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The Heliospheric Modulation of Cosmic Ray Electrons: Rigidity Dependence of the Perpendicular Diffusion Coefficient

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Abstract

The modulation of cosmic ray electrons in the heliosphere provides a useful tool in understanding and estimating the diffusion tensor applicable to heliospheric modulation. Using a comprehensive modulation model including all major mechanisms to study electron modulation, especially at energies below 500 MeV, we found that perpendicular diffusion is very important to electron modulation at these energies. Electrons respond directly to the energy dependence of the diffusion coefficients below 500 MeV, in contrast to protons which experience large adiabatic energy losses below this energy. As a result of this and because drifts become unimportant for electrons at these low energies, important conclusions can be made about the absolute values, spatial and especially the rigidity dependence of the diffusion coefficients.

1. Introduction

Diffusion perpendicular to the heliospheric magnetic field (HMF) plays an important role in the modelling of the heliospheric modulation of galactic cosmic rays (CRs). This followed directly from the simulation of latitude dependent modulation, first studied 23 years ago with a two-dimensional model by Fisk (1976). Even with the introduction of global and neutral sheet drifts in models of increasing complexity (Kóta & Jokipii, 1983; Potgieter & Moraal, 1985, le Roux & Potgieter, 1991, Burger & Hattingh, 1998) the importance of the perpendicular diffusion coefficient has remained and is arguably the most important element of the diffusion tensor. But because no comprehensive theory exists for it, the best that can be done at this stage is to make reasonable assumptions about its value, spatial and rigidity dependence.

Fortunately, the modulation of cosmic ray electrons in the heliosphere provides a useful tool in understanding and in determining the diffusion coefficients. Computed electron modulation responds directly to what is assumed for the energy dependence of the diffusion coefficients below 500 MeV, in contrast to protons which experience large adiabatic energy changes below this energy and which consequently obscure the effects of changing the energy dependence of any of the diffusion coefficients. Another aspect is that drifts become progressively less important with decreasing electron energy, to have almost no effect on electron modulation below 100 - 200 MeV. For the present work, electron modulation was used to illustrate how important perpendicular diffusion is, in particular its rigidity dependence, to the heliospheric modulation of cosmic ray electrons. This is a continuation of the work presented by Potgieter & Ferreira (1999) during the Cospar Meeting in Nagoya, Japan. For a detailed description of the importance of the various parameters in electron modulation, the reader is referred to Potgieter (1996).

2. The Modulation Model

The model that was used for this study is based on the numerical solution of the Parker's (1965) transport equation (TPE):

$$\frac{\partial f}{\partial t} = -(\mathbf{V} + \langle \mathbf{v_D} \rangle) \cdot \nabla f + \nabla \cdot (\boldsymbol{K}_s \cdot \nabla f) + \frac{1}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ \ln R}, \tag{1}$$

where $f(\boldsymbol{r},R,t)$ is the CR distribution function; R is rigidity, $\boldsymbol{r}$ is position, and t is time. Terms on the right-hand side represent convection, gradient and curvature drifts, diffusion and adiabatic energy changes respectively, with $\mathbf{V}$ the solar wind velocity. The symmetric part of the tensor $\boldsymbol{K}_S$ consists of a parallel diffusion coefficient ($K_\parallel$) and a perpendicular diffusion coefficient ($K_\perp$). The antisymmetric element K_A

describes gradient and curvature drifts in the large scale HMF. The pitch angle averaged guiding centre drift velocity for a near isotropic CR distribution is given by $\langle \mathrm{v}_D \rangle = \mathrm{V}\ \mathrm{x}\ K_A \mathrm{e}_B$, with $\boldsymbol{e}_B = \boldsymbol{B}/B$, where B is the magnitude of the background HMF. The TPE was solved in a spherical coordinate system assuming azimuthal symmetry and $\partial f/\partial t = 0$, that is a steady-state for solar minimum modulation, with the neutral sheet "tilt angle" $\alpha = 5°$ during so-called A > 0 epochs (~1990 to present).

The HMF was modified in the off equatorial regions of the heliosphere according to Jokipii & Kota (1989). Qualitatively, this modification is supported by measurements made of the HMF in the polar regions of the heliosphere by Ulysses (Balogh et al. 1995). The solar wind speed V was assumed to change from 450 km.s^{-1} in the equatorial plane ($\theta = 90^{\mathrm{o}}$) to a maximum of 850 km.s^{-1} when $\theta \leq 60^{\mathrm{o}}$. The outer boundary of the simulated heliosphere was assumed at 100 AU which is a reasonable value. The galactic electron spectrum published from the COMPTEL results (Strong et al., 1994) was used as the local interstellar spectrum (LIS).

For the parallel and perpendicular diffusion coefficients, and the "drift" coefficient, the following general forms were assumed respectively:

$$K_{||} = K_0 \beta\ f_1(R) f_2(\theta, r);\ K_{\perp r} = aK_{||};\ K_{\perp\theta} = bK_{||};\ K_A = (K_A)_0 \frac{\beta R}{3B_m}. \qquad (2)$$

Here β is the ratio of the speed of the CR particles to the speed of light; $f_1(R)$ gives the rigidity dependence in GV; K_0 is a constant in units of 6.0 x 10^{20} cm^2 s^{-1}; a is a constant which determines the value of $K_{\perp \mathrm{r}}$ which contributes to perpendicular diffusion in the radial direction, and b a constant determining the value of $K_{\perp\theta}$ which contributes to perpendicular diffusion in the polar direction. Diffusion perpendicular to the HMF was therefore enhanced in the polar direction by assuming $K_{\theta\theta} = K_{\perp\theta} = bK_{||}$. (See Kota & Jokipii, 1995 and Potgieter, 1996 for arguments supporting this procedure). $(K_A)_0$ specifies the amount of drifts allowed, with $(K_A)_0 = 1.0$ a maximum. The effective radial diffusion coefficient is given by $K_{\mathrm{rr}} = K_{||} \cos^2\psi + K_{\perp \mathrm{r}} \sin^2\psi$, with ψ the angle between the radial direction and the averaged HMF direction. Note that $\psi \rightarrow 90°$ when r > ~10 AU with the polar angle $\theta \rightarrow 90°$, and $\psi \rightarrow 0°$ when $\theta \rightarrow 0°$, which means that $K_{||}$ dominates K_{rr} in the inner and polar regions and $K_{\perp \mathrm{r}}$ dominates in the outer equatorial regions of the heliosphere. Values of the different constants and the function $f_2(\theta, r)$ are given below. The differential intensity which is $\propto R^2 f$, is calculated in units of particles m^{-2} sr^{-1} s^{-1} MeV $^{-1}$.

3. Results and Discussion

The results of Potgieter & Ferreira (1999) are shown in Figure 1 and illustrate in general that when the rigidity dependence of $K_{\perp}$ (that is both $K_{\perp \mathrm{r}}$ and $K_{\perp\theta}$ in Eq. 2) is taken independently from that for $K_{||}$ at energies below ~ 500 MeV, it clearly dominates modulation at these lower energies. We assumed for these results that $f_2(\theta, r) = 1 + r/r_0$ in Eq. 2, with r_0 = 1 AU, K_0 = 25, a = 0.05 and b = 0.15. The HMF magnetic cycle was chosen to be A > 0 (e.g. the present solar polarity cycle). The different rigidity dependencies for $K_{||}$ and $K_{\perp}$ are shown in the inserted graph where $K_{\perp}$ was multiplied by 10 for illustrative purposes. In this case the rigidity dependence for $K_{||}$ was according to the damping model - composite slab - 2D geometry of Bieber et al. (1994); see also Potgieter (1996). It is evident that the computed spectra in the inner heliosphere are still compatible to data at higher energies but below ~ 100 MeV the modulation becomes unreasonably large which is apparently not supported by measurements. However, these results illustrate that although $K_{\perp \mathrm{r}}$ and $K_{\perp\theta}$ is only 5% and 15% of the value of $K_{||}$ respectively, perpendicular diffusion, especially in the polar direction, dominates electron modulation below ~ 100 MeV and that it is as such a very important parameter that should be studied in detail. If the increase in the low energy part of observed electron spectra with decreasing energy was taken as a characteristic of modulated electron spectra, as in Figure 4 (see also Potgieter & Ferreira, 1999; their Figure 1) then $K_{\perp} \propto \beta R$, as shown in our Figure 1, is not a workable option.

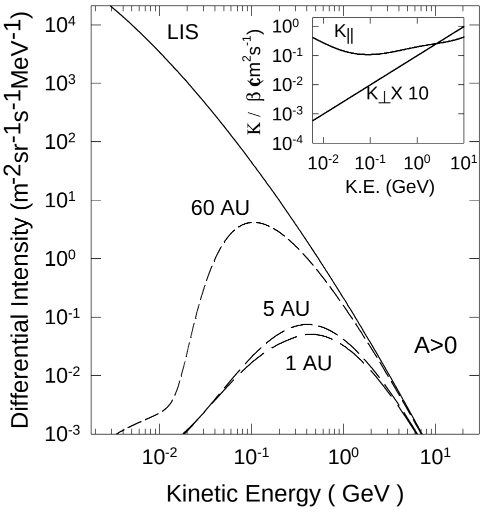

Figure 1: Computed electron modulation at 1, 5 & 60 AU in the equatorial plane with the LIS at 100 AU for an A > 0 epoch. Insert shows the values of $K_{\|}$ and $K_{\perp r}$ in units of 6 x 10^{20}; note that $K_{\perp\theta}$ = 3

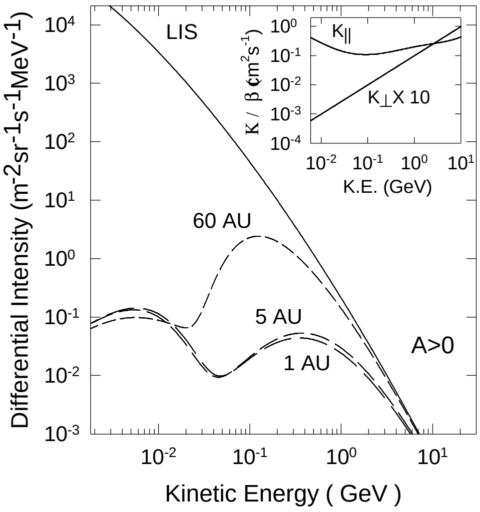

Figure 2: Similar to Fig. 1 but with $K_{\perp\theta}$ = 8 $K_{\perp r}$, in units of 6 x 10^{20}. Note how the situation changed at the low energies, especially in the inner heliosphere.

Recently, it was argued by Kota & Jokipii (1995) that $K_{\perp\theta}$ plays a crucial role in CR modulation which is the assumption for this work, because perpendicular diffusion enhanced in the polar direction seems a necessity for getting computed latitude dependencies compatible to the Ulysses observations (see also Potgieter, 1999). Because $K_{\perp\theta}$ may be considerably larger than $K_{\perp r}$ in the heliospheric polar regions, Figure 2 illustrates whether a further enhancement of $K_{\perp\theta}$ may change the features shown in Figure 1 by taking b = 0.40 instead of 0.15 in Eq. 2. This increase caused additional low energy electrons to reach the equatorial

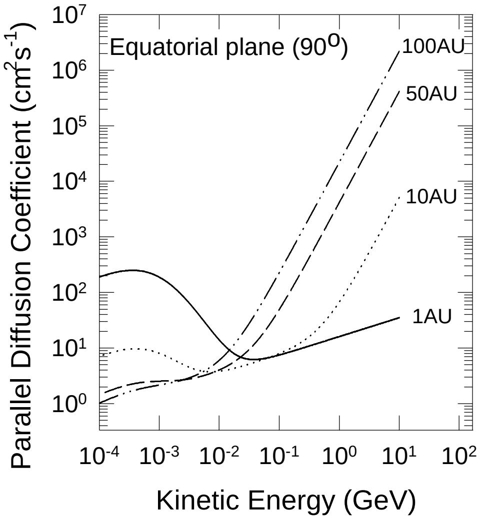

Figure 3: The parallel diffusion coefficient $K_{\|}$ for cosmic ray electrons in units of 6 x 10^{20} at 1 AU, 10AU, 50AU and 100AU in the equatorial plane as a function of kinetic energy. Note the changes from 1 AU to 100 AU.

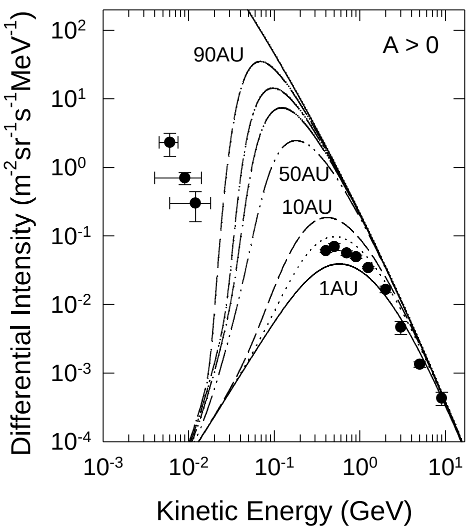

Figure 4: Computed electron spectra in the equatorial plane at 1, 5, 10, 50, 70, 80 & 90 AU, corresponding to the function shown for $K_{\|}$ in Figure 3 and based on the assumption that $K_{\perp} \propto K_{\|}$. Data are at ~5AU for 1997 from the Ulysses/KET experiment.

plane, compared to Figure 1. A further increase in *b* had little additional effect at these low energies, so that there is clearly a limit to what his approach can do.

To extend the study on the modulation aspects shown in Figures 1 and 2, Ferreira (1999) constructed an analytical expression for $K_{||}$, applicable to electrons, using the theoretical work of Hattingh (1998) and Burger & Hattingh (1998) where they on their part used the formalism of Bieber et al. (1994), especially the random sweeping model for dynamical turbulence with pure slab geometry. (See also Zank et al. 1998). This expression is depicted in Figure 3 as a function of kinetic energy for 1 AU, 10 AU, 50 AU and 100 AU in the equatorial plane. No explicit latitude dependence was assumed. It is evident that the radial dependence of $K_{||}$ is much more sophisticated with main feature the changing slopes of the function and that $K_{||}$ is much larger in the outer heliosphere at high energies than at low energies, with the opposite at 1 AU.

The corresponding computed spectra are shown in Figure 4. In this case $a = 0.05$ and $b = 0.15$ in Eq. 2. The electron data from the Ulysses/KET experiment for 1997 are shown to provide a reference for inner heliospheric electron intensities during minimum modulation. The compatibility between the data and the model is reasonable at energies > 400 MeV, but not at energies < 100 MeV. Although we used a very sophisticated function for $K_{||}$ and $K_{\perp}$ it does not give electron modulation compatible to Ulysses data at low energies. In this case the only way to assure compatibility is to make $K_{\perp}$ almost independent of kinetic energy at low energies because as shown before it dominates electron modulation at these low energies – see also Ferreira (1999); Ferriera & Potgieter (SH3.1.07, this volume). It should be kept in mind, however, that measured low energy electrons might contain a Jovian contribution. For another perspective on the topic of this paper, but in support of this work, see the independent study by Burger et al. (SH3.3.02), and for implications of observed cosmic ray electrons in the outer heliopshere, see Potgieter & Ferreira (SH3.1.06).

4. Conclusions

Electron modulation was used to illustrate how important $K_{\perp r}$, but especially $K_{\perp\theta}$ and its rigidity dependence is to electron modulation below 100 - 300 MeV. It was illustrated that although $K_{\perp r}$ and $K_{\perp\theta}$ was only 5% and 15% of the value of $K_{||}$ respectively, perpendicular diffusion dominates electron modulation below ~ 100 MeV. It was argued that if the increasing intensity with decreasing energy below ~ 100 MeV in the observed electron spectra in the inner heliosphere were taken as a characteristic of modulated cosmic ray electron spectra, as in Figure 4, then, to assure reasonable compatibility with data below ~ 100 MeV, $K_{\perp r}$ and certainly $K_{\perp\theta}$ must be nearly independent of kinetic energy below ~ 100 MeV.

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