \propto 1/r_g^2$ contributes mainly to λ_{rr} beyond ~ 20 AU so that λ_{rr} has a strong r-dependence upstream beyond ~ 30 AU. (2) The drop in λ_{rr} across the termination shock is reduced ($\sin^2\psi$ is less sensitive to the shock jump than $\cos^2\psi$). (3) The R-dependence of λ_{rr} is determined by $\lambda_{\|}$, giving it the same dependence as for the MQLT model. (4) The negative r-dependence of λ_{rr} below R < 1 GV is weakened by the important contribution of $\lambda_\perp$ to λ_{rr}.

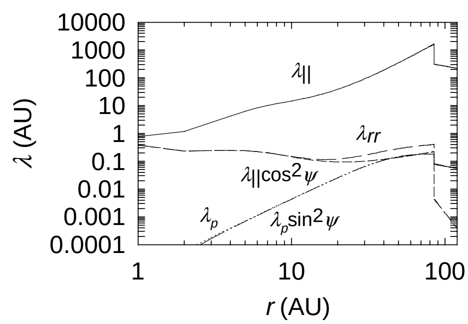

Figure 3(a): As Figure 1, but for the nonperturbative (NP) model of $\kappa_\perp$.

In Figure 3 calculations of mean free paths are presented on the basis of the NP model. The key results produced by this model are the following: (1) $\lambda_\perp$ contributes significantly to λ_{rr} beyond ~ 20 AU from the Sun below ~ 3 GV so that the r-dependence of λ_{rr} upstream is reduced compared to the MAD case for intermediate R-values (Figure 3(a)). This is because $\lambda_\perp$ is independent of r_g ($\omega\tau$ >> 1 in equation (5)). (2) For R << 3 GV beyond ~ 20 AU, $\lambda_\perp$ and therefore λ_{rr} is strongly dependent on r ($\omega\tau$ << 1 in equation (5)). (3) Above ~ 3 GV beyond ~ 20 AU $\lambda_{\|}$ contributes the most to λ_{rr}. (4) Consequently, λ_{rr} features a three interval R-dependence in the outer heliosphere with the weakest dependence $\lambda_{rr} \propto R$ for intermediate values of R < 3 GV in the middle interval, and $\lambda_{rr} \propto R^2$ for R > 3 GV and R << 3 GV in the other two intervals (Figure 3(b)). It was proposed and demonstrated first by Moraal et al. (1999), in an empirical approach to the CR diffusion tensor, that a similar three interval R-dependence for λ_{rr}, with the weakest R-dependence in the center interval, is necessary for the simulation of both observed galactic and anomalous CR spectra. In the companion paper (le Roux et al., 1999a) we will show that our CR modulation model does the best in reproducing observed CR spectra when using the NP model for $\kappa_\perp$. The NP model tentatively provides a theoretical basis for the work of Moraal et al. (1999).

4 Summary:

The parallel, perpendicular and radial mean free paths for CRs were determined theoretically on the basis of 3 plausible theories for $\kappa_\perp$ assuming that field line random walk is more important than resonant perpendicular diffusion. A MHD model for field turbulence transport in the solar wind (Zank et al., 1996, 1998) was used to calculate the spatial dependence of the mean free paths. Concerning the MQLT model for $\lambda_\perp$, $\lambda_\parallel$ contributes solely to λ_{rr}, and consequently a big drop in λ_{rr} across the termination shock implying a strong galactic CR modulation barrier, is predicted. For the MAD model of $\lambda_\perp$, $\lambda_\perp$ produces a strong contribution to λ_{rr} for $r > 20$ AU from the Sun resulting in a large r-dependence for λ_{rr} for $r > 30$ AU upstream. For both models, $\lambda_{rr} \propto R^2$ at large r due to the resonant interaction of CRs with the energy range of the power spectra. Regarding the NP model for $\kappa_\perp$, $\lambda_\perp$ contributes significantly to λ_{rr} for $R < 3$ GV at large r but $\lambda_\parallel$ dominates in λ_{rr} for $R > 3$ GV. This leads to a complex three interval R-dependence for λ_{rr}, with the weakest R-dependence $\lambda_{rr} \propto R$ given by the middle interval, and $\lambda_{rr} \propto R^2$ in the other two intervals. A similar R-dependence was first proposed empirically by Moraal et al. (1999) as a necessary condition for the simulation of both observed galactic and anomalous CR spectra. In the companion paper (le Roux et al., 1999a) we will show that our CR modulation model does the best in reproducing observed CR spectra when using the NP model for $\kappa_\perp$. The NP model tentatively provides a theoretical basis for the work of Moraal et al. (1999).

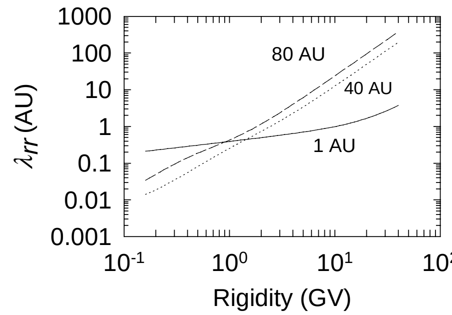

Figure 3(b): The rigidity dependence of the radial mean free path (λ_{rr}) in the ecliptic plane for the NP model of $\kappa_\perp$.

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